Muon-electron backward scattering beyond the double-logarithmic approximation

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Perturbative calculations in quantum field theory can become more involved in the presence of widely separated energy scales. Higher-order corrections can then contain logarithmically enhanced terms depending on ratios of the different scales. If these logarithms become large, they can compensate the suppression by additional powers of the coupling constant and therefore have to be taken into account. In this case, it is useful to identify the origin of the logarithmically enhanced contributions and to determine which momentum regions are responsible for them.

In this work, we consider electron–muon backward scattering in the high-energy limit,

$$m^2 \ll s,$$

where m denotes the lepton mass and s the center-of-mass energy squared. One reason for considering this process in QED is that the logarithmically enhanced contributions can be studied in a relatively simple setting. In this way, complications related to the non-Abelian structure of QCD are avoided, while the process still contains a non-trivial factorization structure. In particular, endpoint divergences and the associated rapidity logarithms appear. Similar structures are also relevant in more complicated processes in QCD, for example in the analysis of heavy-meson form factors.

The leading logarithmic structure of electron–muon backward scattering and its all-order resummation are already known. At n-loop order, the leading contributions are proportional to

$$\alpha^{n+1} \ln^{2n}\left(\frac{m^2}{s}\right).$$

These terms originate from specific momentum configurations in twisted-box topologies. Their structure can be understood using a method based on physical cutoffs, where the relevant logarithmic integration region is isolated directly. In this way, the leading logarithms can be extracted without performing a complete method-of-regions calculation, and the procedure can be generalized to arbitrary orders in perturbation theory.

The main goal of this thesis is to go one step further and investigate the next-to-leading logarithmic contributions at two-loop order. For this purpose, the different diagram classes, including vacuum polarisation, vertex, box, as well as twisted-box corrections, are considered. The corresponding loop integrals are analyzed with the method of regions in order to identify the momentum configurations which can generate the logarithmically enhanced terms. In particular, diagrams containing at least one twisted-box substructure are of special interest, since these configurations are required for the appearance of the double logarithms.

While the leading logarithms can be obtained efficiently with the cutoff method, the determination of the next-to-leading logarithmic terms is considerably more involved. A complete two-loop calculation within the method of regions requires the analysis of several combinations of momentum regions and leads to a significantly more complicated calculation. In order to avoid this, we investigate a hybrid approach in which the cutoff method and the method of regions are combined.

The basic idea is to treat the loop momentum associated with the twisted-box substructure as soft. The corresponding double-logarithmic contribution is then extracted using physical cutoffs. At the same time, the scaling of the second loop momentum changes accordingly and its contribution is analyzed with the method of regions. In this way, the double logarithm is obtained from the cutoff integration, while the remaining single logarithm is determined from the method-of-regions calculation. This allows the next-to-leading logarithmic contribution to be extracted without carrying out the complete two-loop calculation in all momentum regions.

The purpose of this analysis is not only to determine the next-to-leading logarithmic terms at two-loop order, but also to investigate whether this strategy can be generalized to higher orders in perturbation theory. Since the leading-logarithmic cutoff method already admits an all-order extension, a corresponding treatment of the next-to-leading logarithms would provide a useful simplification for future calculations. Furthermore, similar endpoint and rapidity structures appear in QCD processes, such that the methods studied in this thesis may also provide a starting point for related applications in more complicated theories.