Quantum Chromodynamics and Jets at the LHC

Herbstschule HEP

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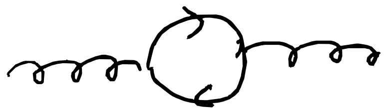
Higher-orders

-> two types correction : real & virtual

Loops

p^2  $\sim C_F \underbrace{\int \frac{d^4 q}{(2\pi)^4}}_{|\vec{q}|^3 \, d|\vec{q}| \, d\Omega} \frac{1}{(p-q)^2} \frac{1}{q^2}$

$$\xrightarrow{|q| \to \infty} \int \frac{d|q|}{|q|} \xrightarrow{\Lambda \text{ cutoff}} \lim_{\Lambda \to \infty} \ln(\Lambda)$$

-> regularization : CDR $d = 4 - 2\varepsilon$

-> renormalization in QCD :

$$\psi \to \psi^B = \underline{Z_\psi^{1/2}} \, \underline{\psi^R} \qquad A^B = Z_A^{1/2} A^R$$

$$g_s^B = \bar{\mu}^{\varepsilon} Z_g \, g^R \qquad m \, , \, \xi$$

$$\hookrightarrow \bar{\mu} = \mu \left(\frac{e^{\gamma_E}}{4\pi} \right)$$

$\overline{MS}$: $$Z = \left(1 + \sum_{n=1}^{\infty} \alpha_s^n \sum_{k=1}^{n} \frac{z^{(n,k)}}{\varepsilon^k} \right)$$

Coupling: $\alpha_s = \frac{g^2}{(4\pi)}$ $\Rightarrow$ $\alpha_s^B(\varepsilon) = \bar{\mu}^{-2\varepsilon} Z(\alpha(\mu), \varepsilon) \alpha(\mu)$

$$\bar{\mu}_0^{-2\varepsilon} Z(\alpha(\mu_0), \varepsilon) \alpha(\mu_0) = \bar{\mu}^{-2\varepsilon} Z(\alpha(\mu), \varepsilon) \alpha(\mu)$$

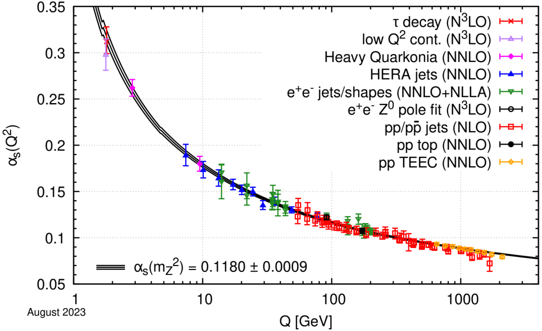
$$\alpha(\mu) = \sum_{i=1}^{\infty} c_i \alpha(\mu_0)$$

$$\alpha(\mu)|_{\varepsilon=0} = \alpha(\mu_0) + 2 z^{(1,1)} \alpha^2(\mu_0) \ln\left(\frac{\mu}{\mu_0}\right) + \mathcal{O}$$

β- function: $$\beta(\alpha(\mu)) = \frac{d\alpha(\mu)}{d\ln(\mu)}$$

$$\beta(\alpha) = \alpha^2 \sum_{k=1} \beta_k \alpha^k \qquad \beta_k = 2(k+1) z^{(k+1,1)}$$

confinement limit



$$\alpha(\mu) = \frac{\alpha(\mu_0)}{1 + \beta_0 \alpha \ln\left(\frac{\mu^2}{\mu_0^2}\right)}$$

$\beta_0 > 0$

← asymptotic freedom

IR divergencies

$$\frac{1}{(p_q+p_g)^2} = \frac{1}{2E_q \underline{E_g}(1-\cos\theta)}$$

$$\Rightarrow \quad \frac{|M_{qqg}|^2 d\Phi_3}{|M_{qq}|^2 d\Phi_2} = \frac{2\alpha_s C_F}{\pi} \underbrace{\frac{dE_g}{E_g}\frac{d\theta}{\sin\theta}}\frac{d\phi}{(2\pi)}$$

→ after reg. $\quad \sim \frac{C_{-2}}{\varepsilon^2} + \frac{C_{-1}}{\varepsilon} + C_0 + \mathcal{O}(\varepsilon)$

→ virtual correction cancel these

$$\int d\sigma^{NLO} = \int_{\Phi_B} d\sigma^V + \int_{\Phi_{B+1}} d\sigma^R$$

$$d\sigma^V + \int_1 d\sigma^R = \text{finite} \quad \text{(KLN theorem)}$$

$$\int d\sigma^R = \int (\underbrace{d\sigma^R - d\sigma^S}_{\text{finite in all IR limits}}) + \int \underbrace{d\sigma^S}_{\text{simple enough}}$$

$$\int_1 d\sigma^S$$

so that $\int_1 d\sigma^S + d\sigma^V = \text{finite}$

state-of-the-art:

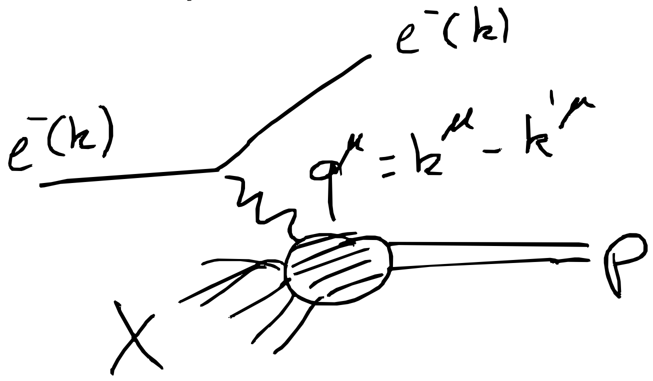
→ automate NNLO QCD

→ first @ N3LO QCD

Hadronic initial states

→ deep-inelastic scattering

$$e^-(k) + P(p) \to e^-(k') + X(P_X)$$



$$Q^2 = -q^2$$

$$x = x_{Bj} = \frac{Q^2}{2P\cdot q} \in [0,1]$$

Inelasticity: $y = \dfrac{P\cdot q}{P\cdot k}$

c.o.E: $s = (k+P)^2$

$$d\sigma = \frac{1}{4ME} \underline{\sum_X} \, d\phi \, \overbrace{\frac{1}{4} |M_{e^-p\to e^- + X}|^2}^{I}$$

$$\hookrightarrow \frac{d^3k'}{(2\pi)^3 2E'} \cdot \underline{d\phi_X}$$

$$I = \frac{e^4}{Q^4} L^{\mu\nu} H_{\mu\nu}$$

$$\to W_{\mu\nu} = \frac{1}{8\pi} \sum_X \int d\phi_X \, H_{\mu\nu} \leftarrow \text{depend } P, q$$

→ just QED: Parity conserved, $q^\mu W_{\mu\nu} = 0$

$$W_{\mu\nu} = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) W_1(x,Q^2) + \hat{p}^\mu \hat{p}^\nu \frac{W_2(x,Q^2)}{P\cdot q} \qquad \hat{p}^\mu = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) P_\nu$$

$$d\sigma = \frac{4\pi\alpha}{y Q^4}\left[y^2 W_1(x,Q^2) + \left(\frac{(1-y)}{x} - \underbrace{xy\frac{M^2}{Q^2}}_{\to 0}\right) W_2(x,Q^2)\right]$$

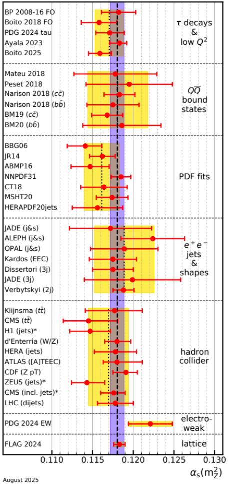
Consider $Q^2 \to \infty$, $x = \text{const}$

$$= \frac{4\pi\alpha}{yQ^4}\Big[\big(1+(1-y)^2\big) \underset{\substack{\uparrow \\ \text{Transverse}}}{F_1} + \frac{1-y}{x} \underbrace{(F_2 - 2xF_1)}_{\text{Longitudinal}}\Big]$$

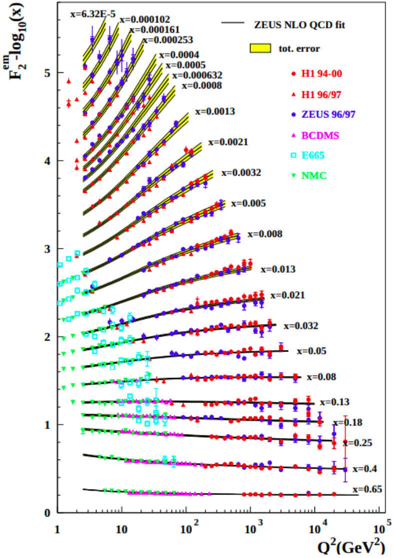
scattering of spin 1/2 particle

$F_2 - 2xF_1 = 0$ Callan-Gross

Strong coupling constant determination



DIS



gluons

sea. quarks

valence

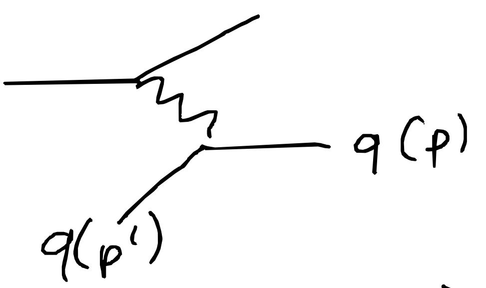
quarks

Parton model:

$$e(k) + q(p) \to e^-(k') + q(p')$$

hadron

$$p = \omega P$$



$$d\sigma = \frac{4\pi}{y Q^4}\Big[\big(1+(1-y)^2\big)\underbrace{\tfrac{1}{2} e_q^2\, \delta(\omega - x)}_{F_1} + \underbrace{0}_{F_2 - 2x F_1}\Big]$$

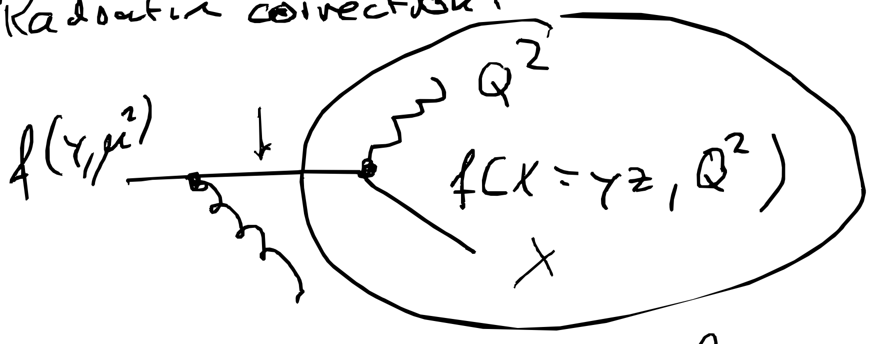
Naive quark model:

$$F_2(x) = \sum_i e_q^2\, f_i(x)\, x$$

parton distribution functions PDFs

$$d\sigma(ep \to e' + X) = \int \frac{d\omega}{\omega} \sum_i f_i(\omega)\, d\hat{\sigma}\left(\frac{x}{\omega}, Q^2\right)$$

Radiative correction:



$$f(x,Q^2) = \underbrace{f(x,\mu^2)}_{\text{no emission}} + \int_x^1 dy\, f(y,\mu^2) \int_0^1 dx\, \delta(x - yz) \int_{\mu^2}^{Q^2} dk^2\, P(z,k^2)$$

$$\Rightarrow \frac{df(x,\mu^2)}{d\mu^2} = \int_x^1 dz\, P(z,\mu^2)\, f\left(\frac{x}{z}, \mu^2\right)$$

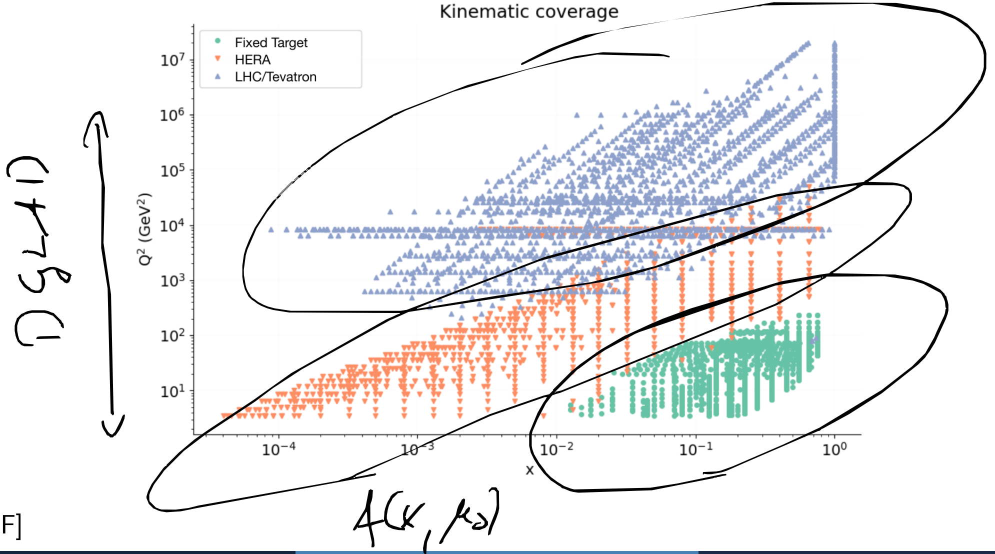
$\Rightarrow$ all flavours: DGLAP

LO:
$$\frac{d f_a(x,\mu^2)}{d\mu^2} = \int_x^1 dz\, \frac{\alpha_s}{2\pi} P_{ab}(z,\mu^2)\, f_b\left(\frac{x}{z},\mu^2\right)$$

$$\Rightarrow F = \sum \int_x^1 \frac{dz}{z}\, C_i\left(\frac{x}{z}, \frac{Q^2}{\mu_F^2}, \alpha_s(\mu)\right) f_i(z, \mu_F^2)$$

proton-proton:

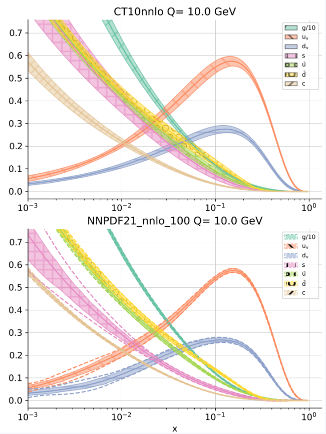
$$d\sigma = \sum_{a,b} \int_0^1 dx_1\, dx_2\, \underbrace{f_a}_{\text{data}}(x_1, \mu_F^2)\, f_b(x_2, \mu_F^2)\, d\hat{\sigma}_{ab}(x_1, x_2, \mu_F^2, \mu_R^2)$$


Kinematic coverage
Fixed Target
HERA
LHC/Tevatron
10^7
10^6
10^5
10^4
10^3
10^2
10^1
Q^2 (GeV^2)
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^0
x
$f(x, \mu_F)$

[NNPDF]

PDFs

[redacted]



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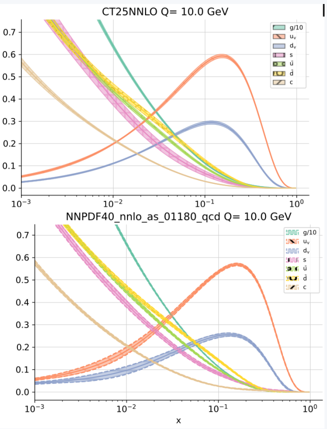
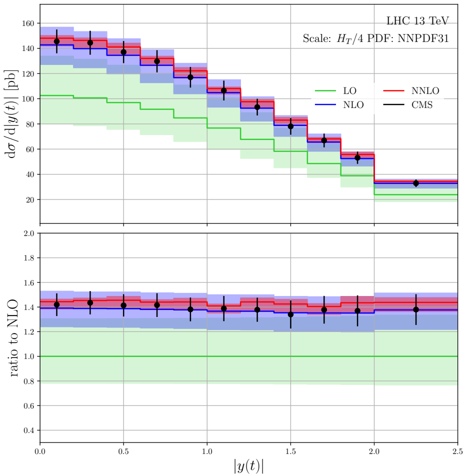


[Juan Cruz Martinez]

LHC top-quark pair at higher orders



(μ_0^R, μ_0^F)

→ scale variation: μ_F, μ_R

7-point variation