Standard Model Precision Physics

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Section 1

Introduction and outline

Experiment and theory

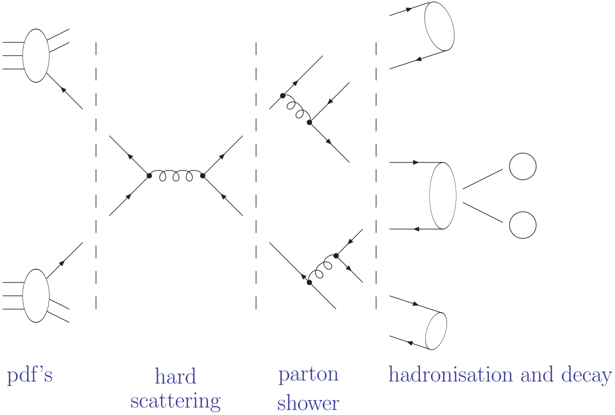
Experiments measure observables $\mathcal{O}$ like cross-sections, jet rates, $p_\perp$-distributions, asymmetries, etc.

Theory specified by a Lagrange density $\mathcal{L}$ with parameters g (coupling), m (masses), etc.

How to **relate** the **observables to the fundamental parameters** of the theory?
We need

- Precise and accurate measurements (see experimental lectures).
- Precise and accurate theory predictions (topic of these lectures)

Scattering events



Underlying event:	Interactions of the proton remnants.
Multiple interactions:	more than one pair of partons undergo hard scattering
Pile-up events:	more than one hadron-hadron scattering within a bunch crossing

Outline

Part I:	**Basics**
Part II:	**Leading-order calculations**
Part III:	**Next-to-leading-order calculations**
Part IV:	**Higher-order calculations**

Aims of these lectures

- Experiment: Understand programs you are using
  - event generators
  - fixed-order N^{k}LO programs
  - combination of the above
- Theory: Learn modern techniques
  - multi-parton calculations
  - multi-loop calculations
  - combination of the above

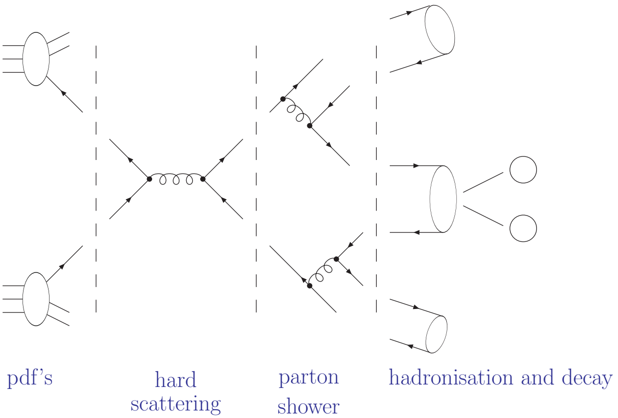
Literature

- Textbooks:
  - M. Peskin und D. Schroeder, *An Introduction to Quantum Field Theory*, Perseus Books, 1995.
  - M. Schwartz, *Quantum Field Theory and the Standard Model*, Cambridge University Press, 2014.
  - R.K. Ellis, W.J Stirling, B.R. Webber, *QCD and Collider Physics*, Cambridge University Press, 1996.
  - S. Weinzierl, *Feynman integrals*, Springer, 2022.
- Reviews and lecture notes:
  - L. Dixon, *Calculating scattering amplitudes efficiently*, Proceedings TASI 95, hep-ph/9601359.
  - R.K. Ellis, Z. Kunszt, K. Melnikov, G. Zanderighi, *One-loop calculations in quantum field theory: from Feynman diagrams to unitarity cuts*, Phys. Rept. 518 (2012) 141, arXiv:1105.4319.

Section 2

Basics

Scattering events



We want **observables**, which are **sensitive** to the **hard scattering**, but insensitive to the details of the parton shower, hadronisation and decay.

Infrared-safe observables

Soft and collinear particles

A **particle detector has a finite resolution**.

- Finite angular resolution: A pair of particles moving in (almost) the same direction can not be resolved below a certain angle.
  We call these particles a **pair of collinear particles**.

  Example: An electron and a photon not resolved by the electromagnetic calorimeter.

- Detection only above a certain threshold: A particle with an energy below a certain threshold will not be detected.
  We call this particle a **soft particle**.

  Example: A low-energy photon.

Infrared-safe observables

Observables which do not depend on long-distance behaviour, are called **infrared-safe observables** and can reliably be calculated in perturbation theory.

In particular, it is required that they do not change value, if infinitessimal soft or collinear particles are added.

For example:

$$\text{collinear}: \qquad \lim_{p_i || p_j} \mathcal{O}_n(p_1, ..., p_i, ..., p_j, ..., p_n) = \mathcal{O}_{n-1}(p_1, ..., p_{ij}, ..., p_n)$$

$$\text{soft}: \qquad \lim_{p_j \to 0} \mathcal{O}_n(p_1, ..., p_j, ..., p_n) = \mathcal{O}_{n-1}(p_1, ..., p_{j-1}, p_{j+1}, ..., p_n)$$

Event shapes

Event shapes are observables calculated from all particles in an event.

Typical examples of **infrared-safe event shapes** in electron-positron annihilation are thrust, heavy jet mass, wide jet broadening, total jet broadening, C parameter, etc.

Example (Thrust)

$$T = \max_{\hat{n}} \frac{\sum\limits_i |\vec{p}_i \cdot \hat{n}|}{\sum\limits_i |\vec{p}_i|}$$

For two particles back-to-back one has

$$T = 1$$

For many particles, isotropically distributed we have

$$T = \frac{1}{2}$$

Spherocity versus sphericity

Spherocity:

$$S = \left(\frac{4}{\pi}\right)^2 \min_{\hat{n}} \left(\frac{\sum\limits_i |\vec{p}_i^{\perp}|}{\sum\limits_i |\vec{p}_i|} \right)^2$$

Sphericity:

$$\left(\frac{4}{\pi}\right)^2 \min_{\hat{n}} \frac{\sum\limits_i |\vec{p}_i^{\perp}|^2}{\sum\limits_i |\vec{p}_i|^2}$$

Sphericity is not infrared-safe !

Jet algorithms

The most fine-grained look at hadronic events consistent with infrared safety is given by classifying the particles into jets.

Sequential recombination algorithm:

- *a* **resolution variable** *y_{ij} where a smaller y_{ij} means that particles i and j are "closer";*

$$y_{ij}^{\text{Durham}} = \frac{2(1-\cos\theta_{ij})}{Q^2} \, min(E_i^2, E_j^2)$$

- *a* **combination procedure** *which combines two four-momenta into one;*

$$p^\mu_{(ij)} = p^\mu_i + p^\mu_j.$$

- *a* **cut-off** *y_{cut} which provides a stopping point for the algorithm.*

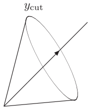
Jet algorithms

- In electron-positron annihilation one uses mainly **exclusive jet algorithms**, where each particle in an event is assigned uniquely to one jet.
- In hadron-hadron collisions one uses mainly **inclusive jet algorithms**, where each particle is either assigned uniquely to one jet or to no jet at all.
- One distinguishes further **sequential recombination algorithm** and **cone algorithms**. Infrared-safe cone algorithm: SISCone.
- Once jets are defined we can look at cross sections, $p_{\perp}$ distributions, rapidity distributions, etc.

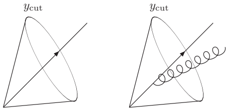
Modeling of jets

In a perturbative fixed-order calculation jets are modeled by only a few partons. This improves with the order to which the calculation is done.

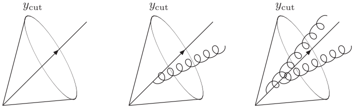
At leading order:



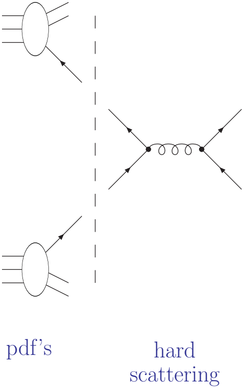
At next-to-leading order:



At next-to-next-to-leading order:



Fixed-order calculations



- Done at the **parton level** with quarks, gluons, photons, electrons, etc..
- Usually **only a few partons** in the final state.
- Method of choice to describe **well-separated jets**.

The master formula for the calculation of observables

$$\langle O_N \rangle =$$

$$\underbrace{O(p_1, \dots, p_n)}_{\text{observable}}$$

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The master formula for the calculation of observables

$$\langle O_N \rangle = \underbrace{\frac{1}{2K(\hat{s})}}_{\text{flux factor}} \underbrace{\frac{1}{n_a^{\text{spin}} n_b^{\text{spin}} n_a^{\text{colour}} n_b^{\text{colour}}}}_{\text{average over initial spins and colours}}$$

$$\times \sum_{n \geq N} \underbrace{\int d\phi_n}_{\text{integral over phase space}} \underbrace{O(p_1, \dots, p_n)}_{\text{observable}} \underbrace{|\mathcal{A}_{n+2}|^2}_{\text{amplitude}}$$

The master formula for the calculation of observables

$$\langle O_N \rangle = \underbrace{\sum_{a,b} \int dx_1 \, \mathbf{f_a(x_1)} \int dx_2 \, \mathbf{f_b(x_2)}}_{\text{pdf's}} \; \underbrace{\frac{1}{2K(\hat{s})}}_{\text{flux factor}} \; \underbrace{\frac{1}{n_a^{\text{spin}} n_b^{\text{spin}} n_a^{\text{colour}} n_b^{\text{colour}}}}_{\text{average over initial spins and colours}}$$

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Ingredients:

- Infrared-safe observables
- Scattering amplitude
- Integration over the phase space
- Parton distribution functions

Scattering amplitudes

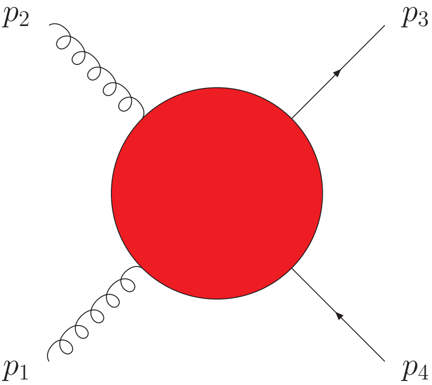
Scattering amplitudes

- We would like to make precise predictions for observables in scattering experiments from quantum field theory.
- Any such calculation will involve a scattering amplitude.
- Unfortunately we cannot calculate scattering amplitudes exactly.
- If we have a small parameter like a small coupling, we may use perturbation theory.
- We may organise the perturbative expansion of a scattering amplitude in terms of Feynman diagrams.

Scattering amplitude = sum of all Feynman diagrams

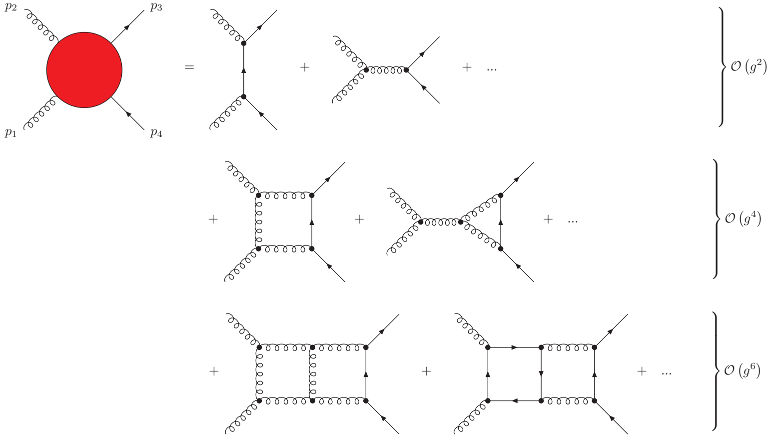
Scattering amplitudes

Example: **Scattering amplitude** for a $2 \to 2$-process:



Scattering amplitudes

We may compute the scattering amplitude within perturbation theory:



Perturbation theory

We write the perturbative expansion of an amplitude as

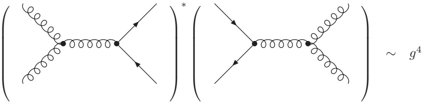
$$\mathcal{A}_n = g^{n-2}\left(\mathcal{A}_n^{(0)} + g^2 \mathcal{A}_n^{(1)} + g^4 \mathcal{A}_n^{(2)} + g^6 \mathcal{A}_n^{(3)} + \dots\right),$$

where $\mathcal{A}_n^{(l)}$ denotes an amplitude with n external particles and l loops.

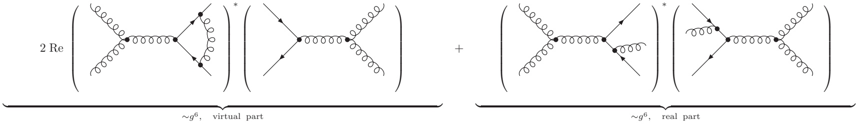
Perturbation theory

In the master formula for the computation of an observable the amplitude squared enters:

At leading order (LO) only **Born amplitudes** contribute:



At next-to-leading order (NLO): **One-loop amplitudes** and **Born amplitudes with an additional parton**.



Real part contributes whenever the additional parton is not resolved.

Perturbation theory

Perturbative expansion of the amplitude squared (**LO**, NLO, **NNLO**):

$$|\mathcal{A}_n|^2 = g^{2n-4} \underbrace{\left|\mathcal{A}_{\mathbf{n}}^{(0)}\right|^2}_{\text{Born}} + g^{2n-2} \underbrace{2 \,\mathrm{Re}\ \mathcal{A}_n^{(0)\,*}\, \mathcal{A}_n^{(1)}}_{\text{virtual}} + g^{2n} \underbrace{\left(\left|\mathcal{A}_{\mathbf{n}}^{(1)}\right|^2 + 2 \,\mathrm{Re}\ \mathcal{A}_{\mathbf{n}}^{(0)\,*}\, \mathcal{A}_{\mathbf{n}}^{(2)} \right)}_{\text{one-loop squared and two-loop}}$$

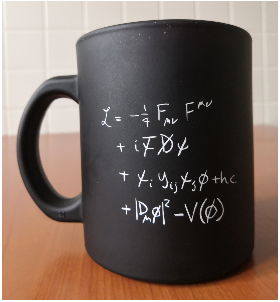
$$|\mathcal{A}_{n+1}|^2 = g^{2n-2} \underbrace{\left|\mathcal{A}_{n+1}^{(0)}\right|^2}_{\text{real}} + g^{2n} \underbrace{2 \,\mathrm{Re}\ \mathcal{A}_{\mathbf{n+1}}^{(0)\,*}\, \mathcal{A}_{\mathbf{n+1}}^{(1)}}_{\text{loop+unresolved}}$$

$$|\mathcal{A}_{n+2}|^2 = g^{2n} \underbrace{\left|\mathcal{A}_{\mathbf{n+2}}^{(0)}\right|^2}_{\text{double unresolved}}$$

The Standard Model

Having a cup of coffee is always a good idea!

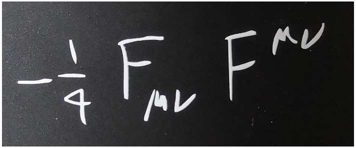
1. What is written on the coffee mug?
2. How do we get from there to predictions for the experiments at CERN?



The left-hand side: Lagrange density

- classical mechanics: Lagrange function
- fields: Lagrange density
- classical: principle of stationary action:
  $S = \int d^4x \mathcal{L}$.
- quantum: weight each configuration with $e^{\frac{i}{\hbar}S}$.

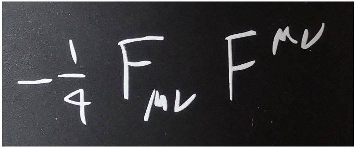
Gauge bosons



- compact notation for all three gauge groups
- $F_{\mu\nu}$: field strength tensor
- μ, ν: Lorentz indices
- in the case of electrodynamics:

$$F_{\mu\nu} F^{\mu\nu} \quad = \quad 2 \left(\vec{B}^2 - \vec{E}^2 \right)$$

Gauge bosons



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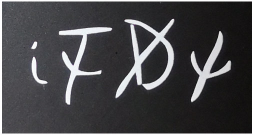
In more detail:

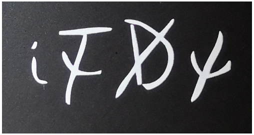
$$\mathcal{L}_{\mathrm{gauge}} \quad = \quad \underbrace{-\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu}}_{SU(3)} \underbrace{-\frac{1}{4} W^a_{\mu\nu} W^{\mu\nu a}}_{SU(2)} \underbrace{-\frac{1}{4} B_{\mu\nu} B^{\mu\nu}}_{U(1)} + \mathcal{L}_{\mathrm{GF}} + \mathcal{L}_{\mathrm{FP}}$$

Fermions



- compact notation for all fermions.
- ψ: spinor field,
  $\bar{\psi}$: adjoint spinor field.
- $\slashed{D} = \gamma^\mu D_\mu$: covariant derivative contracted with Dirac matrices γ^μ.
- covariant derivative: standard derivative plus coupling to gauge fields;
  "minimal substitution".

Fermions



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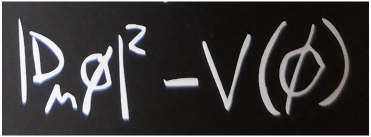
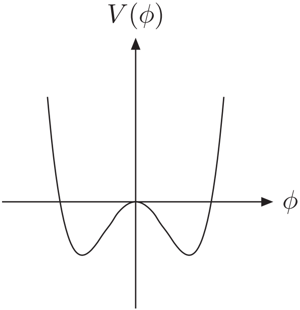
In more detail:

$$\mathcal{L}_{\text{fermions}} = \sum_{\text{families}} \left\{ i \left(\bar{u}_L, \bar{d}'_L \right) \not{D} \begin{pmatrix} u_L \\ d'_L \end{pmatrix} + i \bar{u}_R \not{D} u_R + i \bar{d}'_R \not{D} d'_R + i \left(\bar{\nu}'_L, \bar{e}_L \right) \not{D} \begin{pmatrix} \nu'_L \\ e_L \end{pmatrix} + i \bar{\nu}'_R \not{D} \nu'_R + i \bar{e}_R \not{D} e_R \right\}$$

The Higgs sector



- ϕ: Higgs field
- D_μ: covariant derivative
- $V(\phi)$: Higgs potential



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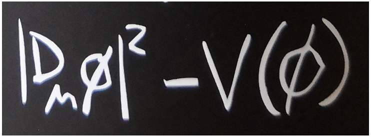

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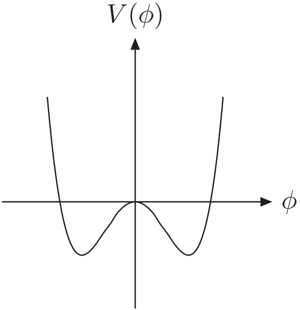
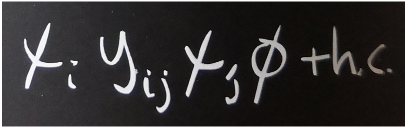


In more detail:

$$\mathcal{L}_{\text{Higgs}} = \left(D_\mu \phi\right)^\dagger \left(D^\mu \phi\right) + \mu^2 \phi^\dagger \phi - \frac{1}{4} \lambda \left(\phi^\dagger \phi\right)^2$$

Yukawa couplings



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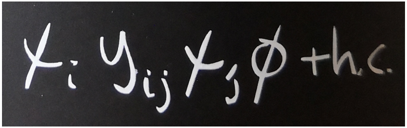
- i, j: indices in family space
- gives masses to the fermions

Yukawa couplings



- compact notation
- y_{ij}: Yukawa couplings
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- gives masses to the fermions

In more detail (with my conventions):

$$\mathcal{L}_{\mathrm{Yukawa}} = \sum_{\mathrm{families}} \left\{ - \left(\bar{u}_L, \bar{d}_L\right)_i \phi \left(y_d\right)_{ij} d_{R,j} - \left(\bar{u}_L, \bar{d}_L\right)_i \phi^C \left(y_u\right)_{ij} u_{R,j} - \left(\bar{\nu}_L, \bar{e}_L\right)_i \left(y_e\right)_{ij} \phi e_{R,j} - \left(\bar{\nu}_L, \bar{e}_L\right)_i \phi^C \left(y_\nu\right)_{ij} \nu_{R,j} \right.$$
$$\left. + \text{h.c.} \right\}$$

Feynman rules

From the Lagrange density to Feynman rules

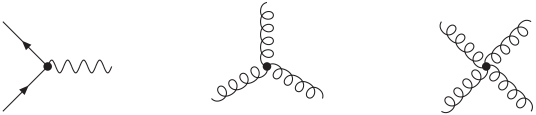
1. Sort the terms in the Lagrangian according to the number of fields involved.

   What about

   - Terms without any fields in the Lagrangian?
   - Terms with a single field in the Lagrangian?

2. Terms bilinear in the fields determine the **propagators**:

3. Terms with three or more fields determine the **vertices**:



Propagators

1. Start from

$$\mathcal{L}_{\text{bilinear}} = \frac{1}{2}\phi_j P_{jk}(x)\phi_k$$

2. Define the **inverse** of P by

$$\sum_j P_{jl}(x)P^{-1}_{lk}(x-y) = \delta_{jk}\delta^D(x-y)$$

3. Define the **Fourier transform** of the inverse by

$$P^{-1}_{jk}(x) = \int \frac{d^Dq}{(2\pi)^D} e^{-iq\cdot x}\tilde{P}^{-1}_{jk}(q)$$

4. Then

$$\text{Propagator} = i\left(\tilde{P}^{-1}(q)\right)_{jk}$$

Example: The photon propagator in a covariant gauge

- We have

$$P^{\mu\nu}(x) \quad = \quad \Box g^{\mu\nu} - \left(1 - \frac{1}{\xi}\right) \partial^\mu \partial^\nu$$

- We determine $\tilde{P}^{-1}_{\mu\nu}(q)$ from

$$P^{\mu\lambda}(x) \int \frac{d^4q}{(2\pi)^4} e^{-iq\cdot(x-y)} \tilde{P}^{-1}_{\lambda\nu}(q) \quad = \quad g^\mu_{\ \nu} \int \frac{d^4q}{(2\pi)^4} e^{-iq\cdot(x-y)}$$

We have for

$$M^{\mu\nu} = -g^{\mu\nu} + \left(1 - \frac{1}{\xi}\right) \frac{q^\mu q^\nu}{q^2} \quad \text{and} \quad N_{\mu\nu} = -g_{\mu\nu} + (1-\xi) \frac{q_\mu q_\nu}{q^2}$$

the relation

$$M^{\mu\lambda} N_{\lambda\nu} \quad = \quad g^\mu_{\ \nu}$$

- Propagator:

$$= \quad \frac{i}{q^2} \left(-g_{\mu\nu} + (1-\xi) \frac{q_\mu q_\nu}{q^2} \right)$$

Vertices

1. Start from

$$\mathcal{L}_{\mathrm{int}}(x) \quad = \quad O_{j_1 \ldots j_n}(\partial_1, \ldots, \partial_n)\, \phi_{j_1}(x) \ldots \phi_{j_n}(x),$$

with the notation that ∂_k acts only on the k-th field $\phi_{j_k}(x)$.

2. The vertex is then given by

$$V \quad = \quad i \sum_{\text{permutations}} (-1)^{P_F}\, O_{j_1 \ldots j_n}(iq_1, \ldots, iq_n)$$

- Momenta are taken to flow outward.
- Summation is over all permutations of indices and momenta of identical particles.
- In the case of identical fermions there is in addition a minus sign for every odd permutation of the fermions, indicated by $(-1)^{P_F}$.

Example: The three-gluon vertex

- We have

$$\mathcal{L}_{ggg} = -g f^{abc} g^{\mu\rho} \left(\partial^{\nu} A^a_{\mu}(x)\right) A^b_{\nu}(x) A^c_{\rho}(x).$$

- Thus

$$O^{abc,\mu\nu\rho}(\partial_1, \partial_2, \partial_3) = -g f^{abc} g^{\mu\rho} \partial_1^{\nu}, \qquad O^{abc,\mu\nu\rho}(iq_1, iq_2, iq_3) = -g f^{abc} g^{\mu\rho} iq_1^{\nu}.$$

- Hence

$$\begin{aligned} V_{ggg} &= i \sum_{\text{permutations}} \left(-g f^{abc} g^{\mu\rho} iq_1^{\nu}\right) \\ &= \mathbf{-g f^{abc} \left[g^{\mu\nu} \left(q_1^{\rho} - q_2^{\rho}\right) + g^{\nu\rho} \left(q_2^{\mu} - q_3^{\mu}\right) + g^{\rho\mu} \left(q_3^{\nu} - q_1^{\nu}\right) \right]} \end{aligned}$$

Feynman rules

Each piece of a Feynman diagram corresponds to a mathematical expression:

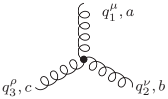
External edge:

$$= \quad \varepsilon_\mu^a(q)$$

Internal edge:

$$= \quad -\frac{i}{q^2} g_{\mu\nu} \delta^{ab}$$

Vertex:



$$= \quad -g f^{abc} \left[g^{\mu\nu} \left(q_1^\lambda - q_2^\lambda \right) + g^{\nu\lambda} \left(q_2^\mu - q_3^\mu \right) + g^{\lambda\mu} \left(q_3^\nu - q_1^\nu \right) \right]$$

Feynman rules

- **Integrate** over any momentum not constrained by momentum conservation with measure

$$\int \frac{d^D k}{(2\pi)^D},$$

where D denotes the dimension of space-time.

- Multiply by a factor (-1) for each **closed fermion loop**.

- **Symmetry factor**: Multiply the diagram by a factor $1/S$, where S is the order of the permutation group of the internal lines and vertices leaving the diagram unchanged when the external lines are fixed.

Take-home messages

1. Consider **infrared-safe** observables!
2. The scattering amplitude is given as the sum of all **Feynman diagrams**.
3. Feynman rules follow from the **Lagrangian**.