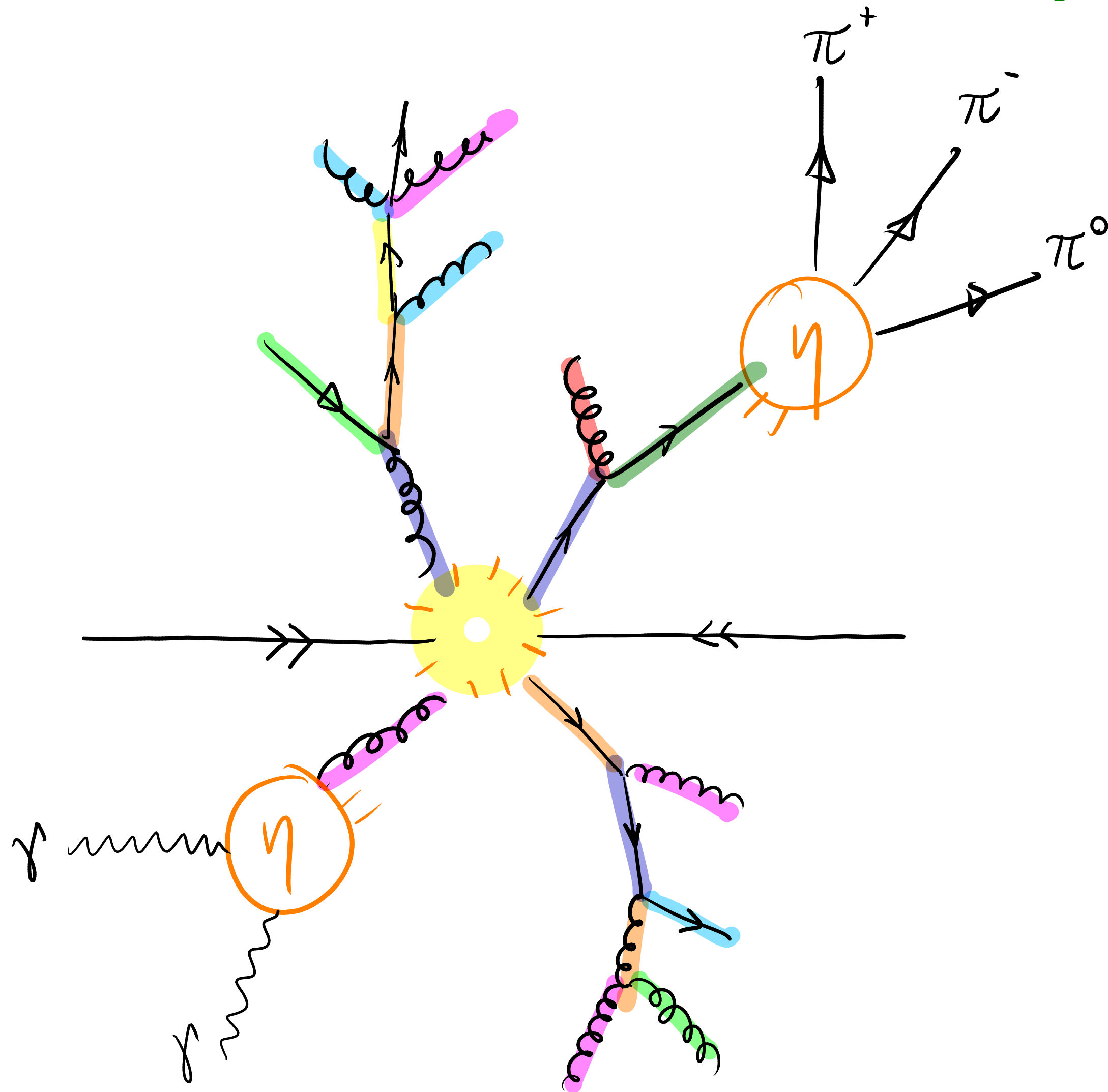


Eta meson Fragmentation Functions

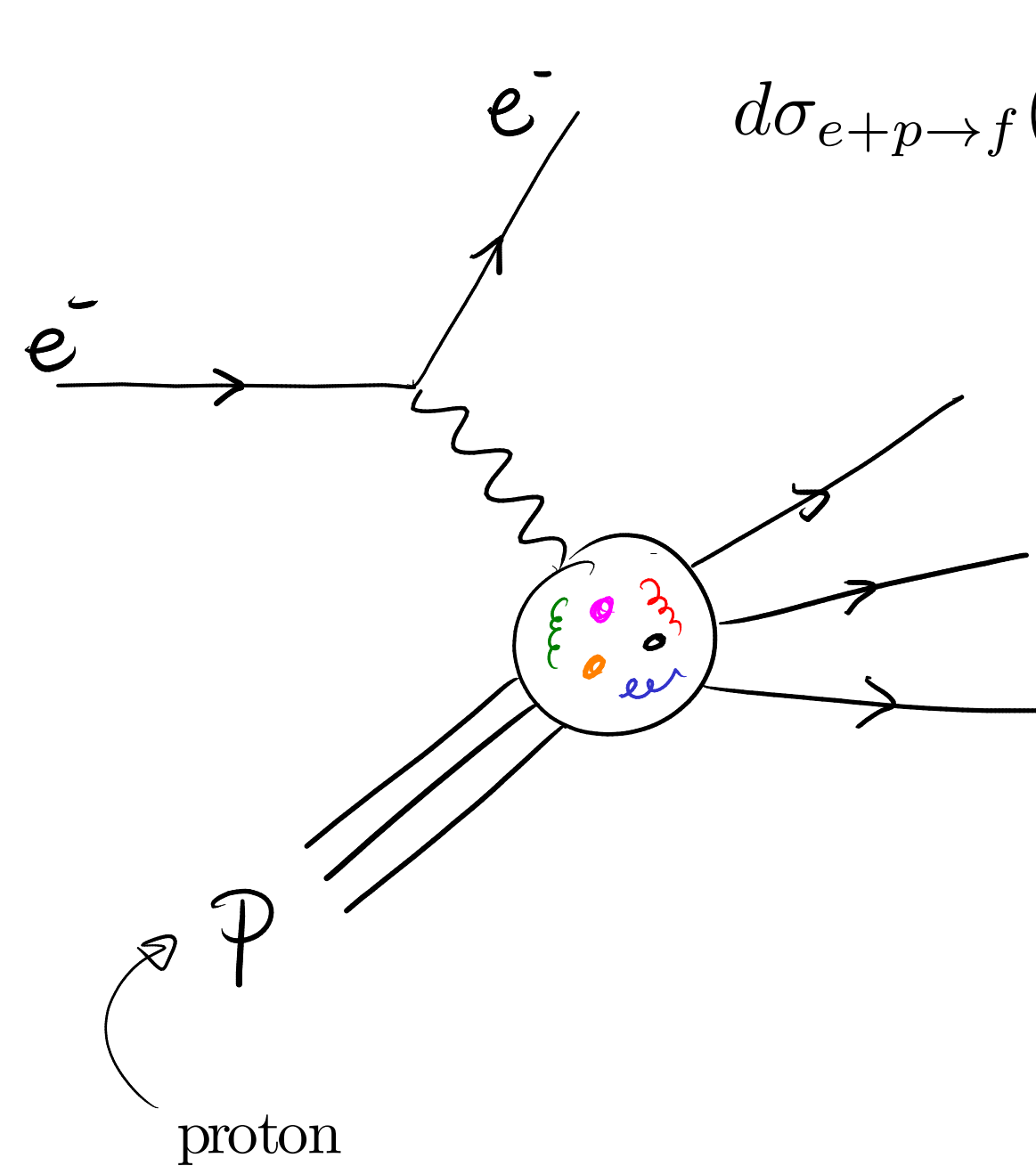


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University of Tübingen

Calculating hadronic cross sections

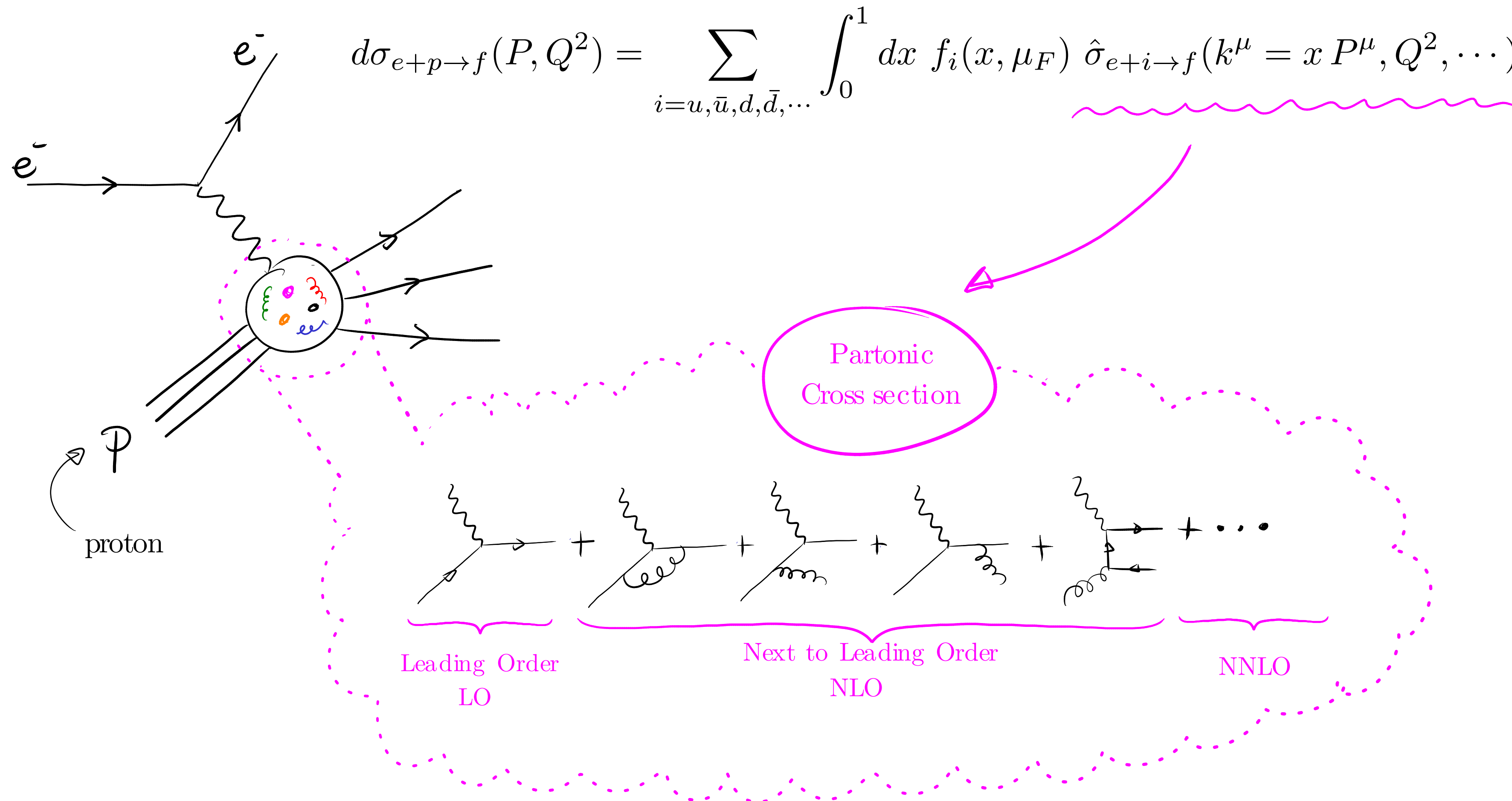
Deep Inelastic Scattering (DIS) $e^- p \rightarrow e^- + X$



$$d\sigma_{e+p \rightarrow f}(P, Q^2) = \sum_{i=u, \bar{u}, d, \bar{d}, \dots} \int_0^1 dx f_i(x, \mu_F) \hat{\sigma}_{e+i \rightarrow f}(k^\mu = x P^\mu, Q^2, \dots)$$

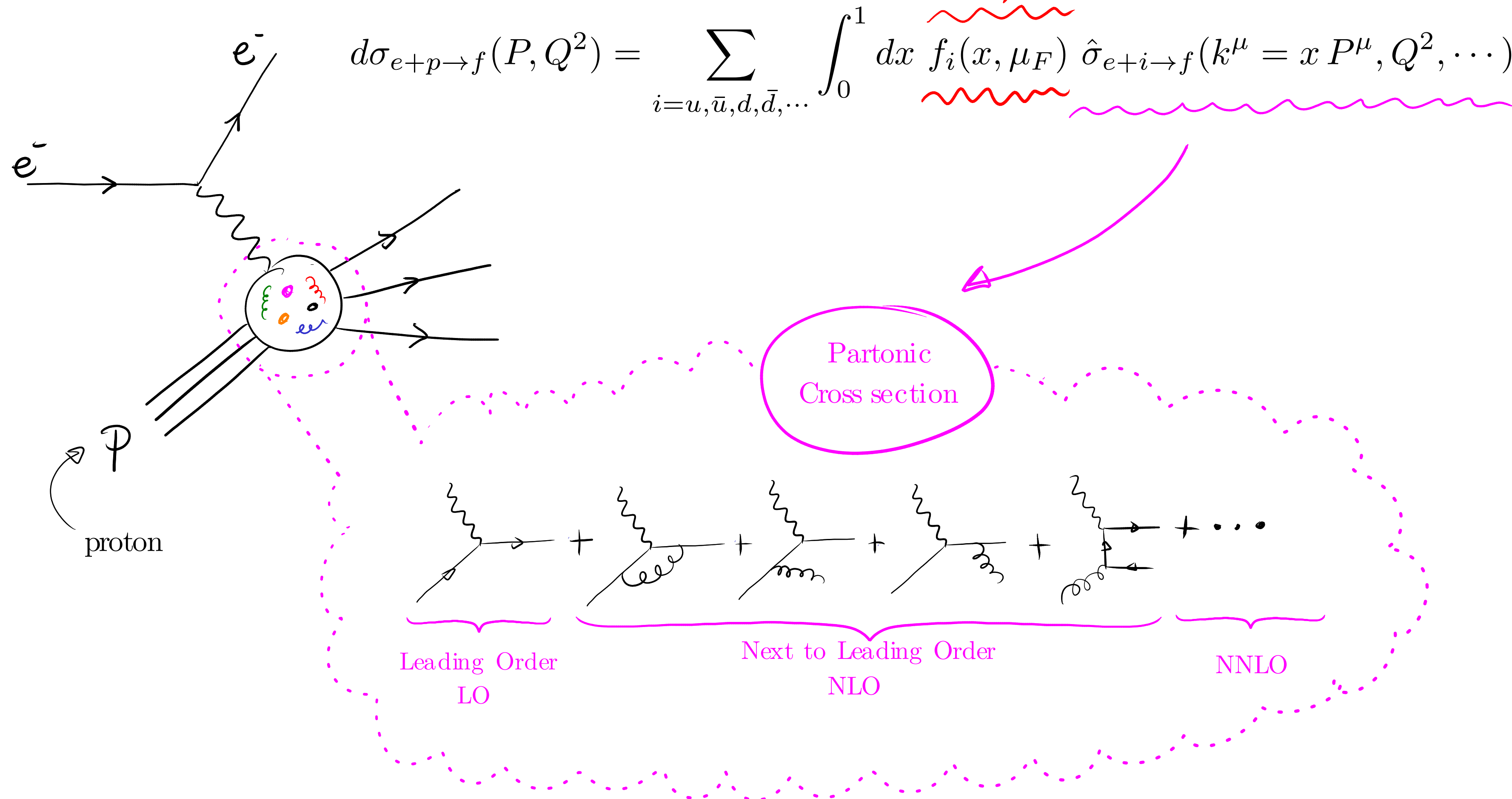
Calculating hadronic cross sections

Deep Inelastic Scattering (DIS) $e^- p \rightarrow e^- + X$



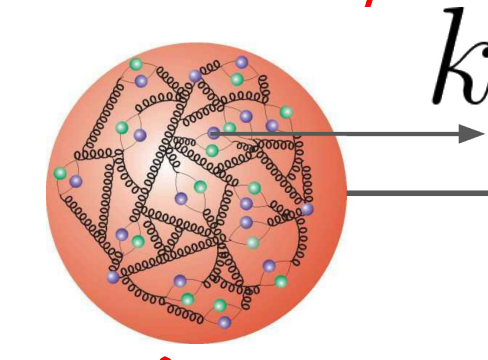
Calculating hadronic cross sections

Deep Inelastic Scattering (DIS) $e^- p \rightarrow e^- + X$



Parton Distribution functions (PDFs)

parton (quark or gluon)



k

P

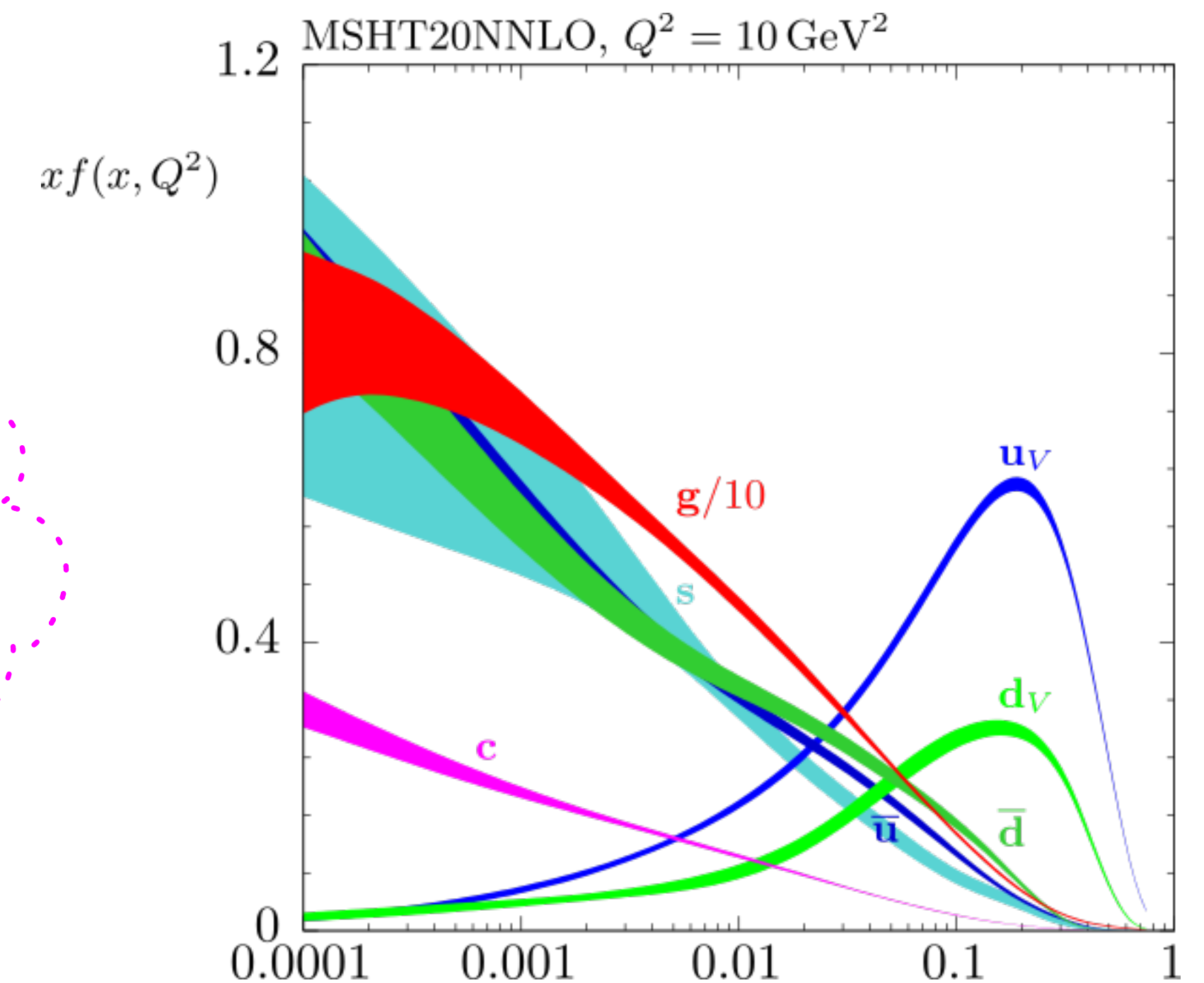
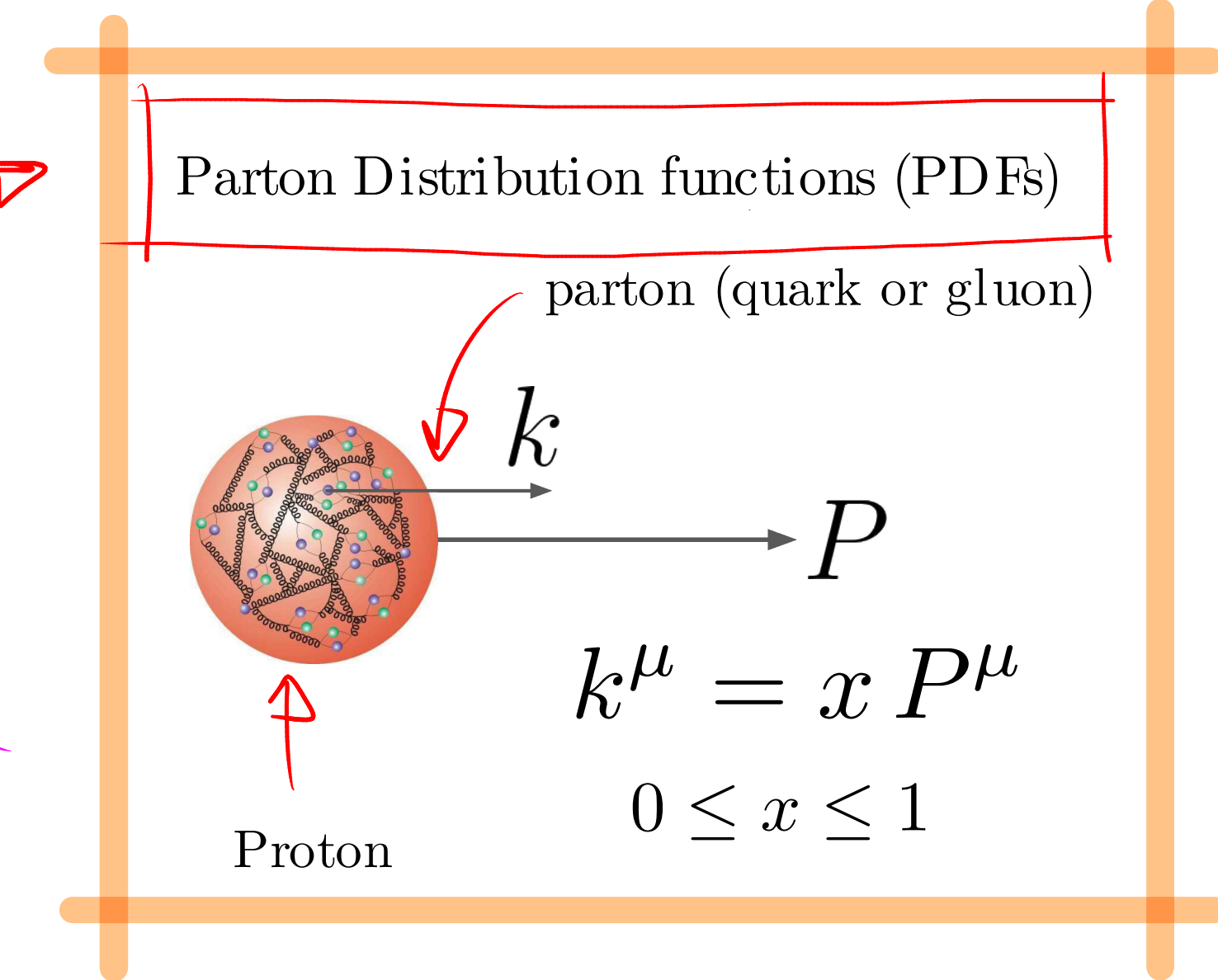
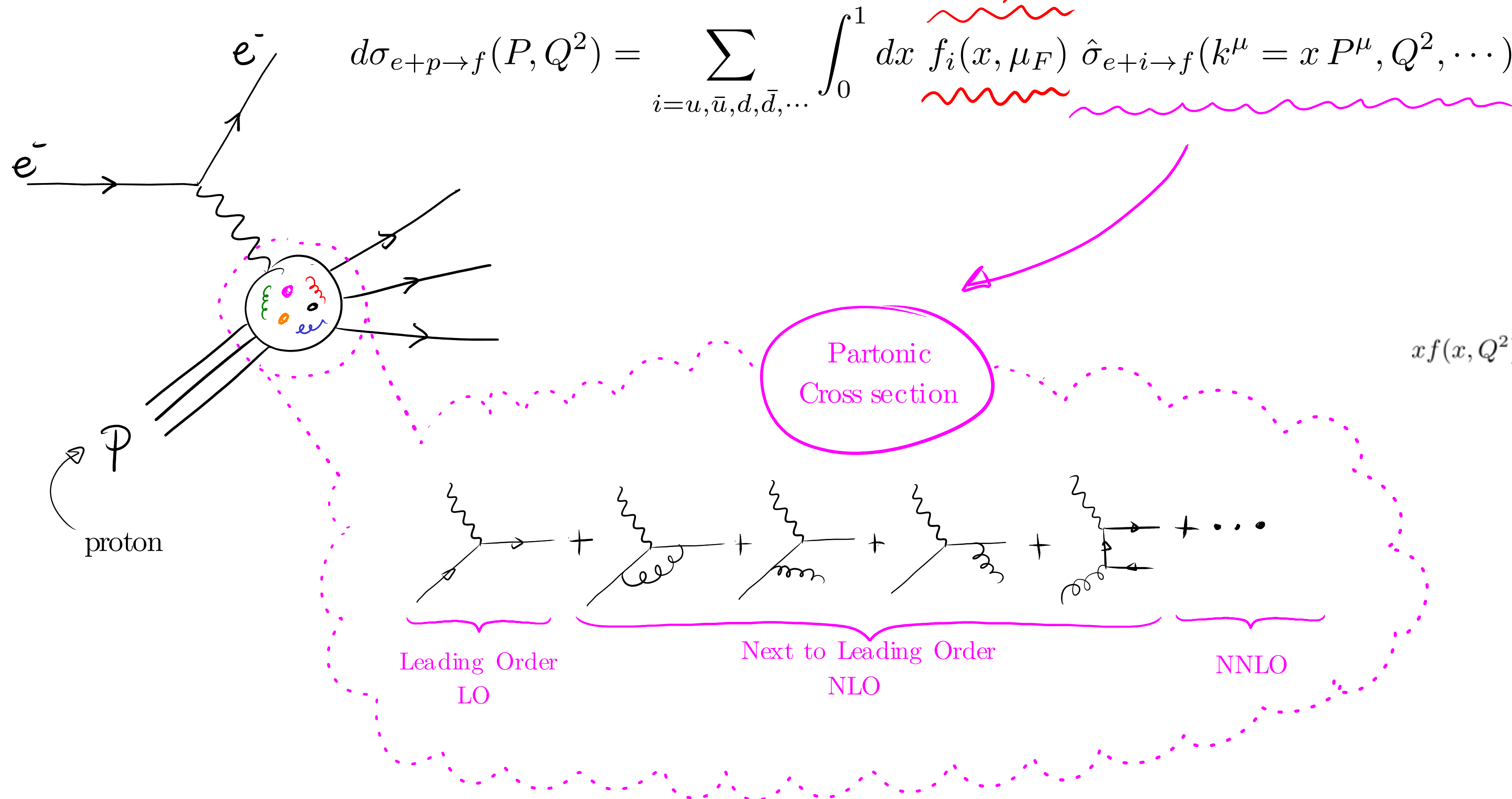
$$k^\mu = x P^\mu$$

$$0 \leq x \leq 1$$

Proton

Calculating hadronic cross sections

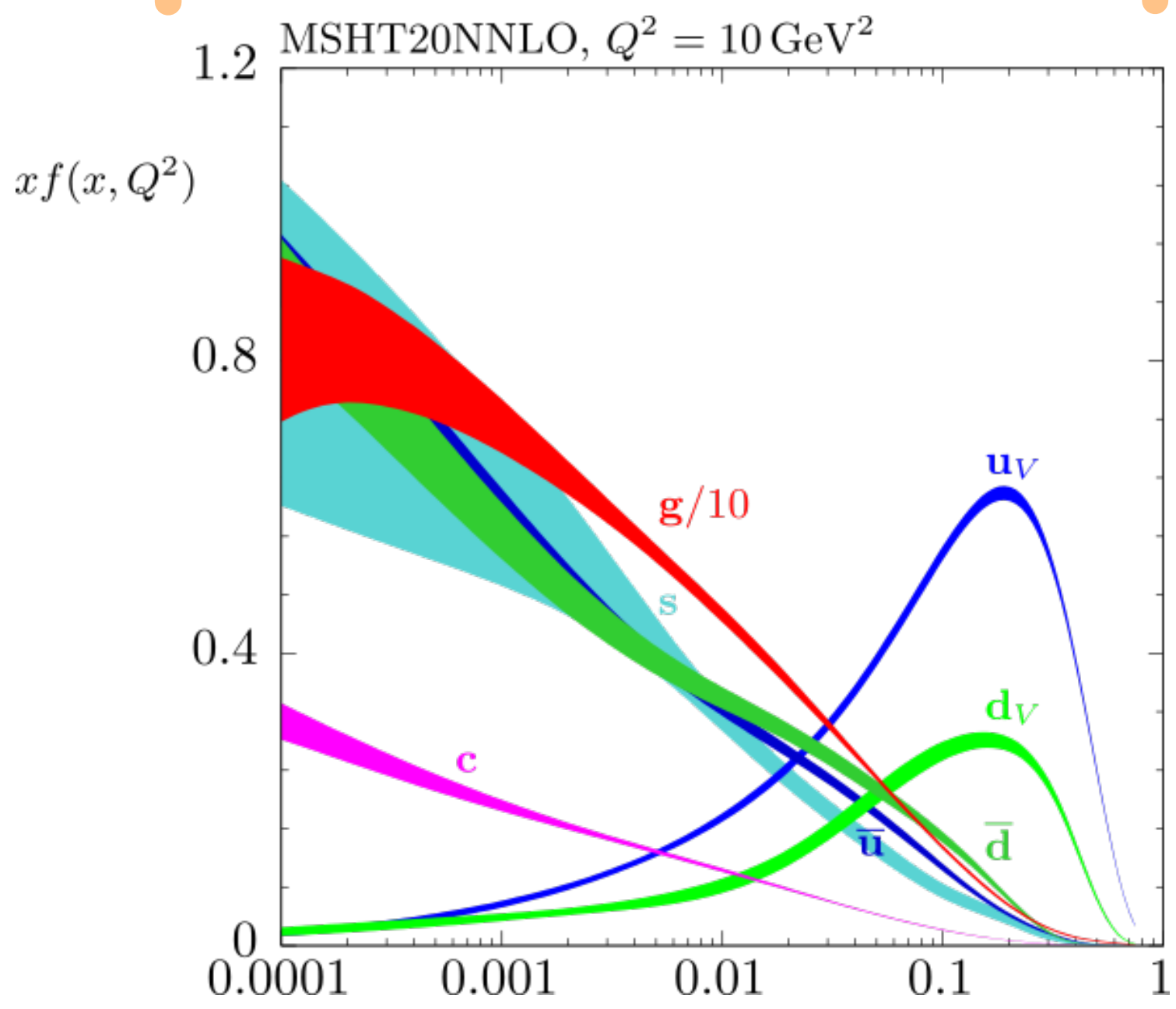
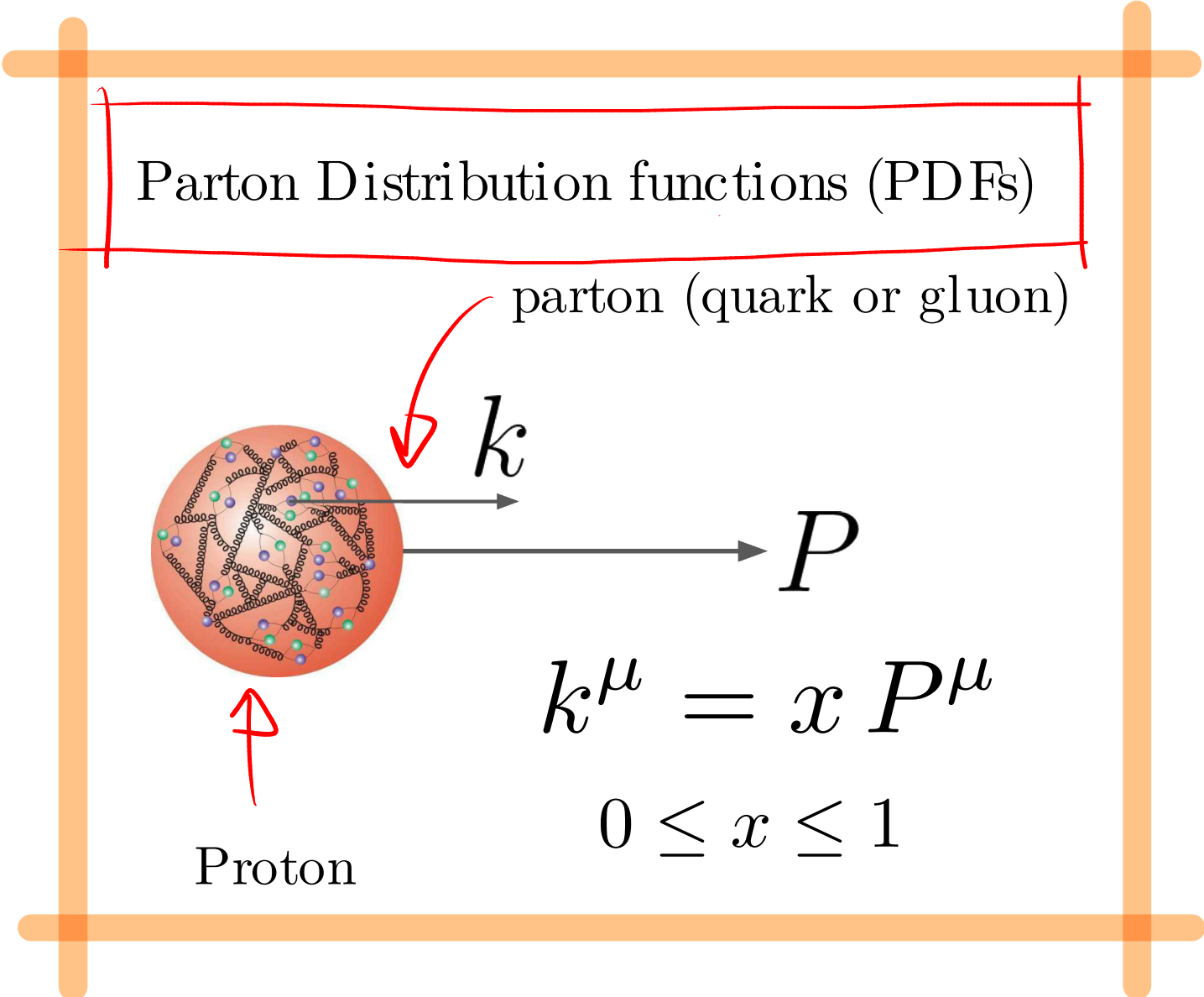
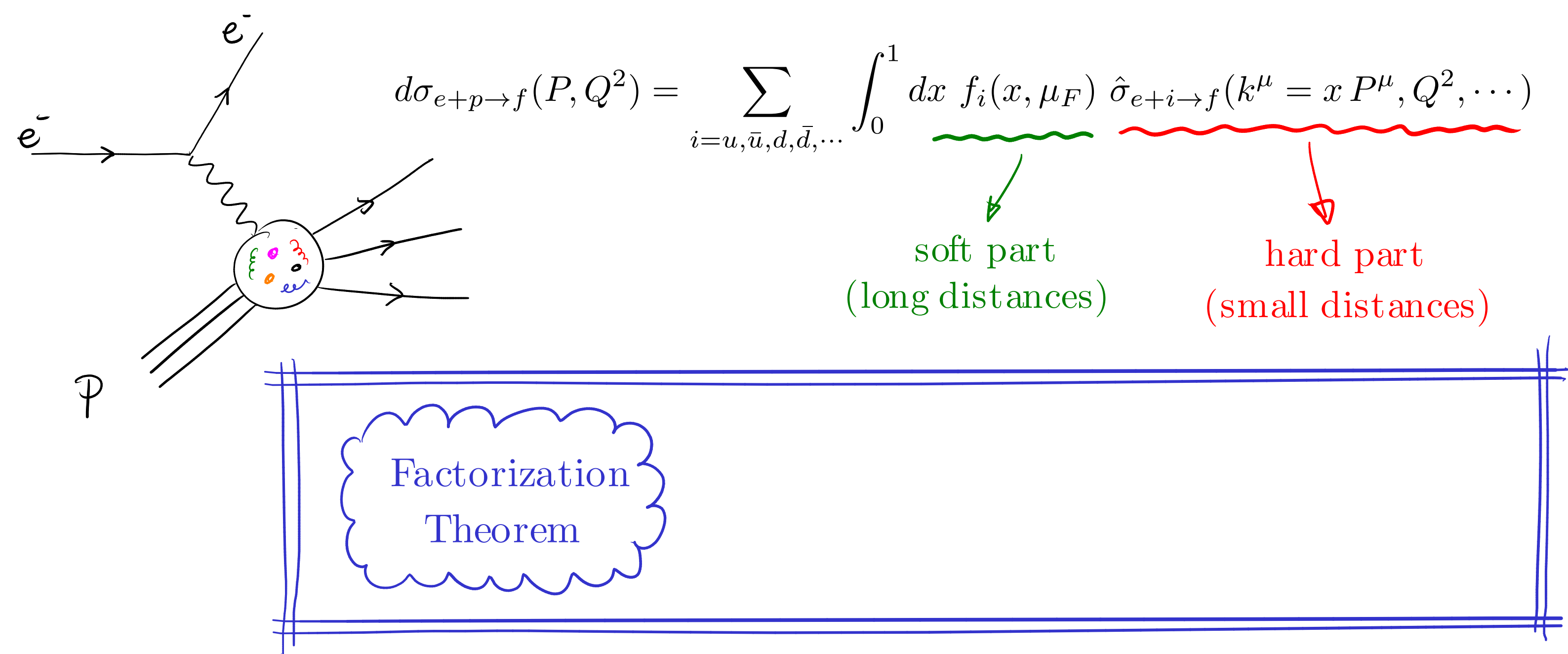
Deep Inelastic Scattering (DIS) $e^- p \rightarrow e^- + X$



arXiv: 2012.04684 - Eur.Phys.J.C 81 (2021) 341

Calculating hadronic cross sections

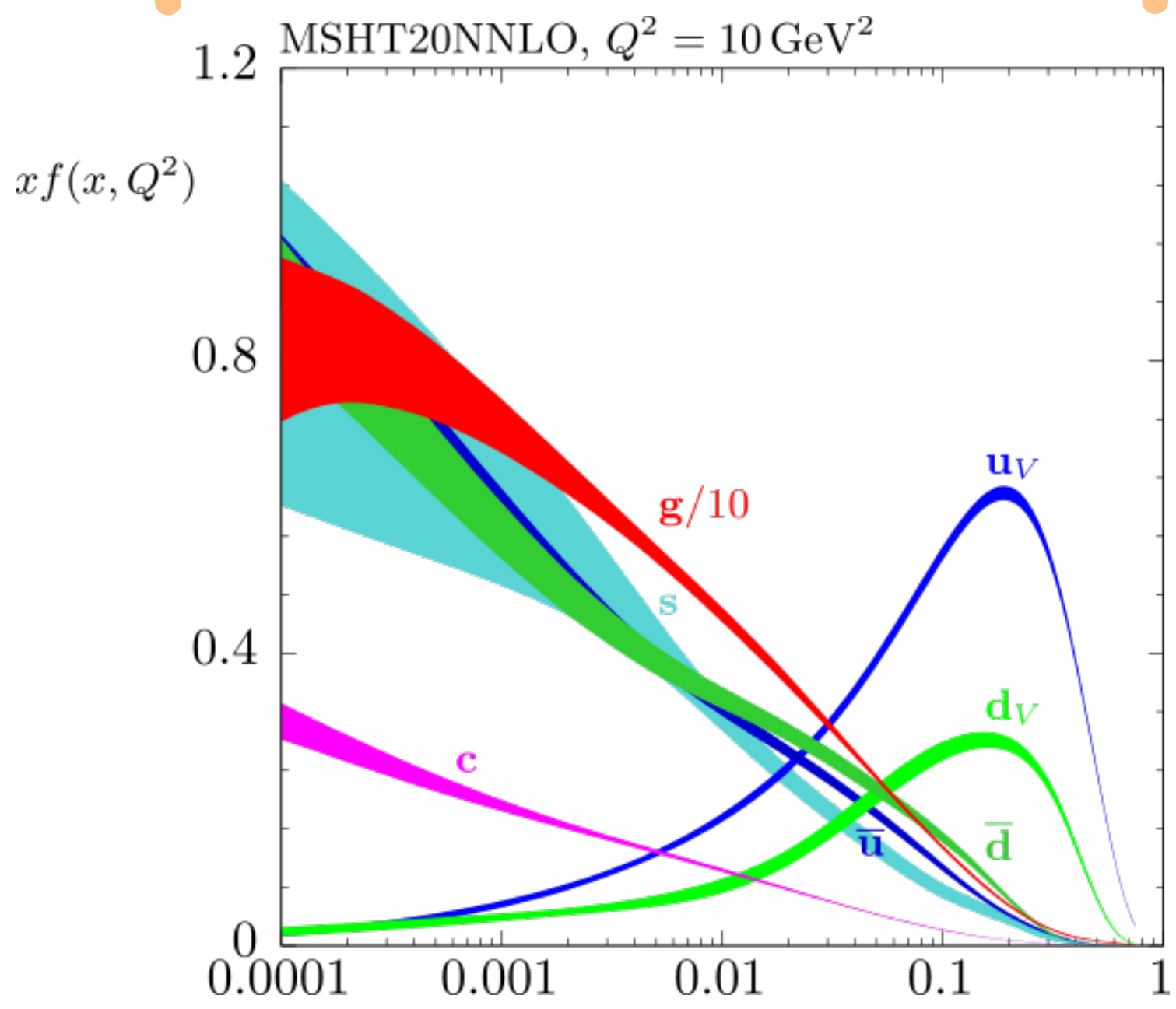
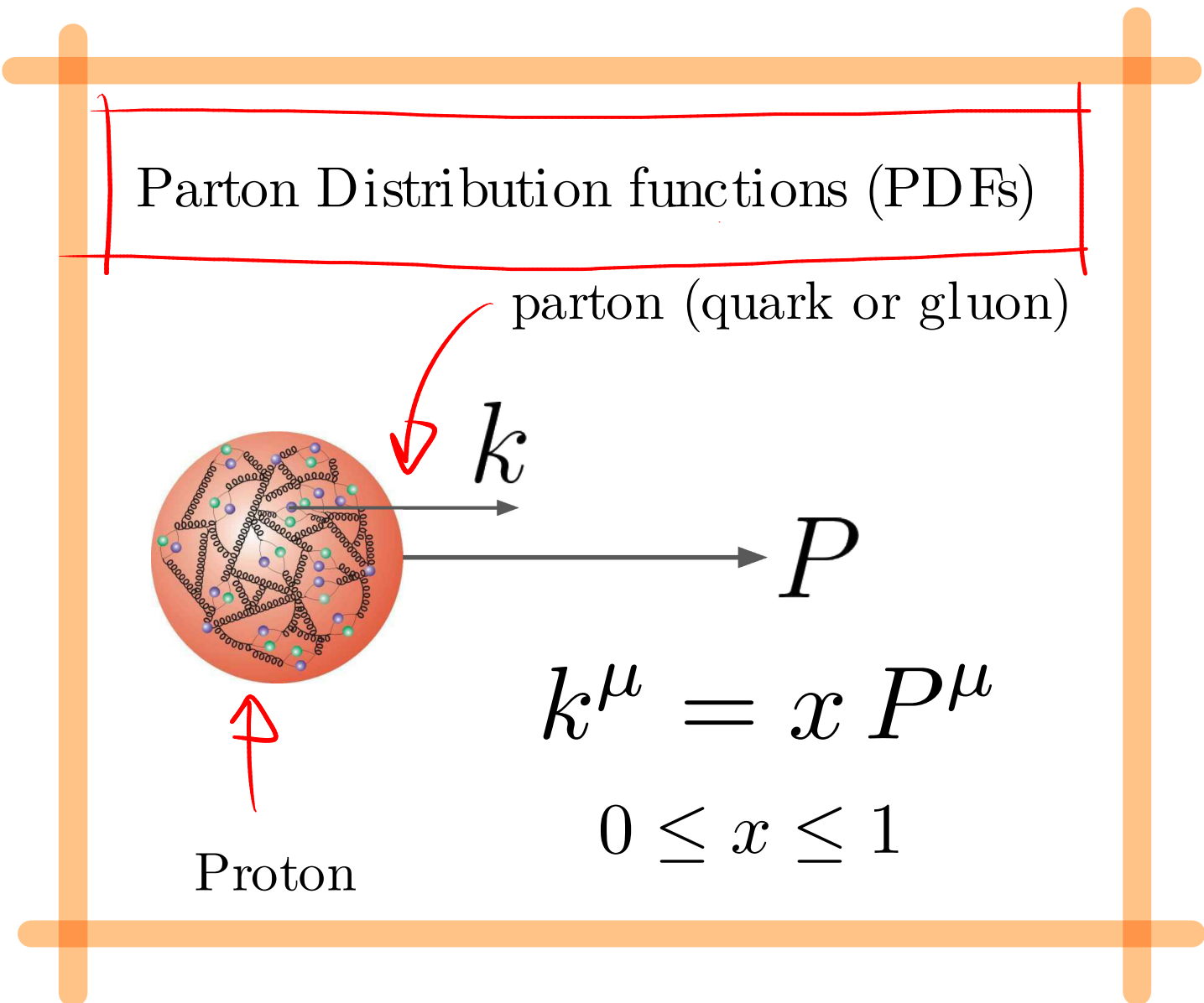
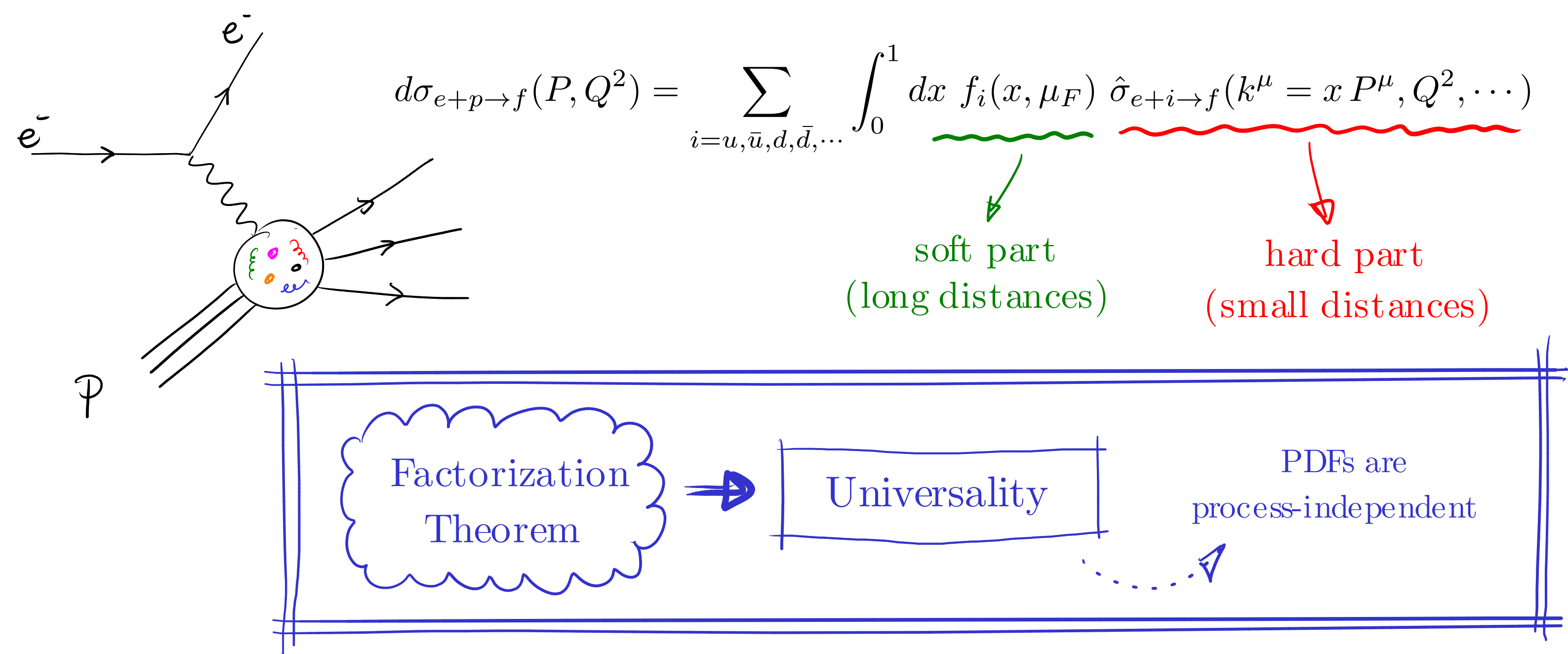
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Calculating hadronic cross sections

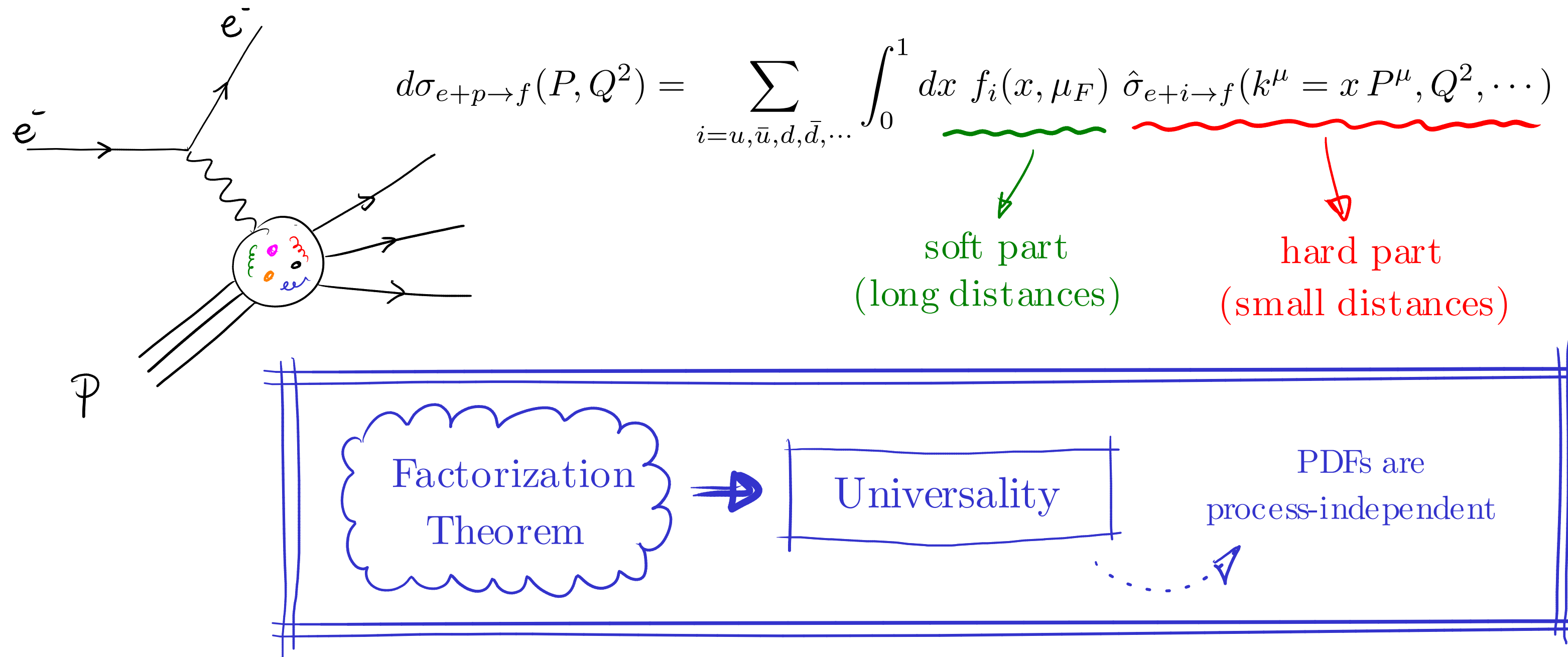
Deep Inelastic Scattering (DIS) $e^- p \rightarrow e^- + X$



arXiv: 2012.04684 - Eur.Phys.J.C 81 (2021) 341

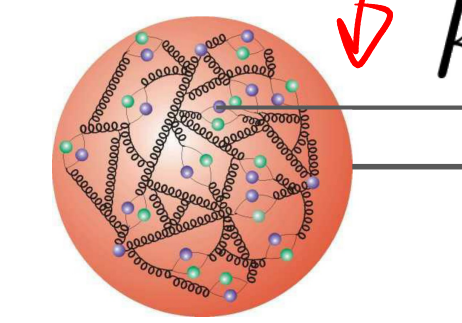
Calculating hadronic cross sections

Deep Inelastic Scattering (DIS) $e^- p \rightarrow e^- + X$



Parton Distribution functions (PDFs)

parton (quark or gluon)

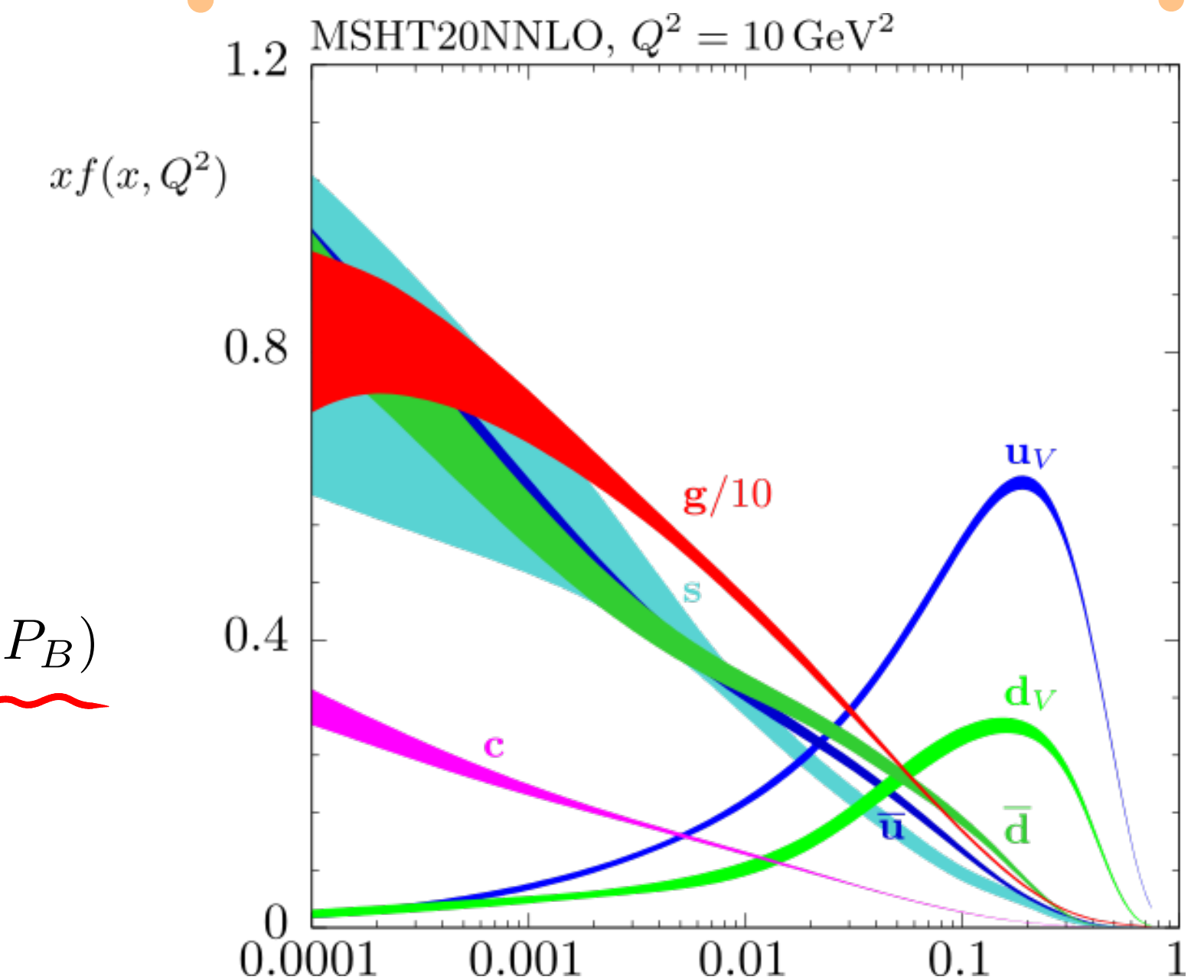
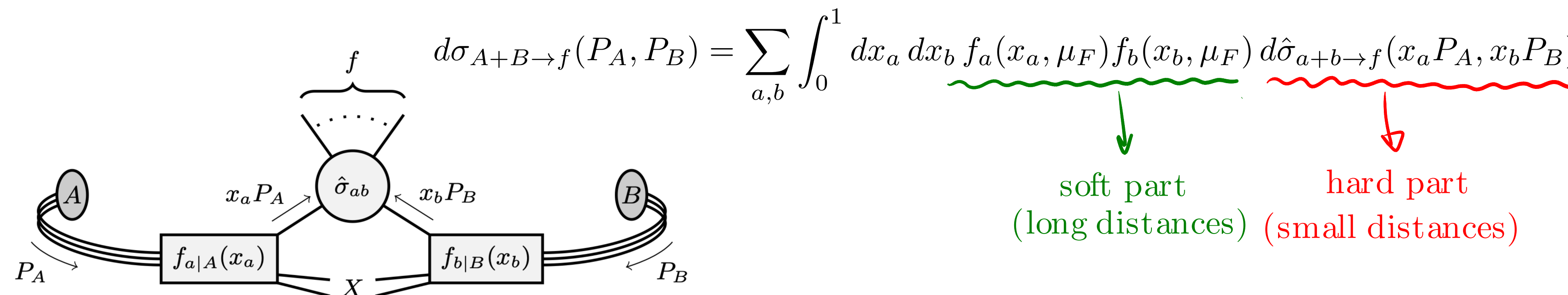


$$k^\mu = x P^\mu$$

$$0 \leq x \leq 1$$

Proton

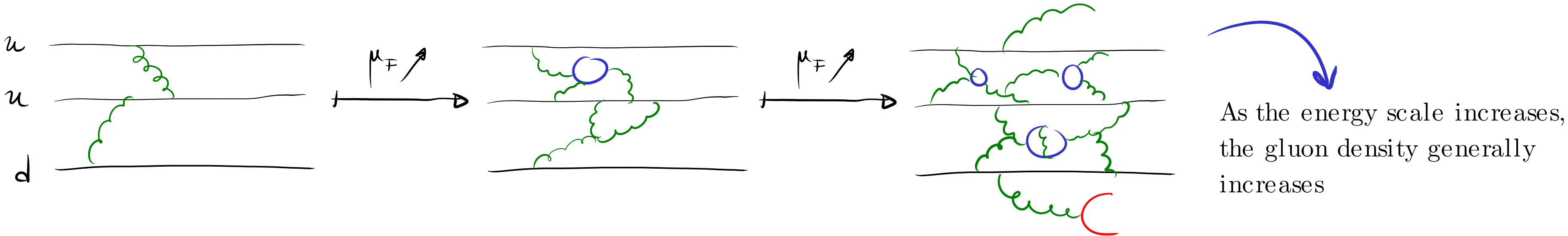
Proton-Proton collision - $pp \rightarrow X$



arXiv: 2012.04684 - Eur.Phys.J.C 81 (2021) 341

Scale evolution of PDFs *(and FFs)*

The scale evolution is calculable using pQCD



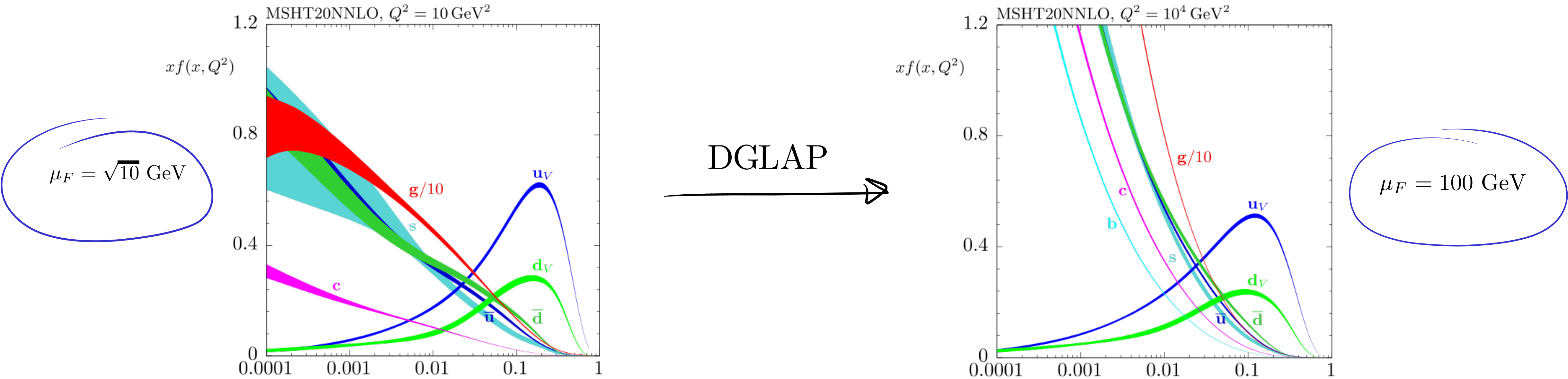
Dokshitzer–Gribov–Lipatov–**Altarelli–Parisi** (DGLAP) equations (1970s)

$$\mu \frac{\partial}{\partial \mu} f_i(x, \mu) = \sum_j \int_x^1 \frac{d\xi}{\xi} P_{ij} \left(\frac{x}{\xi} \right) f_j(\xi, \mu)$$

The kernels are calculated using pQCD

$$P_{qq}(x) = C_F \frac{1+x^2}{1-x} \qquad P_{qg}(x) = T_R [x^2 + (1-x)^2]$$

$$P_{gq}(x) = C_F \frac{1+(1-x)^2}{x} \qquad P_{gg}(x) = \dots$$

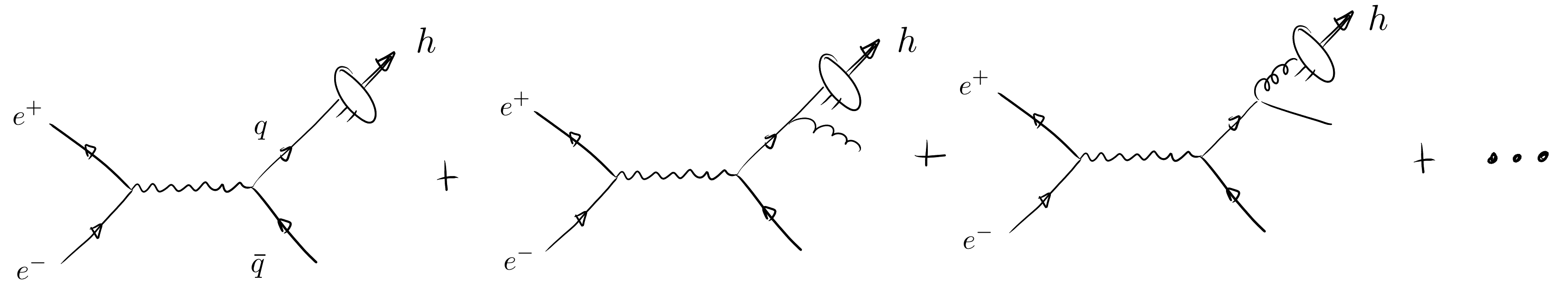


Processes with hadrons in the final state

e.g. $e^+ + e^- \rightarrow h + X$

h could be any hadron

$h = \pi^0, \pi^\pm, K's, \eta, p, \Lambda^0, \dots$

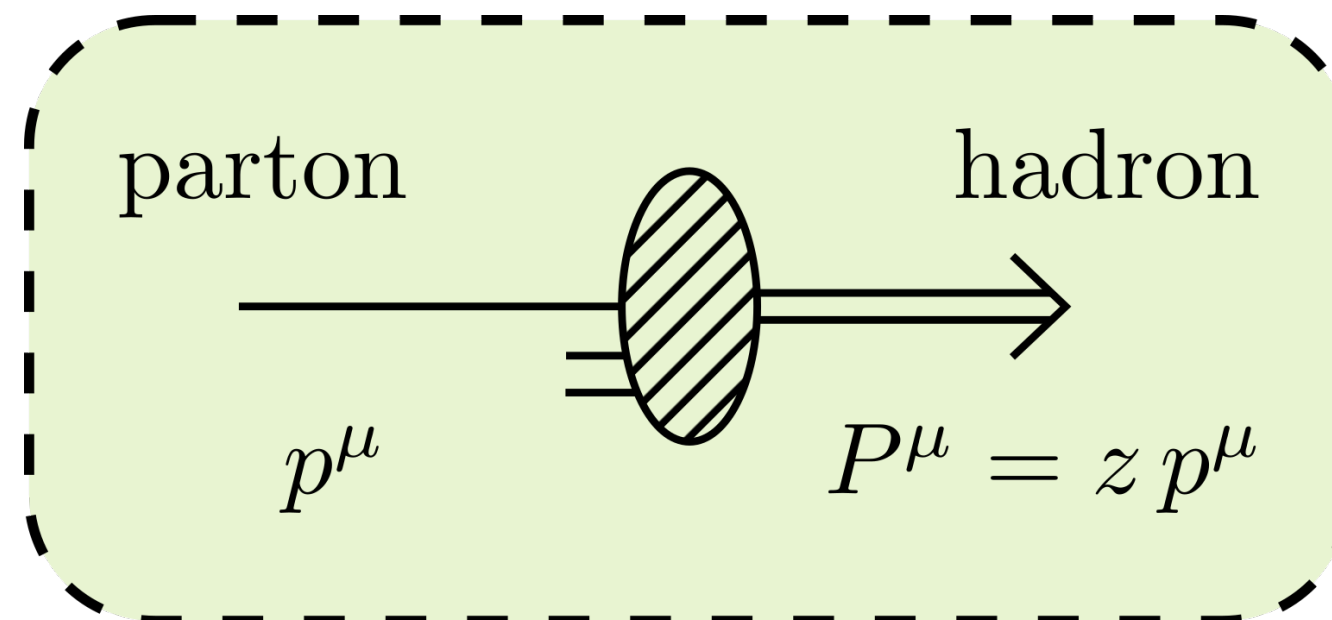


$$d\sigma(Q) = \sum_i \int_0^1 dz \hat{\sigma}_i(Q/\mu_F) D_i(z, \mu_F)$$

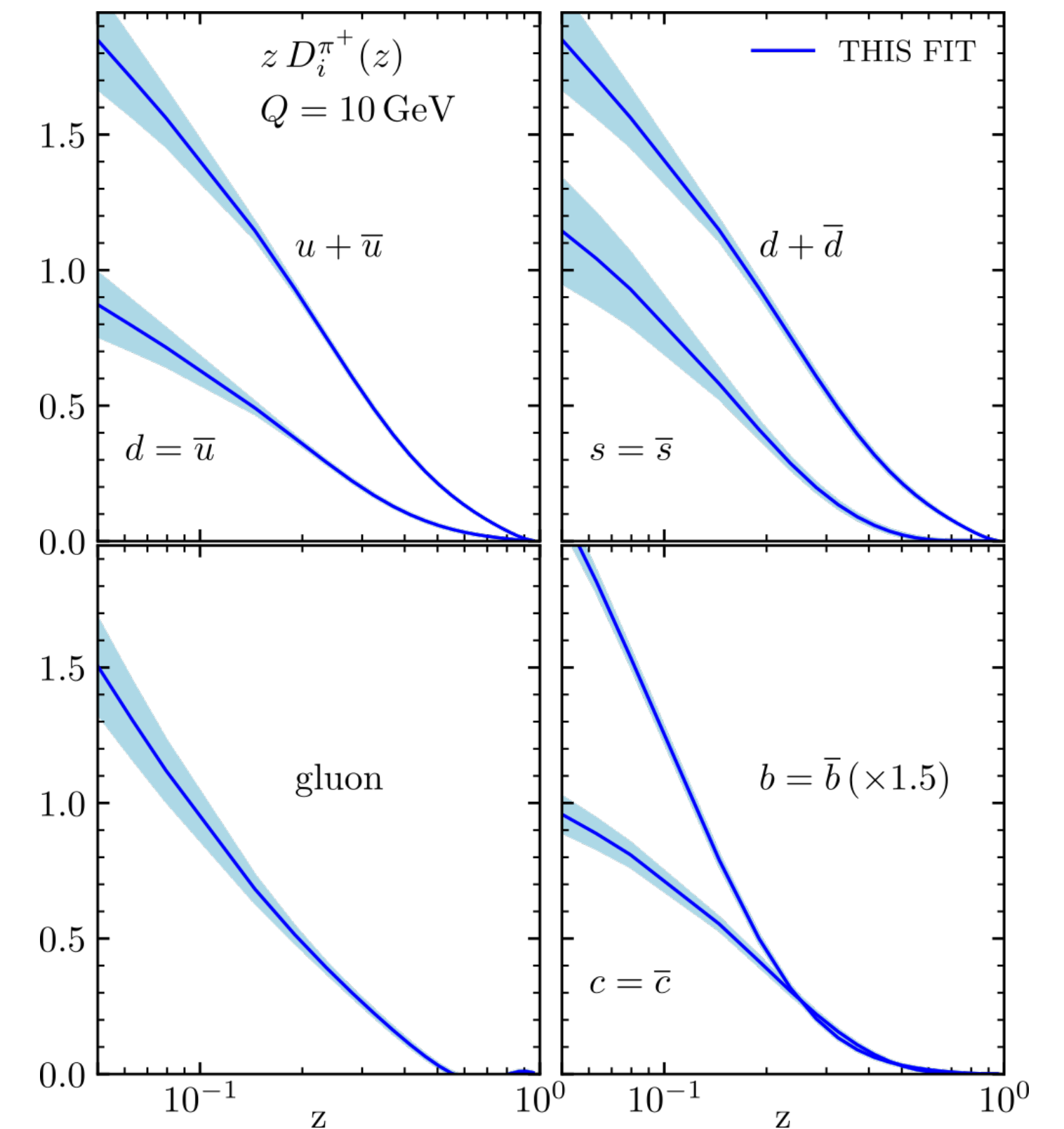
Partonic
Cross section

Fragmentation Functions (FFs)

$$0 \leq z \leq 1$$



Fragmentation Functions for Pions



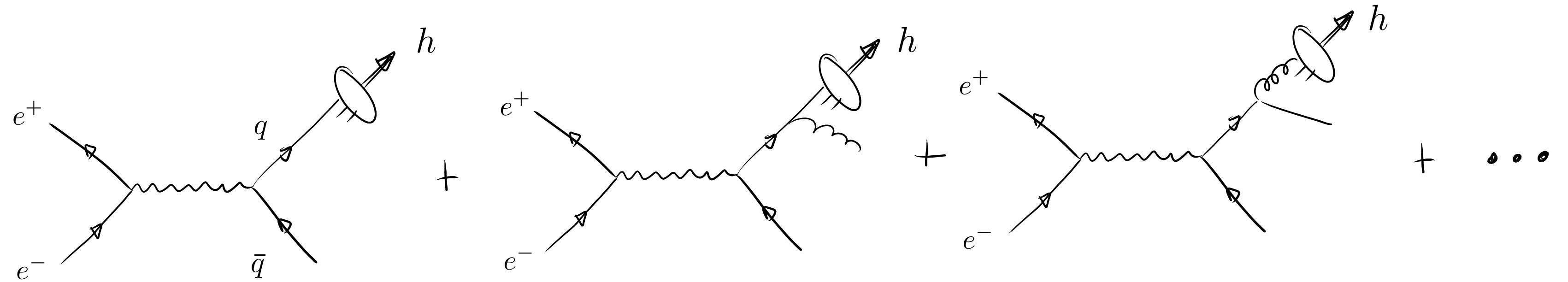
arXiv: 2110.14015 - Phys.Rev.D 105 (2022) 3

Processes with hadrons in the final state

e.g. $e^+ + e^- \rightarrow h + X$

h could be any hadron

$h = \pi^0, \pi^\pm, K's, \eta, p, \Lambda^0, \dots$

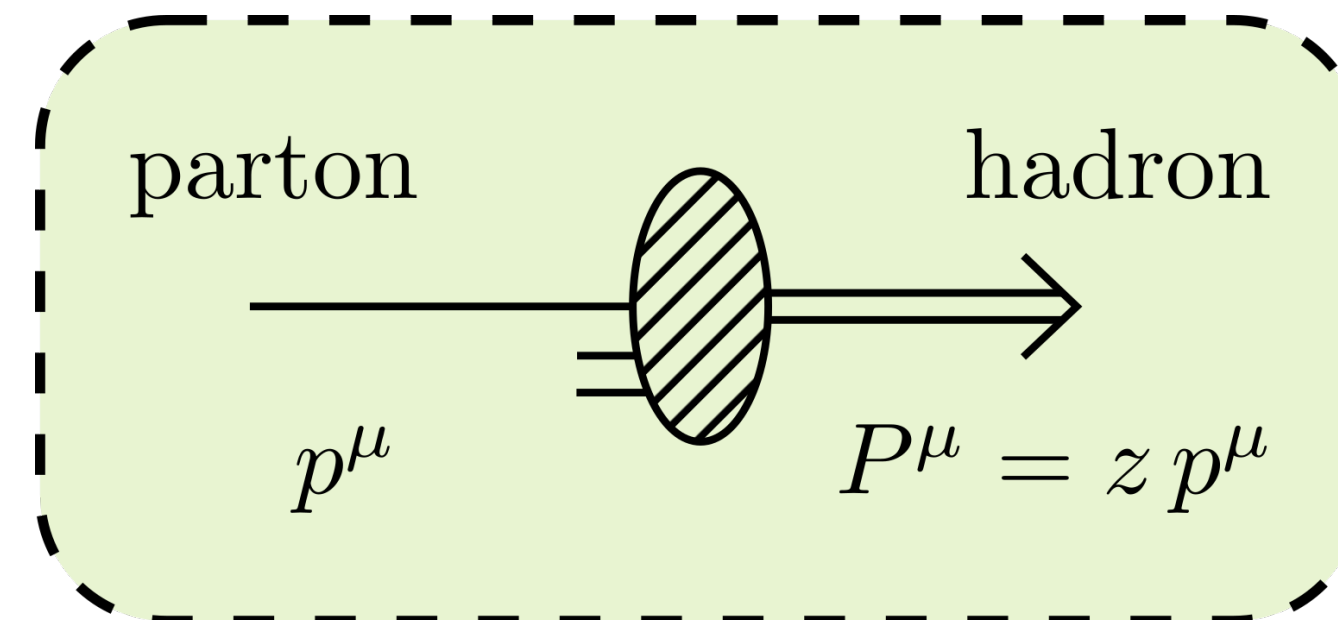


$$d\sigma(Q) = \sum_i \int_0^1 dz \hat{\sigma}_i(Q/\mu_F) D_i(z, \mu_F)$$

Partonic
Cross section

Fragmentation Functions (FFs)

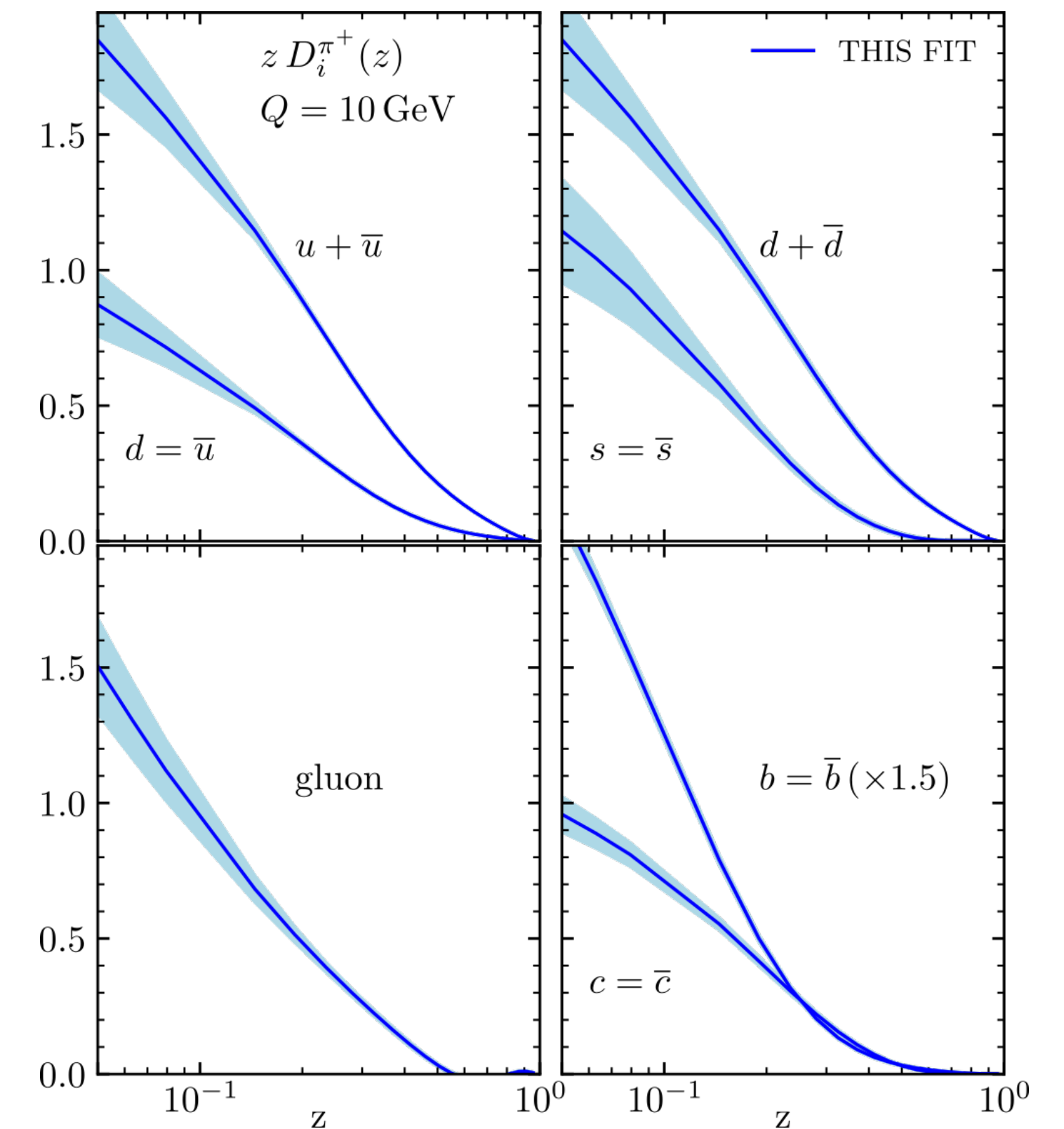
$$0 \leq z \leq 1$$



Scale evolution described by
the Altarelli–Parisi equations

$$\mu \frac{\partial}{\partial \mu} D_i(z, \mu) = \sum_j \int_z^1 \frac{d\zeta}{\zeta} P_{ij} \left(\frac{x}{\zeta} \right) D_j(\zeta, \mu)$$

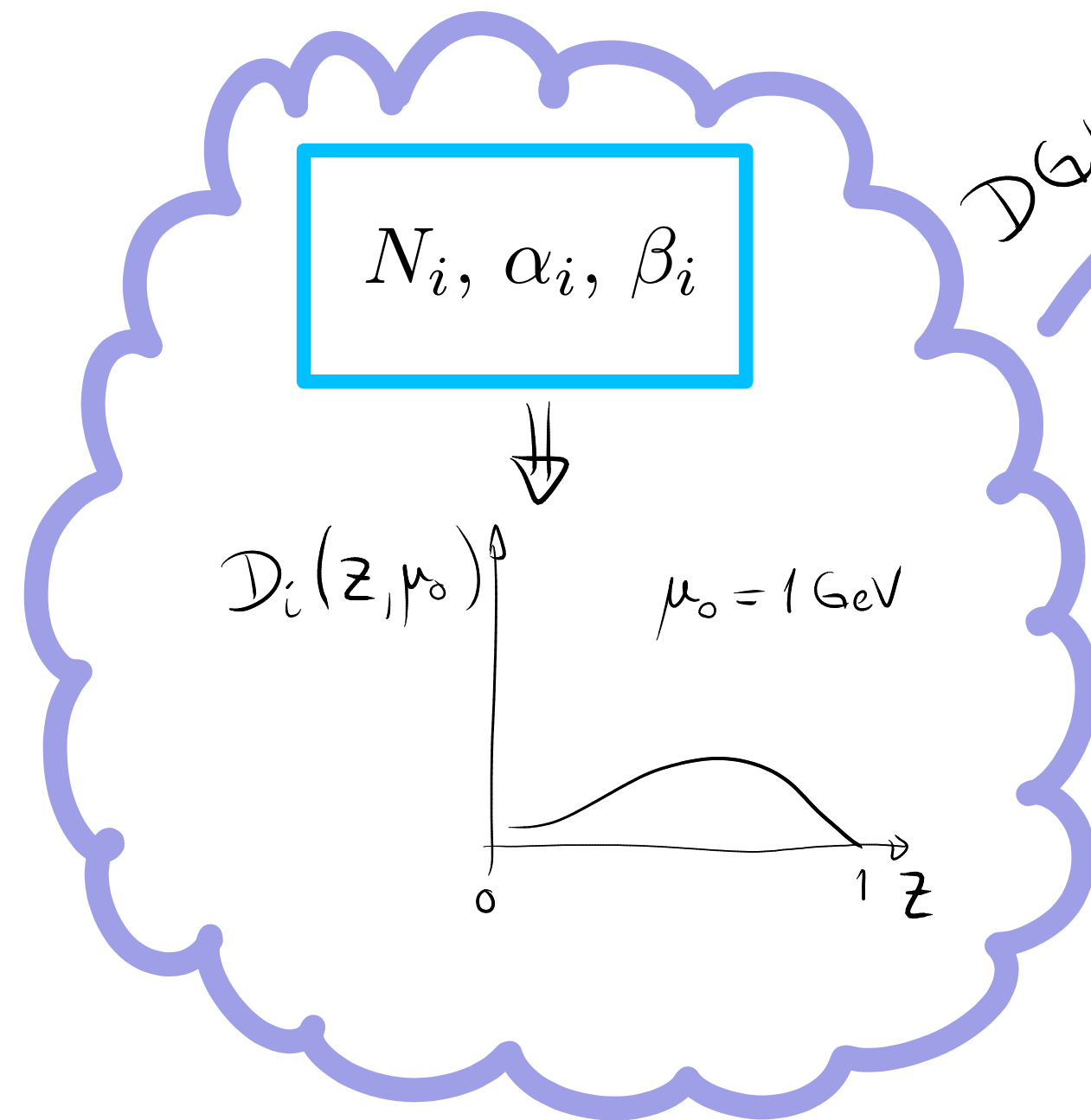
Fragmentation Functions for Pions



arXiv: 2110.14015 - Phys.Rev.D 105 (2022) 3

How do we determine fragmentation functions from data?

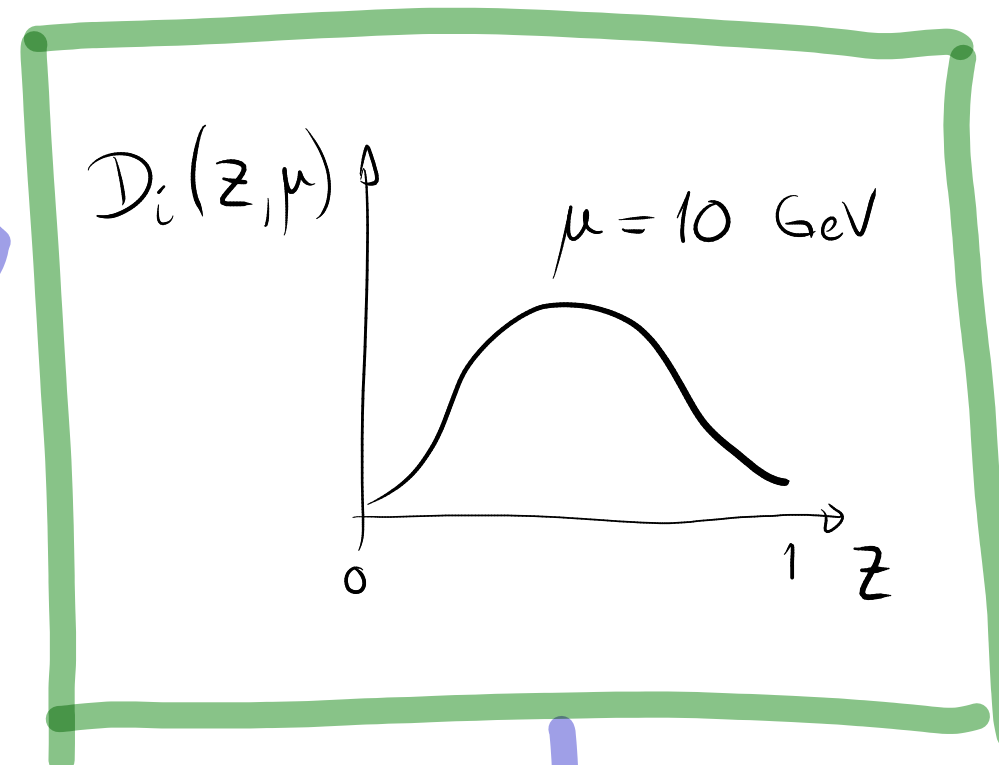
e.g. Experimental data from $\sigma(e^+ + e^- \rightarrow h + X)$ $\nearrow \sqrt{s} = 10 \text{ GeV}$
 $\searrow \sqrt{s} = 30 \text{ GeV}$



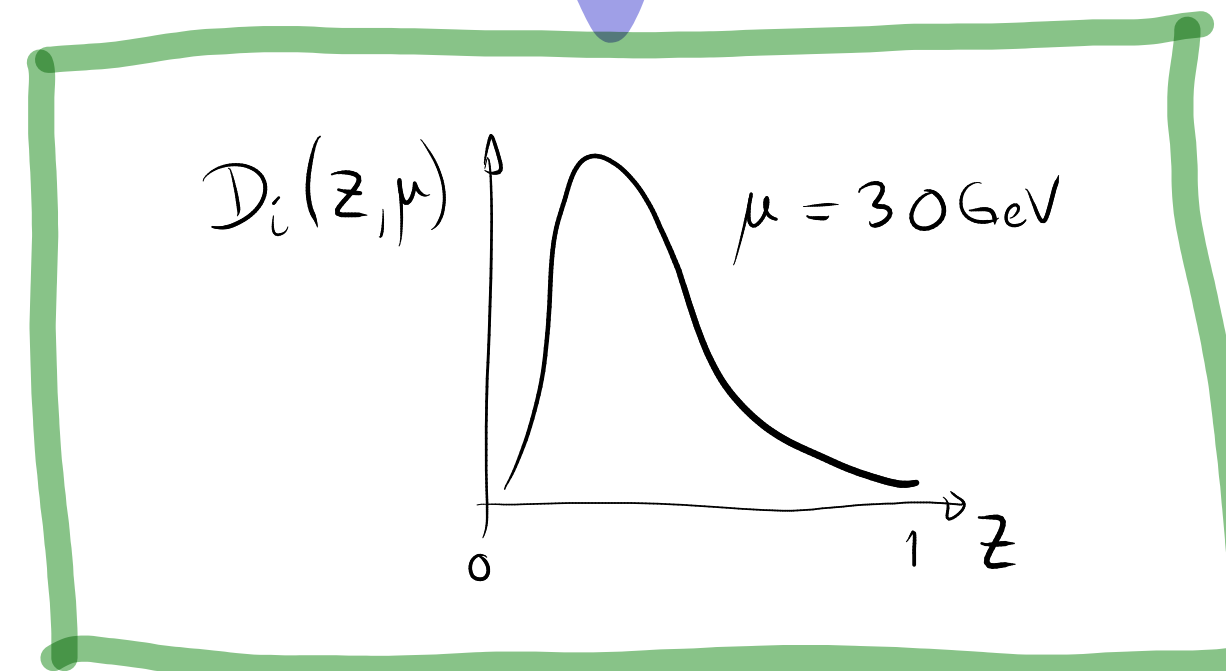
$$D_i(z, \mu_0) = N_i z^{\alpha_i} (1 - z)^{\beta_i}$$

Ansatz for the FFs at initial scale μ_0

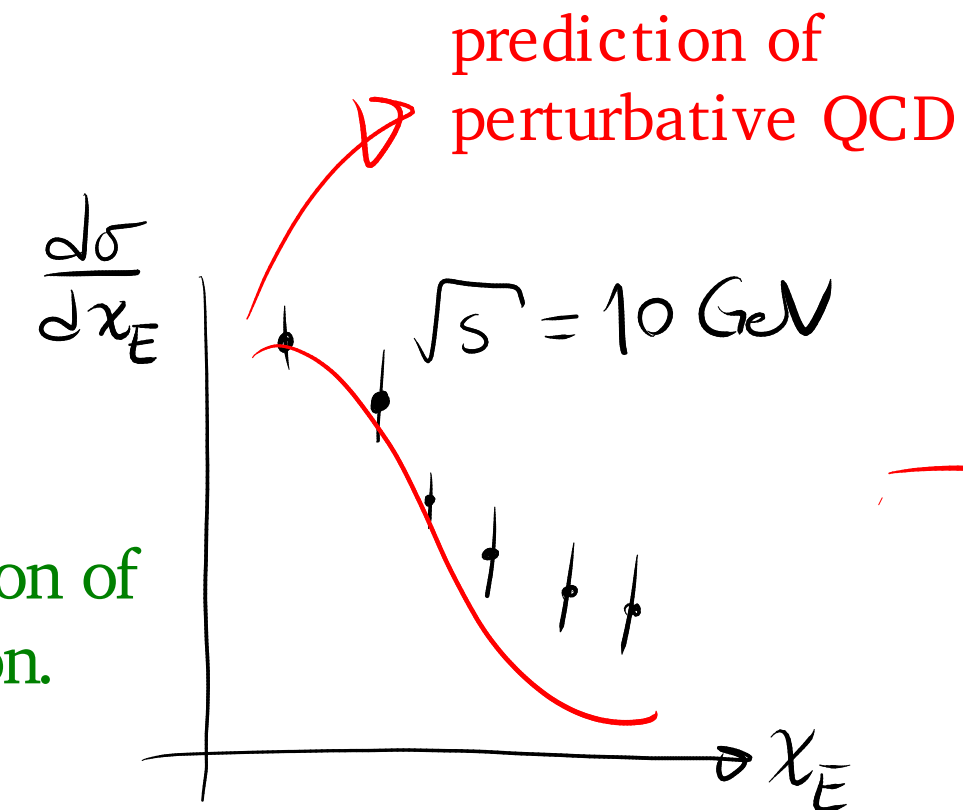
DGLAP



DGLAP



pQCD calculation of the cross section.



prediction of perturbative QCD

$$\chi^2 = \sum_i \frac{(D_i - T_i)^2}{\sigma_i^2}$$

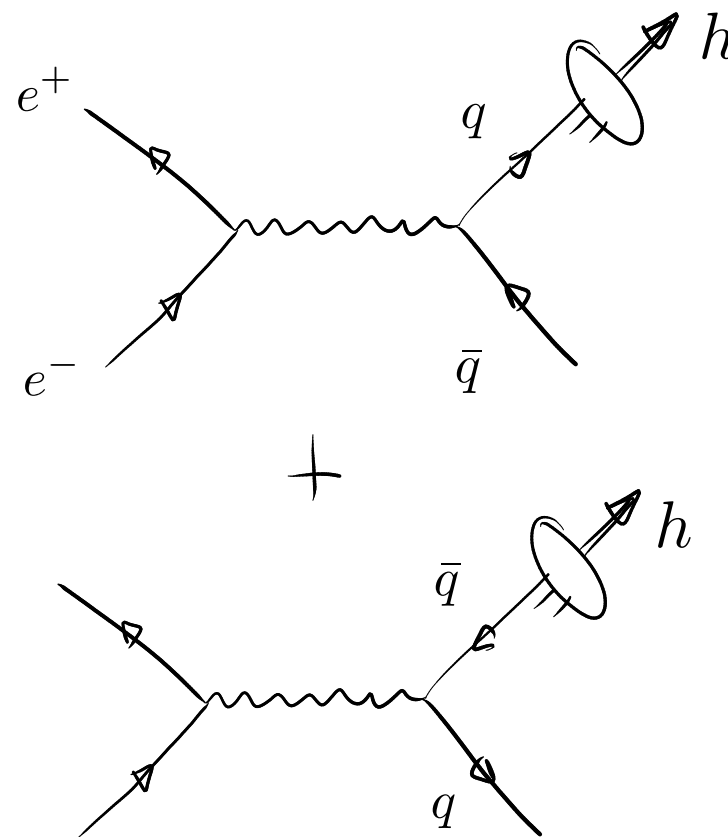
Function to be minimized

Typical observables involving hadron production

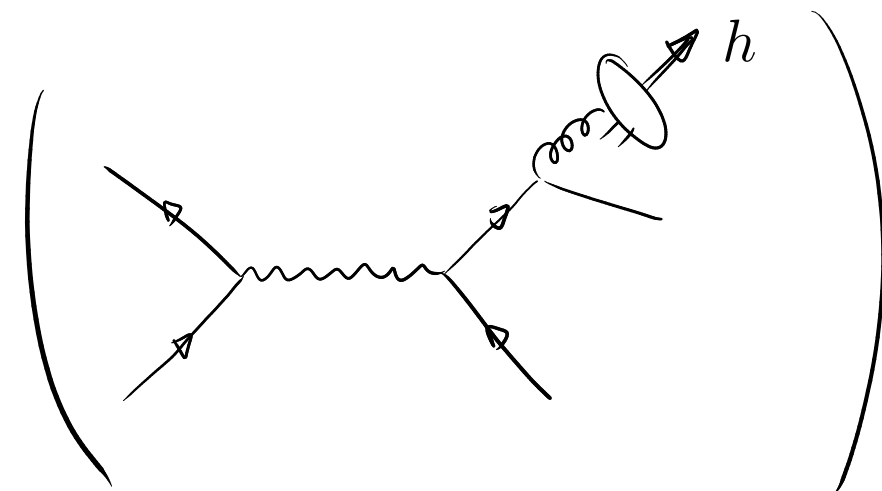
h could be any hadron
 $h = \pi^0, \pi^\pm, K's, \eta, p, \Lambda^0, \dots$

$$e^+ + e^- \rightarrow h + X$$

Sensitive to $q + \bar{q}$ combinations

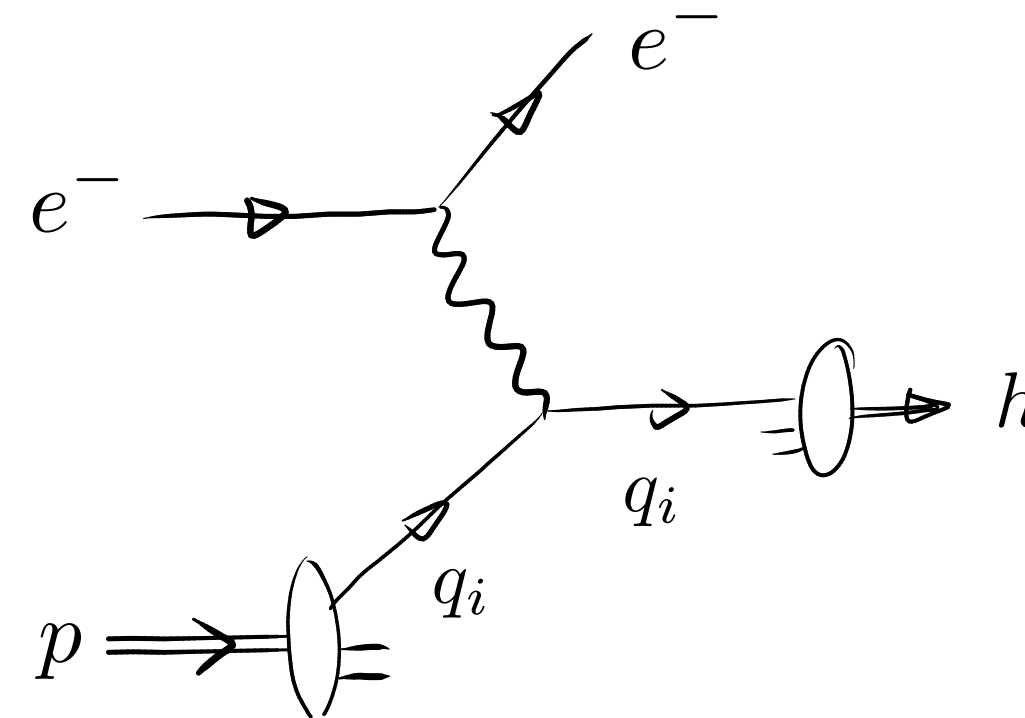


$$d\sigma = \sum_{i=u,\bar{u},d,\dots} d\hat{\sigma}_i \otimes D_i$$

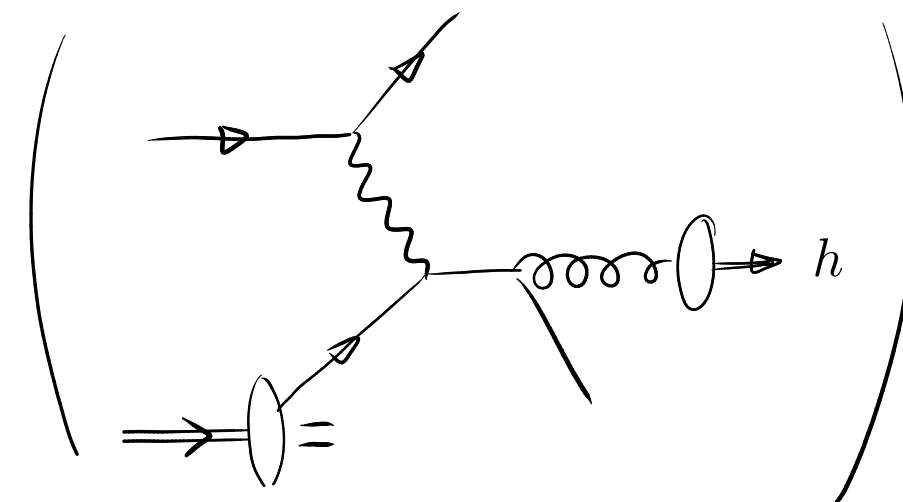


$$e^- + p \rightarrow e^- + h + X$$

Sensitive to different quark flavors

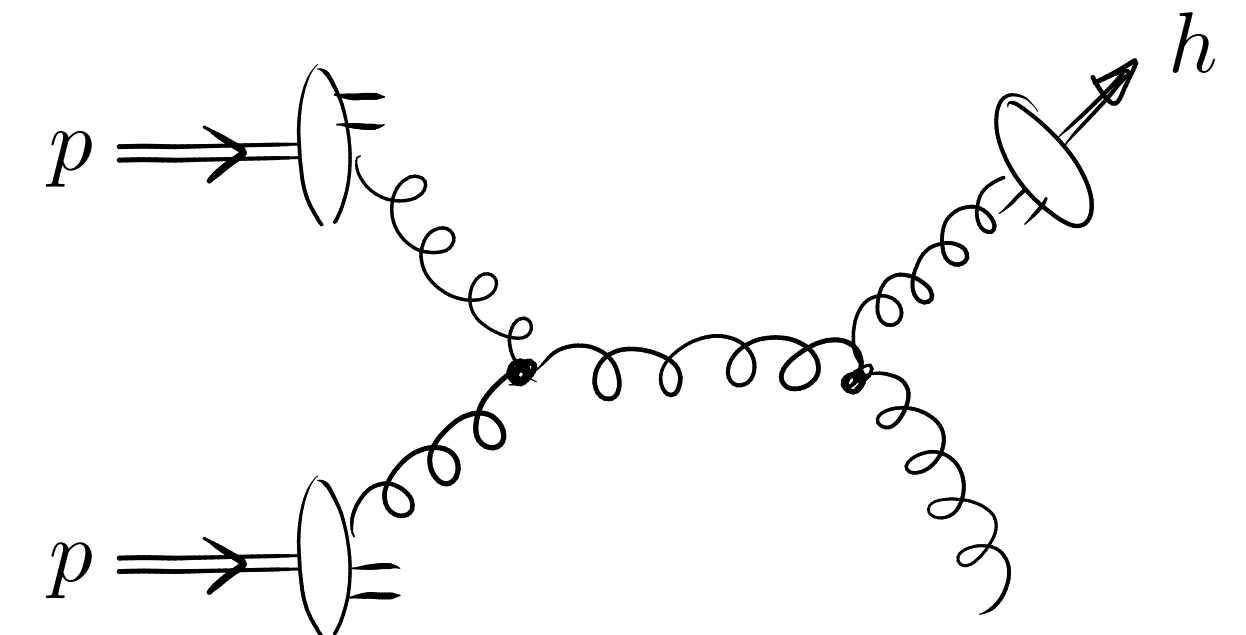


$$d\sigma = \sum_{i=u,\bar{u},d,\dots} f_i \otimes d\hat{\sigma}_i \otimes D_i$$

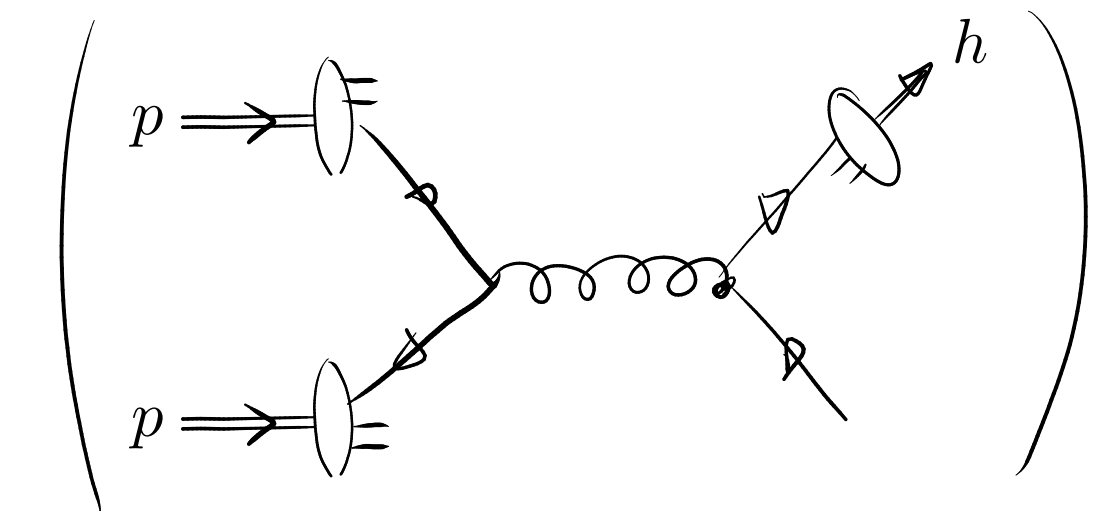


$$p + p \rightarrow h + X$$

Sensitive to gluons



$$d\sigma = \sum_{i,j,k} f_i \otimes f_j \otimes d\hat{\sigma}_{ij \rightarrow k} \otimes D_k$$



Our Work...

❖ *C. A. Aidala, D. A. Loomis, R. T. Martinez, R. Sassot, M. Stratmann ()*

Eta Fragmentation Functions Revisited

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Ramiro Tomas Martinez‡ and Rodolfo Sassot§
Universidad de Buenos Aires, Facultad de Ciencias Exactas y Naturales,
Departamento de Física and IFIBA-CONICET, Ciudad Universitaria, (1428) Buenos Aires, Argentina

Marco Stratmann¶
Institute for Theoretical Physics, University of Tübingen,
Auf der Morgenstelle 14, 72076 Tübingen, Germany

We revisit the extraction of parton-to-eta meson fragmentation functions at next-to-leading order accuracy in QCD in the light of the recent hadroproduction measurements in proton-proton collisions obtained by the PHENIX, LHCb, and ALICE collaborations. In addition to an increased precision, the data explore complementary rapidity ranges and center-of-mass system energies. The analysis exploits the theoretical scale dependence to ease tensions among the data sets at different energies that are potentially caused by QCD corrections beyond the next-to-leading order. The resulting set of fragmentation functions yields a consistent description of all available data. Estimates of uncertainties are obtained with the Monte Carlo replica method.

I. INTRODUCTION

Fragmentation functions (FFs) are evolving in recent years into increasingly precise tools for both unveiling the path to confinement of quarks and gluons into hadrons [1], as well as for testing key properties of Quantum Chromodynamics (QCD) such as factorization and universality [2, 3]. This remarkable improvement is being made possible by vigorous experimental programs pursuing greater precision and kinematic coverage, together achieving milestones in higher order perturbative QCD both combined in global QCD analyses [4] and as parton distribution functions [5] as often

our understanding of proton-nucleus and heavy-ion collisions by studying modifications of their yields due to medium-induced effects, see, e.g., [11]. Fifteen years ago, the first global QCD analysis of eta meson (η) FFs at next-to-leading order (NLO) accuracy was presented in reference [12] (AESSS). Back then it combined world data on single inclusive electron-positron annihilation (SIA) experiments [13–24] with the proton-proton collision data [25, 26] available at that time, finding a good agreement within uncertainties. Since then, other groups have also addressed η fragmentation. In ref. [27] only SIA data were included, which on their own do not suffice to determine all aspects of parton-sality property assumed in perturbative QCD. The fit described in [28] (NPC23) considers in addition results of proton-proton cross section ratios to pions, which pose a more subtle, however, issues with the individual absolute values as we shall discuss later. Clearly, the results from various different analyses that we are

arXiv: 2507.04887v1

Phys. Rev. D 113, 092012 (2026)

Ramiro Tomas Martinez, University of Buenos Aires &
University of Tübingen

Our Work...

❖ *C. A. Aidala, D. A. Loomis, R. T. Martinez, R. Sassot, M. Stratmann ()*

❖ Extraction of η -meson fragmentation functions (FFs) at Next to Leading Order (NLO)

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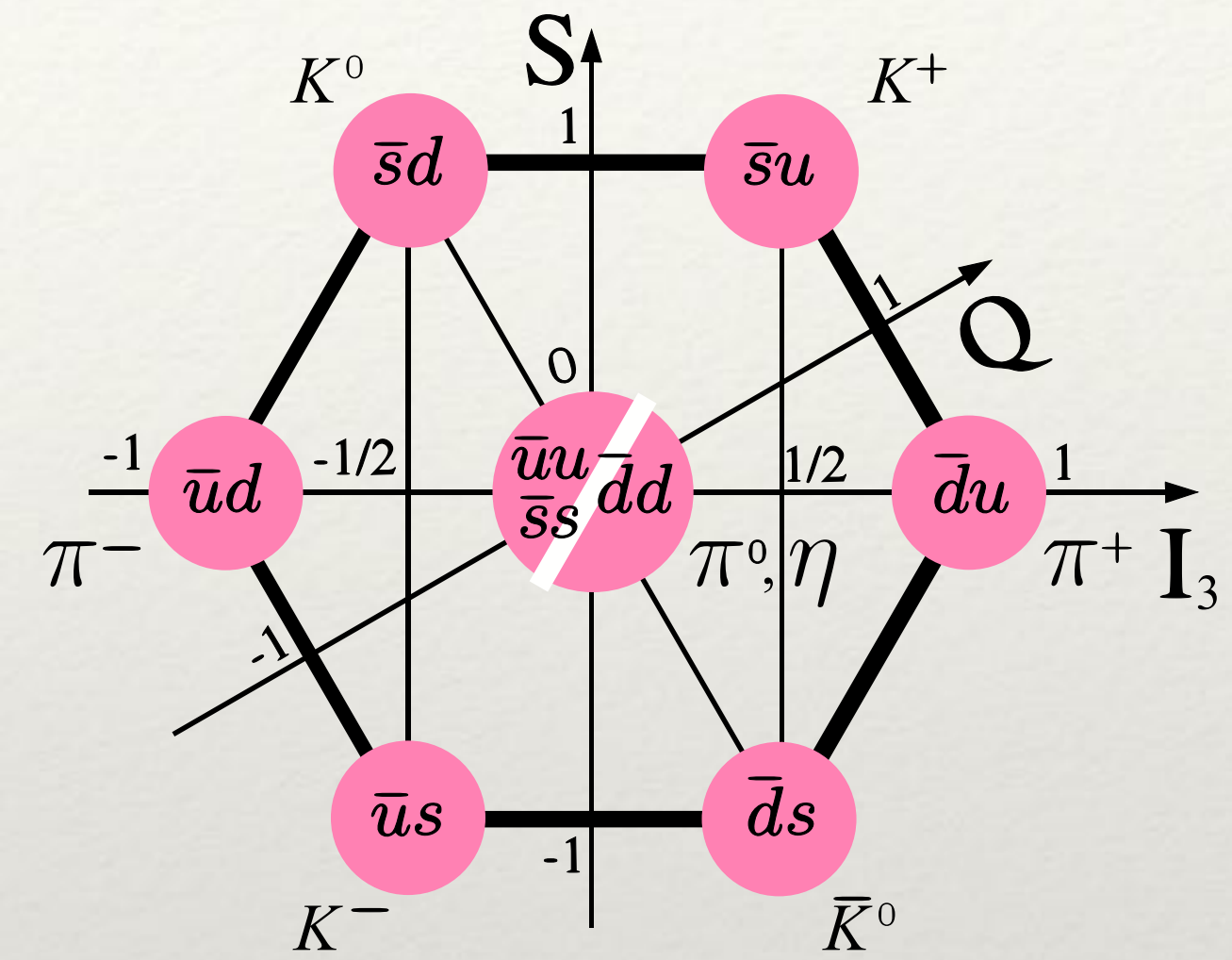
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❖ C. A. Aidala, D. A. Loomis, R. T. Martinez, R. Sassot, M. Stratmann ()

❖ Extraction of η -meson fragmentation functions (FFs) at Next to Leading Order (NLO)



$$|\eta\rangle = |u\bar{u}\rangle + |d\bar{d}\rangle - 2|s\bar{s}\rangle$$

arXiv: 2507.04887v1

Phys. Rev. D 113, 092012 (2026)

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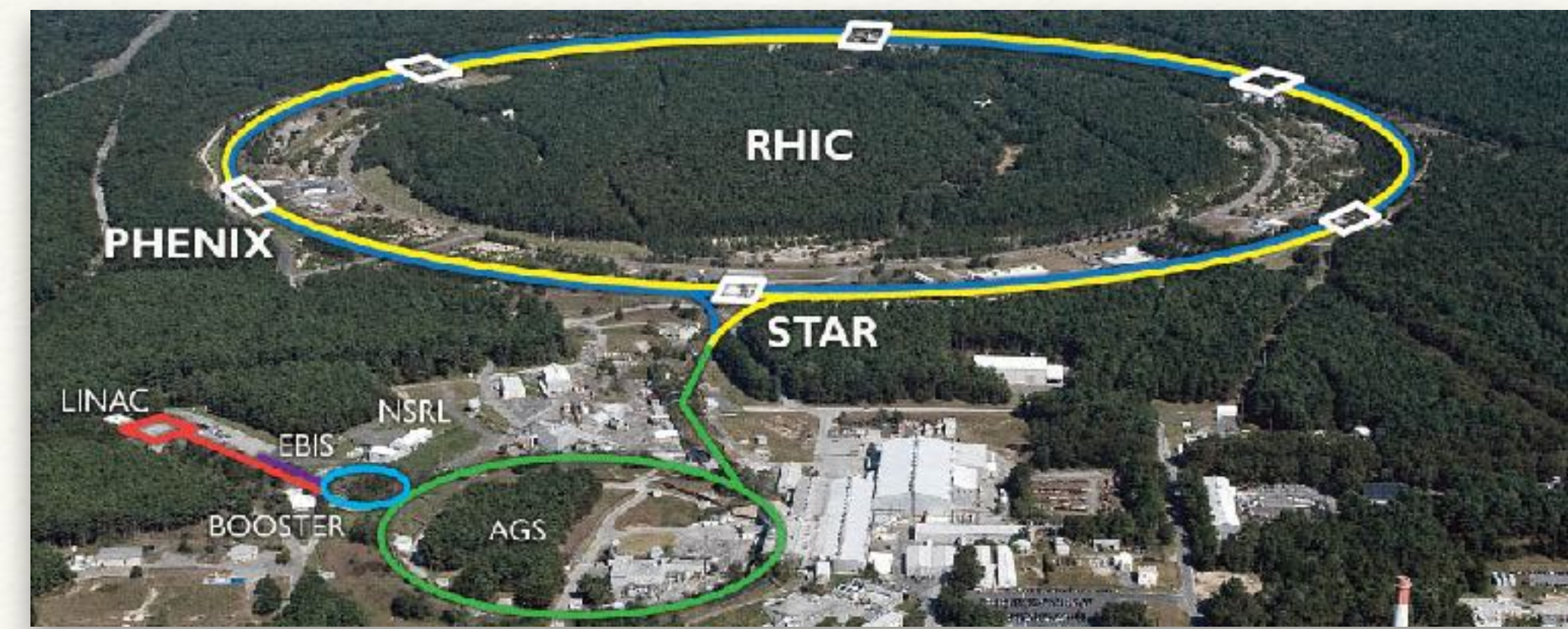
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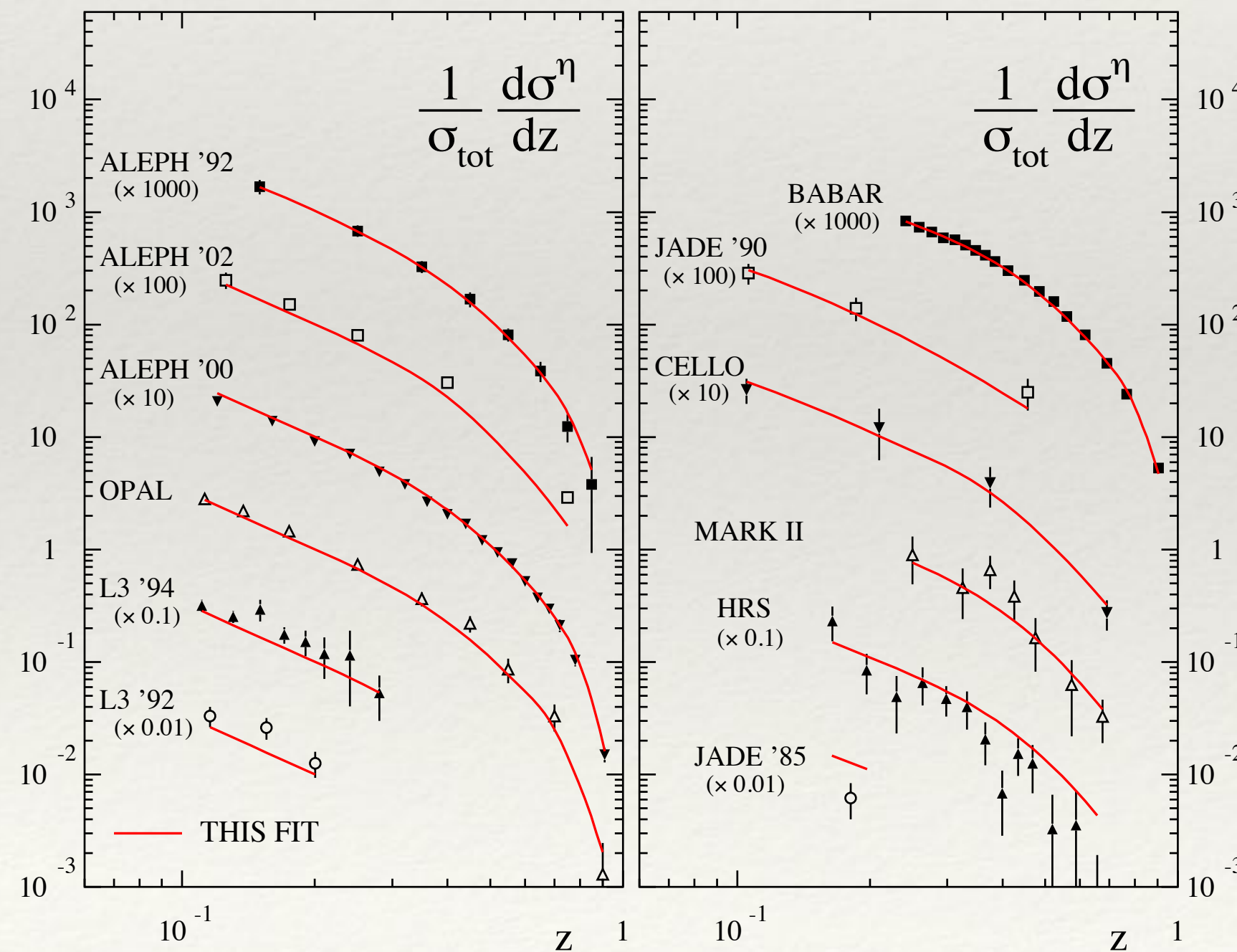
❖ 2010 —> First extraction of η FFs (AESSS)

- ❖ C. A. Aidala, F. Ellinghauss, R. Sassot, J. P. Seele, M. Stratmann
- ❖ *Phys. Rev. D* 83, 034002 (2011) arXiv: 1009.6145v2
- ❖ Used the data available at the time

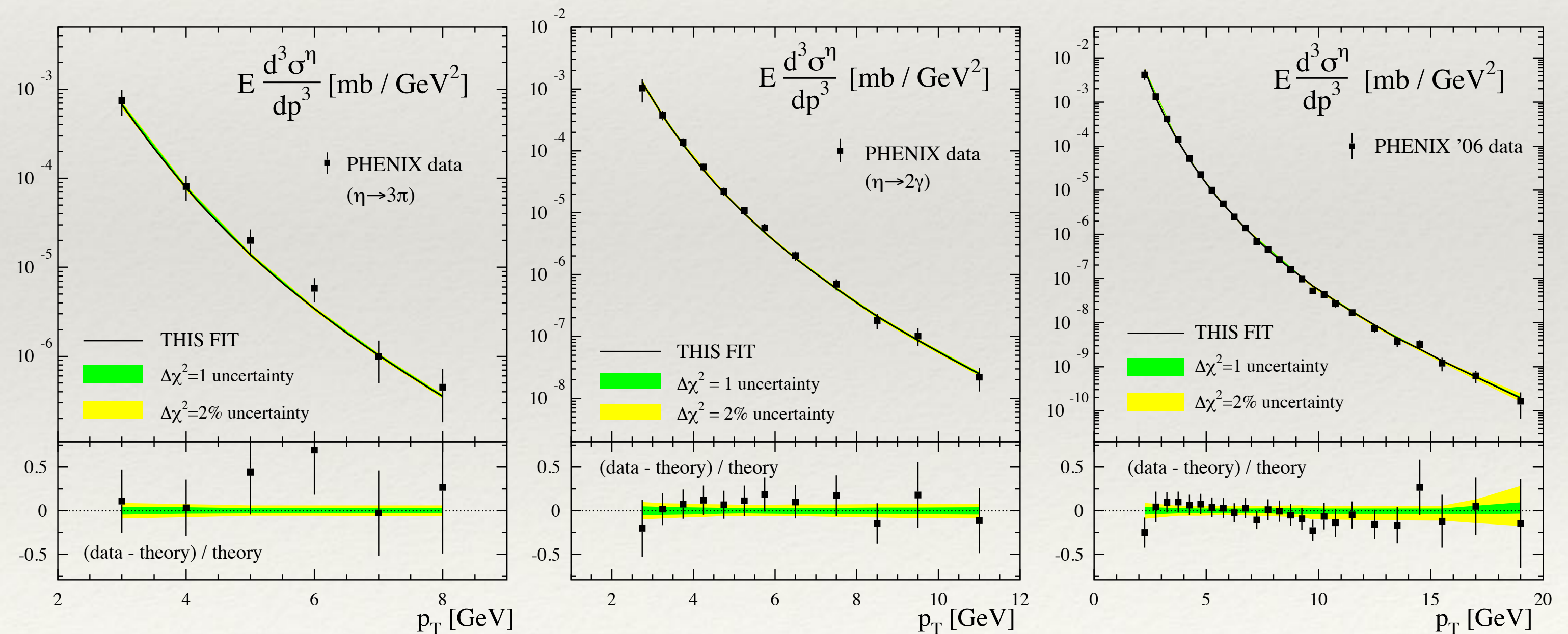


Relativistic Heavy Ion Collider (RHIC)
Brookhaven National Laboratory (BNL)

e+e- data



pp data $\sqrt{s} = 200$ GeV



After a decade and a half of LHC data...

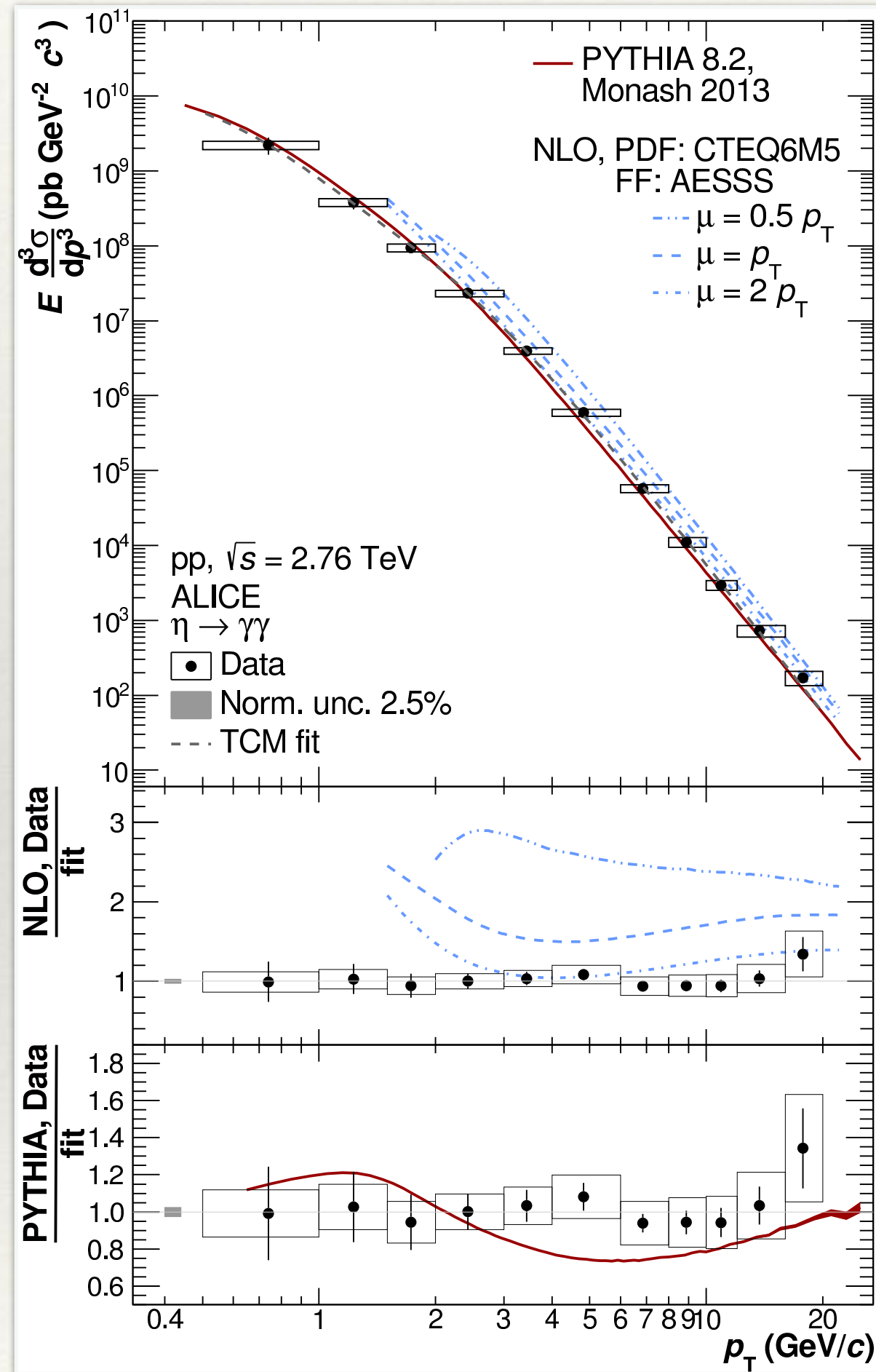
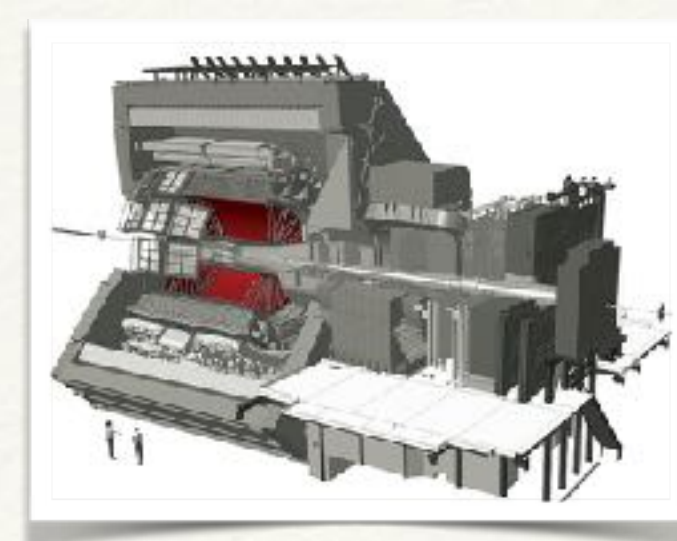
Data from the ALICE Collaboration $pp \rightarrow \eta + X$

$\sqrt{s} = 2.76$ TeV

$\sqrt{s} = 7$ TeV

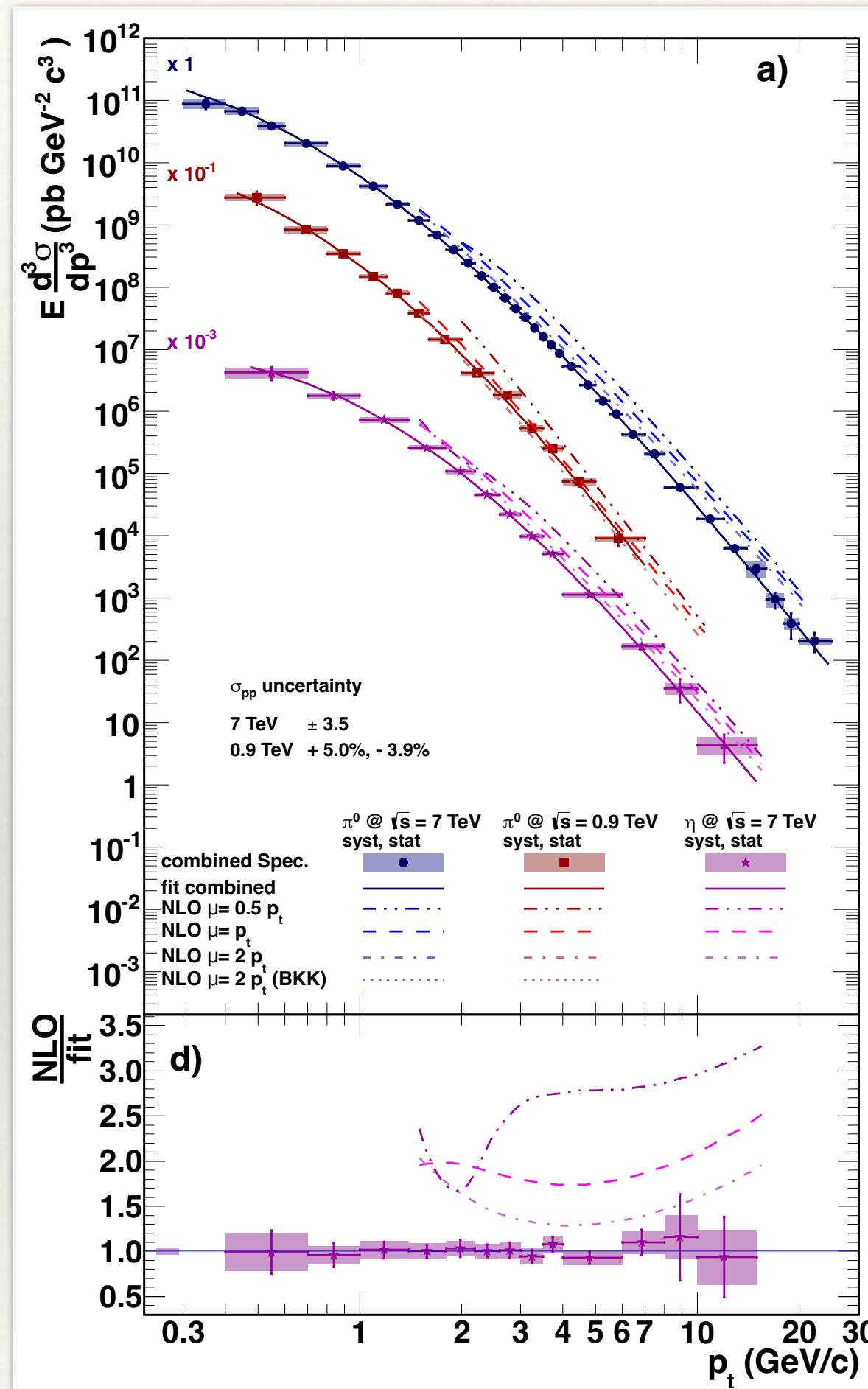
$\sqrt{s} = 8$ TeV

$\sqrt{s} = 13$ TeV



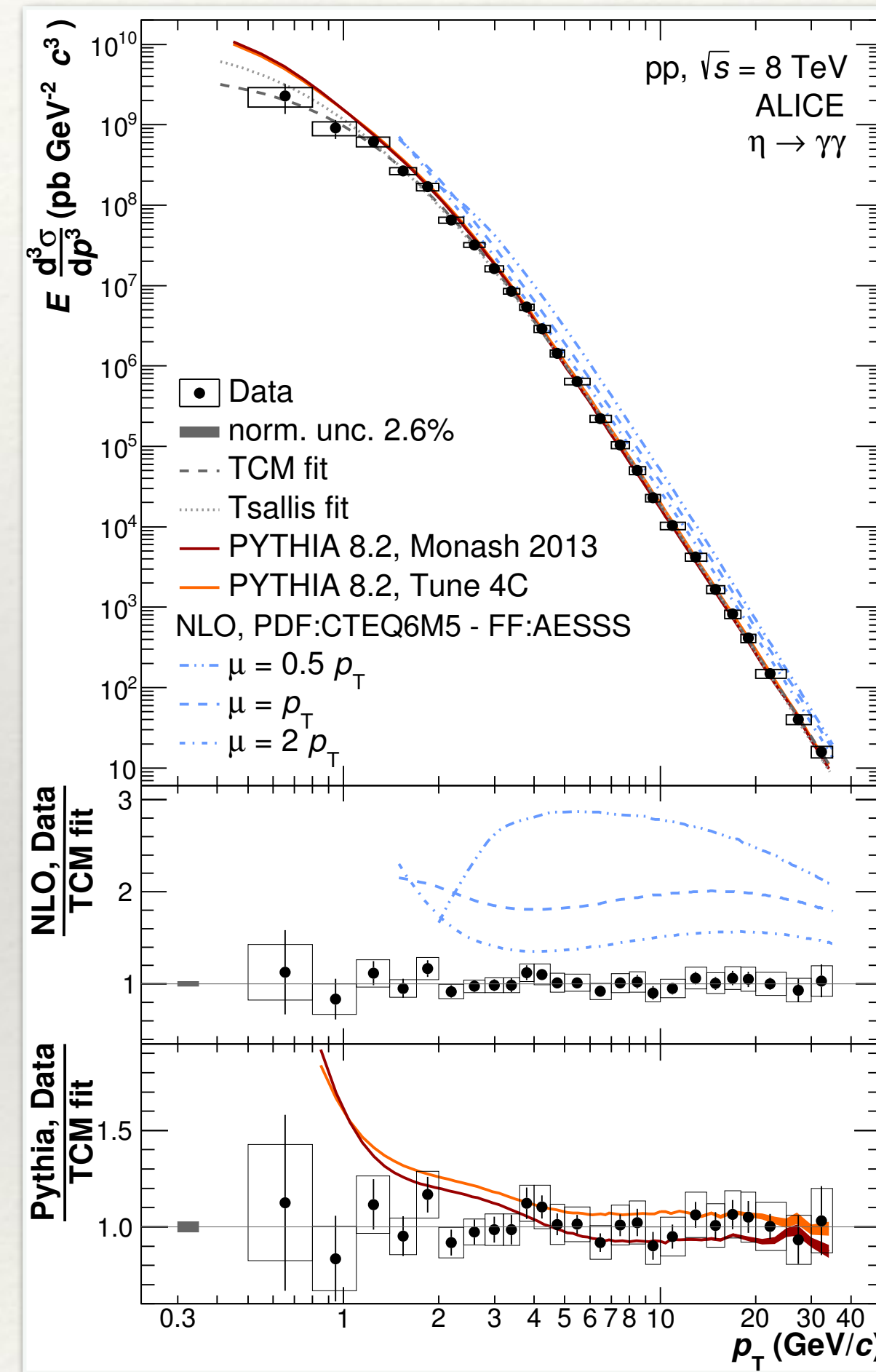
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Eur.Phys.J.C 77 (2017) 5, 339



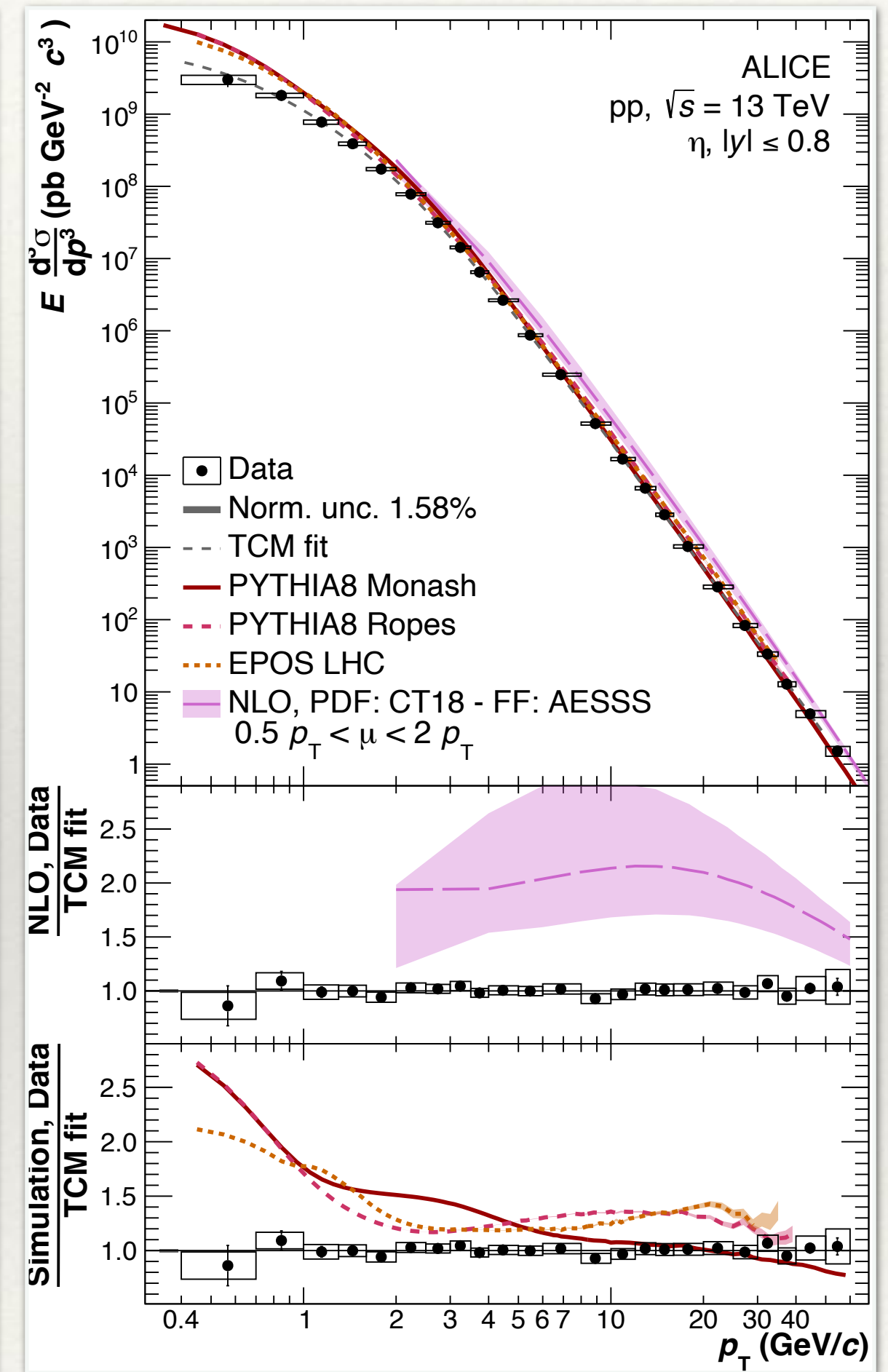
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Phys.Lett.B 717 (2012) 162



arXiv: 1708.08745v2

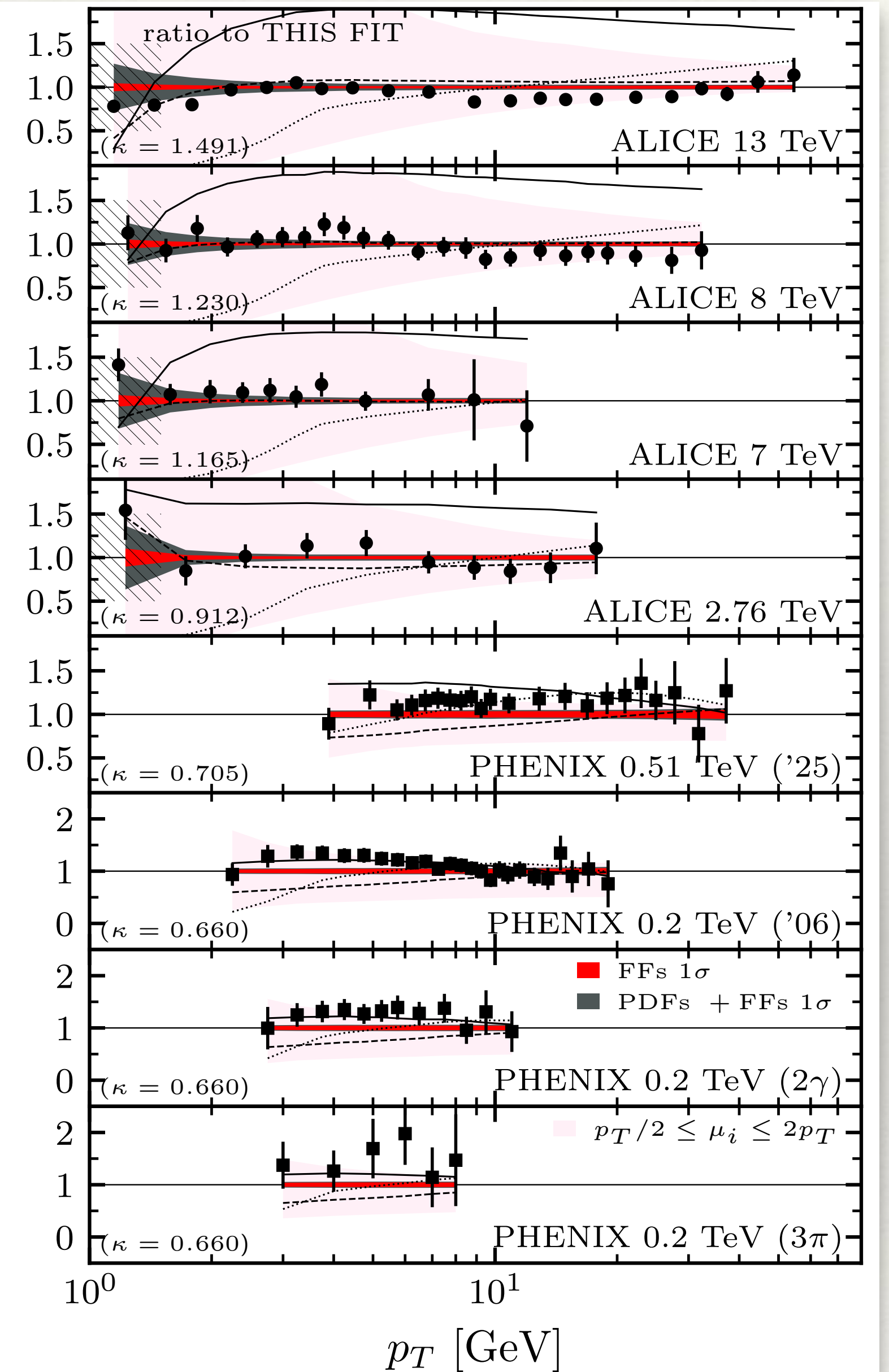
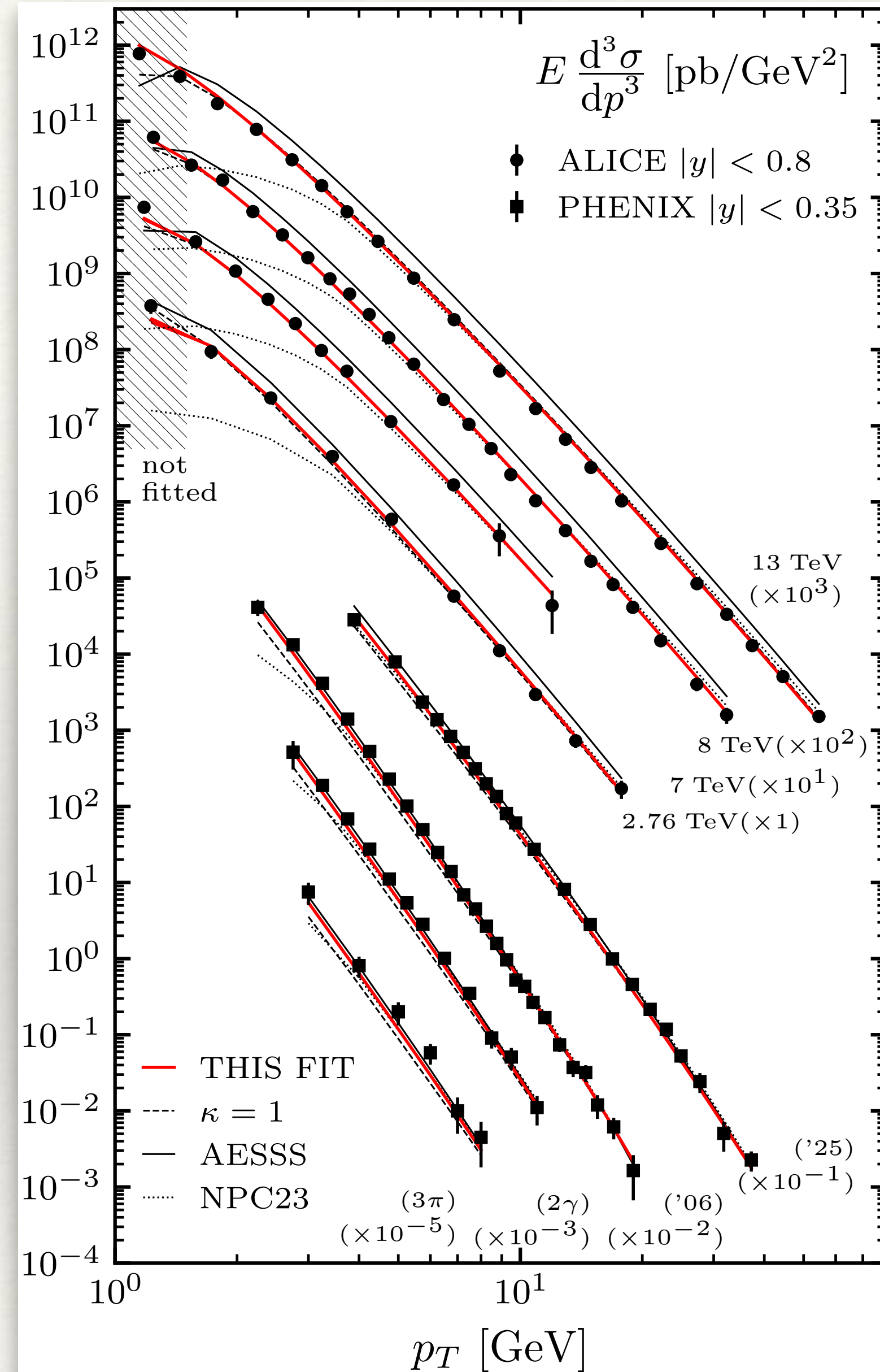
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arXiv: 2411.09560v1

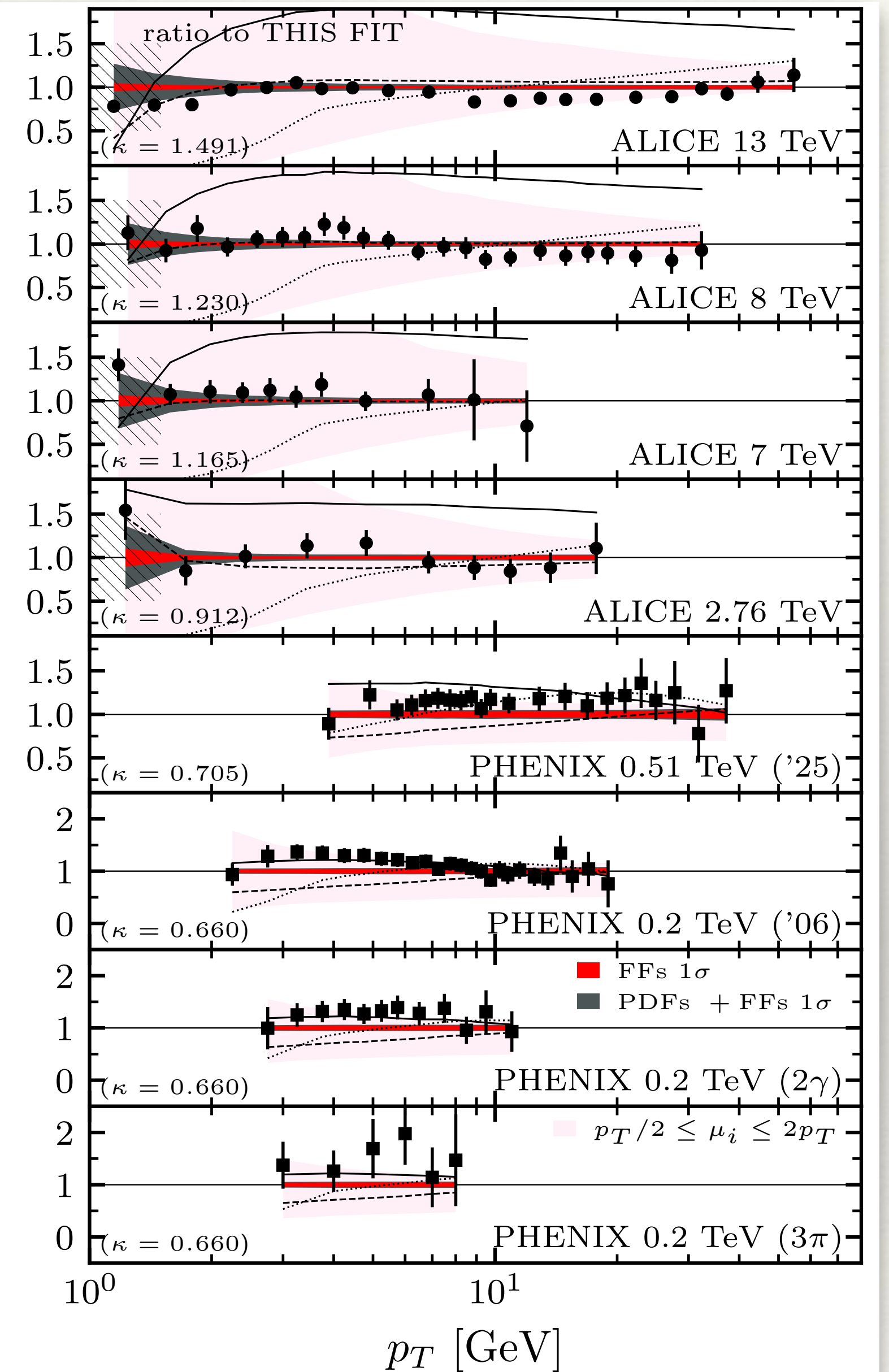
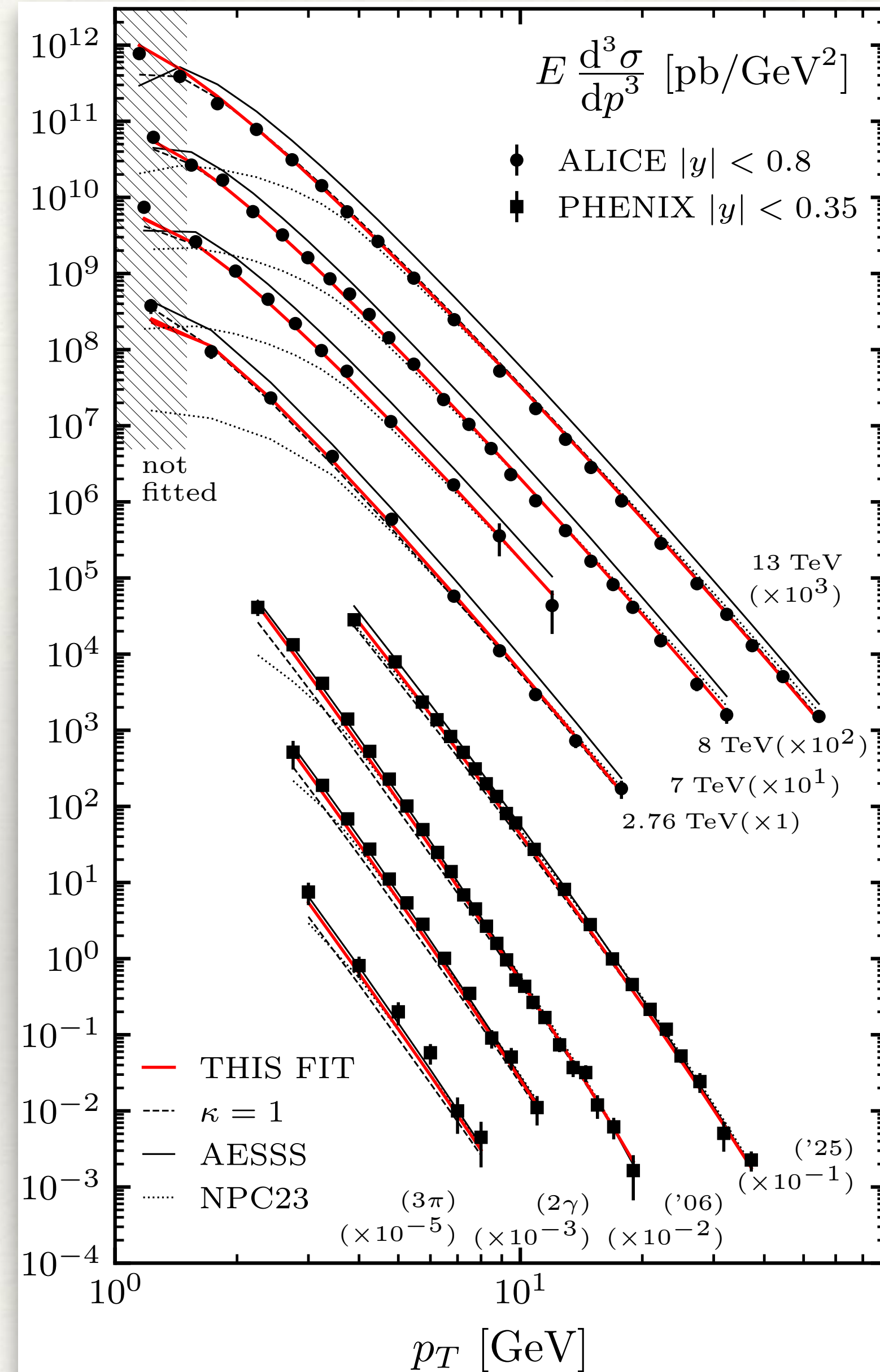
JHEP 08 (2025) 035

What happens if we
perform a new fit
using the new data?



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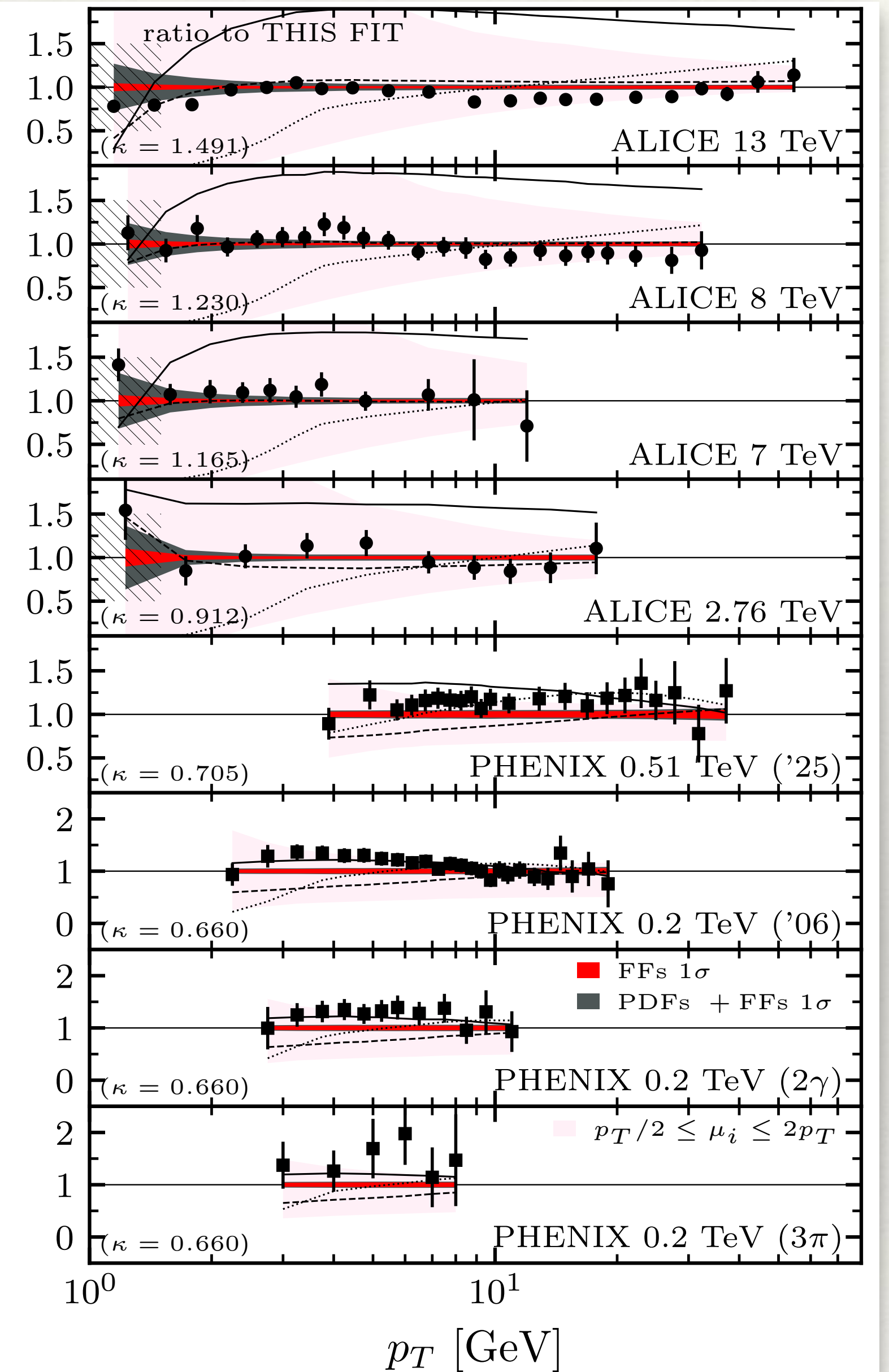
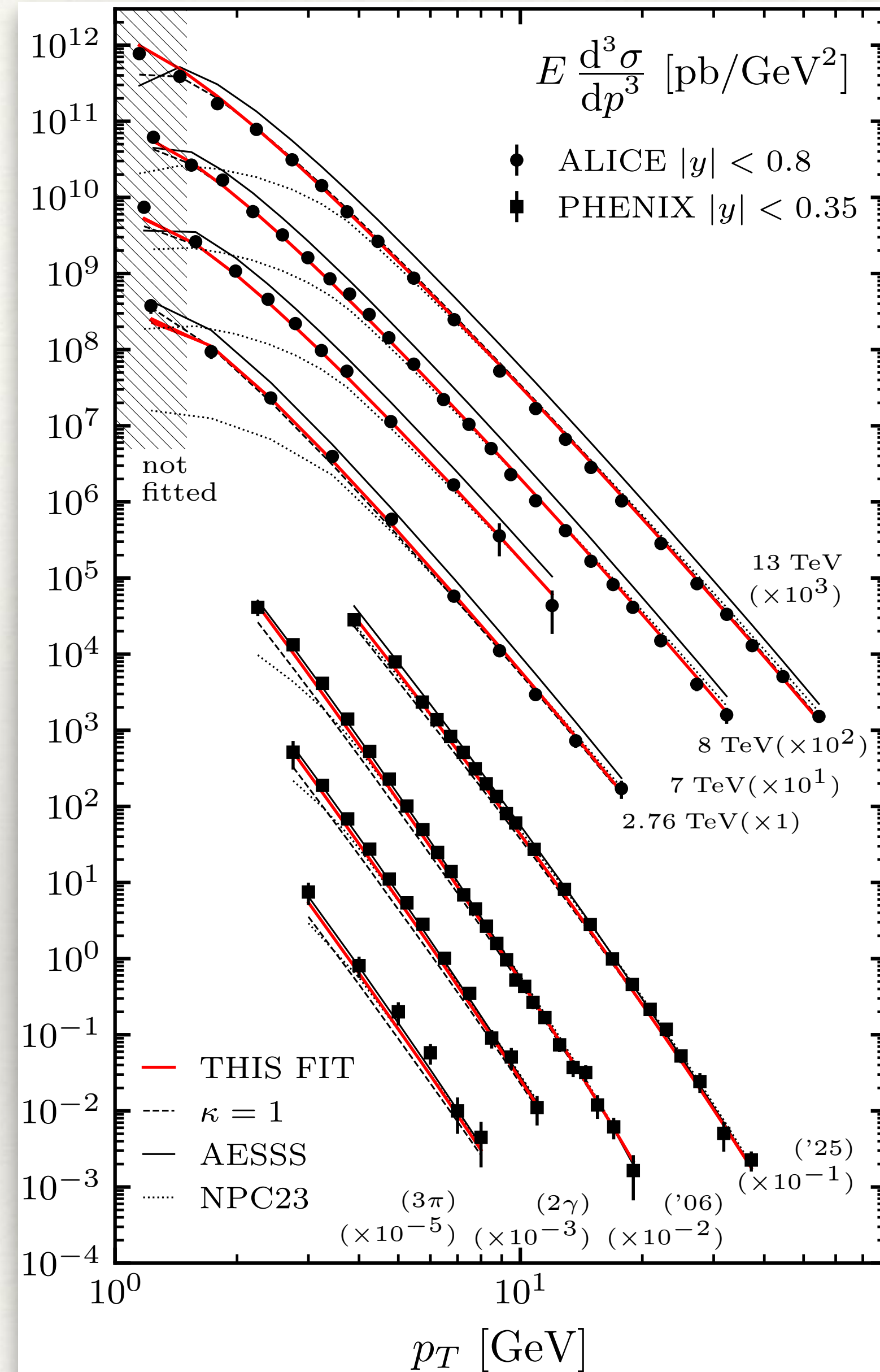
Result of a new fit using all
the new data



What happens if we perform a new fit using the new data

Result of a new fit using all the new data

The new fit gives a consistent description of the data sets over the full energy range.

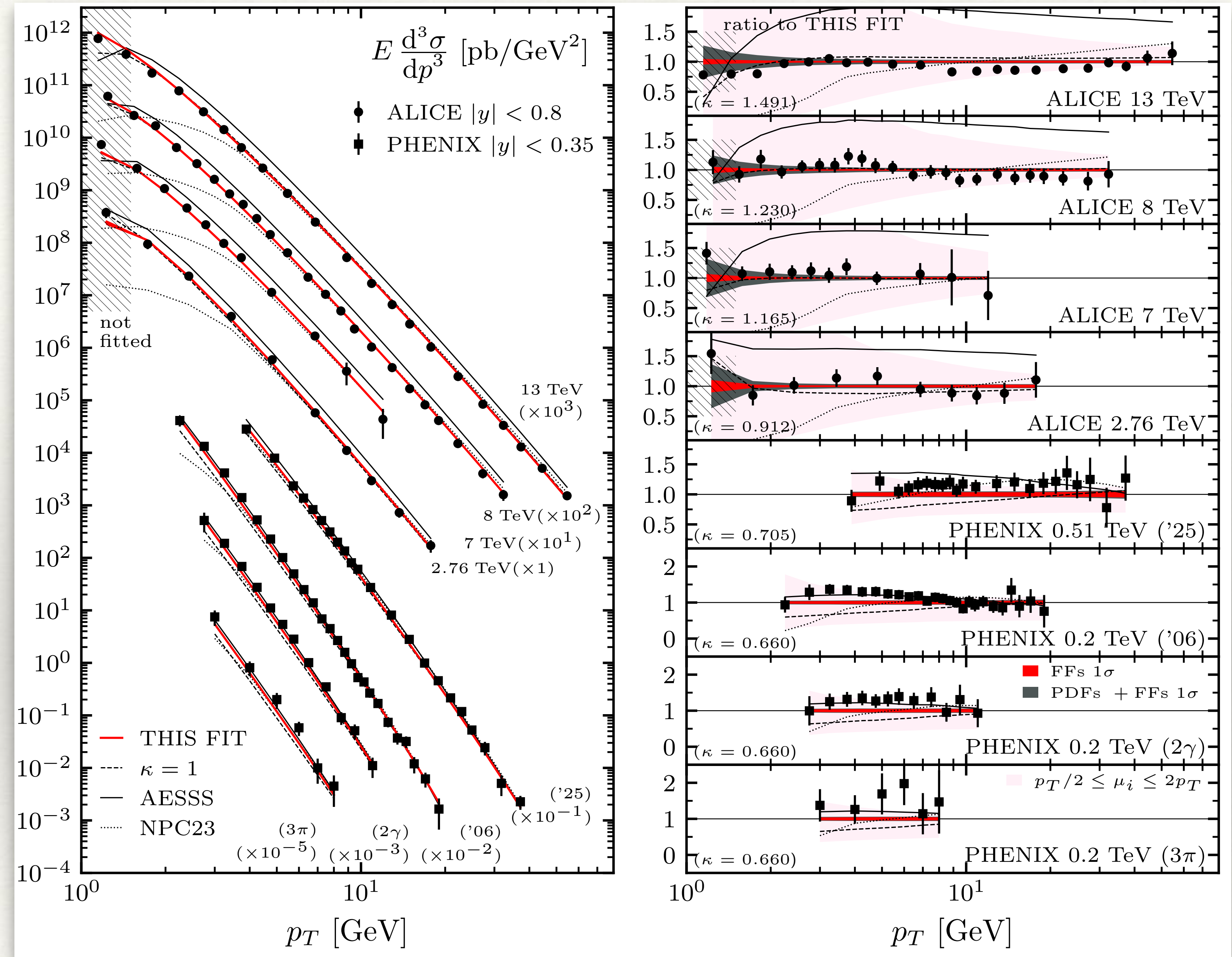


What happens if we perform a new fit using the new data?

Result of a new fit using all the new data

The new fit gives a consistent description of the data sets over the full energy range.

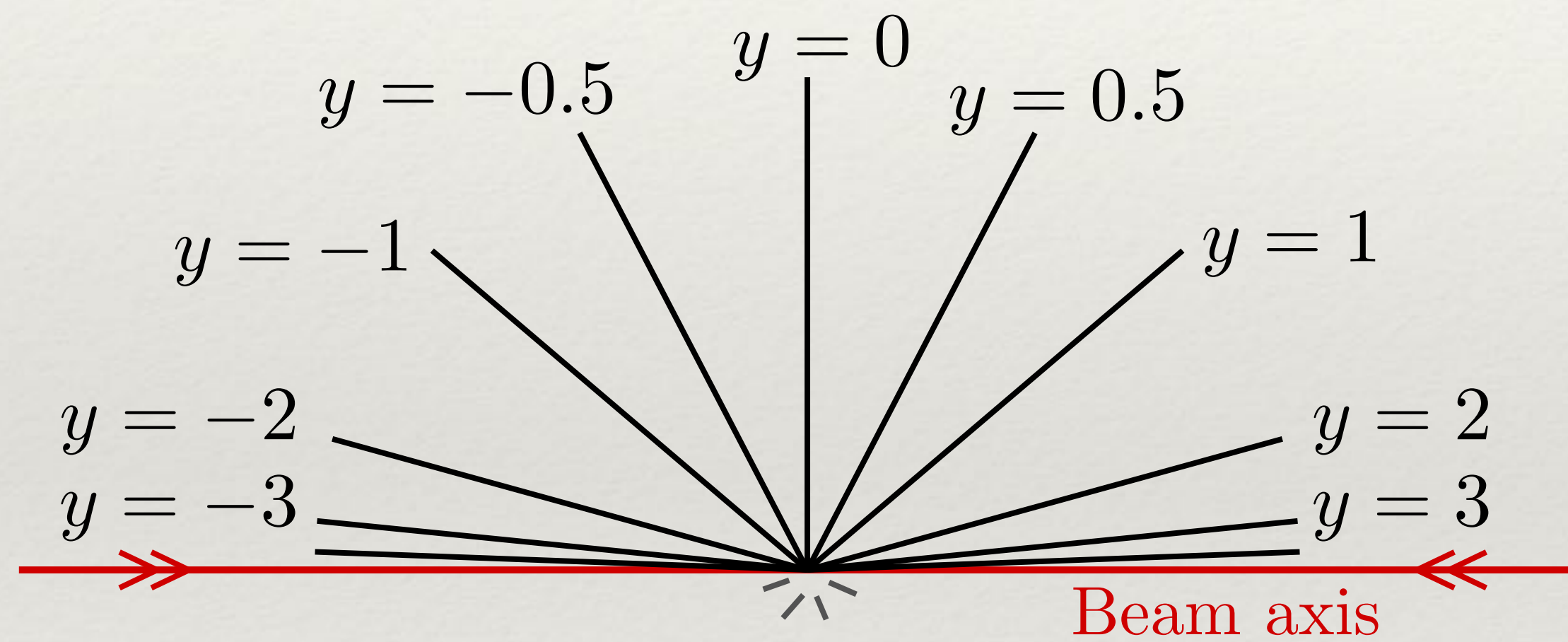
This is not the end of QCD :)



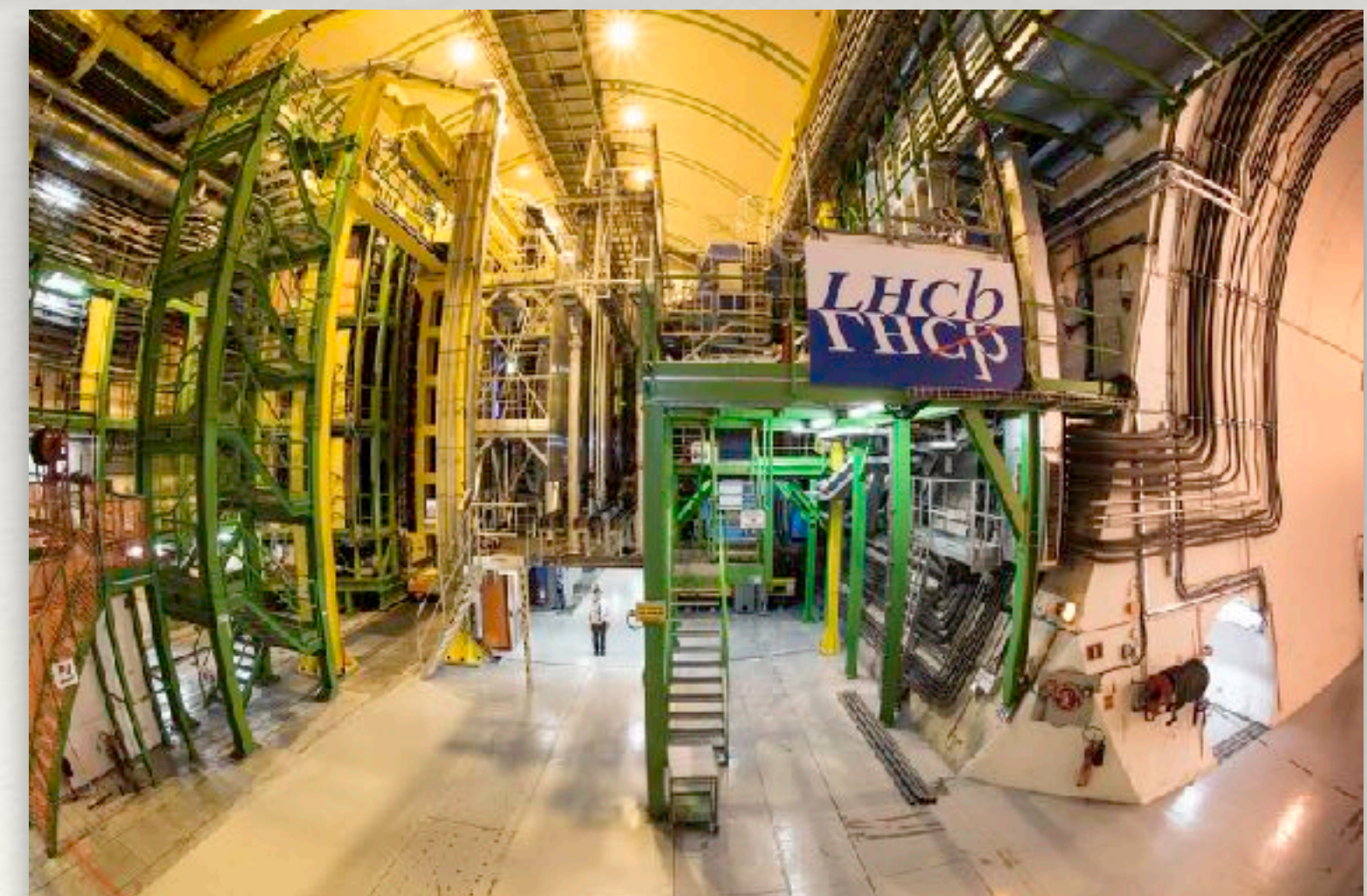
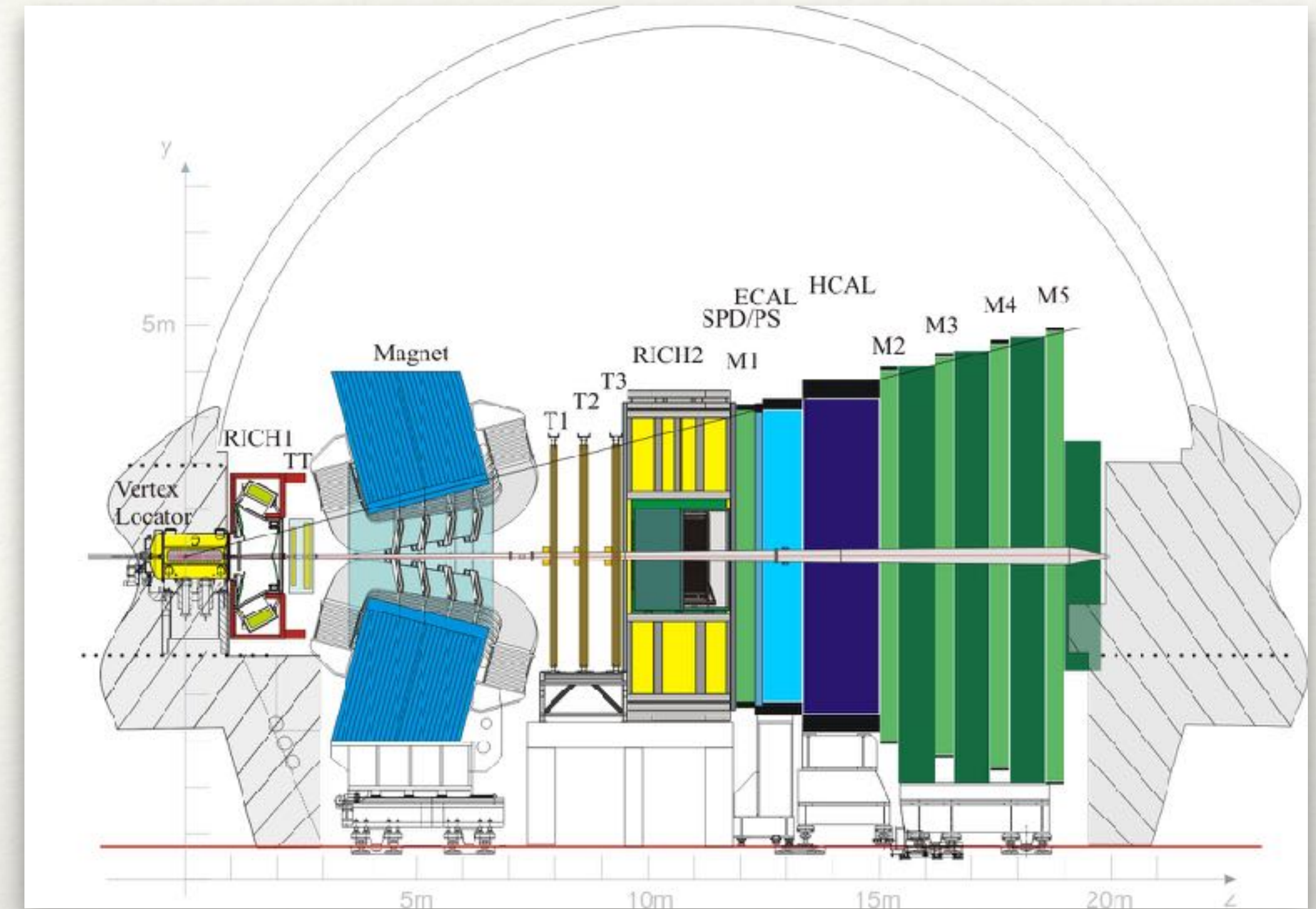
Observables in the forward region

Pseudorapidity

$$y = -\ln \left[\tan(\theta/2) \right]$$



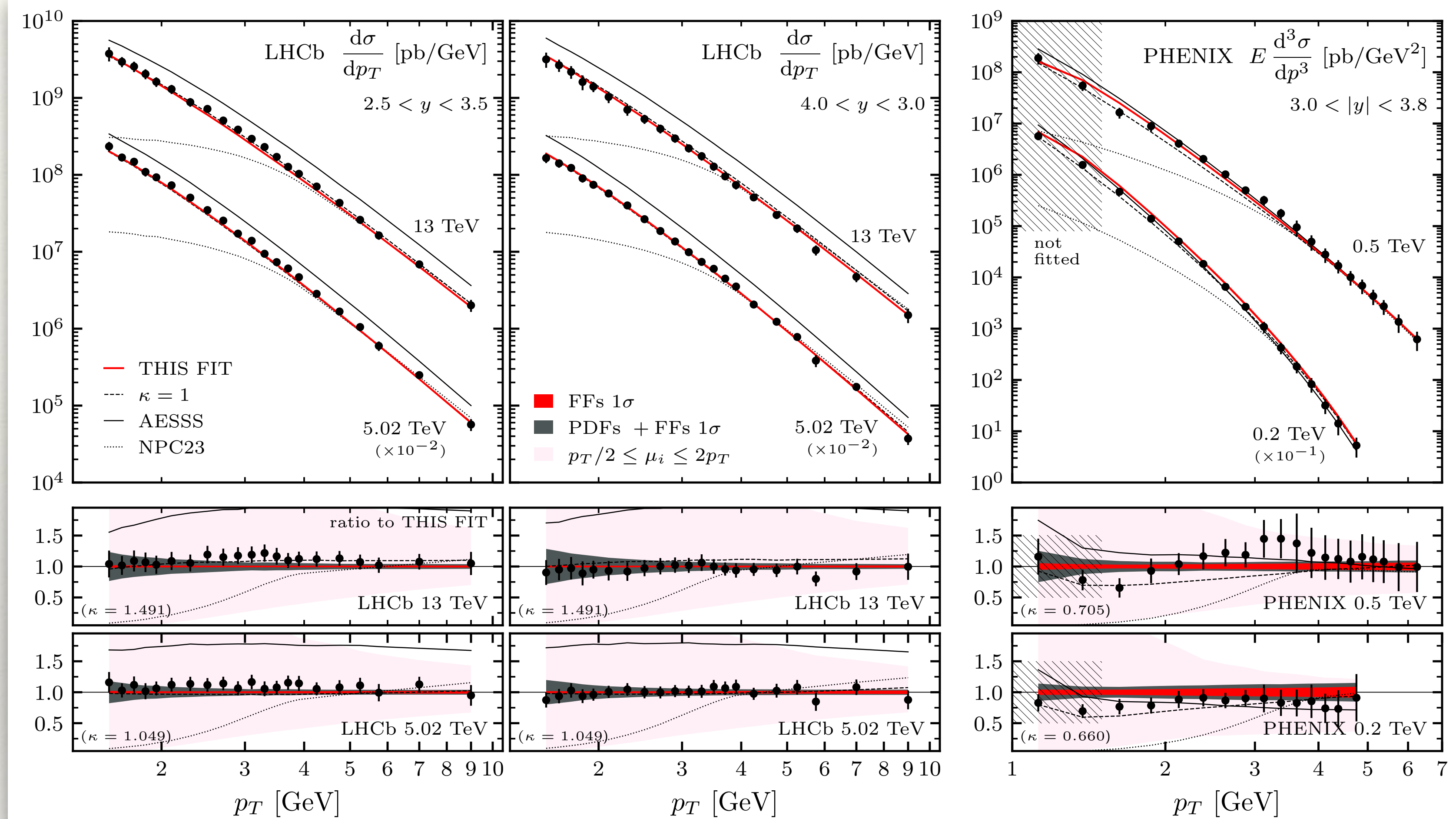
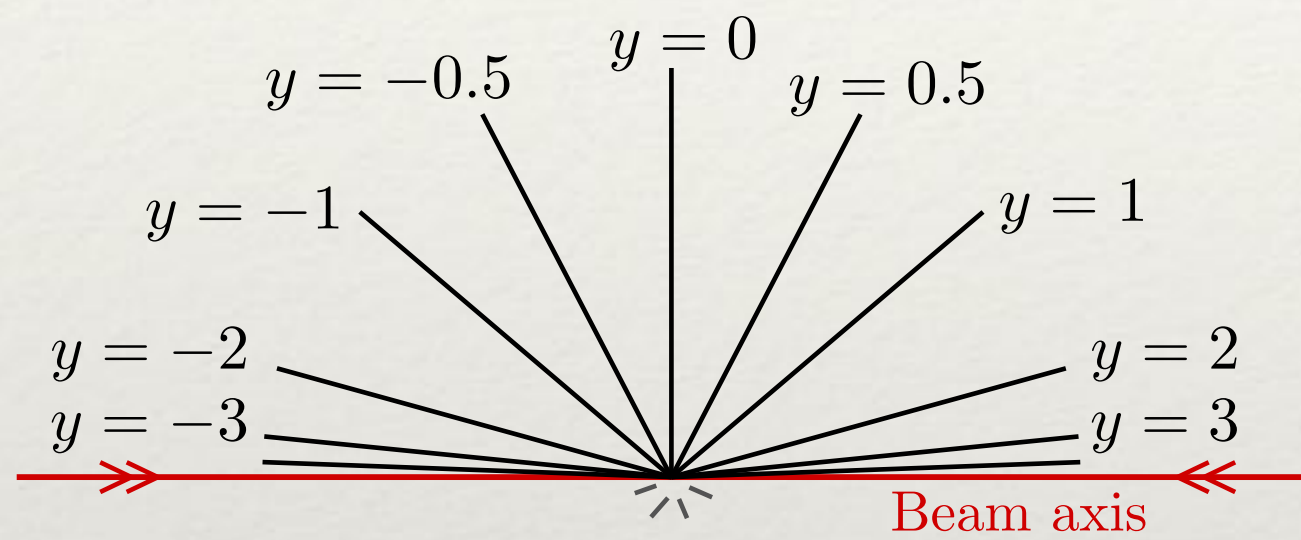
LHCb detector



Observables in the forward region

Pseudorapidity

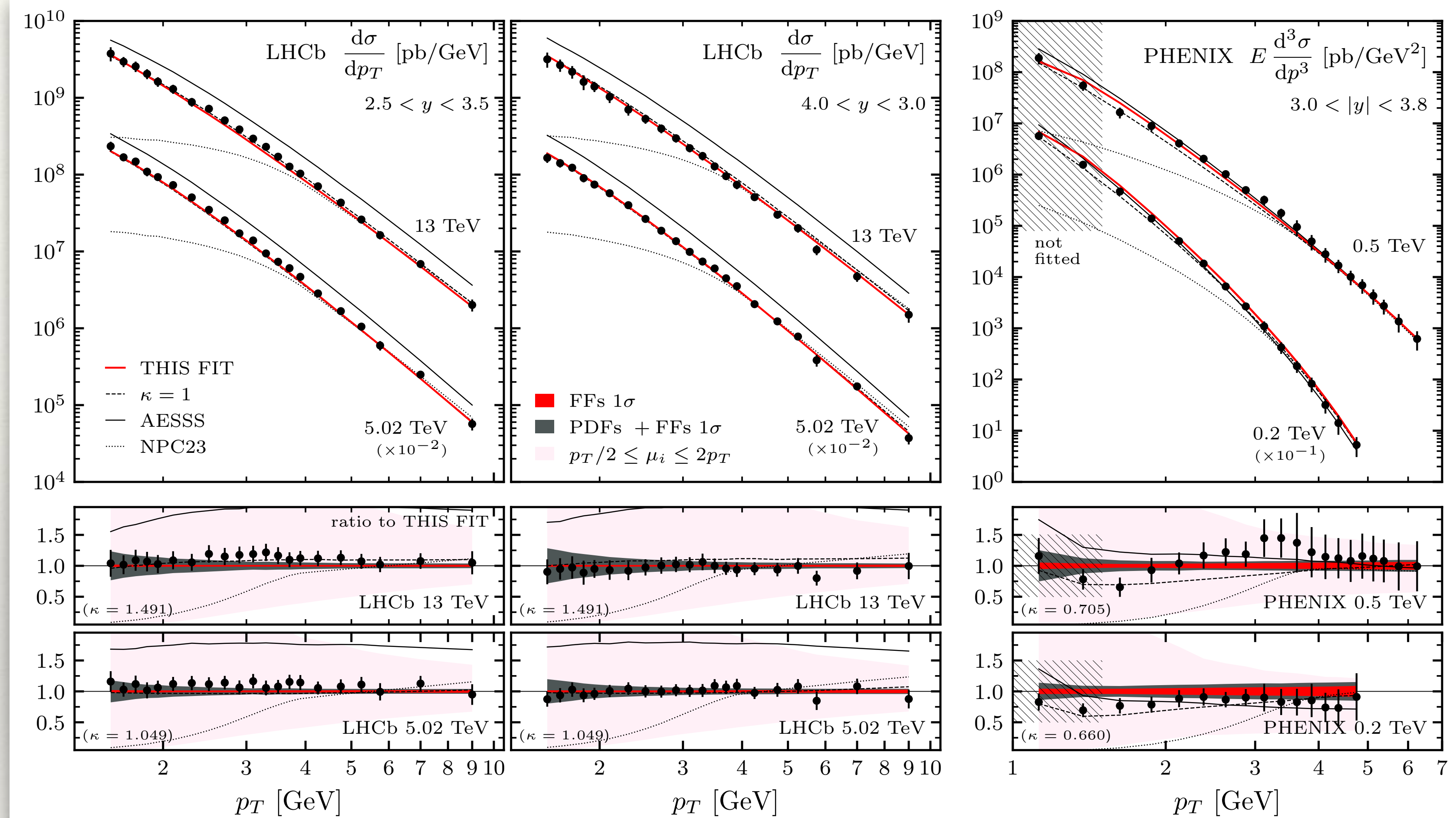
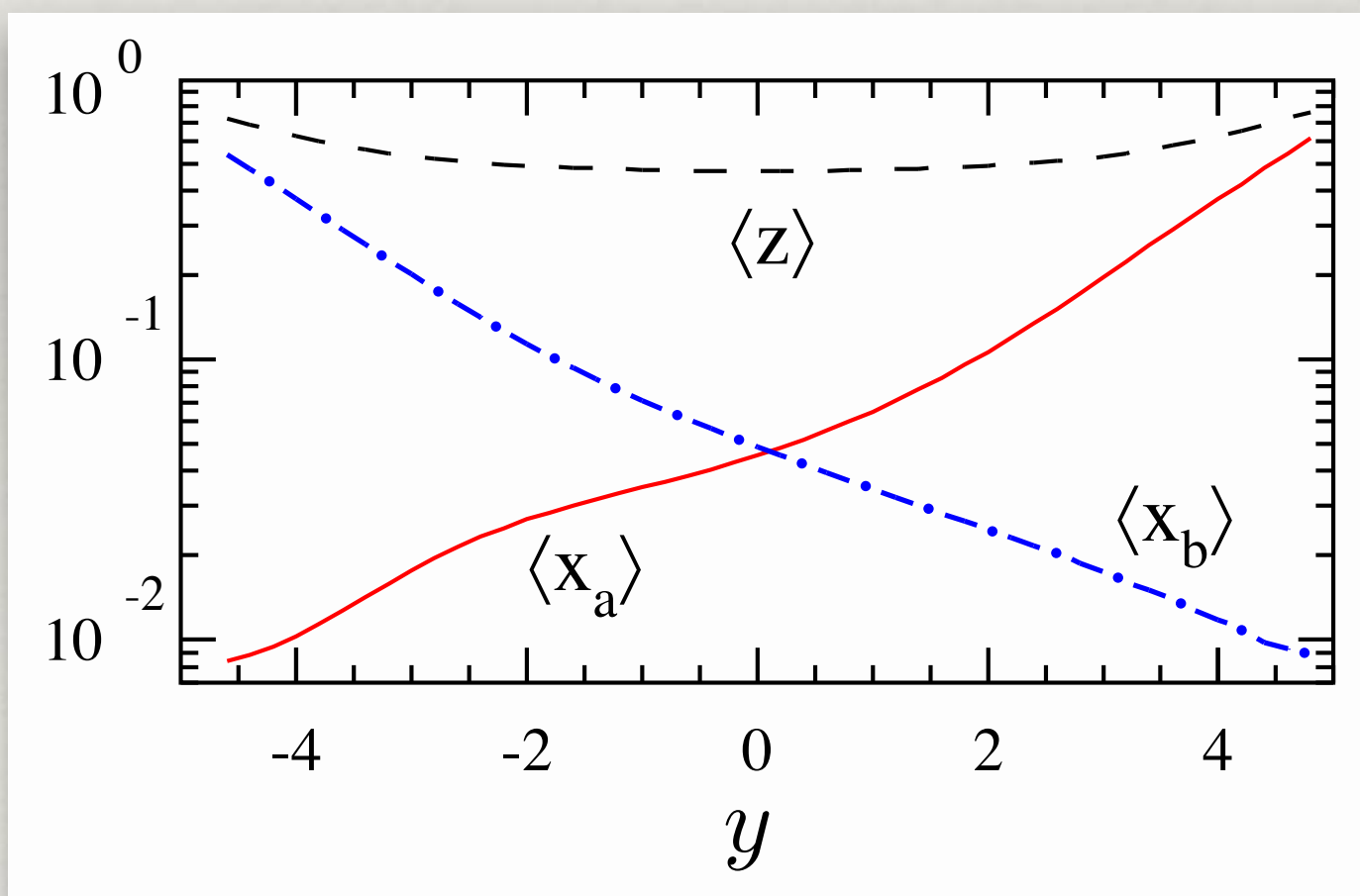
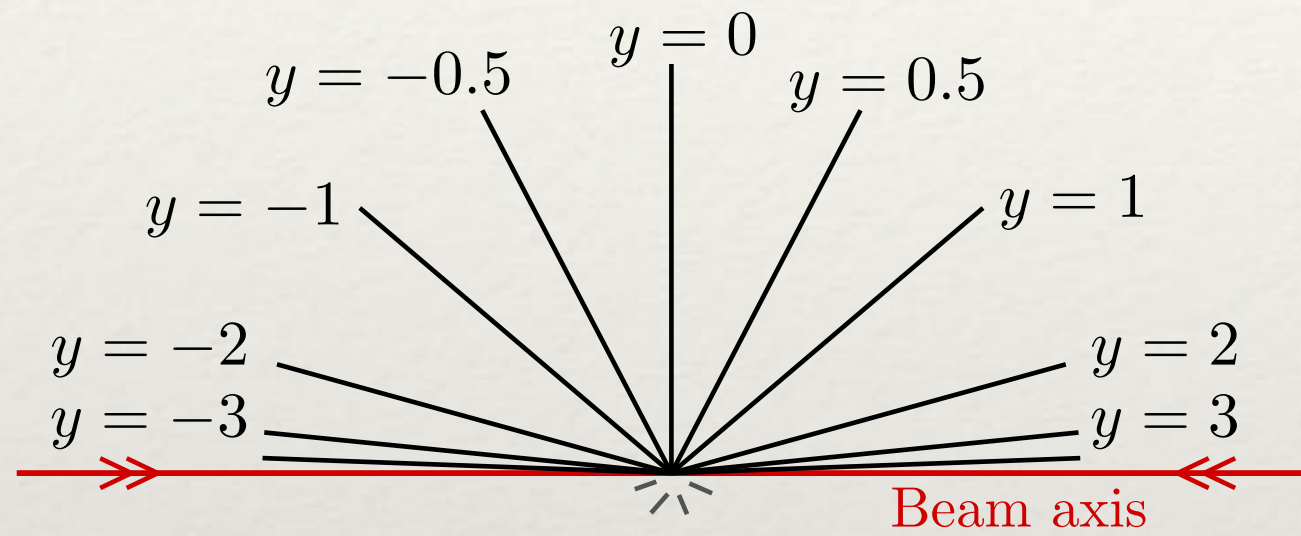
$$y = -\ln [\tan(\theta/2)]$$



Observables in the forward region

Pseudorapidity

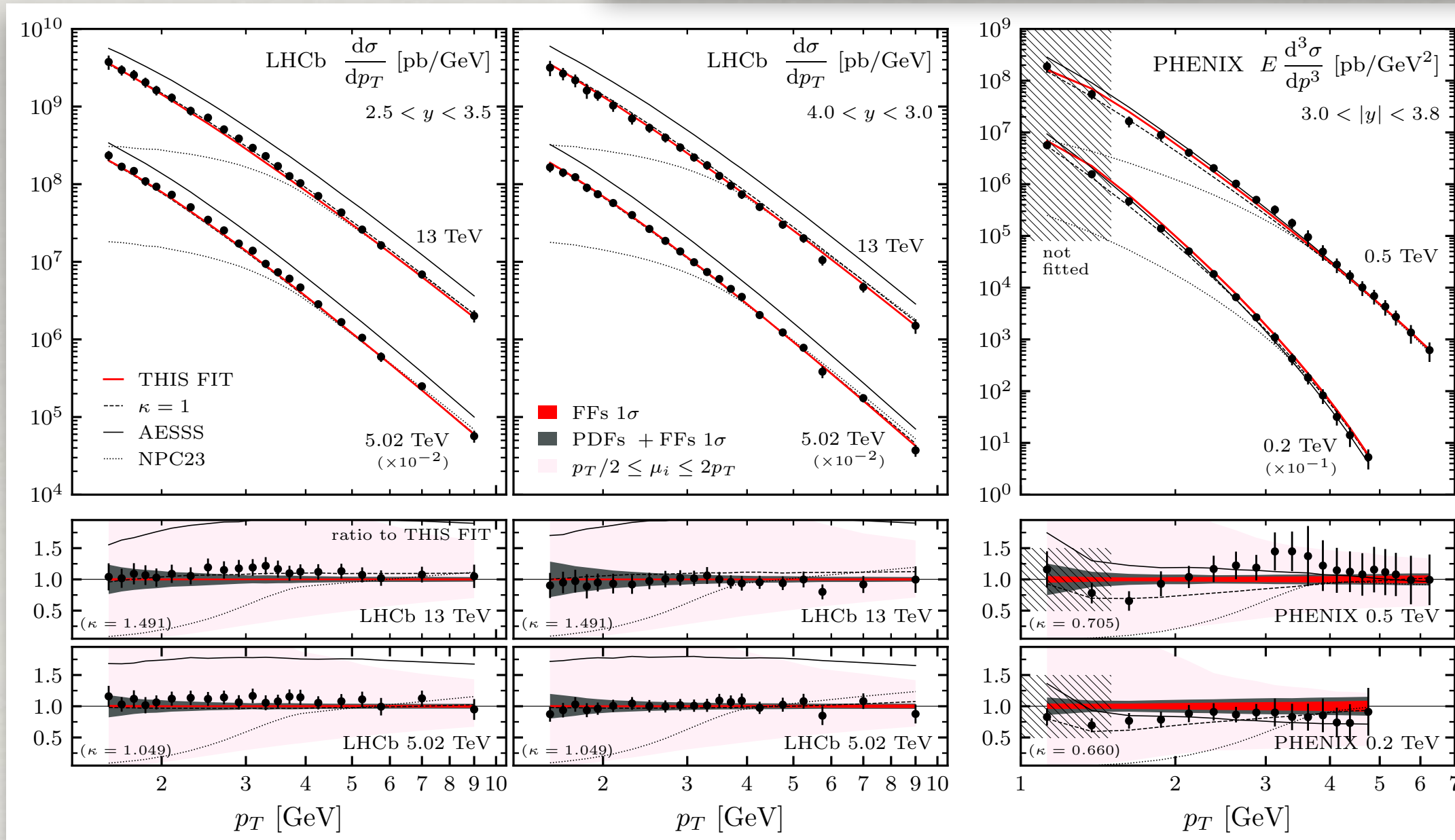
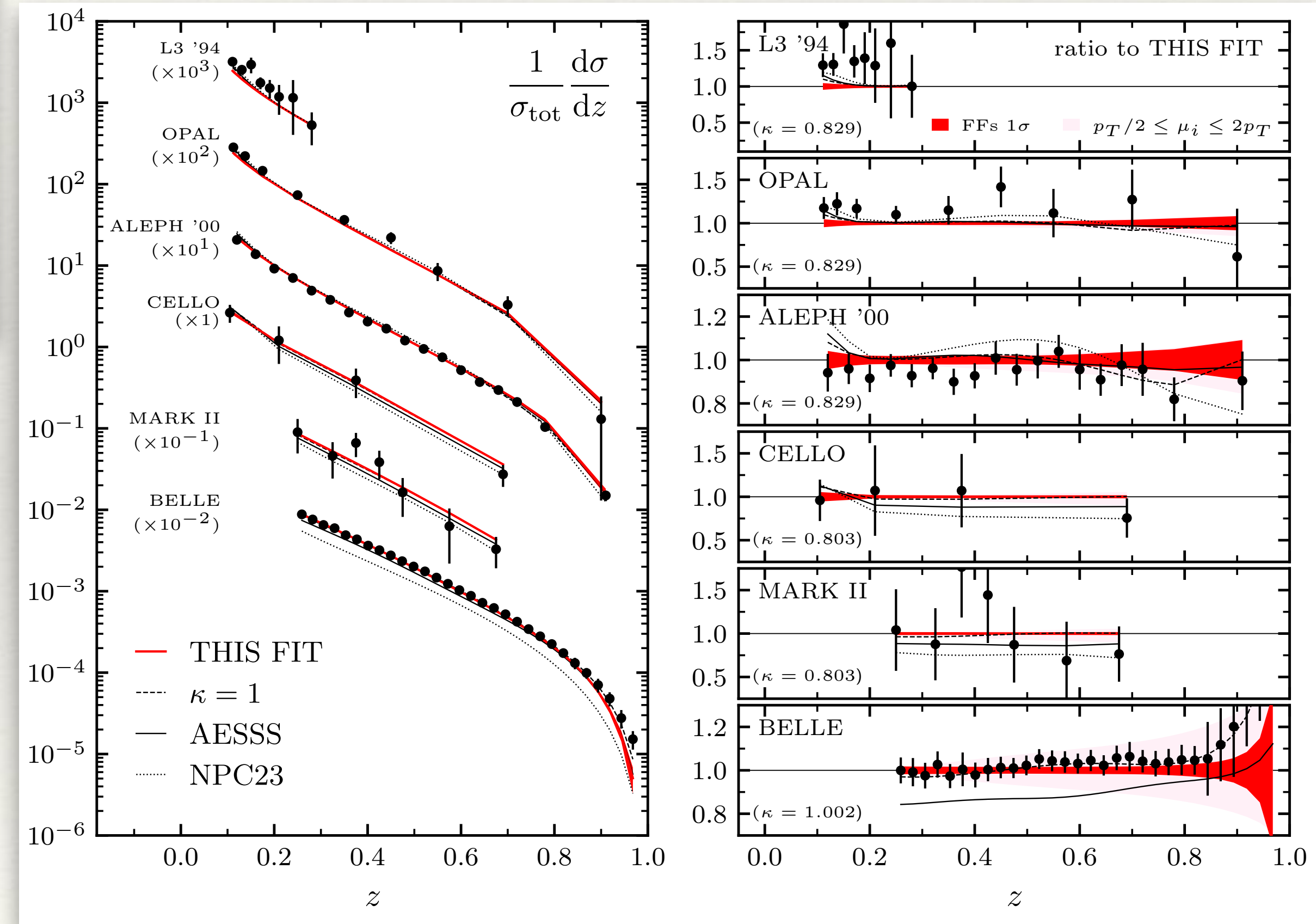
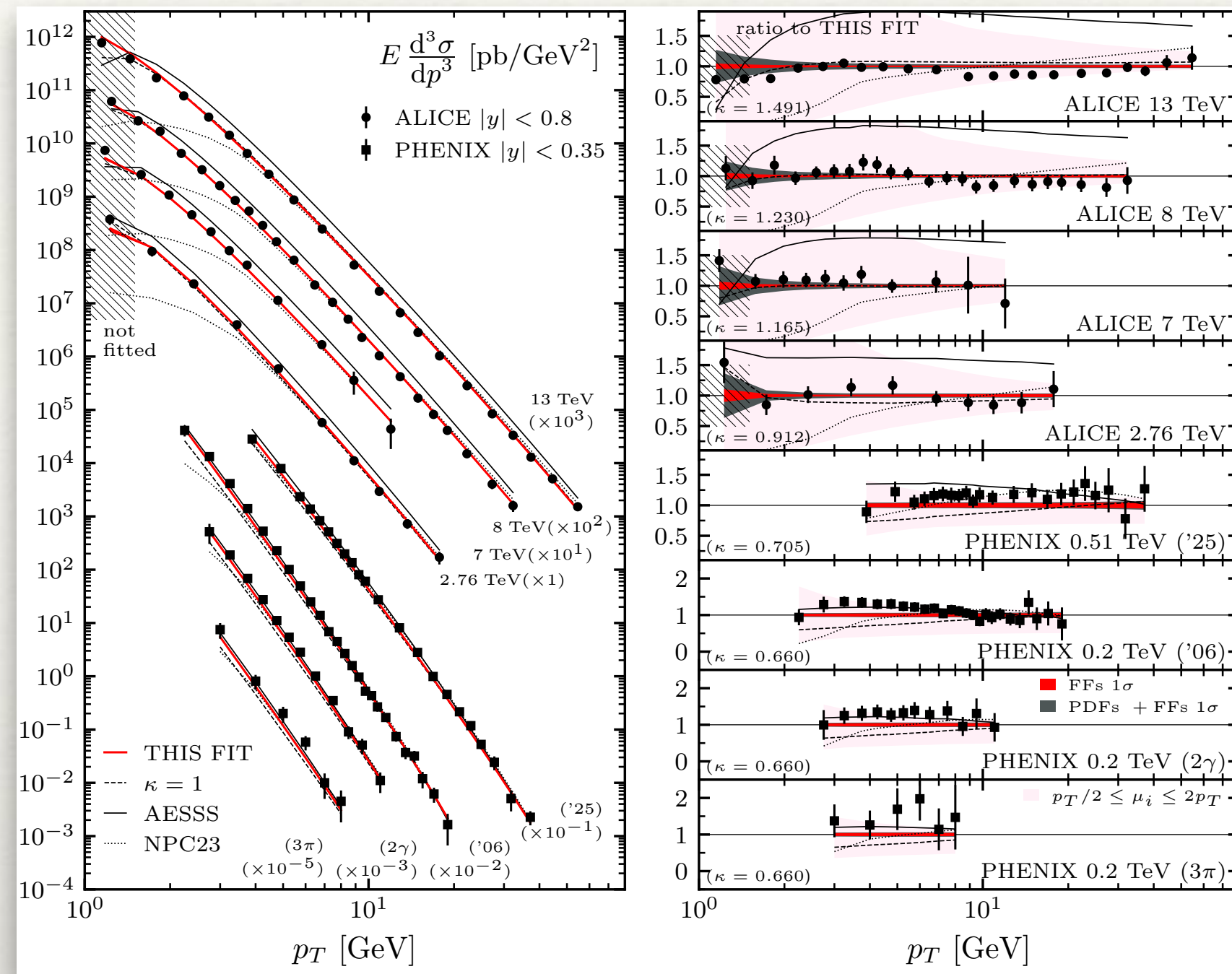
$$y = -\ln [\tan(\theta/2)]$$



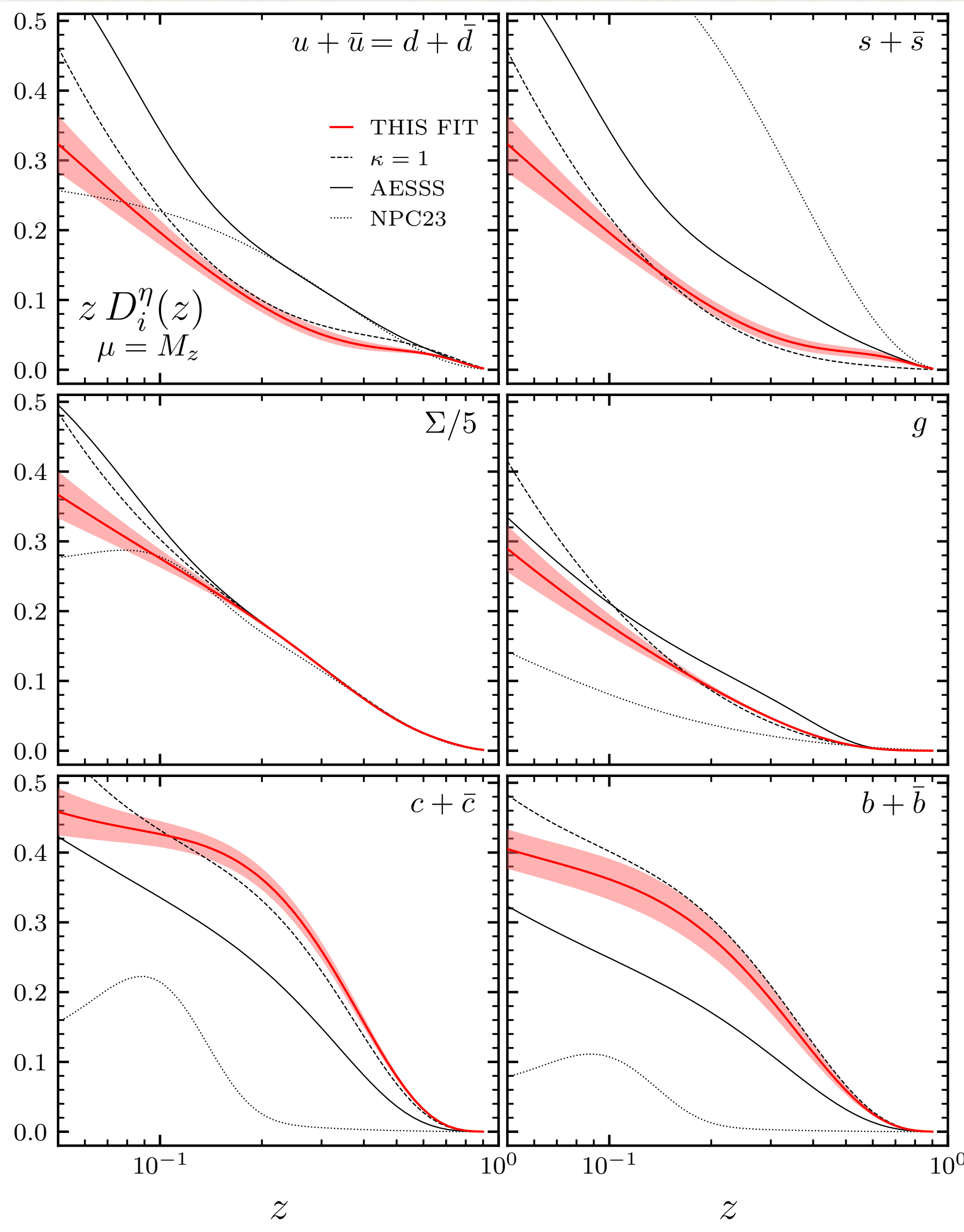
R. Sassot, P. Zurita, M. Stratmann
Phys.Rev.D82(2010) 074011

$$\langle x_i \rangle \equiv \frac{\int dx_i x_i \frac{d\sigma}{dx_i dp_T}}{\int dx_i \frac{d\sigma}{dx_i dp_T}}, \quad \langle z \rangle \equiv \frac{\int dz z \frac{d\sigma}{dz dp_T}}{\int dz \frac{d\sigma}{dz dp_T}}$$

All data included in the fit



Extracted η fragmentation functions



Our FF set is publicly available in LHAPDF format.

https://github.com/rt-martinez/ALMSS25_ETA_nlo

The screenshot shows the GitHub repository page for `rt-martinez / ALMSS25_ETA_nlo`. The repository is a public repository with a main branch. The file list shows the following files:

Name	Last commit message	Last commit date
..		
ALMSS25_ETA_nlo.info	Eta meson fragmentation functi...	3 weeks ago
ALMSS25_ETA_nlo_0000.dat	added FFs for eta meson	3 months ago
ALMSS25_ETA_nlo_0001.dat	added FFs for eta meson	3 months ago
ALMSS25_ETA_nlo_0002.dat	added FFs for eta meson	3 months ago
ALMSS25_ETA_nlo_0003.dat	added FFs for eta meson	3 months ago
ALMSS25_ETA_nlo_0004.dat	added FFs for eta meson	3 months ago
ALMSS25_ETA_nlo_0005.dat	added FFs for eta meson	3 months ago
ALMSS25_ETA_nlo_0006.dat	added FFs for eta meson	3 months ago
ALMSS25_ETA_nlo_0007.dat	added FFs for eta meson	3 months ago

Conclusions

- ❖ The previous extraction tended to overestimate the η meson production cross sections measured by ALICE at high center-of-mass energies.
- ❖ By performing a new global fit including all currently available data, this tension can be significantly reduced.
- ❖ The main change in the new extraction is a softer gluon fragmentation function, which leads to a better description of the high-energy measurements.
- ❖ The resulting FF set provides a consistent description of the available data over a broad range of center-of-mass energies.

Thank you :)

Backup

Fit setup and data sets

Parametrization

$$D_i^\eta(z,\mu_0) = N_i z^{\alpha_i} (1-z)^{\beta_i} \left[1 + \gamma_i (1-z)^{\delta_i}\right] \qquad i = u, \bar{u}, d, \cdots, g$$

$\hookrightarrow$

$\mu_0 = 1 \text{ GeV}$ initial scale

TABLE I: Data sets, normalizations $\mathcal{N}_i$, and the partial and total χ^2 values obtained in our best fit based on energy dependent scale factors $\kappa(\sqrt{S})$.

experiment	$\sqrt{s}$	data type	$\mathcal{N}_i$	data fitted	χ^2
PHENIX mid y	200 GeV	2γ [25]	1.145	12	7.13
		3π [25]	1.069	6	4.31
		2006 [26]	1.050	25	30.92
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PHENIX fwd y	510 GeV				
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	7 TeV [32]		1.027	10	3.84
	8 TeV [31]		0.985	22	19.55
	13 TeV [32]		0.949	19	54.46
LHCb [33]	5.02 TeV	very fwd	1.021	21	8.76
		fwd	1.021	21	11.55
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PP data (sum)				240	192.70
BELLE [39]	10.58 GeV		1.015	30	9.04
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JADE 90 [16]	34.9 GeV		-	3	0.43
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ALEPH02 [20]	91.2 GeV		-	5	67.03
L3 92 [21]	91.2 GeV		-	3	6.32
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OPAL [23]	91.2 GeV		-	9	12.40
JADE85 [15]	34.4 GeV		-	1	12.30
SIA data (sum)				109	170.00
TOTAL:				349	362.70

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Light quarks

$|\eta\rangle = |u\bar{u}\rangle + |d\bar{d}\rangle - 2|s\bar{s}\rangle$

[

$D_u^\eta = D_{\bar{u}}^\eta = D_d^\eta = D_{\bar{d}}^\eta$

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└─ $\mu_0 = 1 \text{ GeV}$ initial scale

Light quarks $|\eta\rangle = |u\bar{u}\rangle + |d\bar{d}\rangle - 2|s\bar{s}\rangle$

$$\begin{cases} D_u^\eta = D_{\bar{u}}^\eta = D_d^\eta = D_{\bar{d}}^\eta \\ D_s^\eta = D_{\bar{s}}^\eta = N_s D_u^\eta \end{cases}$$

Heavy quarks

$$\begin{cases} D_c^\eta(z, m_c) = D_{\bar{c}}^\eta(z, m_c) = N_c z^{\alpha_c} D_c^{res}(z, m_c) \\ D_b^\eta(z, m_b) = D_{\bar{b}}^\eta(z, m_b) = N_b z^{\alpha_b} D_b^{res}(z, m_b) \end{cases}$$

└─ $D_i^{h^+} = D_i^{\pi^+} + D_i^{K^+} + D_i^p + D_i^{res^+}$

15 parameters
to be determined

PDFs: MSHT20NLO - Eur.Phys.J.C 81 (2021) 341 (2012.04684)

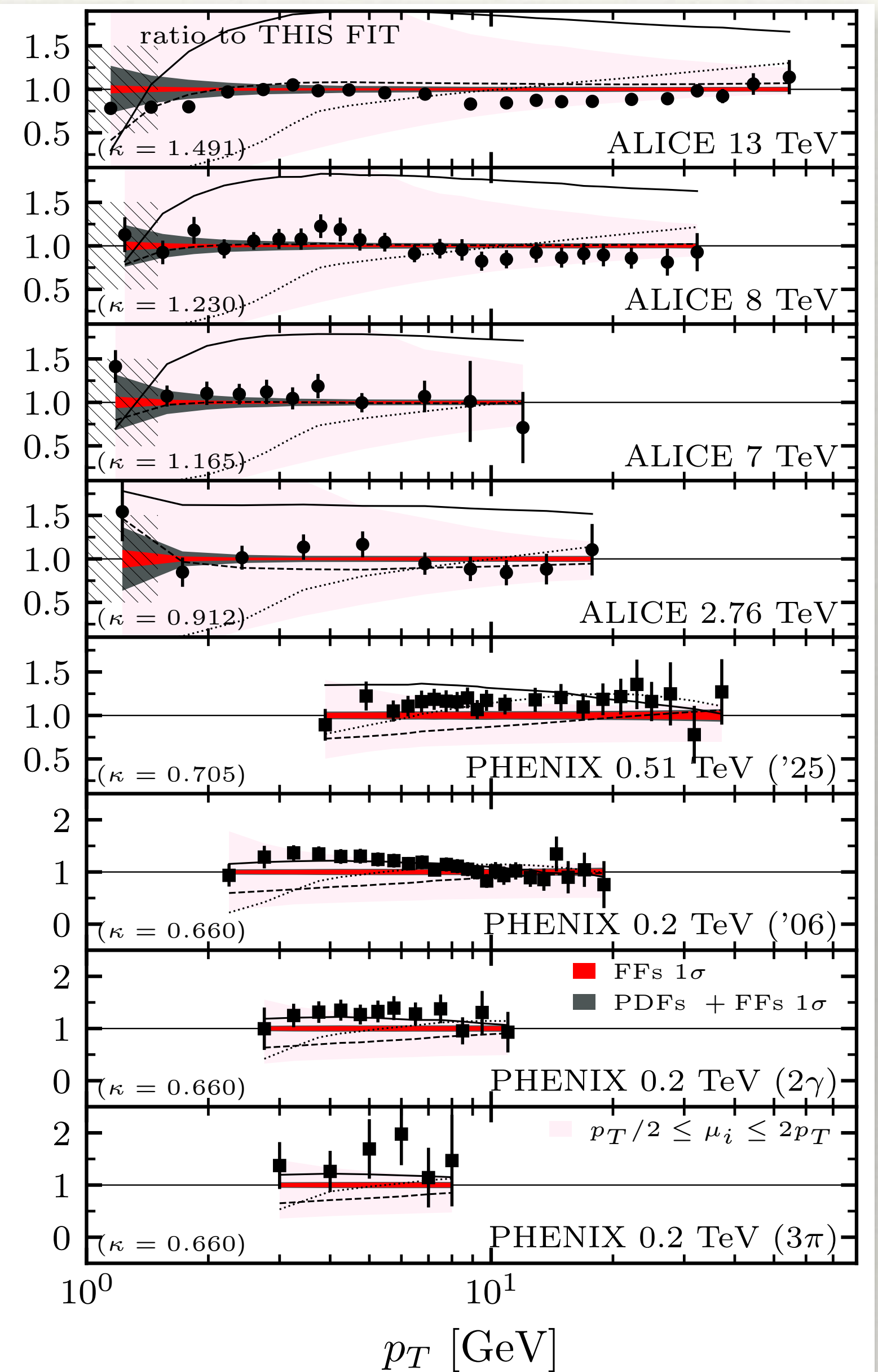
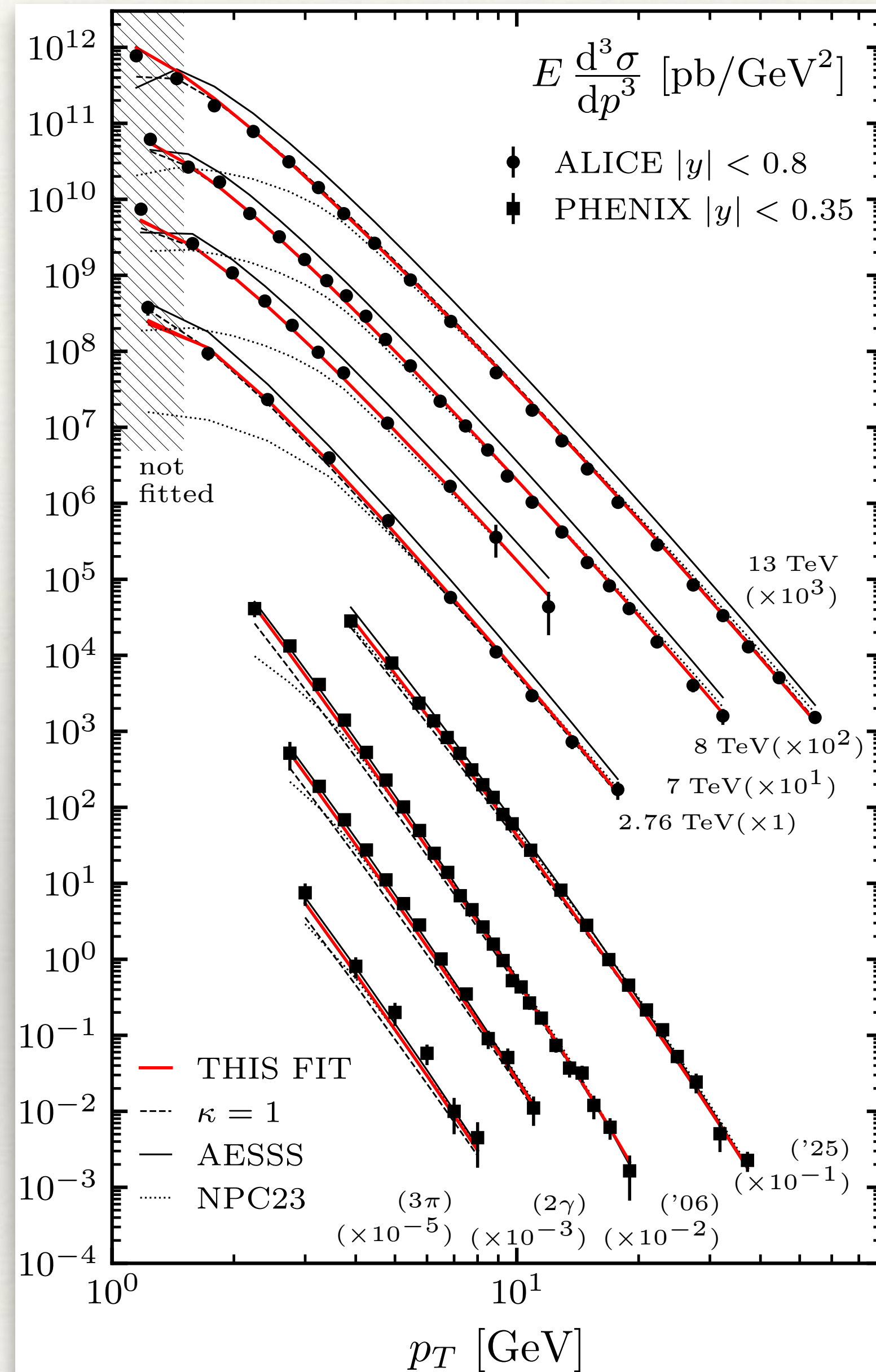
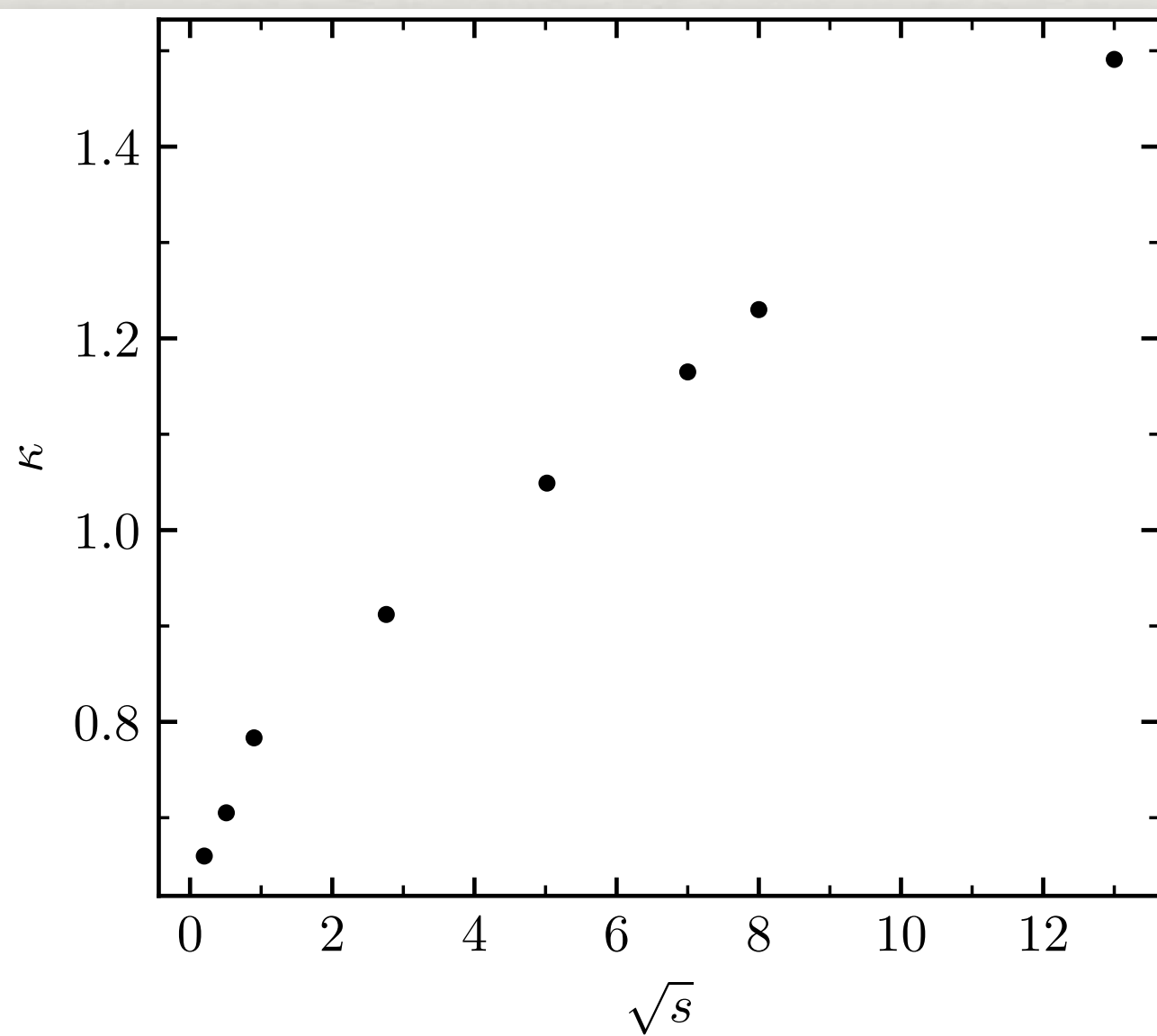
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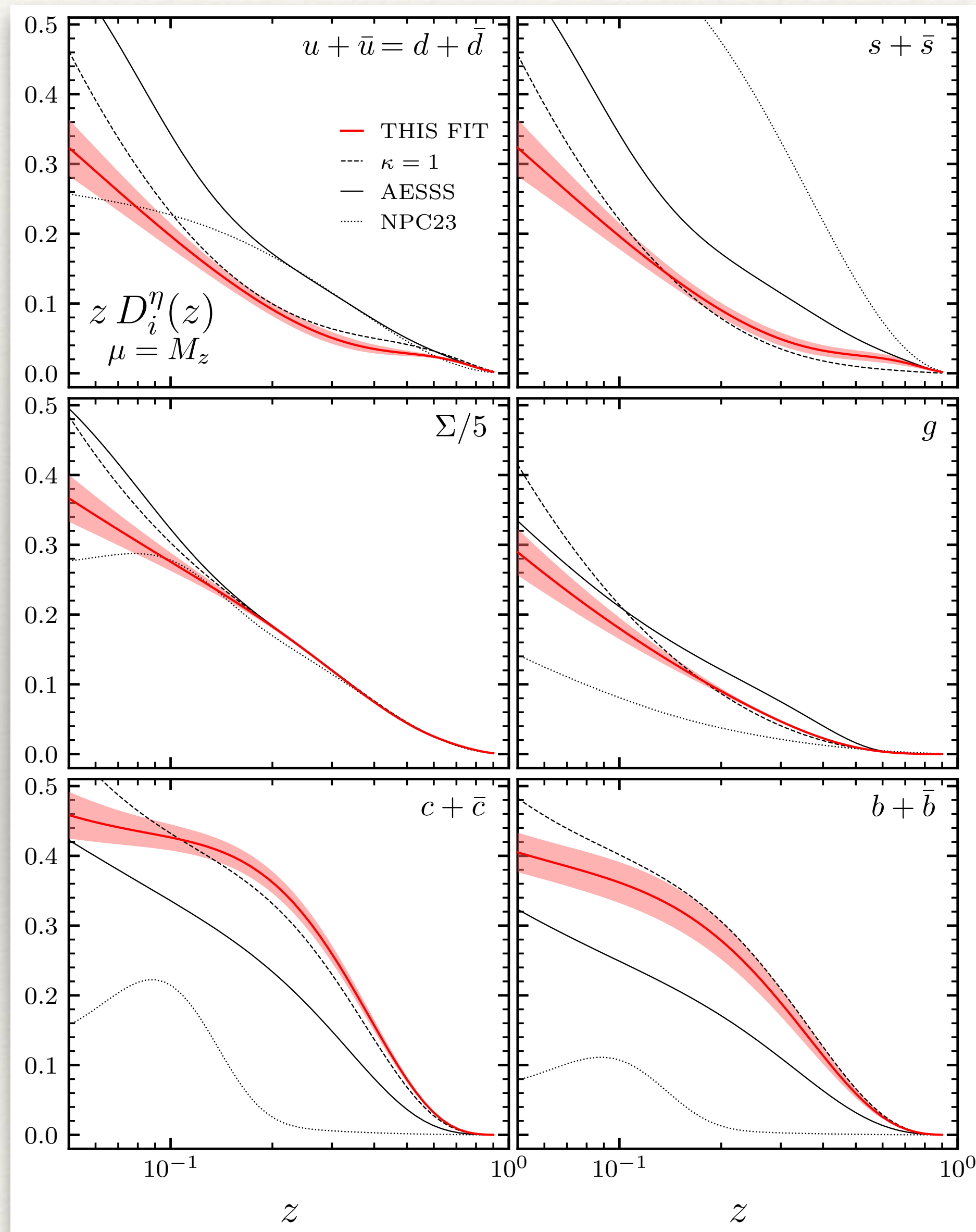
What happens if we perform a new fit using the new data?

Modified scale prescription
 $\mu = \kappa(\sqrt{s}) p_T$

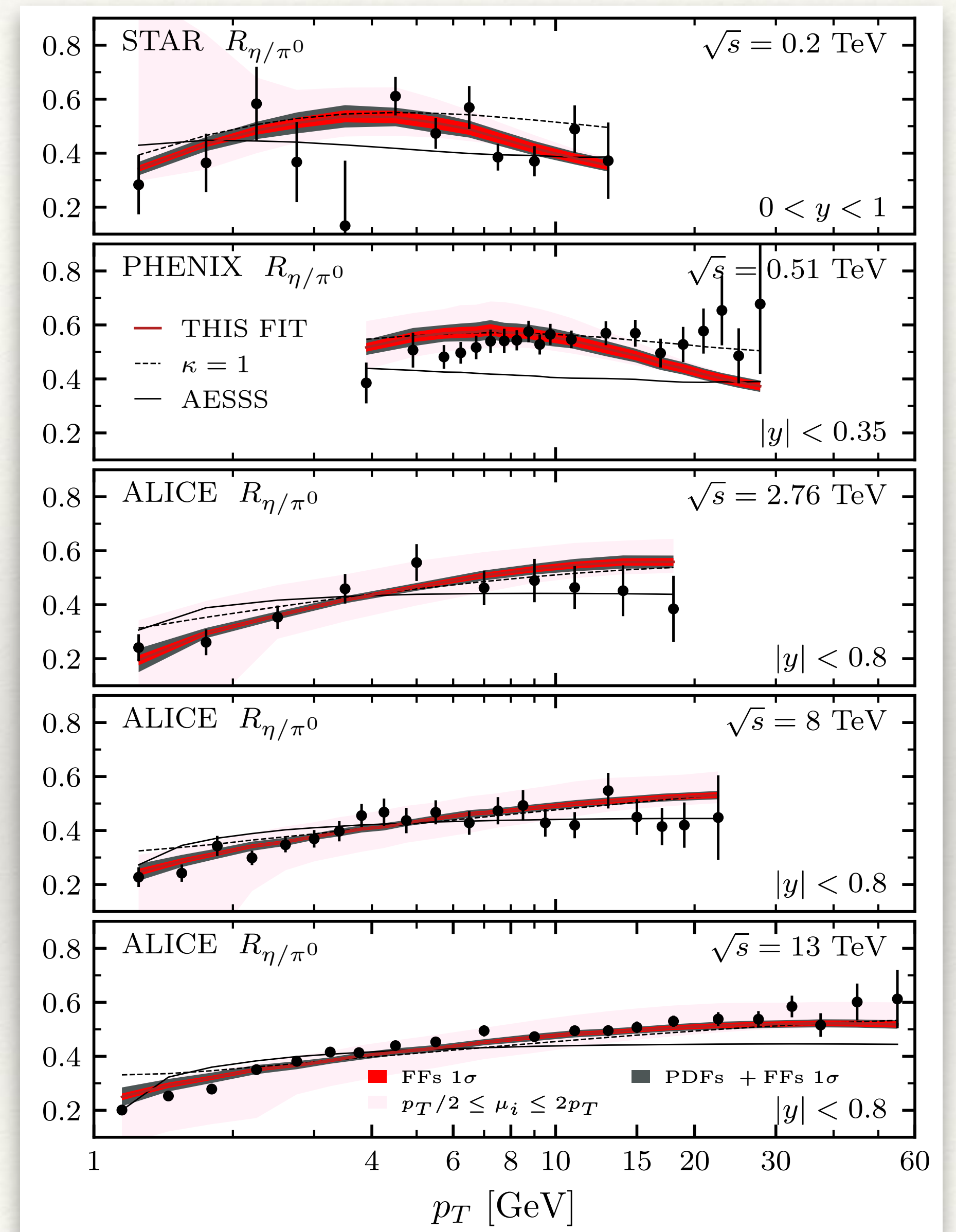
Fitted κ
 Factors



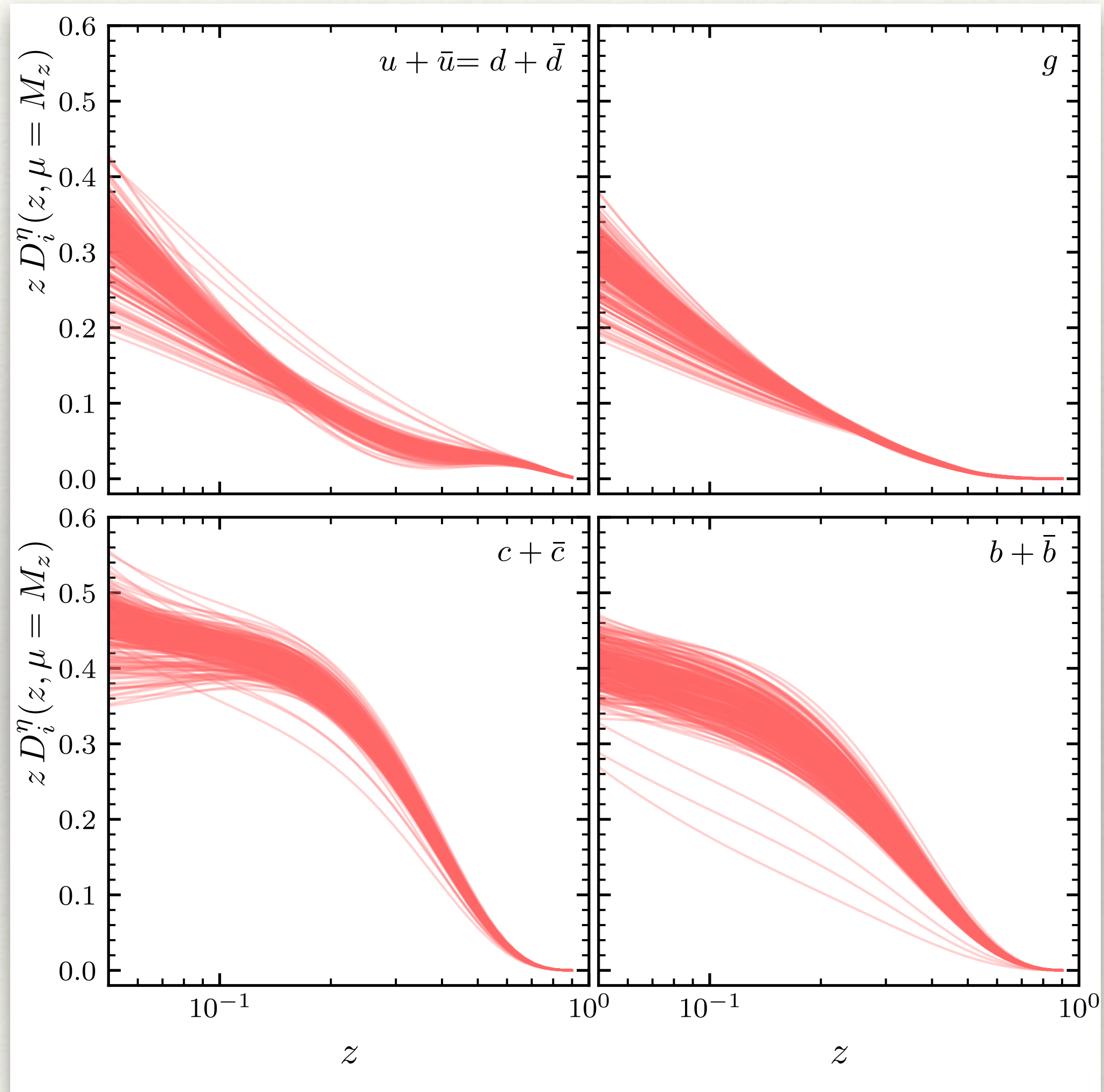
Extracted η fragmentation functions



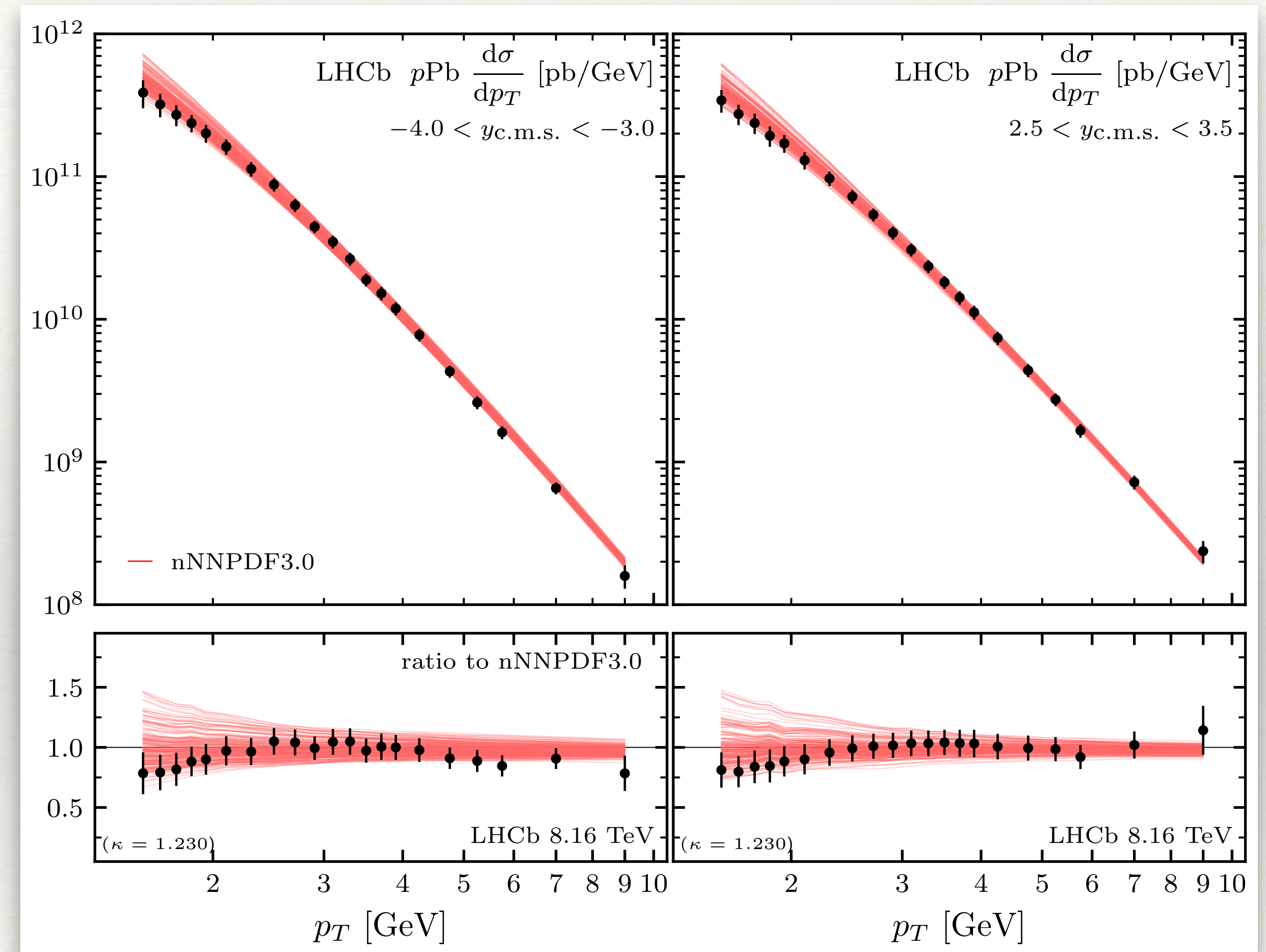
η/π^0 ratio prediction



300 FFs replicas



Example of a cross-section calculation with uncertainties



Key ingredients in the calculation of a cross section

- 1) The theory Lagrangian $\mathcal{L}_{\text{QCD}} \rightarrow$ Feynman rules are derived using quantum field theory (QFT)
- 2) Calculate the relevant diagrams for the process under consideration. e.g. $\sigma(e^+e^- \rightarrow X)$

$$\sigma \sim \underbrace{\alpha_s^0 \left| \begin{array}{c} e^+ \\ \searrow \\ \text{---} q \\ \nearrow \\ e^- \end{array} \right|^2}_{\text{Leading order (LO)}} + \underbrace{\alpha_s^1 \left(\begin{array}{c} e^+ \\ \searrow \\ \text{---} q \\ \nearrow \\ e^- \end{array} + \begin{array}{c} e^+ \\ \searrow \\ \text{---} g \\ \nearrow \\ e^- \end{array} + \dots \right)}_{\text{Next to leading order (NLO)}} + \underbrace{\alpha_s^2 \left(\begin{array}{c} e^+ \\ \searrow \\ \text{---} g \\ \nearrow \\ e^- \end{array} + \begin{array}{c} e^+ \\ \searrow \\ \text{---} g \\ \nearrow \\ e^- \end{array} + \dots \right)}_{\text{Next to next to leading order (NNLO)}} + \dots \quad \text{N}^3\text{LO}$$

- 3) Divergences and the evolution of α_s

Perturbative expansion

$$\sigma(Q) \sim \sum_n \alpha_s^n(\mu_R) C_n(Q/\mu_R)$$

μ_R : Renormalization scale

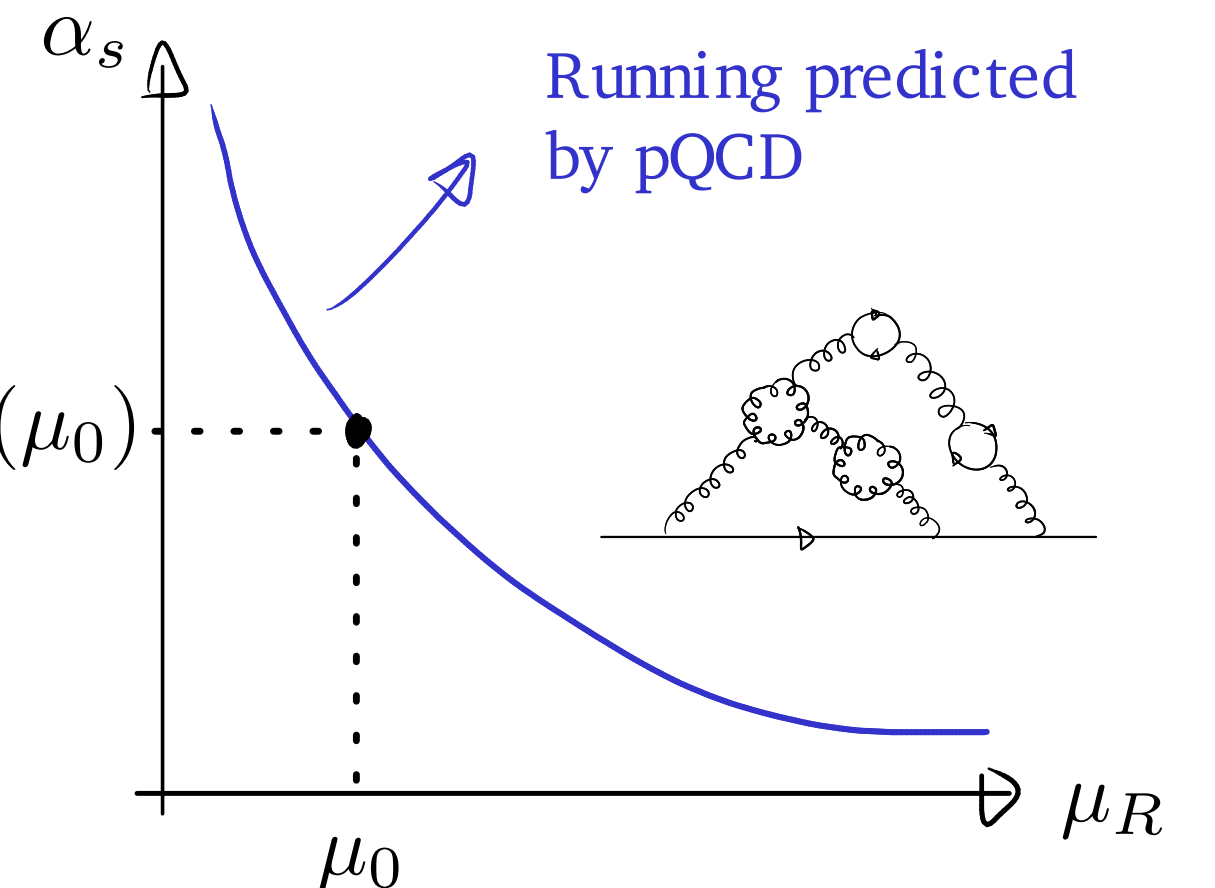
RGE - Renormalization Group Equations

$$\mu_R \frac{d\alpha_s}{d\mu_R} = \beta(\alpha_s) = \beta_0 \frac{\alpha_s^2}{2\pi} - \beta_1 \frac{\alpha_s^3}{(2\pi)^2} + \dots$$

LO solution

$$\alpha_s(\mu_R) = \frac{\alpha_s(\mu_0)}{1 + \beta_0 \frac{\alpha_s(\mu_0)}{2\pi} \ln\left(\frac{\mu^2}{\mu_0^2}\right)}$$

Value to be determined

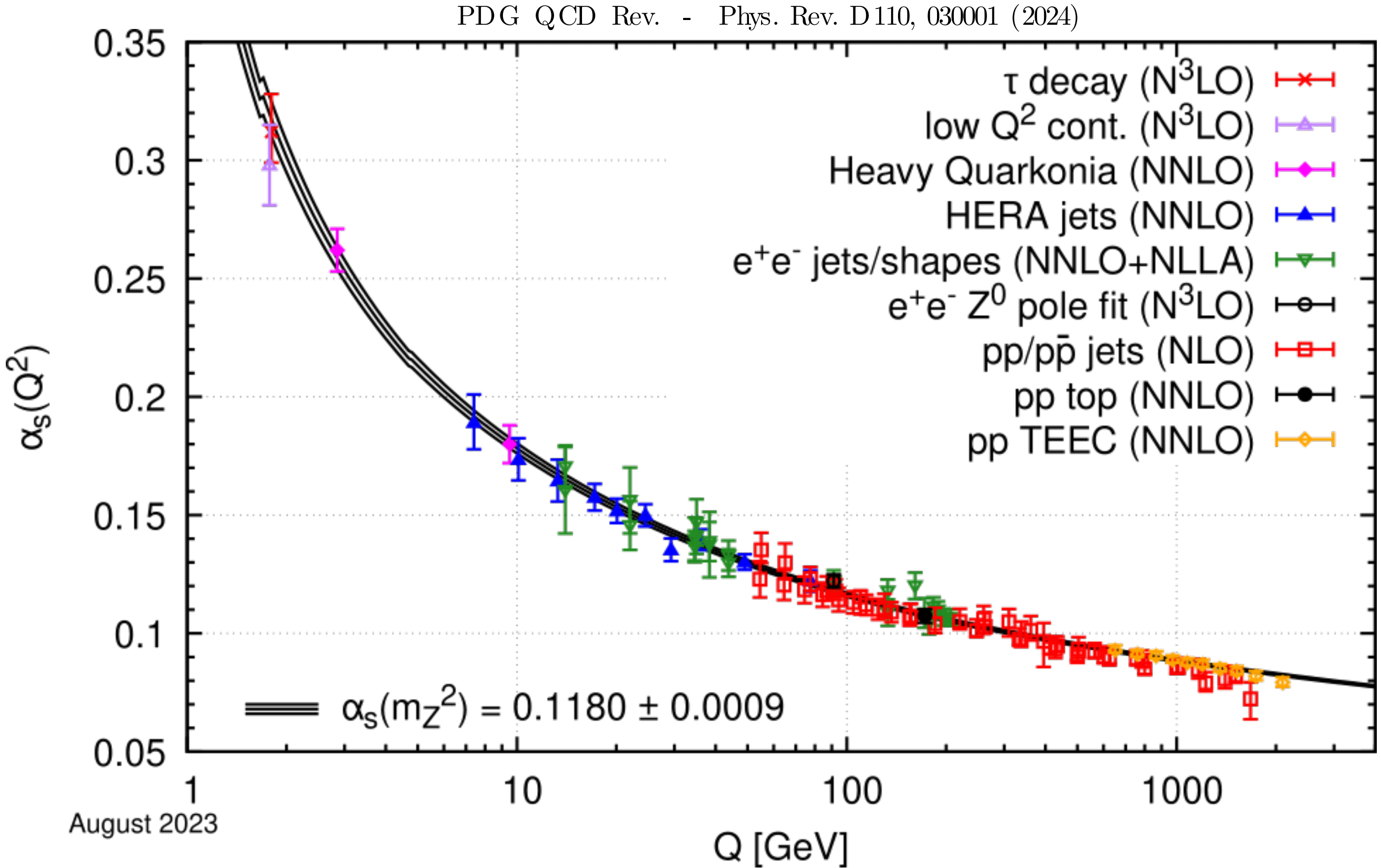
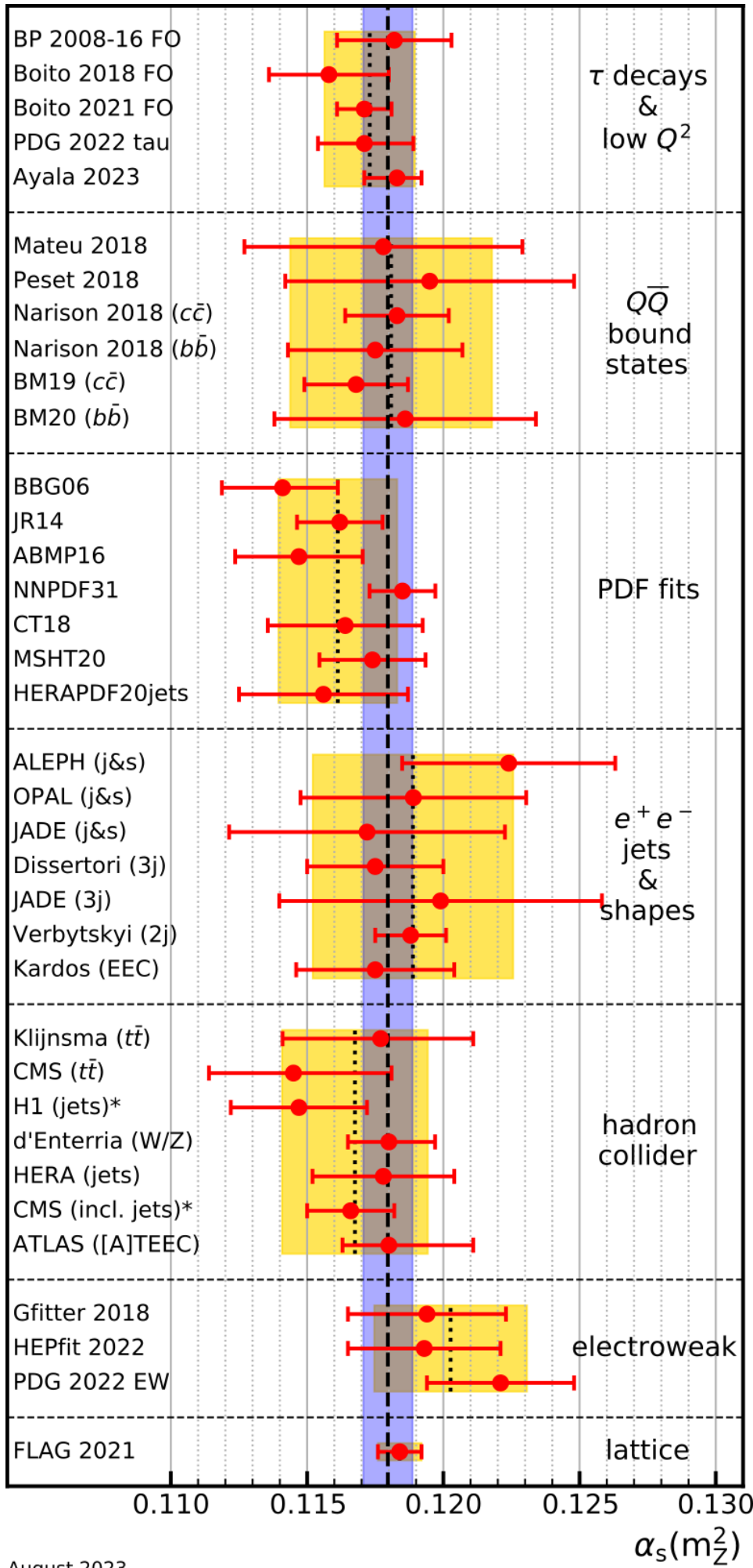


Reference scale

typically chosen as $\mu_0 = m_Z = 91.2 \text{ GeV}$

Fitting procedure

- A set of observables is considered
 - * Experimental measurements
 - * Theoretical calculations at a given perturbative order
- A renormalization scale is chosen. e.g. $\mu_R = Q = \sqrt{s}$



Perturbative expansion

$$\sigma(Q) \sim \sum_n \alpha_s^n(\mu_R) C_n(Q/\mu_R)$$

μ_R : Renormalization scale

RGE - Renormalization Group Equations

$$\mu_R \frac{d\alpha_s}{d\mu_R} = \beta(\alpha_s) = \beta_0 \frac{\alpha_s^2}{2\pi} - \beta_1 \frac{\alpha_s^3}{(2\pi)^2} + \dots$$

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The non-perturbative nature of FFs

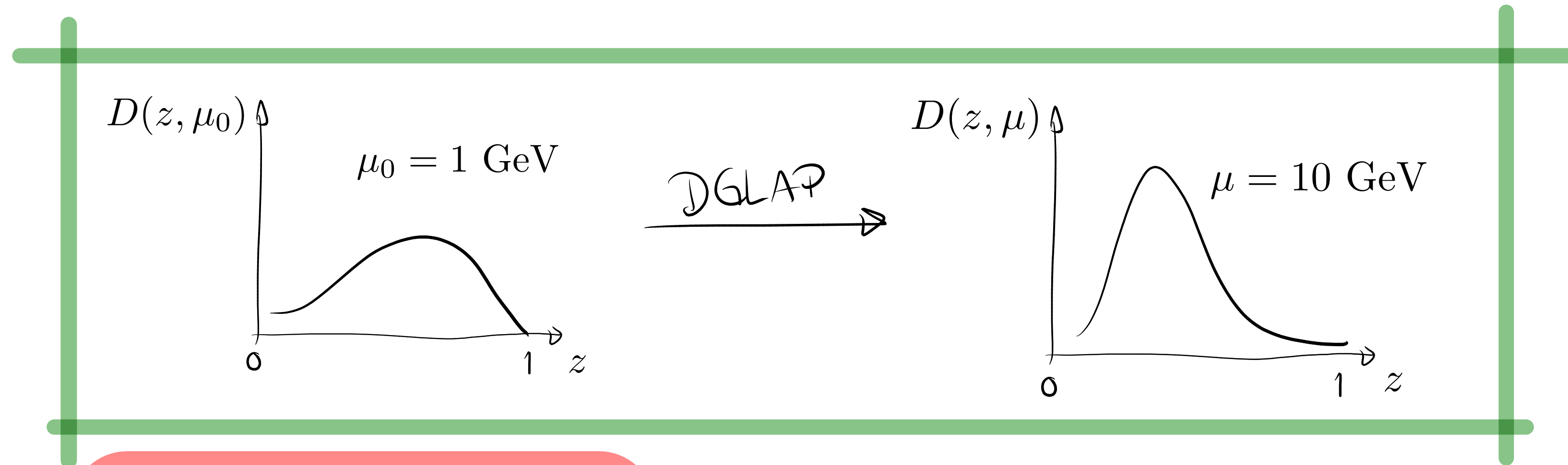
Evolution governed by
the Altarelli–Parisi equations

$$\mu \frac{\partial}{\partial \mu} D_i(z, \mu) = \sum_j \int_z^1 \frac{d\zeta}{\zeta} P_{ij} \left(\frac{x}{\zeta} \right) D_i(\zeta, \mu)$$

PDFs and FFs at a given scale can, in principle, be calculated using QCD

$$D_q^H(z) \propto \int dx^- e^{-iP_H^+ x^- / z} \text{Tr} \left[\gamma^+ \langle 0 | \Psi(0) \mathcal{P} | H(P_H^+) X \rangle \langle H(P_H^+) X | \mathcal{P}' \bar{\Psi}(x) | 0 \rangle \right]$$

It has no perturbative
expansion



We propose an ansatz
(parametrization)
at an initial scale

$$D_i(z, \mu_0) = N_i z^{\alpha_i} (1 - z)^{\beta_i}$$

N_i, α_i, β_i

Parameters to
be determined

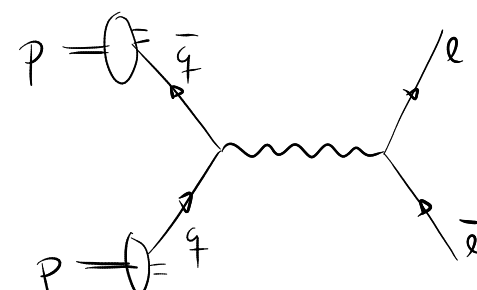
Dependence of the cross section σ on the scales μ_R, μ_F

At all orders,
the cross section
would be independent
of μ_R, μ_F

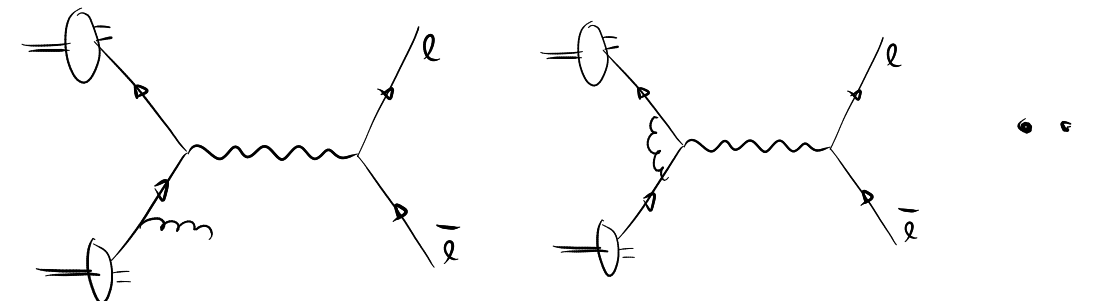


$$\sigma^{\text{NLO}}(Q, \mu_R, \mu_F) = \sum_{ab} f_a(\mu_F) \otimes f_b(\mu_F) \otimes \hat{\sigma}_{ab}^{(0)} + \frac{\alpha_s(\mu_R)}{2\pi} \sum_{ab} f_a(\mu_F) \otimes f_b(\mu_F) \otimes \hat{\sigma}_{ab}^{(1)}$$

LO



NLO



$$\hat{\sigma}_{ab}^{(1)}(Q, \mu_R, \mu_F)$$

Truncated
perturbative
series

Residual
dependence on
 μ_R, μ_F

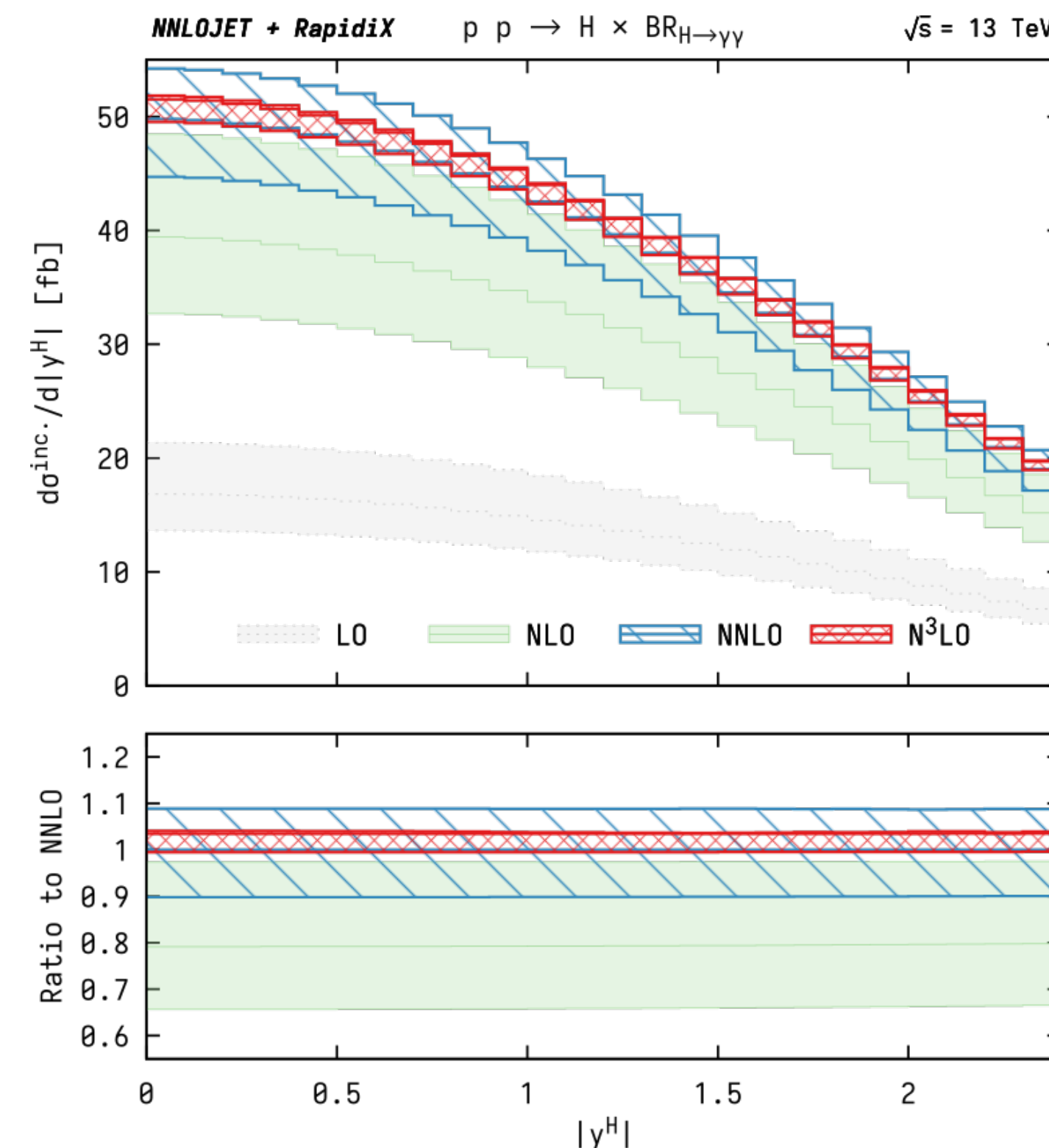
We have to choose
values for the scales

$$\text{e.g. } \mu_R = \mu_F = Q$$

We can ask what
happens if...

$$\frac{Q}{2} \leq \mu_i \leq 2Q$$

Scale uncertainty
band



Scale dependence (bands)
decreases as the
perturbative order
increases