Track Based Software Alignment of the Mu3e Pixel Detector Using the General Triplet Track Fit

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3rd Sept. 2026 | Herbstschule HEP

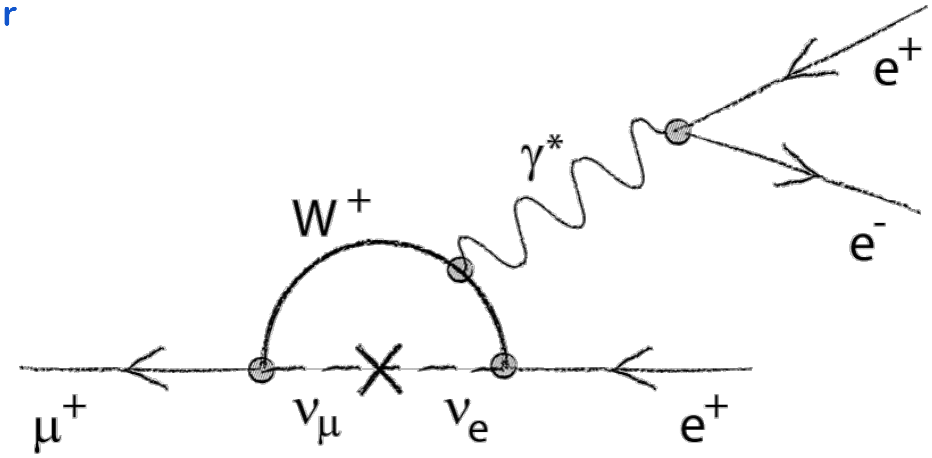
The Mu3e Experiment

aims to observe the extremely rare **lepton flavour violating** decay $\mu^+ \rightarrow e^+e^-e^+$

→ **strongly suppressed** (BR ~ 10^{-54}) in the Standard Model

→ when including neutrino oscillations

Goal: reach an ultimate sensitivity of one in 10^{16} muon decays



Standard Model $\mu^+ \rightarrow e^+e^+e^-$ process via neutrino oscillation [1]

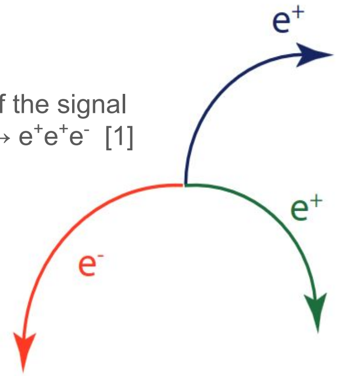
[1] Mu3e TDR: arXiv:2009.11690

The Mu3e Experiment

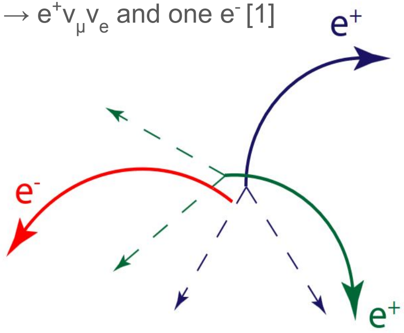
topology of the decay

→ common vertex and coincident in time
→ 4-momenta sum $(\sum P_e) \approx (m_\mu,0,0,0)$

topology of the signal decay $\mu^+ \rightarrow e^+e^+e^-$ [1]



combinatorial background: two Michel decays $\mu^+ \rightarrow e^+\nu_\mu\nu_e$ and one e^- [1]



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requirements for **precise event selection**

→ momentum resolution : < 0.5 MeV
→ timing resolution : < 0.5 ns
→ vertex resolution : < 200 µm

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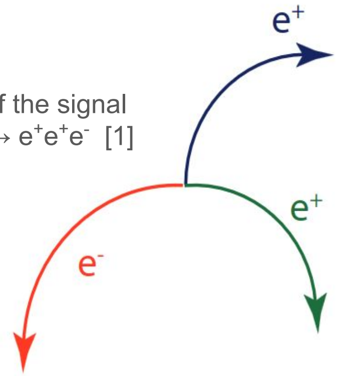

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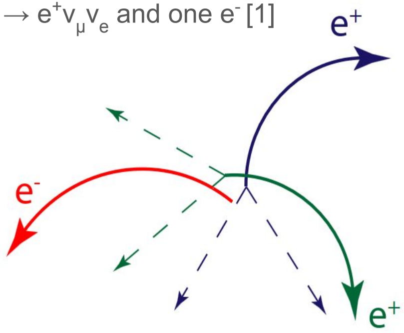

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minimal detector configuration successfully commissioned in June 2025 at PSI

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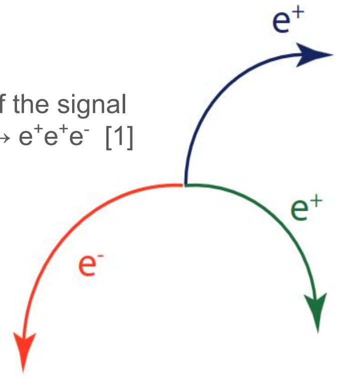

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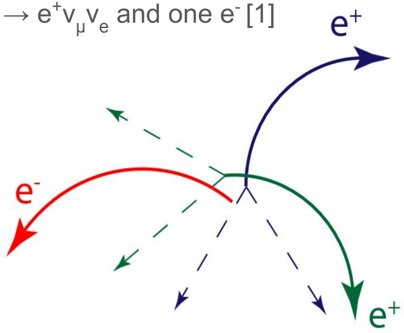

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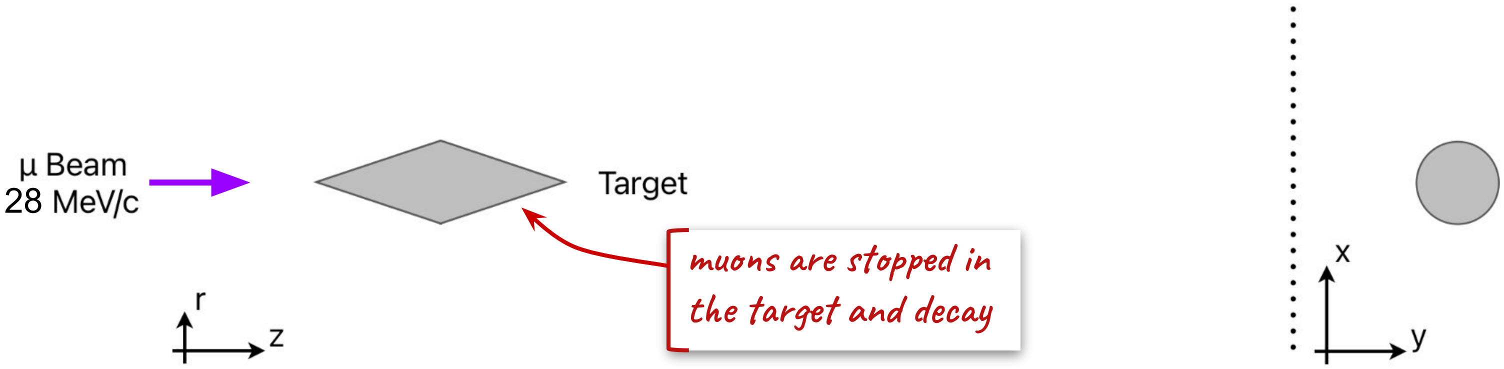
The Mu3e Detector

Minimal Detector Configuration



cross section of the Mu3e minimal detector configuration, adapted from [1]

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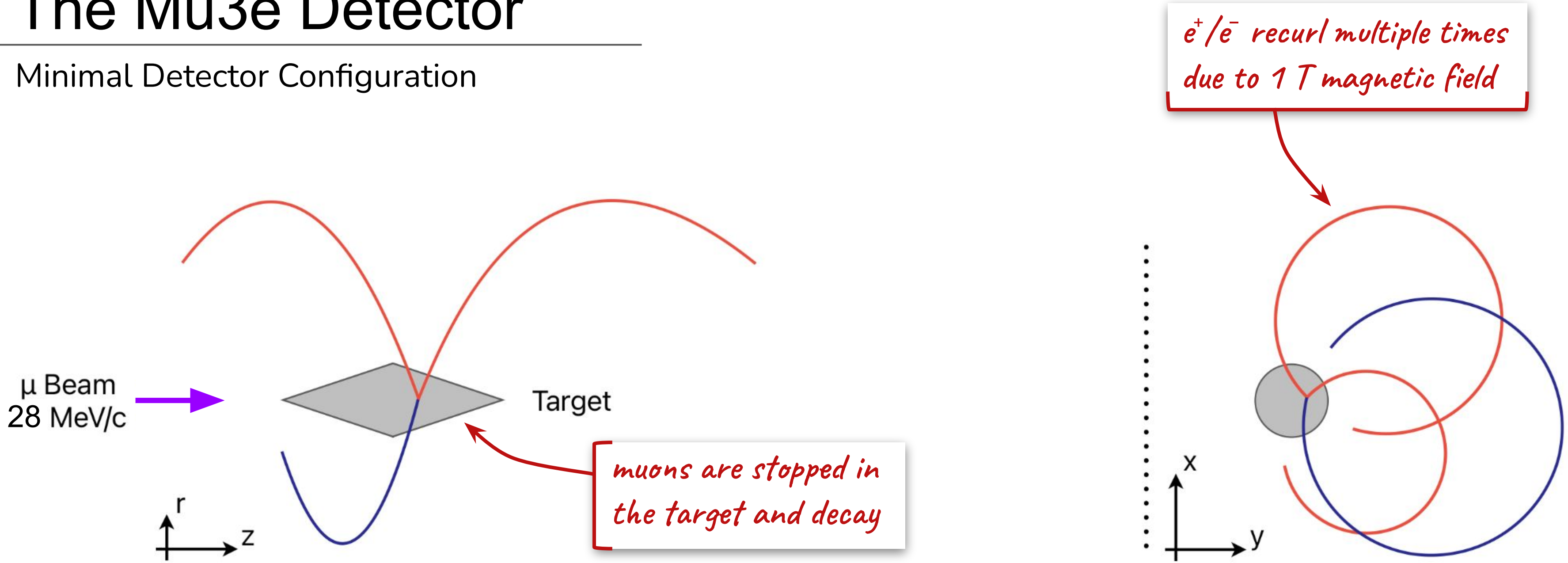

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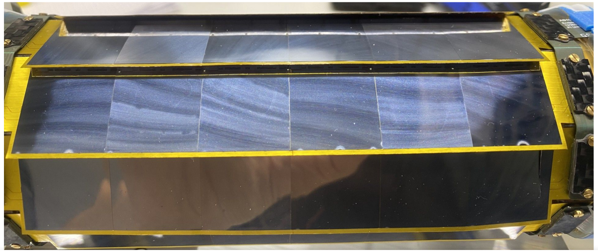
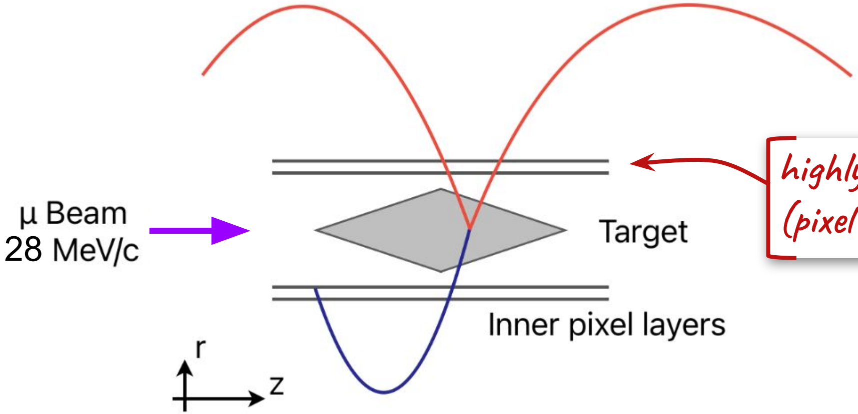
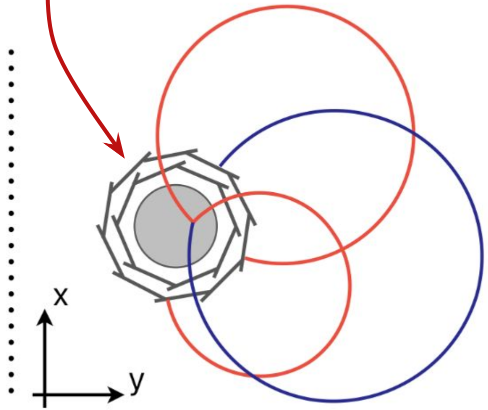


ultra light layers (0.1% x/X0 p. layer)



highly granular vertex detector
(pixel size: 80 x 80 μm²)



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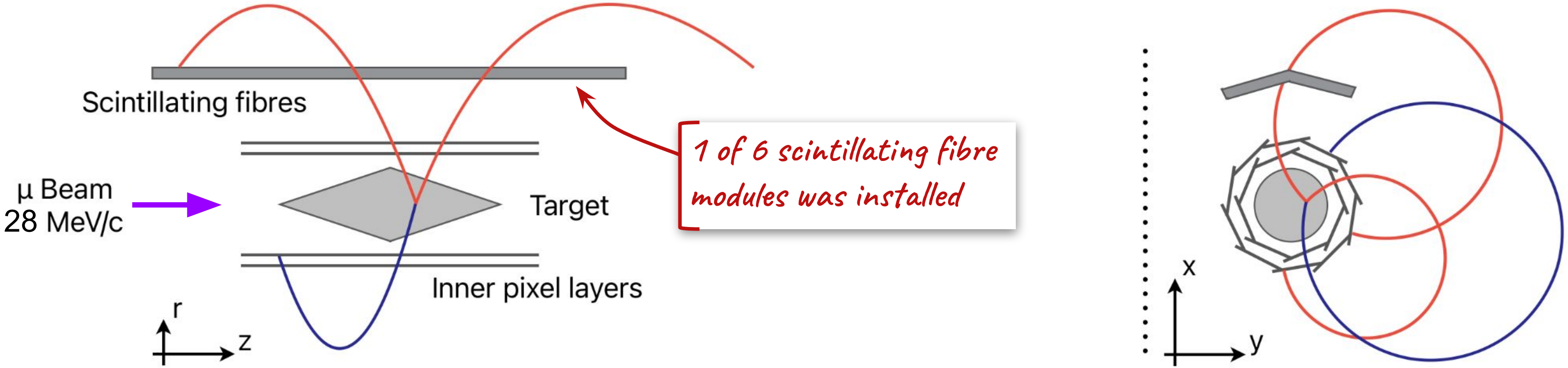

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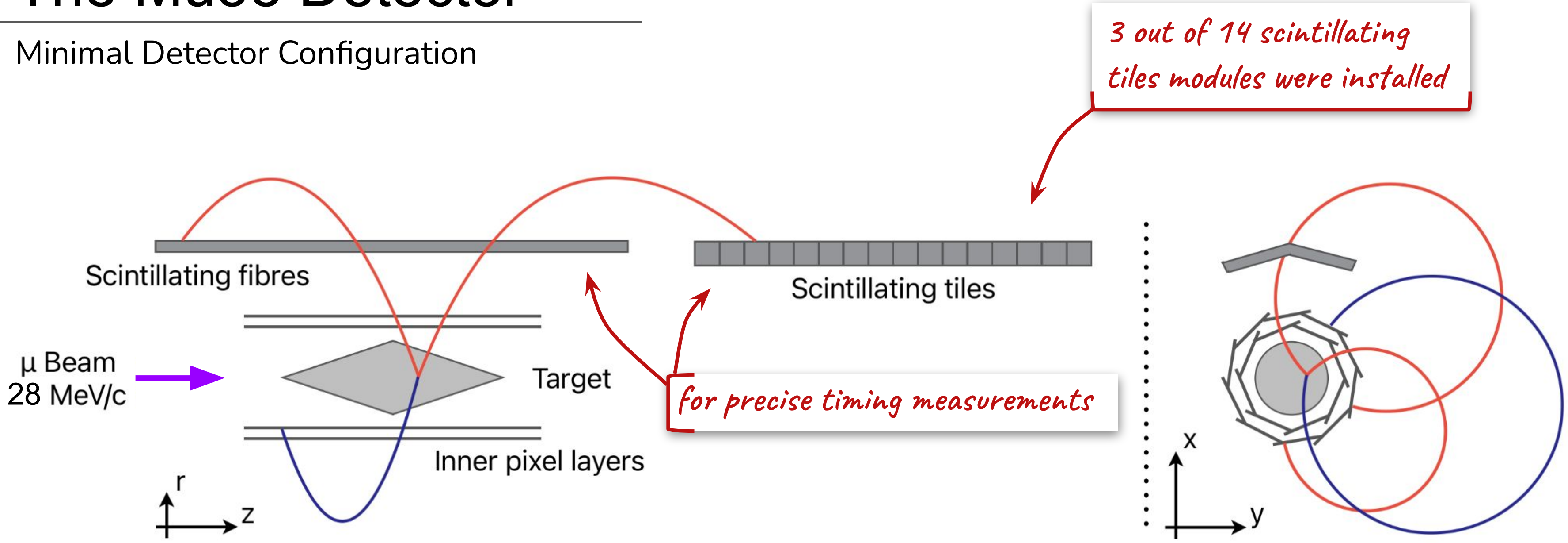

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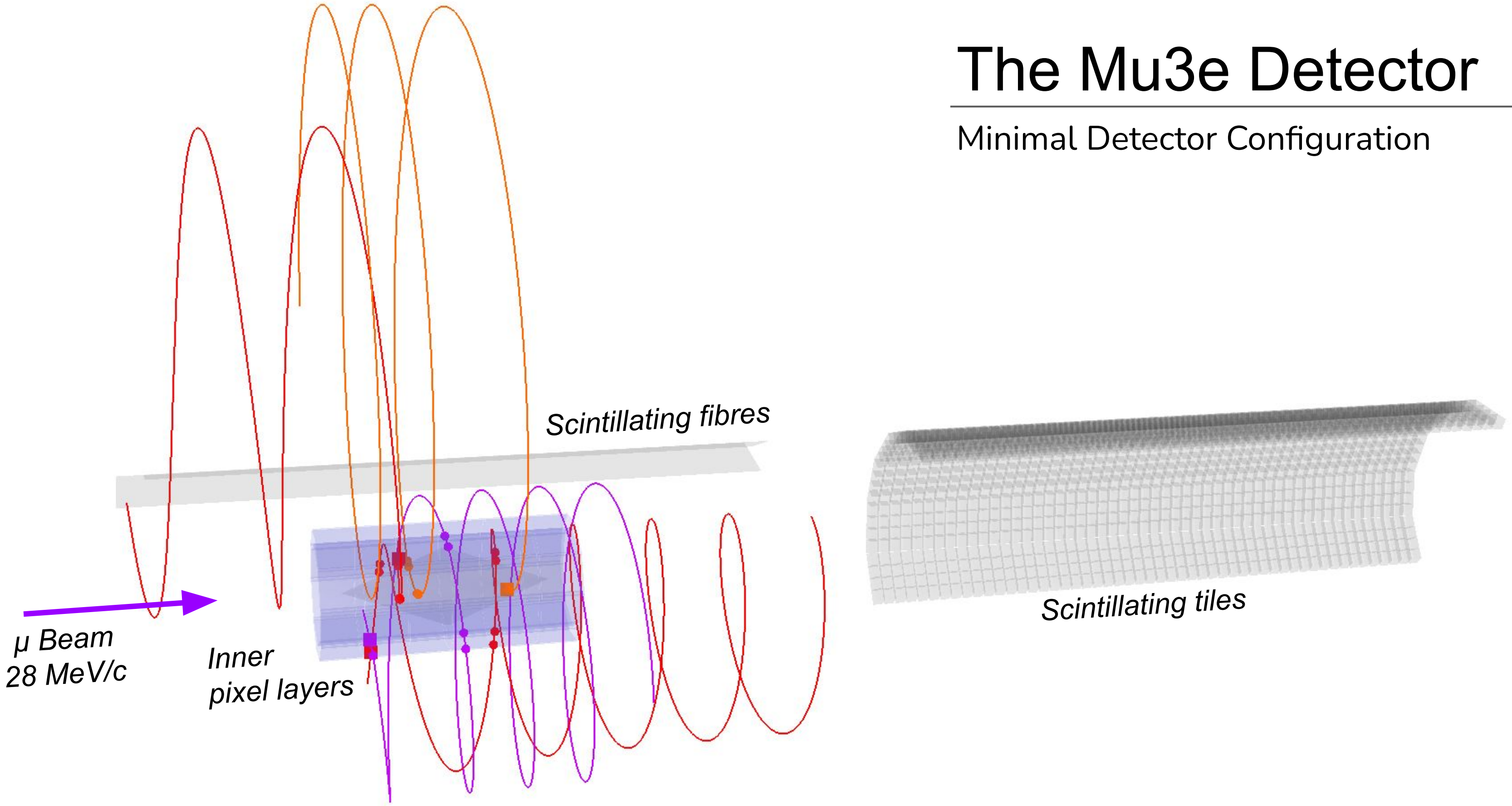
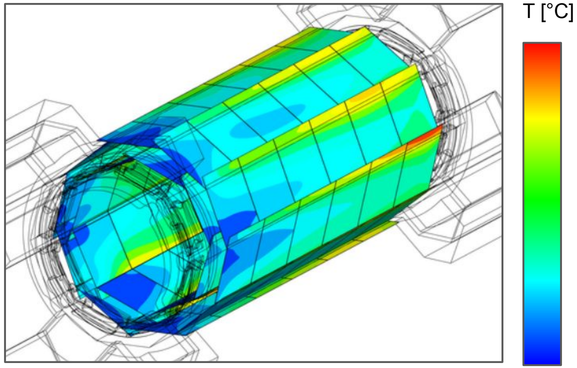


event display showing extrapolated tracks from simulation in the minimal detector configuration

Misalignment in the Mu3e detector

designed to minimize multiple scattering uncertainties using

- ***ultra light pixel layers*** (0.1% X/X_0 per layer)
- ***gaseous helium cooling***



heat dissipation of the inner two pixel layers, from [1]

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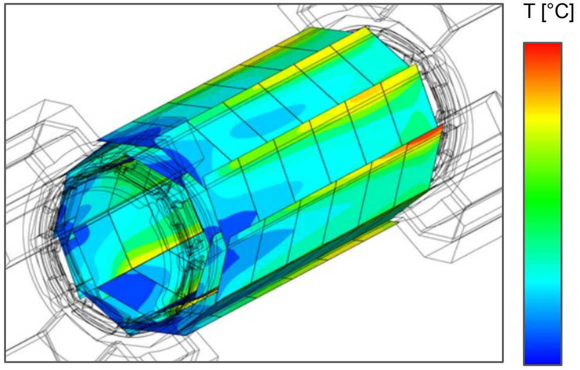
Misalignment in the Mu3e detector

designed to minimize multiple scattering uncertainties using

- ***ultra light pixel layers*** (0.1% X/X_0 per layer)
- ***gaseous helium cooling***

makes the detector more susceptible to misalignment

- shifts, rotations and deformations of thin sensors
- thermal expansion caused by the cooling system



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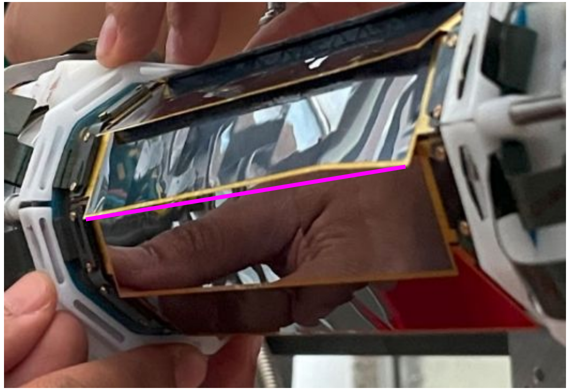

picture of the second pixel layer during installation

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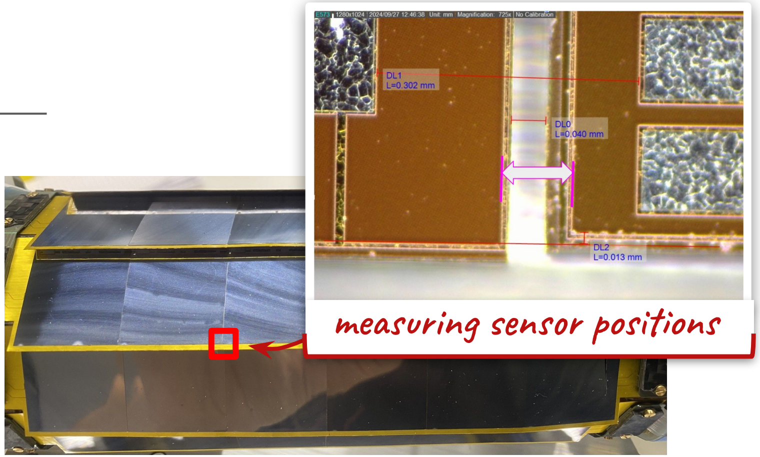
Alignment Strategies for Mu3e

Mechanical alignment

- performed during ladder assembly

Optical alignment

- uses high resolution metrology data (post construction)



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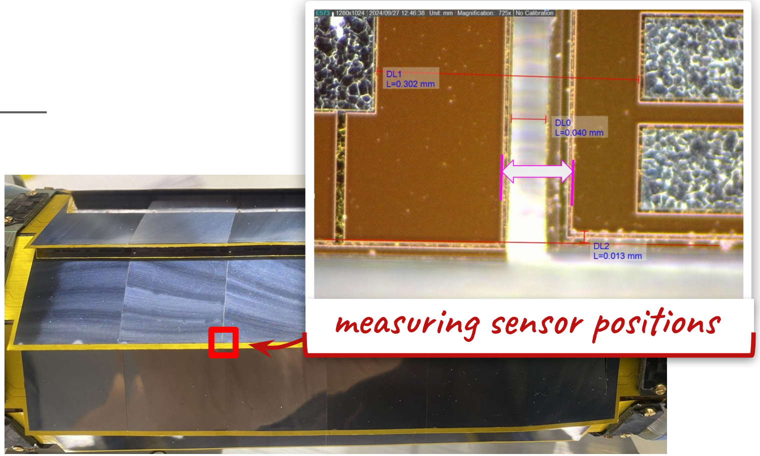

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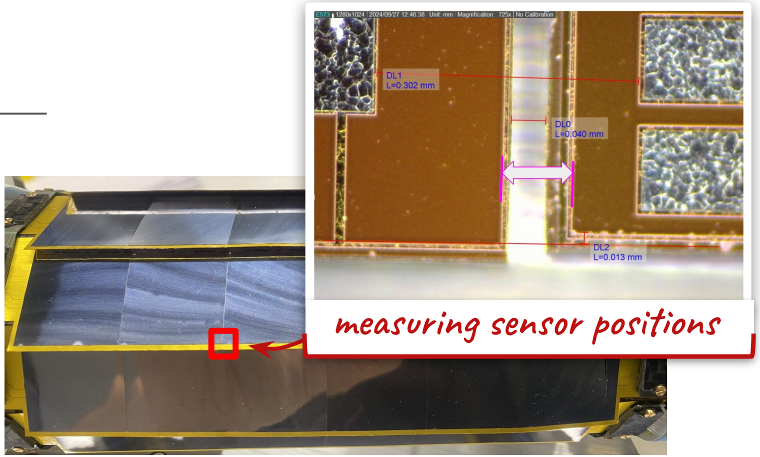

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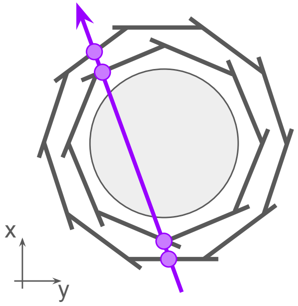
Cosmic muons (origin outside of detector)

- known curvature (straight tracks)
- rather low rate ~ 1 muon / cm² / min



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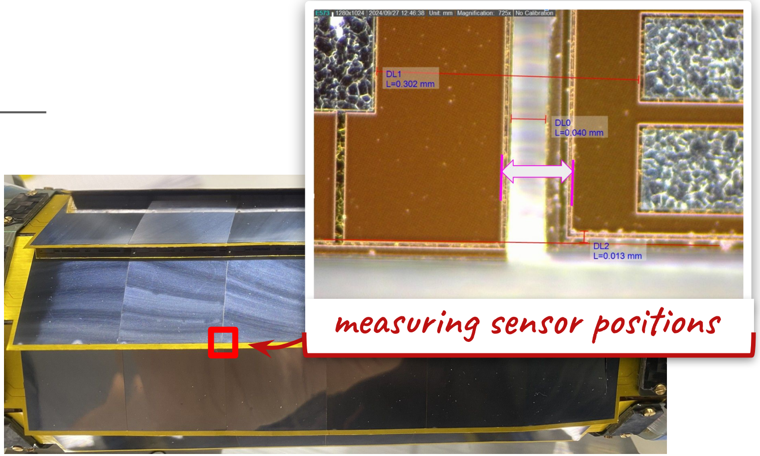
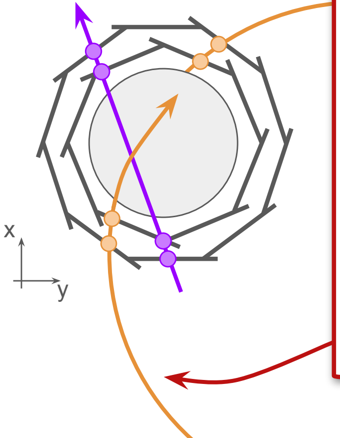
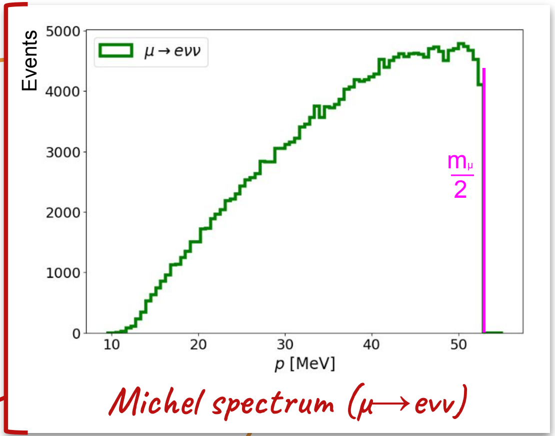
Michel positrons (from Michel decay: μ→eνν)

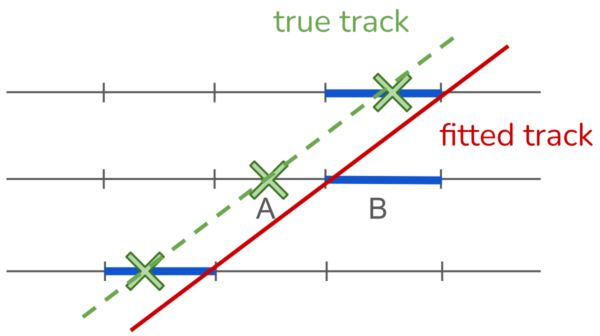
- unknown curvature
- high rate ~ 10^8 muons stops / s



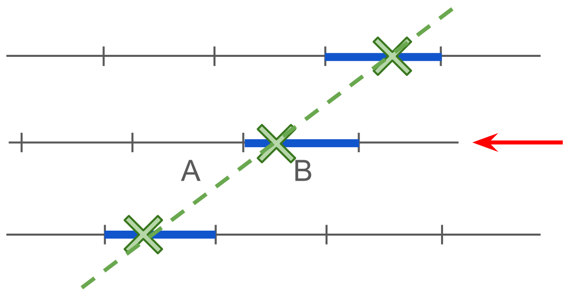




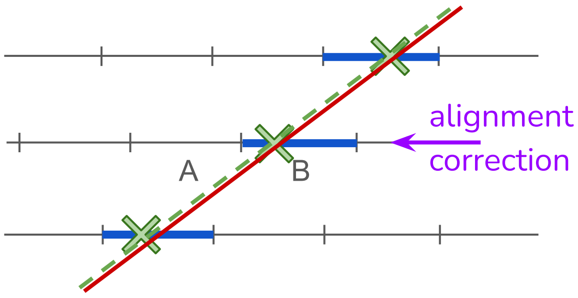
Why is detector alignment important?



Digital reconstruction of the data

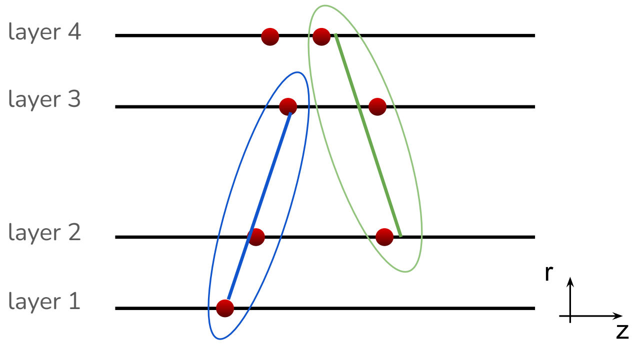


Physical situation in the detector
(misalignment in the longitudinal plane)



Digital correction of the data to
reflect physical situation

General **T**riplet **T**rack **F**it (GTTF)[1]



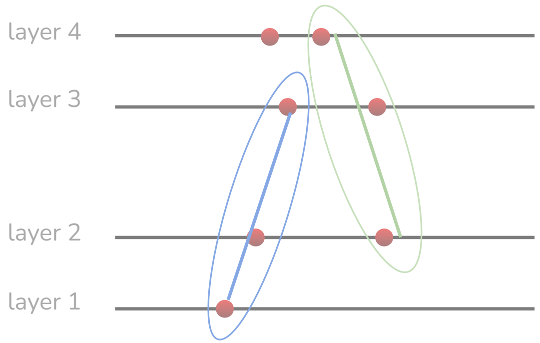
Find and process triplets

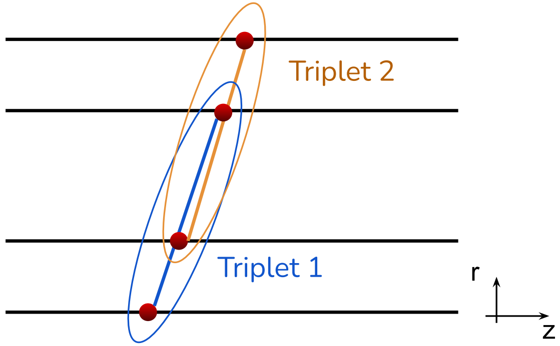
three hits form a triplet:

- highly **parallelizable**
- triplets can be fitted to **filter bad combinations**

[1] Schöning, *A General Track Fit Based on Triplets*, 2025

General Triplet Track Fit (GTTF)[1]





Find and process triplets → **Link triplets**

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triplets which share two common hits are linked to form a track

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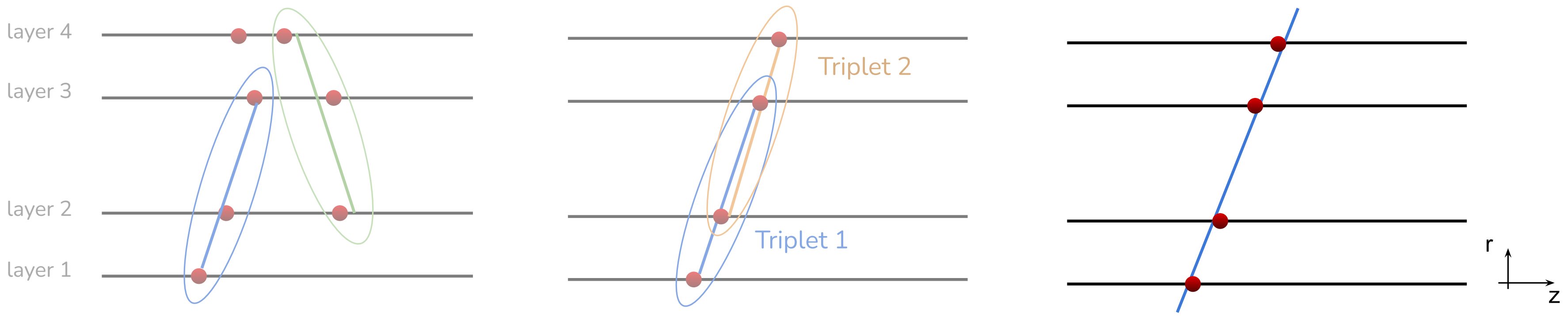

Find and process triplets | Link triplets | Global Track Fit

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fits momentum, hit position shifts **without iteration!**

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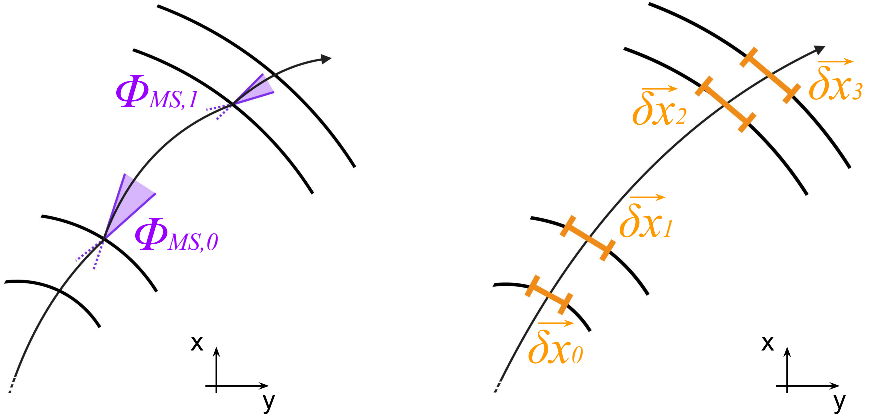
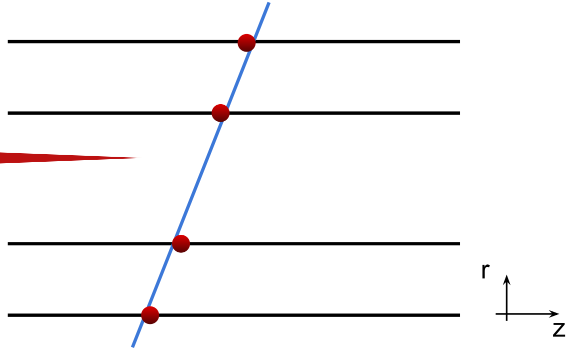
General Triplet Track Fit (GTTF)[1]

fit quality of one track

$$\chi^2 = \underbrace{\boxed{\sum_{j=0}^{n_{\mathrm{scatt}}-1}\left(\frac{\Theta^2_{MS,j}}{\sigma^2_{\Theta_{MS,j}}}+\frac{\Phi^2_{MS,j}}{\sigma^2_{\Phi_{MS,j}}}\right)}}_{\text{Multiple scattering term}} + \underbrace{\boxed{\sum_{k=0}^{n_{\mathrm{hit}}-1}\vec{\delta x}_k^{\top}\,\vec{\vec{V}}_k^{-1}\,\vec{\delta x}_k}}_{\text{Hit uncertainty term}}$$





Global Track Fit

fits momentum, hit position shifts **without iteration!**

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Global Track Fit with Alignment

introduce global alignment parameters $\mathfrak{a}$ in χ^2 function

$\rightarrow$ κ ($\propto$ 1/p) and δ are **track dependent**,
whereas $\mathfrak{a}$ is **track independent**

$$\chi_i^2(\boldsymbol{\kappa}_{i,3D}, \vec{\boldsymbol{\delta}}_i)$$

$$\downarrow$$

$$\boxed{\chi_i^2(\boldsymbol{\kappa}_{i,3D}, \vec{\boldsymbol{\delta}}_i}, \boxed{\mathfrak{a}})$$

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$$\chi^2_i(\boldsymbol{\kappa}_{i,3D}, \vec{\boldsymbol{\delta}}_i)$$

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$$\chi^2_{tot} = \boxed{\sum_{i=1}^{N} \boxed{\chi^2_i(\boldsymbol{\kappa}_{i,3D}, \vec{\boldsymbol{\delta}}_i}, \boxed{\mathfrak{a}})}$$

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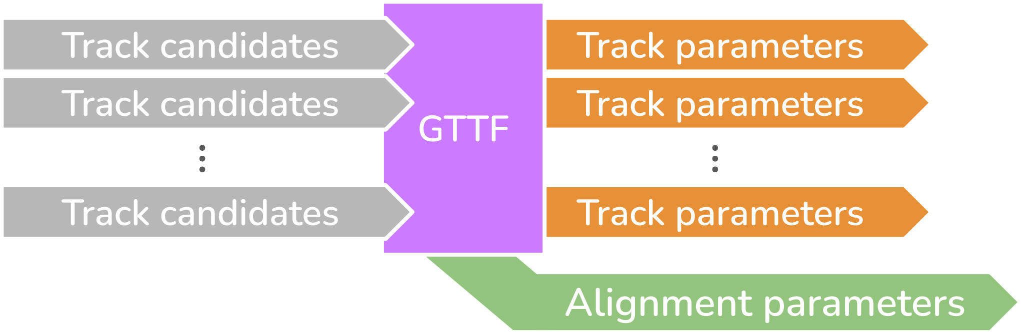
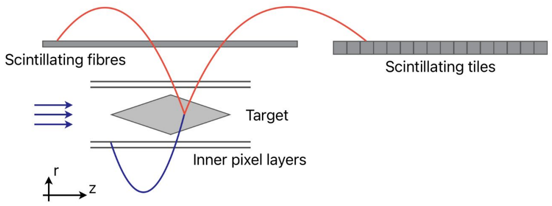


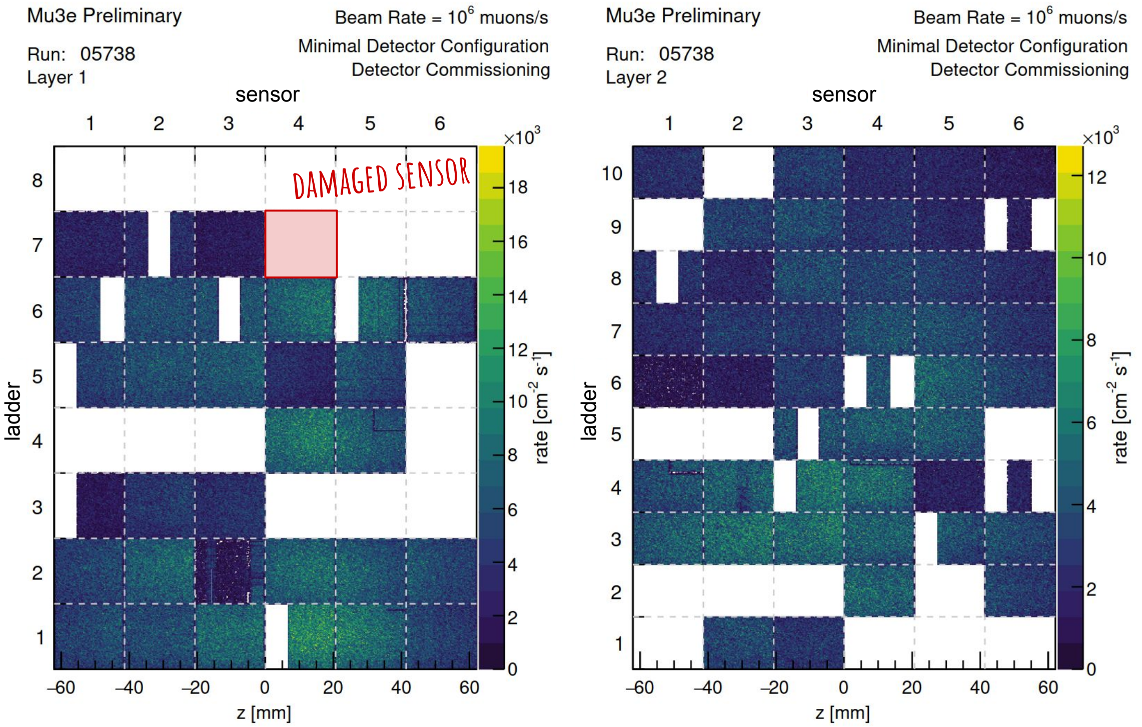
Data Taking with the 2025 Detector Configuration

108 sensors (18 ladders on 2 layers) were installed

in total 30 sensors had **mechanical damage or were lost** before/during operation



cross section of the minimal detector configuration



hit maps of layer 1 and 2 of the vertex detector

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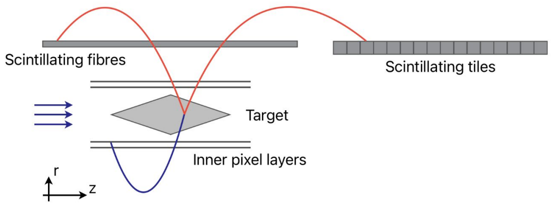
additional sensors showed **inefficiencies** and **missing links**



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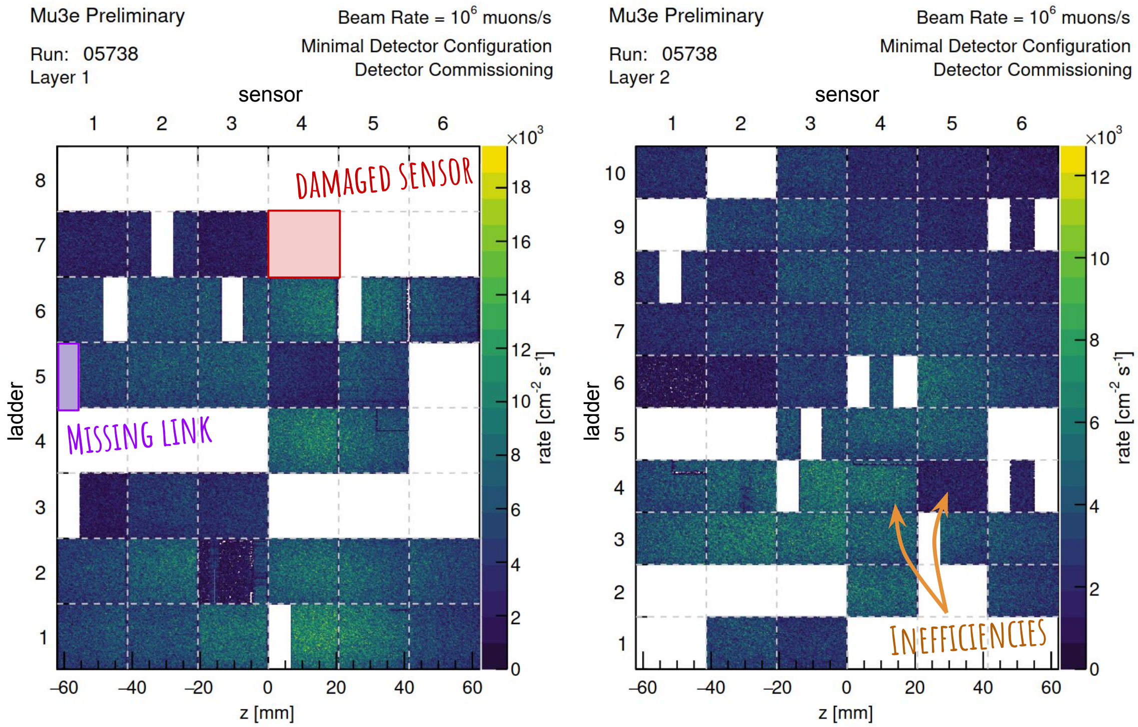

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Aligning the 2025 Detector Configuration

beam data

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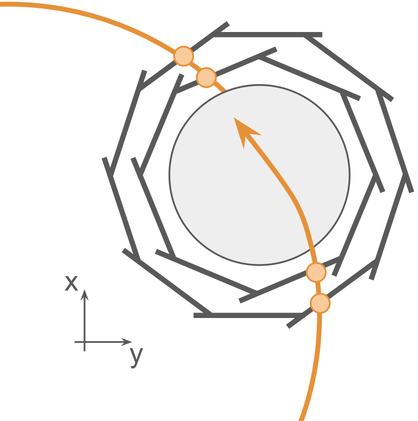
GTTF

~160,000 (6.23%) **4-hit tracks**

with p > 40 MeV



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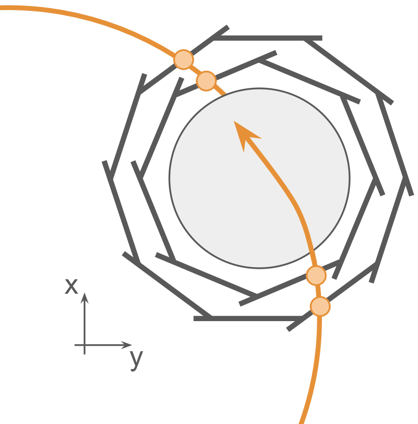
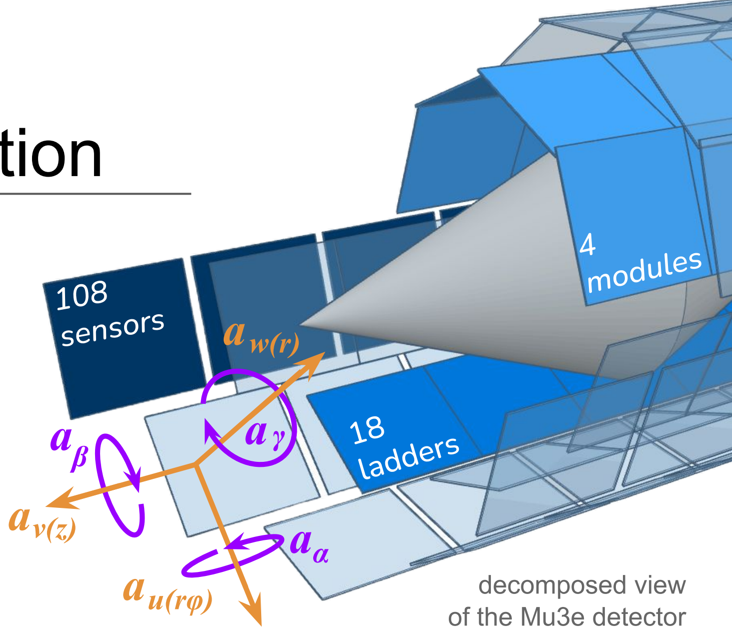
Alignment
6 alignment parameters per
ladder & layer (**a_α, a_β, a_γ, a_u, a_v, a_w**)





decomposed view
of the Mu3e detector

Aligning the 2025 Detector Configuration

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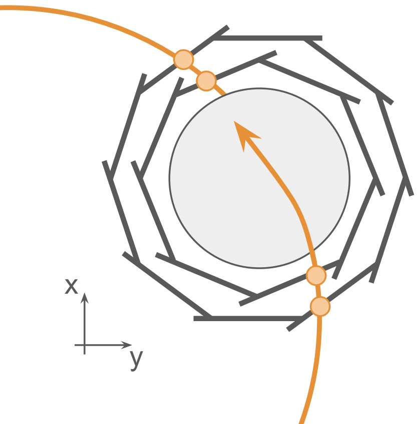
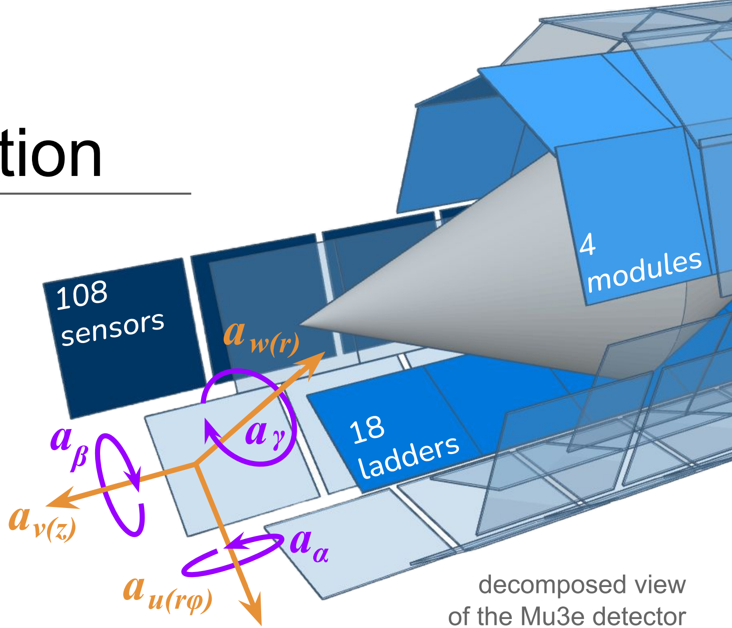
corr. detector alignment

beam data

refit tracks **(GTTF)**





decomposed view of the Mu3e detector

Aligning the 2025 Detector Configuration

beam data

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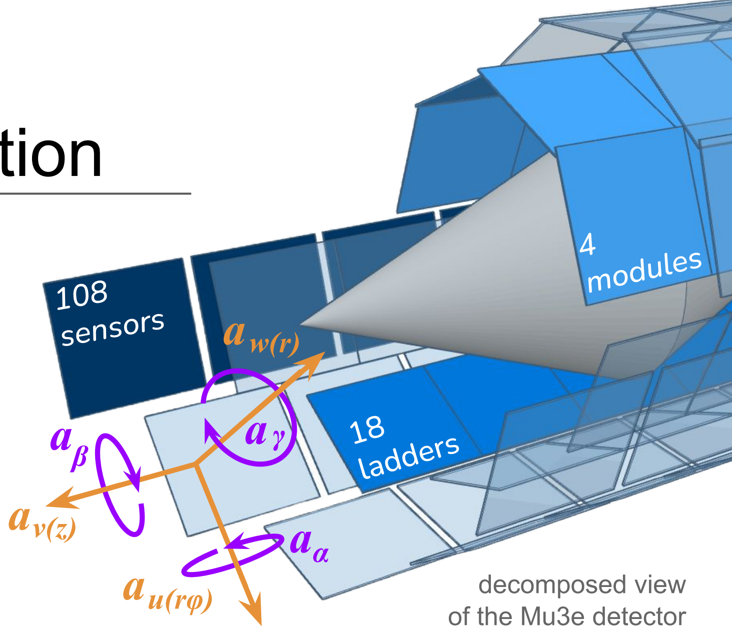
Alignment
6 alignment parameters per ladder & layer (a_α, a_β, a_γ, a_u, a_v, a_w)

corr. detector alignment

beam data

refit tracks (GTTF)

fit quality χ^2 comparison



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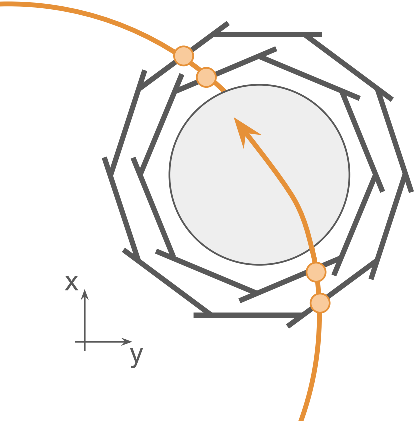
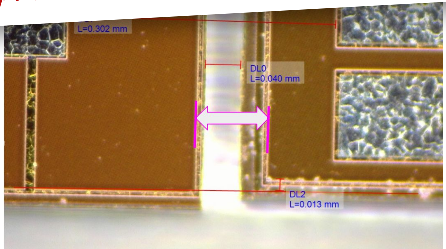


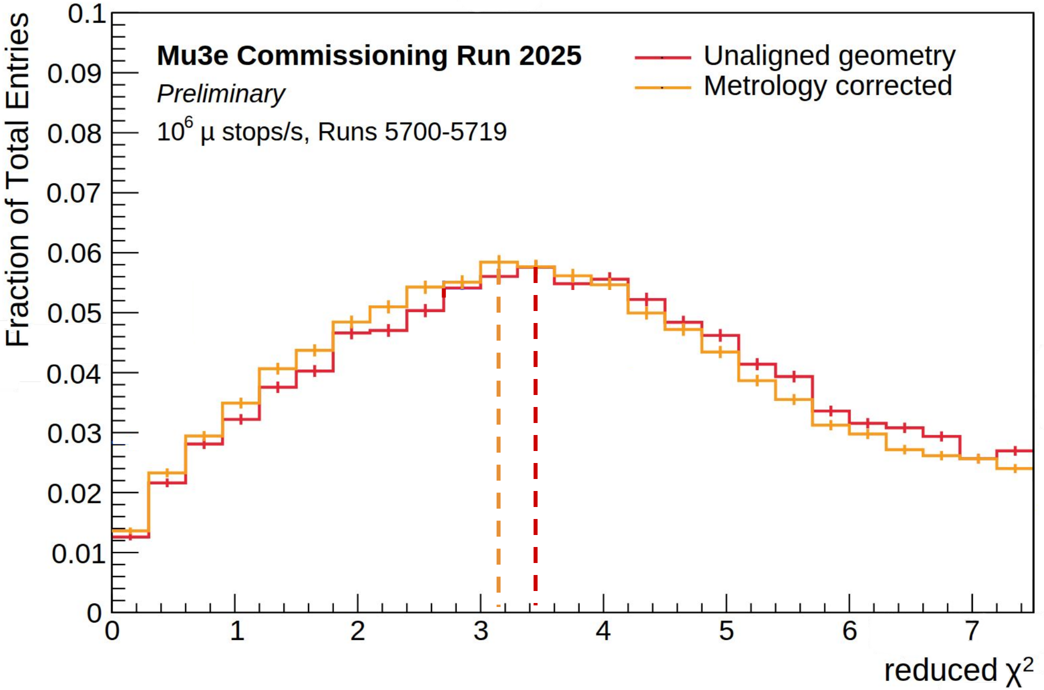
Aligning the 2025 Detector Configuration

→ tracks are assigned fit quality χ^2
- → normalized by number of degrees of freedom
- → peak is expected at ~1.0

including sensor to sensor (metrology) corrections does not improve track reconstruction



sensor positioning on a assembled ladder



reduced χ^2 of tracks reconstructed with different alignments

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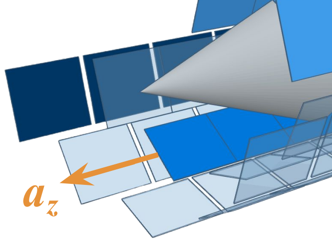
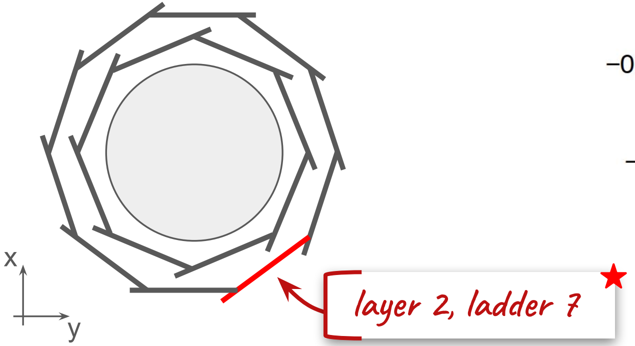
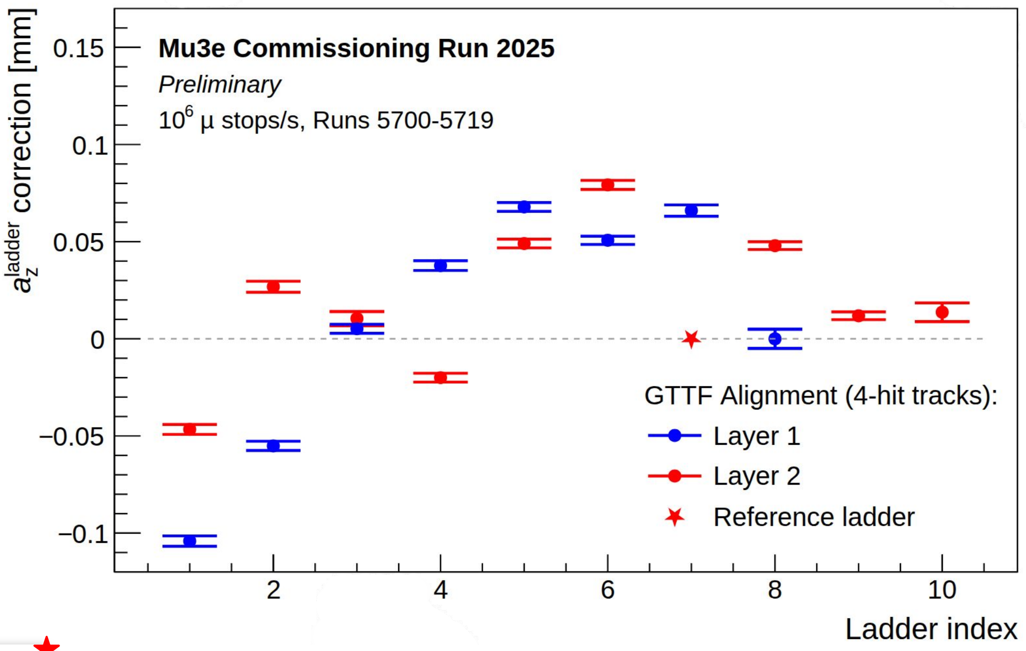


→ aligning ladders <u>and</u> layers

→ corrections of $O(10 - 100\ \mu m)$

alignment reference:





computed alignment correction by the GTTF along z per ladder

Aligning the 2025 Detector Configuration

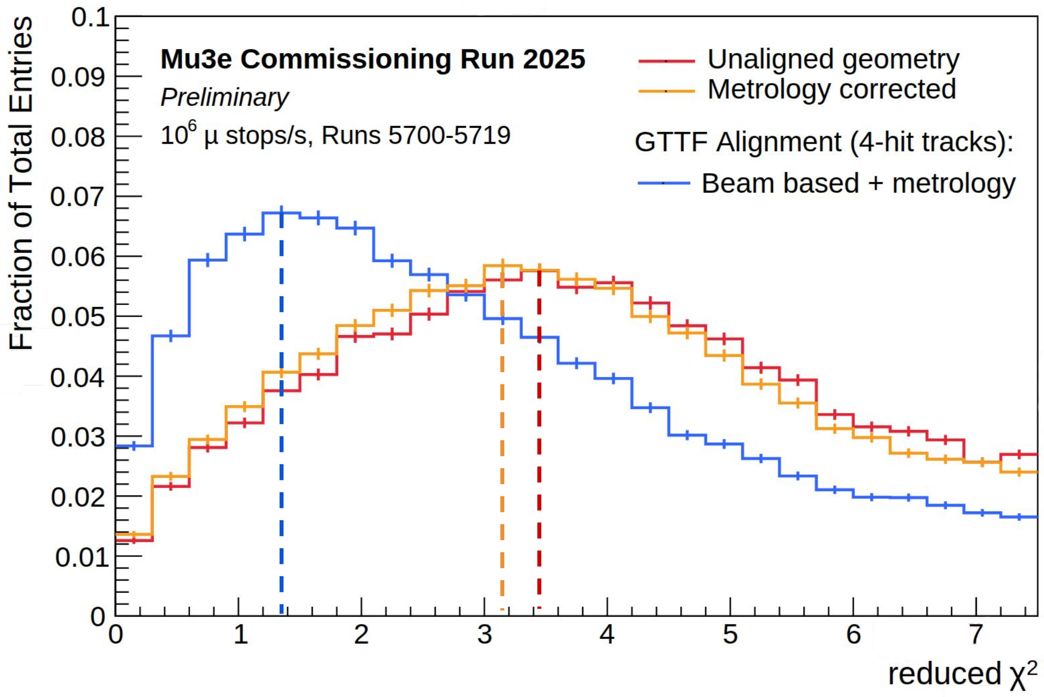
→ aligning ladders and layers

→ corrections of $O(10 - 100\ \mu m)$

→ yet **the peak** is not at 1.0

→ effect of energy loss is currently studied

ladder & layer alignment more impactful than optical alignment



reduced χ^2 of tracks reconstructed with different alignments

Summary & Outlook

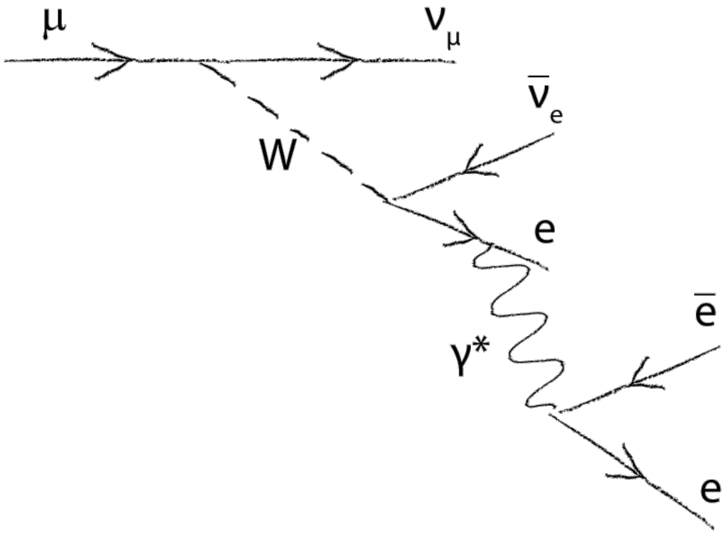
- General Triplet Track Fit (GTTF) provides alignment parameters ***without*** the need of an external track fit

No iteration needed!

- Mu3e requires ***accurate alignment*** due to its light build structure
- global ladder & layer alignment has a higher impact on the reconstruction than sensor level alignment
- ***High parallelizability*** of the method makes it well-suited for online alignment
  - Applicability for Mu3e is under investigation

Backup

The Mu3e Experiment



internal conversion: the main background for the signal decay

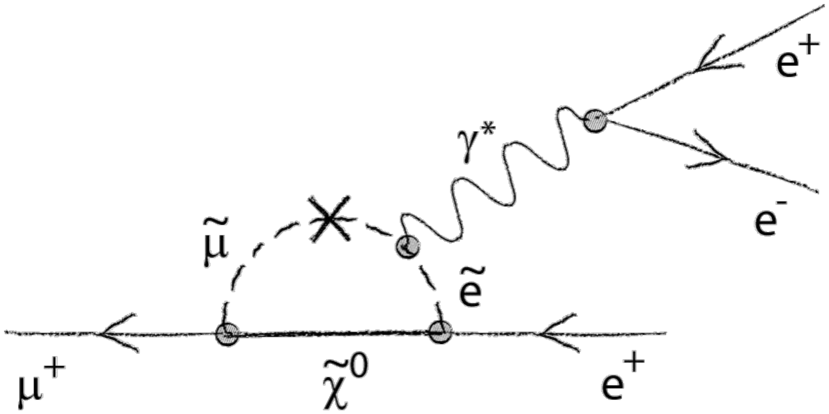


Diagram for SUSY μ → eee process via slepton mixing

[1] Mu3e TDR: arXiv:2009.11690

Theory behind the GTTF

χ² function of one track which has to be minimized

$$\chi_i^2(\boldsymbol{\kappa}_{i,3D}, \vec{\boldsymbol{\delta}}_i) = (\boldsymbol{\Psi}_i + \boxed{\boldsymbol{\rho}_i \boldsymbol{\kappa}_{i,3D}} - \boxed{\vec{\boldsymbol{H}}_i \vec{\boldsymbol{\delta}}_i})^\top \boldsymbol{D}_{i,MS} (\boldsymbol{\Psi}_i + \boxed{\boldsymbol{\rho}_i \boldsymbol{\kappa}_{i,3D}} - \boxed{\vec{\boldsymbol{H}}_i \vec{\boldsymbol{\delta}}_i}) + \boxed{\vec{\boldsymbol{\delta}}_i^\top \vec{\vec{\boldsymbol{D}}}_{i,hit} \vec{\boldsymbol{\delta}}_i}$$

$\boldsymbol{\kappa}$ … curvature of the track

δ … hit position shifts

GTTF with alignment

χ^2 function of one track which has to be minimized

$$\chi_i^2(\boldsymbol{\kappa}_{i,3D}, \vec{\boldsymbol{\delta}}_i) = (\boldsymbol{\Psi}_i + \boldsymbol{\rho}_i \boldsymbol{\kappa}_{i,3D} - \vec{\boldsymbol{H}}_i \vec{\boldsymbol{\delta}}_i)^\top \boldsymbol{D}_{i,MS} (\boldsymbol{\Psi}_i + \boldsymbol{\rho}_i \boldsymbol{\kappa}_{i,3D} - \vec{\boldsymbol{H}}_i \vec{\boldsymbol{\delta}}_i) + \vec{\boldsymbol{\delta}}_i^\top \vec{\vec{\boldsymbol{D}}}_{i,hit} \vec{\boldsymbol{\delta}}_i$$

$$\chi_{tot}^2 = \sum_{i=1}^{N} \chi_i^2(\boldsymbol{\kappa}_{i,3D}, \vec{\boldsymbol{\delta}}_i, \mathfrak{a})$$

- Introduce global alignment parameters, $\mathfrak{a}$, in the χ^2 function
- $\boldsymbol{\kappa}$ and δ are track dependent, whereas $\mathfrak{a}$ is track independent

$$\boldsymbol{\zeta}_i = \boldsymbol{\Psi}_i + \boldsymbol{\rho}_i \boldsymbol{\kappa}_{i,3D} - \vec{\boldsymbol{H}}_i \vec{\boldsymbol{\delta}}_i - \boxed{\vec{\boldsymbol{H}}_i \vec{\mathfrak{J}}_i \mathfrak{a}}$$

$$\chi_i^2(\boldsymbol{\kappa}_{i,3D}, \vec{\boldsymbol{\delta}}_i, \mathfrak{a}) = \boldsymbol{\zeta}_i^\top \boldsymbol{D}_{i,MS} \boldsymbol{\zeta}_i + \vec{\boldsymbol{\delta}}_i^\top \vec{\vec{\boldsymbol{D}}}_{i,hit} \vec{\boldsymbol{\delta}}_i + \boxed{\mathfrak{a}^\top \vec{\vec{\boldsymbol{D}}}_{i,ali} \mathfrak{a}}$$

GTTF with alignment

Solution for alignment parameters:

Alignment parameters: $$\mathfrak{a} = \boldsymbol{S}^{-1} \sum_i^N \vec{\boldsymbol{H}}'^{\top}_i \boldsymbol{K}_{i,\rho} \boldsymbol{\Psi}_i \qquad \text{with} \;\; \boldsymbol{K}_{i,\rho} = \boldsymbol{K}_i - \frac{\boldsymbol{K}_i \boldsymbol{\rho}_i \boldsymbol{\rho}_i^{\top} \boldsymbol{K}_i}{\boldsymbol{\rho}_i^{\top} \boldsymbol{K}_i \boldsymbol{\rho}_i}$$

Covariance of the alignment parameters: $$Cov_{\mathfrak{a}} = \boldsymbol{S}^{-1} = \Bigg(\underbrace{\sum_i^N}_{\text{Sum run over all tracks}} \underbrace{\vec{\vec{\boldsymbol{D}}}'_{i,ali} + \vec{\boldsymbol{H}}'^{\top}_i \boldsymbol{K}_{i,\rho} \vec{\boldsymbol{H}}'_i}_{\text{can be calculated parallely for each track}} \Bigg)^{-1}$$

GTTF with alignment

Solution for alignment parameters:

$$\mathfrak{a} = \boldsymbol{S}^{-1} \sum_i^N \vec{\boldsymbol{H}'}_i^T \boldsymbol{K}_{i,\rho} \boldsymbol{\Psi}_i$$

$$Cov_{\mathfrak{a}} = \boldsymbol{S}^{-1} = \left(\sum_i^N \vec{\vec{\boldsymbol{D}}}'_{i,ali} + \vec{\boldsymbol{H}'}_i^T \boldsymbol{K}_{i,\rho} \vec{\boldsymbol{H}'}_i \right)^{-1}$$

using: $K = D_{MS} - D_{MS} H F^{-1} H^T D_{MS}$

$$K_\rho = K - \frac{K \rho \rho^T K}{\rho^T K \rho}$$

Can be calculated *parallely* for N tracks!

New formulas contain same results as GTTF

$$\boldsymbol{\kappa}_i = -\frac{\boldsymbol{\rho}_i^T \boldsymbol{K}_i \boldsymbol{\Psi}_i}{\boldsymbol{\rho}_i^T \boldsymbol{K}_i \boldsymbol{\rho}_i} + \frac{\boldsymbol{\rho}_i^T (\boldsymbol{K}_i \boldsymbol{\rho}_i \boldsymbol{\rho}_i^T \boldsymbol{K}_i \vec{\boldsymbol{H}'}_i \boldsymbol{S}^{-1} \vec{\boldsymbol{H}'}_i^T \boldsymbol{K}_i - \boldsymbol{K}_i \vec{\boldsymbol{H}'}_i \boldsymbol{S}^{-1} \vec{\boldsymbol{H}'}_i^T \boldsymbol{K}_i \boldsymbol{\rho}_i \boldsymbol{\rho}_i^T \boldsymbol{K}_i) \boldsymbol{\Psi}_i}{(\boldsymbol{\rho}_i^T \boldsymbol{K}_i \boldsymbol{\rho}_i)^2}$$
$$+ \frac{1}{\boldsymbol{\rho}_i^T \boldsymbol{K}_i \boldsymbol{\rho}_i} \left(\boldsymbol{\rho}_i^T \boldsymbol{K}_i \vec{\boldsymbol{H}'}_i \boldsymbol{S}^{-1} \sum_{\substack{j=1,\\ j \neq i}}^N \vec{\boldsymbol{H}'}_j^T \boldsymbol{D}_{j,MS} \boldsymbol{\Psi}_j \right)$$

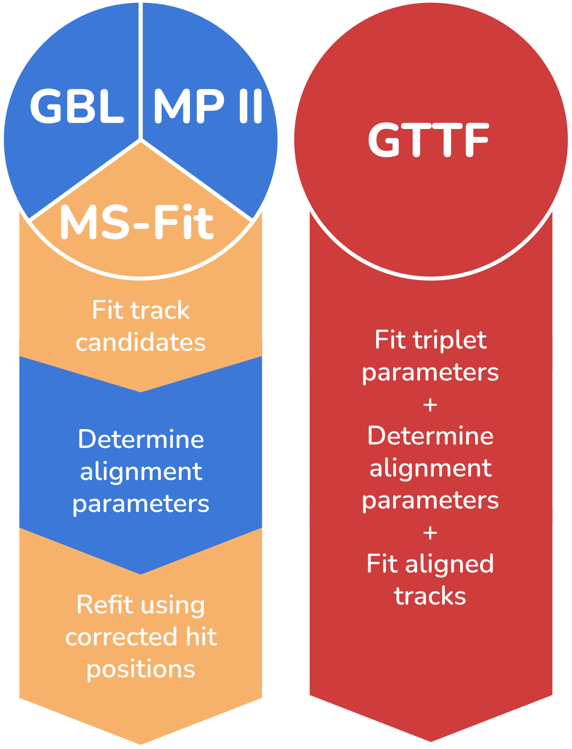
$$\sigma_{\boldsymbol{\kappa}_i} = \frac{1}{\boldsymbol{\rho}_i^T \boldsymbol{K}_i \boldsymbol{\rho}_i} + \frac{\boldsymbol{\rho}_i^T (\boldsymbol{K}_i \vec{\boldsymbol{H}'}_i \boldsymbol{S}^{-1} \vec{\boldsymbol{H}'}_i^T \boldsymbol{K}_i) \boldsymbol{\rho}_i}{(\boldsymbol{\rho}_i^T \boldsymbol{K}_i \boldsymbol{\rho}_i)^2}$$

$$\vec{\boldsymbol{\delta}}_i = \vec{\vec{\boldsymbol{D}}}_{i,hit}^{-1} \vec{\boldsymbol{H}}_i^T \boldsymbol{K}_{i,\rho} \boldsymbol{\Psi}_i - \vec{\vec{\boldsymbol{D}}}_{i,hit}^{-1} \vec{\boldsymbol{H}}_i^T (\boldsymbol{K}_{i,\rho} \vec{\boldsymbol{H}'}_i \boldsymbol{S}^{-1} \vec{\boldsymbol{H}'}_i^T \boldsymbol{K}_{i,\rho}) \boldsymbol{\Psi}_i$$
$$- \vec{\vec{\boldsymbol{D}}}_{i,hit}^{-1} \vec{\boldsymbol{H}}_i^T \boldsymbol{K}_{i,\rho} \vec{\boldsymbol{H}'}_i \boldsymbol{S}^{-1} \sum_{\substack{j=1,\\ j \neq i}}^N \vec{\boldsymbol{H}'}_j^T \boldsymbol{D}_{j,MS} \boldsymbol{\Psi}_j$$

$$\mathrm{Cov}_{\vec{\boldsymbol{\delta}}_i} = \vec{\vec{\boldsymbol{D}}}_{i,hit}^{-1} - \vec{\vec{\boldsymbol{D}}}_{i,hit}^{-1} \vec{\boldsymbol{H}}_i^T \boldsymbol{K}_{i,\rho} \vec{\boldsymbol{H}}_i \vec{\vec{\boldsymbol{D}}}_{i,hit}^{-1} + \vec{\vec{\boldsymbol{D}}}_{i,hit}^{-1} \vec{\boldsymbol{H}}_i^T (\boldsymbol{K}_{i,\rho} \vec{\boldsymbol{H}'}_i \boldsymbol{S}^{-1} \vec{\boldsymbol{H}'}_i^T \boldsymbol{K}_{i,\rho}) \vec{\boldsymbol{H}}_i \vec{\vec{\boldsymbol{D}}}_{i,hit}^{-1}$$

Track Based Alignment

- Former study [1] was done for Mu3e using *Millepede II (MP II)*
    - Track candidates fitted using the *Multiple Scattering (MS) Triplet Fit* [2]
    - Uses *General Broken Lines Fit & MP II* to obtain the alignment parameters of track candidate
- *General Triplet Track Fit (GTTF)* [3] fits track and alignment parameters in one single "*global*" step
    - Not detector specific
    - Includes hit and MS uncertainties



[1] U. Hartenstein, PhD. Thesis, 2019 [2] N. Berger et. al., MS-Fit, 2017: arXiv:1606.04990 [3] A. Schöning, GTTF, 2025: arXiv:2406.05240

Global Track Fit with Alignment

introduce global alignment parameters $\mathfrak{a}$

$$\chi^2_{tot} = \boxed{\sum_{i=1}^{N} \boxed{\chi_i^2(\boldsymbol{\kappa}_{i,3D}, \vec{\boldsymbol{\delta}}_i,} \boxed{\mathfrak{a}})}$$

→ κ ($\propto$ 1/p) and δ are **track dependent**,

whereas $\mathfrak{a}$ is **track independent**

→ the χ^2_{tot} function becomes a **sum of many** individual track χ^2_i

alignment parameters:

$$\mathfrak{a} = \boldsymbol{S}^{-1} \boxed{\sum_i^N \boxed{\vec{\boldsymbol{H}}'^{\top}_i \boldsymbol{K}_{i,\rho} \boldsymbol{\Psi}_i}}$$

with

$$\boxed{\boldsymbol{K}_{i,\rho} = \boldsymbol{K}_i - \frac{\boldsymbol{K}_i \boldsymbol{\rho}_i \boldsymbol{\rho}_i^{\top} \boldsymbol{K}_i}{\boldsymbol{\rho}_i^{\top} \boldsymbol{K}_i \boldsymbol{\rho}_i}}$$

covariance of the alignment parameters:

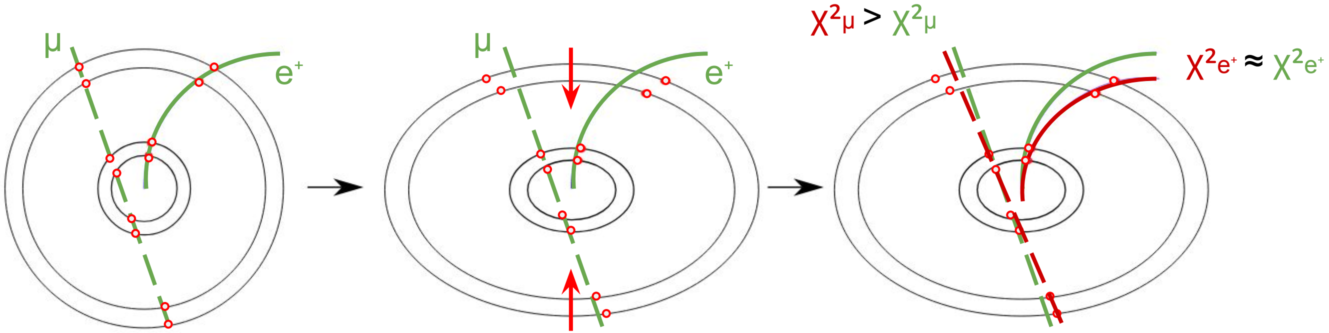
$$Cov_{\mathfrak{a}} = \boldsymbol{S}^{-1} = \boxed{\left(\sum_i^N \boxed{\vec{\vec{\boldsymbol{D}}}'_{i,ali} + \vec{\boldsymbol{H}}'^{\top}_i \boldsymbol{K}_{i,\rho} \vec{\boldsymbol{H}}'_i} \right)}^{-1}$$

Sum runs over all tracks

can be calculated parallelly for each track

Weak Modes

- Weak mode in the transverse plane
  - Michel-positrons have unknown curvature → momentum is reconstructed incorrectly + good fit quality
  - cosmic muons have known curvature → fit quality is affected

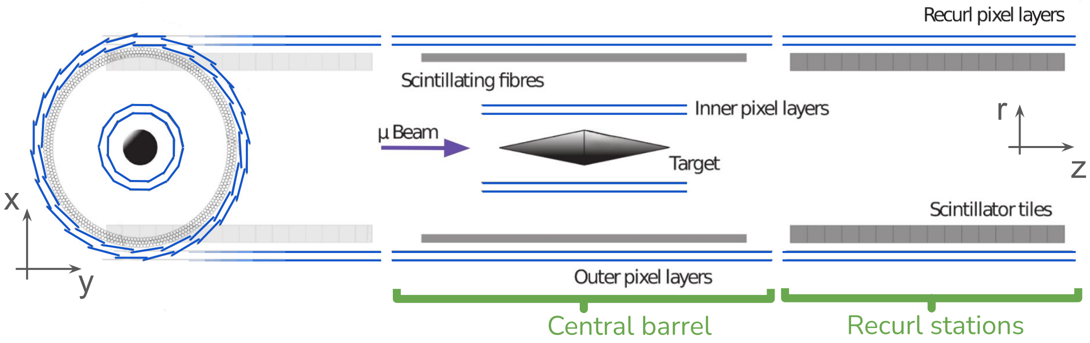


Misalignment mode keeps χ² invariant, but momentum changes (weak mode) [2]

[2] U. Hartenstein, PhD. Thesis, 2019

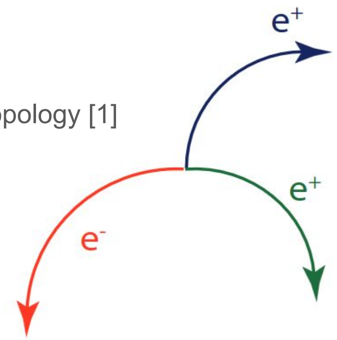
Mu3e Experiment

- Under construction at PSI
- Aims to observe the extremely rare lepton flavour violating decay $\mu \rightarrow eee$ with an ultimate sensitivity of one in 10^{16} muon decays



Cross section of the Mu3e detector, adapted from [1]

Signal Topology [1]



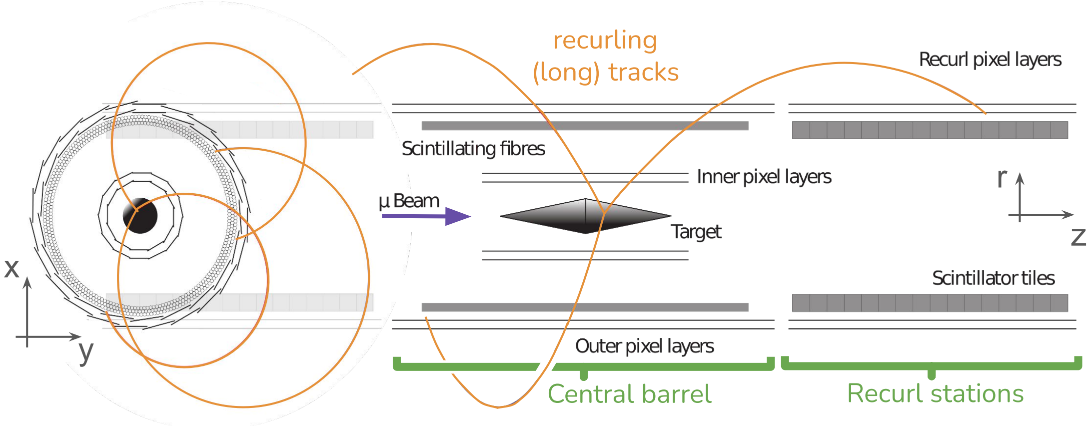
- Requires high momentum and vertex resolution for precise event selection
  - Highly granular detector ($\sigma(x)$ ~ 23 μm)
- Multiple Scattering dominated
  - ***ultra light pixel layers*** (0.1% X_0/layer) and gaseous helium cooling

[1] Mu3e TDR: arXiv:2009.11690

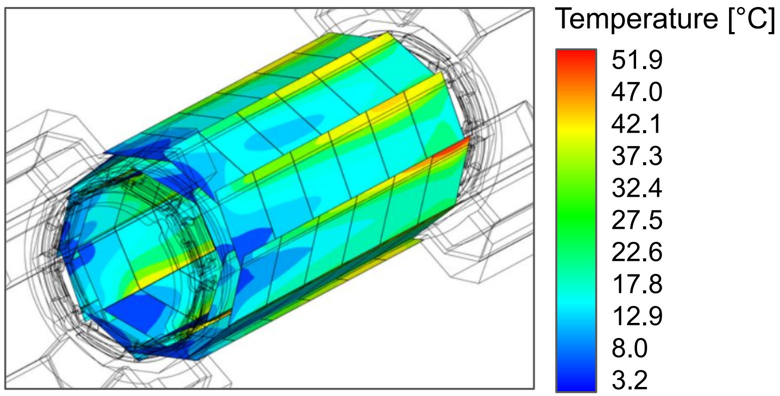
Mu3e Experiment

- Positrons traverse the detector multiple times (**long recurling tracks**)
  - Allows for precise momentum measurements
- Thin sensors and thermal expansion make Mu3e vulnerable to misalignment
  - Detector has to be aligned regularly

⇒ Online alignment



Cross section of the Mu3e detector, adapted from [1]



Heat dissipation of the central pixel tracker, adapted from [1]

[1] Mu3e TDR: arXiv:2009.11690

GTTF vs. MS-fit

GTTF

+ Includes MS and hit position uncertainties
  + Low and high momentum tracks reconstructable
+ Based on triplets
  + Highly parallelizable
  + Includes correlations between triplets

MS-fit

- Only includes MS uncertainties
  - Limited to low energy tracks
+ Based on triplets
  + Highly parallelizable