Logarithmically Enhanced Contributions in Electron–Muon Backward Scattering

Master's Thesis Project

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Motivation

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Soft-Collinear Effective Theory

- Effective field theory for the description of low energetic (soft) and collimated (collinear) QCD radiation
- Systematic expansion in small scale ratios $\Longrightarrow$ introduce small scale parameters λ_i

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Soft-Collinear Effective Theory

- Effective field theory for the description of low energetic (soft) and collimated (collinear) QCD radiation
- Systematic expansion in small scale ratios $\Longrightarrow$ introduce small scale parameters λ_i
- Resummation of large logarithms arising from widely separated scales
- Applications
    - Jet physics and event shape-observables
    - Exclusive charmless B-meson decays
        - $B \to \pi\, l\, \nu_l$, $B \to K^*\, l$, $B \to K\, \pi$, $B \to \gamma\, l\, \nu_l, \ldots$

Motivation

Current status

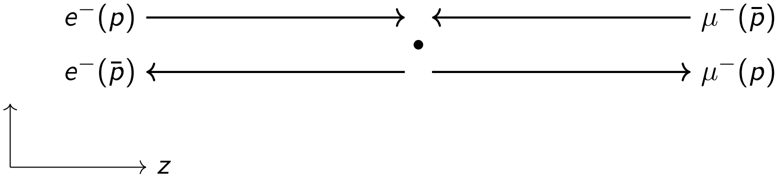
- Leading power in EFT expansion is well understood
- Next-to-leading power predictions face conceptual challenges (in particular endpoint divergences)

$\longrightarrow$ Study aspects in a simple QED process, SCET description insufficient

$\longrightarrow$ $\boxed{\text{Diagrammatic analysis}}$

Electron-Muon Backward scattering

Electron-Muon Backward Scattering



Electron–Muon Backward Scattering

Setup

- Scattering process

$$e^-(p) + \mu^-(\bar{p}) \to e^-(\bar{p}) + \mu^-(p)$$

- Exact backward-scattering kinematics
- Equal lepton masses

$$m_e = m_\mu \equiv m$$

- Strong scale hierarchy

$$m^2 \ll s$$

- Power-counting parameter

$$\lambda = \frac{m}{\sqrt{s}} \ll 1$$

Diagram Topologies

- Perturbative expansion of the scattering amplitude

$$\mathcal{M} = F_1(\lambda)\mathcal{M}^{(0)} + F_2(\lambda)\widetilde{\mathcal{M}} + \mathcal{O}(\lambda)$$

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  - Helicity-conserving Dirac structure:
  $$\mathcal{M}^{(0)} = \frac{4\pi\alpha}{-s}\left[\bar{u}_{\bar{\xi}}^{(e)}\gamma_{\perp}^{\nu}u_{\xi}^{(e)}\right]\left[\bar{u}_{\xi}^{(\mu)}\gamma_{\perp\nu}u_{\bar{\xi}}^{(\mu)}\right]$$

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  - Helicity-flipping Dirac structure:
    $$\widetilde{\mathcal{M}} = \frac{4\pi\alpha}{-s}\left(\left[\bar{u}_{\bar{\xi}}^{(e)}u_\xi^{(e)}\right]\left[\bar{u}_\xi^{(\mu)}u_{\bar{\xi}}^{(\mu)}\right] - \left[\bar{u}_{\bar{\xi}}^{(e)}\gamma_5 u_\xi^{(e)}\right]\left[\bar{u}_\xi^{(\mu)}\gamma_5 u_{\bar{\xi}}^{(\mu)}\right]\right)$$

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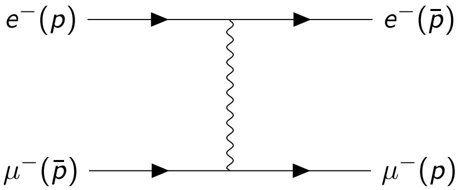
  - Form factor: $F_i(\lambda) = \sum_{i=0}^{\infty}\left(\frac{\alpha}{2\pi}\right)^n F_i^{(n)}(\lambda) = F_i^{(0)}(\lambda) + \frac{\alpha}{2\pi}F_i^{(1)}(\lambda) + \left(\frac{\alpha}{2\pi}\right)^2 F_i^{(2)}(\lambda) + \ldots$

Leading Order

Leading Order

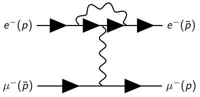


$$F_1^{(0)} = 1$$

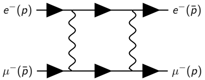
$$F_2^{(0)} = 0$$

Next-to-Leading Order

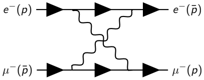
Next to Leading Order



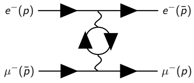
Vertex correction



Box correction



Crossed-box correction



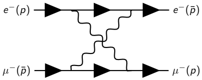
Vacuum-polarisation correction

$$F_1^{(1)} = \frac{1}{2}\ln^2\lambda^2 - \ln\lambda^2 - \frac{2n_l}{3}\ln\frac{\mu^2}{s} - \frac{10n_l}{9} - \frac{\pi^2}{3} - 4 + 2\pi i\left(\frac{1}{\epsilon_{IR}} + \ln\frac{\mu^2}{s}\right),$$

$$F_2^{(1)} = \ln\lambda^2 + 2$$

Structure of logarithmic contributions

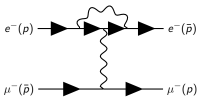
In Feynman gauge, the one-loop contributions can be summarized as follows:



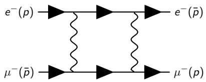
$$F_{1,CB}^{(1)} = \frac{1}{2}\ln^2\lambda^2 + 2\ln\frac{\mu^2}{m^2} + 2\ln\lambda^2 + \frac{2}{\epsilon} + \frac{2\pi^2}{3}$$
$$F_{2,CB} = \ln\lambda^2 + 2$$

- **Crossed box**
    - Leading logarithms fully captured by this topology (also at higher loop orders)

Structure of logarithmic contributions



Vertex correction



Box correction

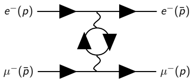
$$F_{1,V}^{(1)} = \frac{1}{\epsilon} - \frac{2}{\epsilon}\ln\lambda^2 - \ln^2\lambda^2 - 2\ln\lambda^2\ln\frac{\mu^2}{m^2} + \ln\frac{\mu^2}{s} - 4\ln\lambda^2 + \frac{\pi^2}{3},$$

$$F_{1,B}^{(1)} = \frac{2}{\epsilon}\ln\lambda^2 + \ln^2\lambda^2 + 2\ln\lambda^2\ln\frac{\mu^2}{m^2} + 2i\pi\left(\ln\frac{\mu^2}{s} + \frac{1}{\epsilon}\right) - \frac{4\pi^2}{3},$$

$$F_{2,V/B}^{(1)} = 0$$

- **Vertex and box corrections**
  - Remaining imaginary IR divergence canceled by real emission $\Rightarrow$ restrict to the real part
  - Cancellation of the corresponding double logarithms $\longrightarrow$ Only NLL contributions

Structure of logarithmic contributions

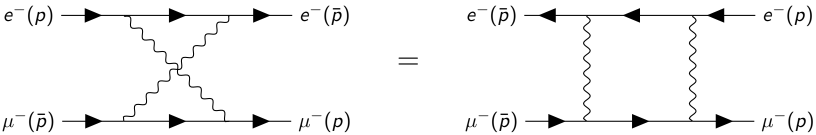


$$F^{(1)}_{1,VP} = -\frac{2}{3\epsilon} - \frac{2}{3}\ln\frac{\mu^2}{s} - \frac{10 n_l}{9}$$
$$F^{(1)}_{2,VP} = 0$$

- **Vacuum-polarisation**
  - Only single poles in $\epsilon \longrightarrow$ Only NLL contribution

Leading Logarithms

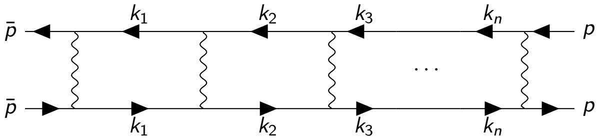
Crossed-Box Notation



- For notational purposes, we twist the electron line in the following.
- The crossed box becomes an ordinary box and vice versa.
- This notation is useful for extracting the double logarithms to all orders.

Leading Logarithms

- Only ladder-type photon exchanges contribute to the leading logarithms



Light-Cone Coordinates

- Any four-vector can be decomposed as

$$p^\mu = \frac{n_+ \cdot p}{2}\, n_-^\mu + \frac{n_- \cdot p}{2}\, n_+^\mu + p_\perp^\mu,$$

  with

$$n_-^2 = n_+^2 = 0, \qquad n_- \cdot n_+ = 2.$$

- Light-cone components

$$p_+ = n_- \cdot p, \qquad p_- = n_+ \cdot p, \qquad n_- \cdot p_\perp = n_+ \cdot p_\perp = 0.$$

- Scalar product

$$p \cdot q = \frac{1}{2}\left(p_+ q_- + p_- q_+\right) + p_\perp \cdot q_\perp .$$

- Decomposition and orthogonality relations remain valid in d dimensions.

Leading Logarithms

- Leading-logarithmic configuration: all lepton propagators become on shell, while the photon propagators become eikonal

$$(k_i - k_{i-1})^2 + i0 \stackrel{!}{=} -(n_+ k_i)(n_- k_{i-1}) + i0$$

- Strong ordering of the longitudinal momentum components

$$\frac{m^2}{\sqrt{s}} \simeq n_- p \ll n_- k_n \ll \ldots \ll n_- k_1 \ll n_- \bar{p} \simeq \sqrt{s}$$

$$\frac{m^2}{\sqrt{s}} \simeq n_- p \ll n_- k_n \ll \ldots \ll n_- k_1 \ll n_- \bar{p} \simeq \sqrt{s}.$$

$\rightarrow$ Cutoffs allow us to work in $d = 4$ dimensions

Leading Logarithms

- Introducing

$$x_i = \frac{n_+ k_i}{\sqrt{s}}, \qquad y_i = \frac{n_- k_i}{\sqrt{s}}, \qquad \lambda = \frac{m}{\sqrt{s}},$$

the leading-logarithmic contribution is

$$F_1^{(n)}(\lambda) \simeq \int_{\lambda^2}^1 \frac{dx_1}{x_1} \int_{x_1}^1 \frac{dx_2}{x_2} \cdots \int_{x_{n-1}}^1 \frac{dx_n}{x_n}$$
$$\times \int_{\lambda^2/x_1}^1 \frac{dy_1}{y_1} \int_{\lambda^2/x_2}^{y_1} \frac{dy_2}{y_2} \cdots \int_{\lambda^2/x_n}^{y_{n-1}} \frac{dy_n}{y_n}$$

- Leading logarithms fully captured by this structure

$$\boxed{F_{1,\mathrm{LL}}^{(n)} = \frac{1}{n!(n+1)!} \ln^{2n} \lambda^2}$$

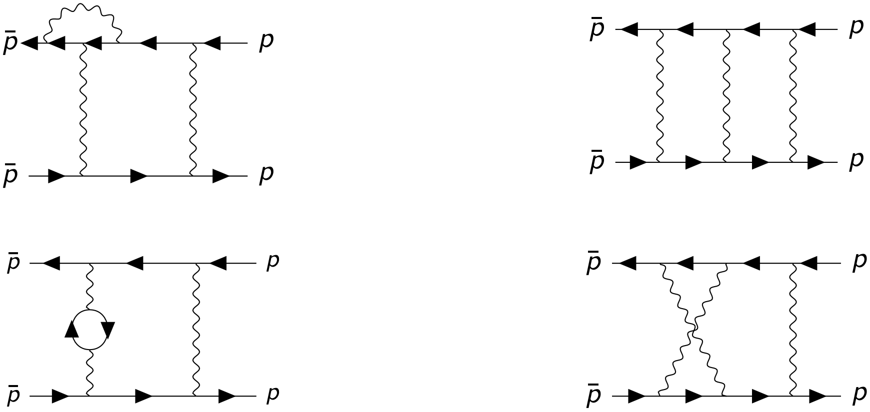
Next-to Leading Logarithms

Next-to-Leading Logarithms

- The goal of my thesis is to study the next-to leading logarithms at NNLO

$$F^{(2)}_{1,\mathrm{NLL}} \sim \ln^3 \lambda^2$$

- Understand how single logs are generated at NLO and attach a double-logarithmic box.
  $\longrightarrow$ Similiar structures as in 1 loop case
- Hybrid method: Double logarithmic contribution from cutoff method and single logarithmic contribution from method of regions (conventional method)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Next-to-Leading Logarithms

$$F^{(2)}_{1,\mathrm{VP}} \simeq -\frac{4}{9} n_l \ln^3 \lambda^2$$

$$F^{(2)}_{1,\mathrm{V+Box}} \simeq -\frac{8}{3} \ln^3 \lambda^2$$

$$F^{(2)}_{\mathrm{CB}} \simeq ?$$

- Agreement of the vacuum-polarisation contribution with the literature

Next-to-Leading Logarithms

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$$F^{(2)}_{1,\mathrm{V+Box}} \simeq -\frac{8}{3} \ln^3 \lambda^2$$

$$F^{(2)}_{\mathrm{CB}} \simeq ?$$

- Agreement of the vacuum-polarisation contribution with the literature
- Double logarithms arise in both the cutoff method and the method of regions in the double-crossed box
- Cutoff method effectively isolates the leading-logarithmic contribution
- Therefore, the NLL term of the double-crossed box cannot be extracted directly

Conclusion and Outlook

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- Electron-muon backward scattering provides a simple QED setup to study logarithmically enhanced contributions.
- Hybrid method does not work for the double-crossed box topology.

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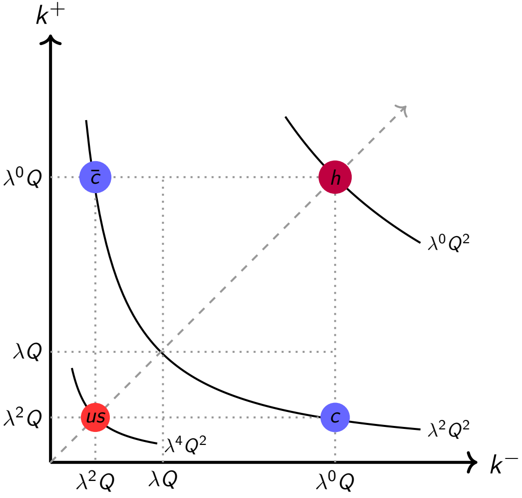
- Hybrid method does not work for the double-crossed box topology.

Outlook:

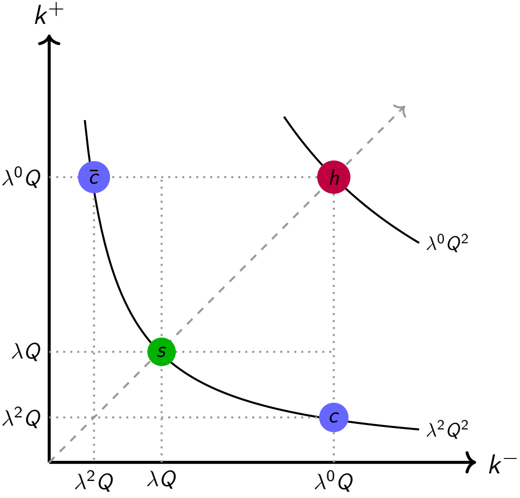
- Determine the NLL contribution of the double-crossed box
  $\longrightarrow$ extract single logarithms using the cutoff method
- Attach further Boxes $\longrightarrow$ Extend to arbitrary order
- Convert to QCD

Backup Slides

Rapidity Divergences

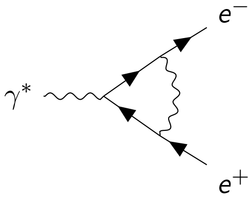


SCET-I

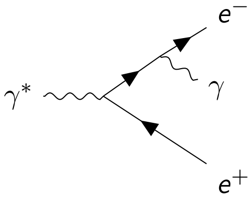


SCET-II

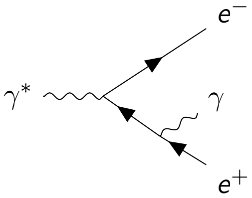
IR Divergences in QED



Virtual correction



Real emission



Real emission

$$\boxed{\text{Virtual correction} + \text{Real emission} \longrightarrow \text{IR-finite observable}}$$