Perturbative Uncertainties in the Back-to-Back Energy-Energy Correlator

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BACKGROUND

- Examining energy-energy correlator (EEC) cross-section in the back-to-back limit
- Improve theory uncertainty analysis via theory nuisance parameters (TNPs) using SCETlib (C^{++} library)
- Ultimate goal: use EEC spectrum to fit α_s
- Work-in-progress => Simplifications:
  - only leading power resummation uncertainties
  - not examining nonsingular contributions

CURRENT STATE OF THEORY UNCERTAINTIES

- Scale variations: vary renormalization scales and take envelope

↳ Issues:

  - arbitrary choice in variation
  - hard to be traced back to (tangible) individual sources

THEORY NUISANCE PARAMETERS

[Tackmann; 2411.18606]

- For a series $f(\alpha) = \hat{f}_0 + \hat{f}_1 \alpha + \hat{f}_2 \alpha^2 + \hat{f}_3 \alpha^3 + \mathcal{O}(\alpha^4)$ with known $\hat{f}_n$
  - approximate unknown part as leading uncertainty f_4

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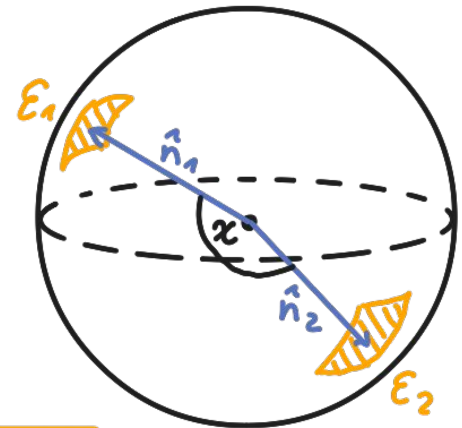
  - approximate unknown part as leading uncertainty f_4
  - f_4 has internal structure $\rightarrow$ $f_n = f_n(\theta_n)$
- θ_n theory nuisance parameter (TNP) with true (but unknown) value $\hat{\theta}_n$ such that $\hat{f}_n = f_n(\hat{\theta}_n)$
- The θ_i are proper parameters:
  - have their own uncertainties $\theta_n = u_n \pm \Delta u_n$
  - can be propagated $\Rightarrow$ correct correlations
  - parametrize: factorization, RGE equations

ENERGY-ENERGY CORRELATORS

[Moult, Zhu; 2506.09119]

- Two detector operators $\mathcal{E}_i(\hat{n}_i)$ in the celestial sphere (energy flux)

  ↳ write in terms of angle x between the detectors:



use variable $z = \frac{1}{2}(1-\hat{n}_1 \cdot \hat{n}_2) = \frac{1}{2}(1-\cos x)$

$$\boxed{EEC(z) = \sum_{i,j} \int d\sigma \, \frac{E_i E_j}{\sigma Q^2} \, \delta\left(z - \frac{1-\cos\theta_{ij}}{2}\right)}$$

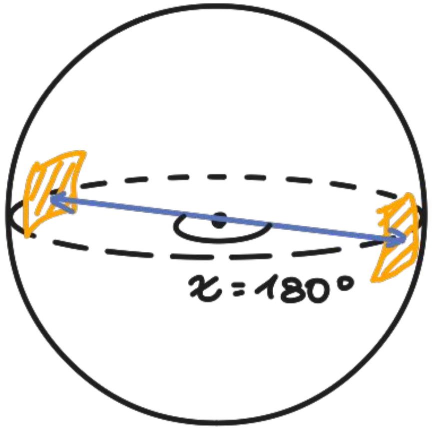
for particles (i,j) with energies E_i, E_j and angle θ_{ij}

ENERGY-ENERGY CORRELATORS

[Moult, Zhu ; 2506.09119]

- Back-to-back limit: $\chi \to 180^\circ$
  $z \to 1$

  ↳ resum large logarithms of $(1-z)$



- Factorization :

$$\boxed{\begin{aligned} EEC_{z\to 1} &= \frac{\sigma_0 Q^2}{4\sigma} \int db_\perp b_\perp J_0\left(\sqrt{1-z}\, b_\perp Q\right) \\ &\times H(Q,\mu)\, S(b_\perp,\mu,\nu) \sum_q J_q(b_\perp, Q,\mu,\nu)\, J_{\bar{q}}(b_\perp, Q, \mu,\nu) \end{aligned}}$$

TNPS FOR THE BACK-TO-BACK EEC

[Moult, Zhu ; 2506.09114]

- Resummation : $\frac{1}{\sigma_0}\frac{d\sigma}{dz} \sim \frac{1}{2}\int_0^\infty \frac{b_T\, db_T}{2} J_0(b_T Q\sqrt{1-z})$

$$\times H(Q,\mu)\, J_a(b_T,\mu,\nu)\, J_b(b_T,\mu,\nu)\, S(b_T,\mu,\nu) + \mathcal{O}(1-z)$$

(scales : $\mu_J^{can} = \mu_S^{can} = \nu_S^{can} = \mu_0^{can} = b_0/b_T$; $\nu_J^{can} = \mu_H^{can} = Q$)

- For general RGE solution $F = \{H, J_i, S\}$ rewrite to logs : $[H \times J_a \times J_b \times S](\alpha_s,\, L \equiv \ln(\sqrt{1-z}))$

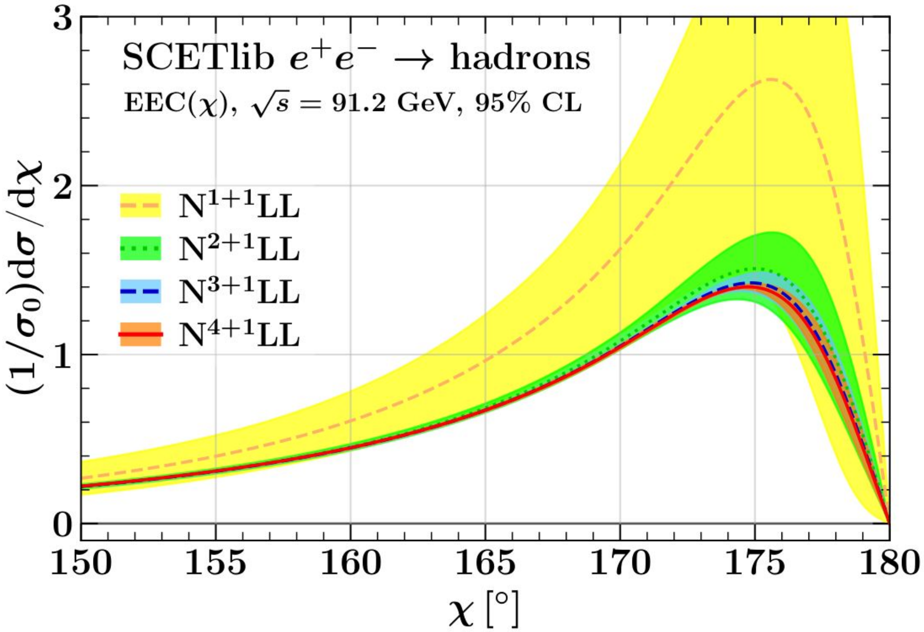
$$F(\alpha_s, L) = F(\alpha_s)\, \exp\left\{\int_0^L dL' \left(\Gamma[\alpha_s(L')]\, L' + \gamma_F[\alpha_s(L')]\right)\right\}$$

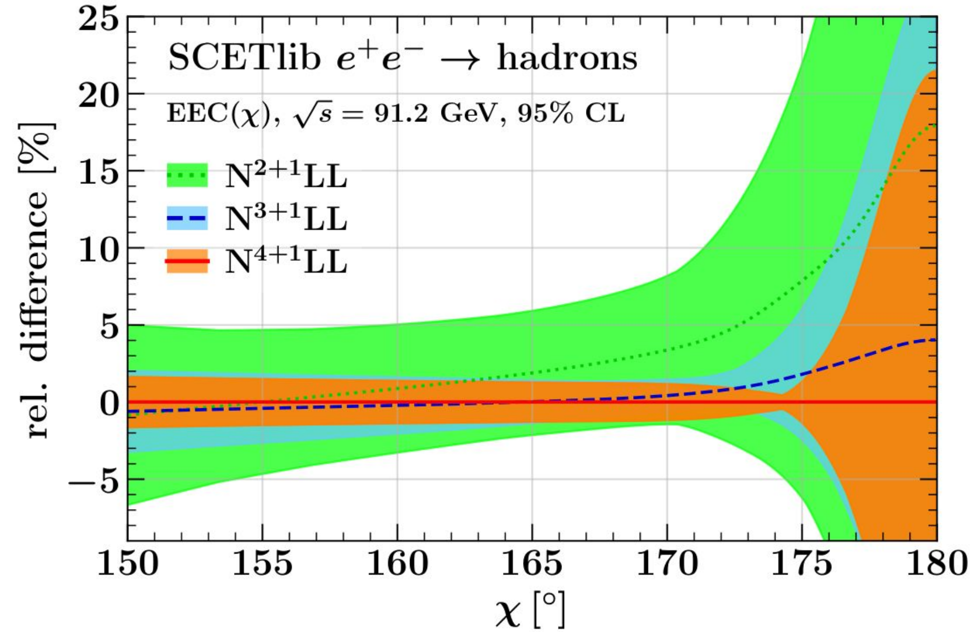
- TNPs:

boundary terms :	$\theta_{Hq\bar{q}V}$, θ_J, θ_S

anom. dims. :	$\theta_{\Gamma_{cusp}}$, θ_{γ_μ}, θ_{γ_ν}

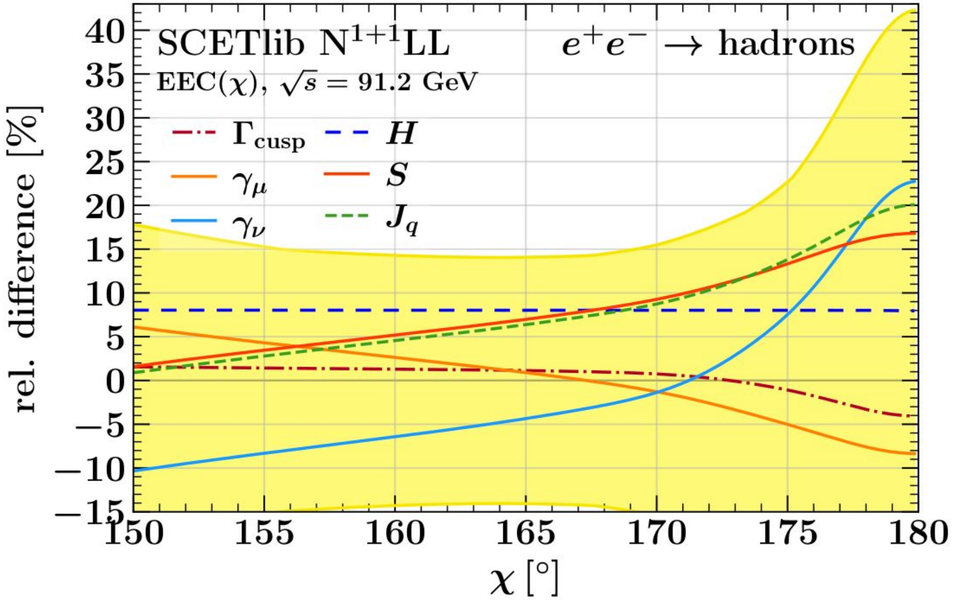
RESULTS: CONVERGENCE PLOTS

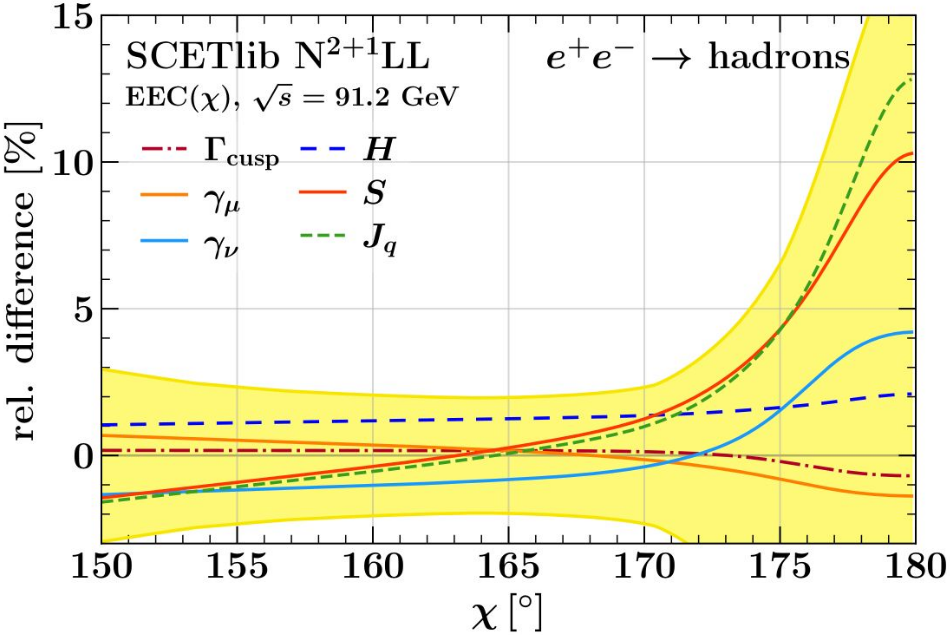




- $\mathrm{N^{k+\ell}LL}$: boundary conditions: $\mathrm{N^{k-1}LO}$, ADMs: $\mathrm{N^{k}LL}$,
  TNPs parametrize $\mathrm{N^{k+\ell}LL}$ ingredients
- Here: 95% CL → $\theta_i = 0 \pm 2$
- TNPs are independent sources of uncertainty
  ↳ summed in quadrature for total uncertainty ($\hat{=}$ colored bands)

RESULTS: TNP BREAKDOWN





- Uncertainty shrinks for higher resummation order
- 68% CL → $\theta_i = 0 \pm 1$
- TNPs are independent sources of uncertainty
  ↳ summed in quadrature for total uncertainty (≙ yellow band)

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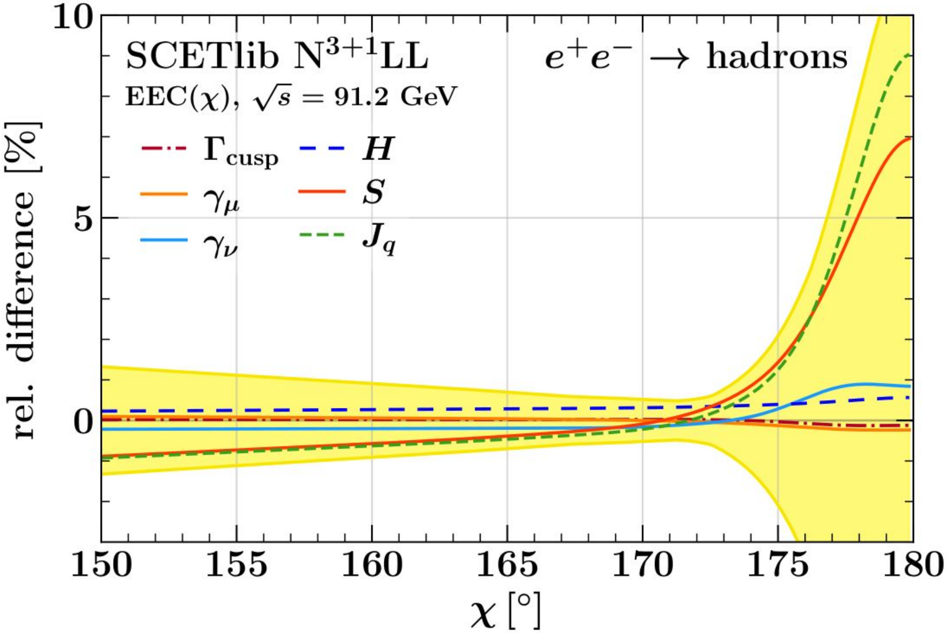

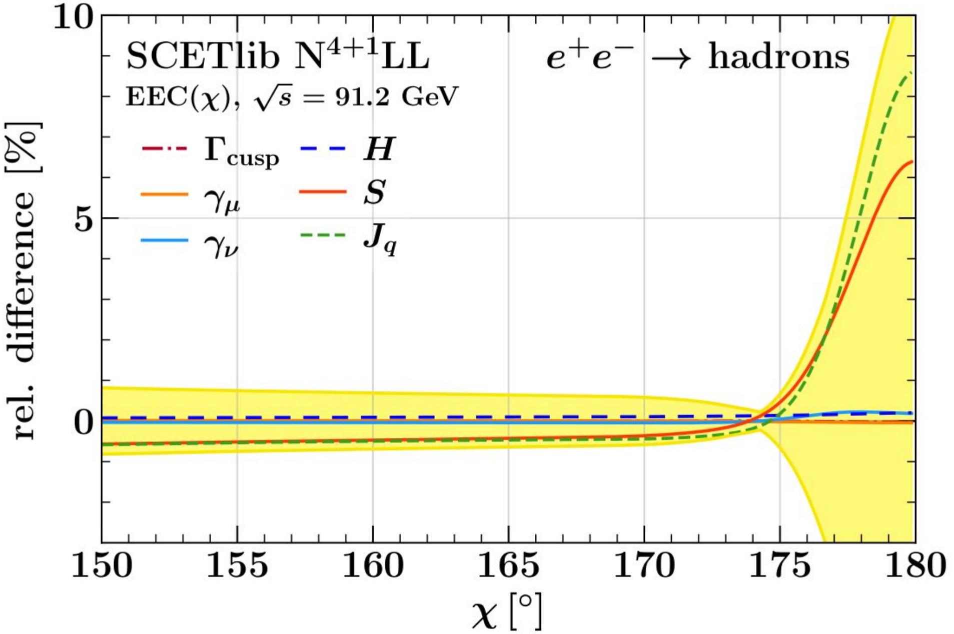

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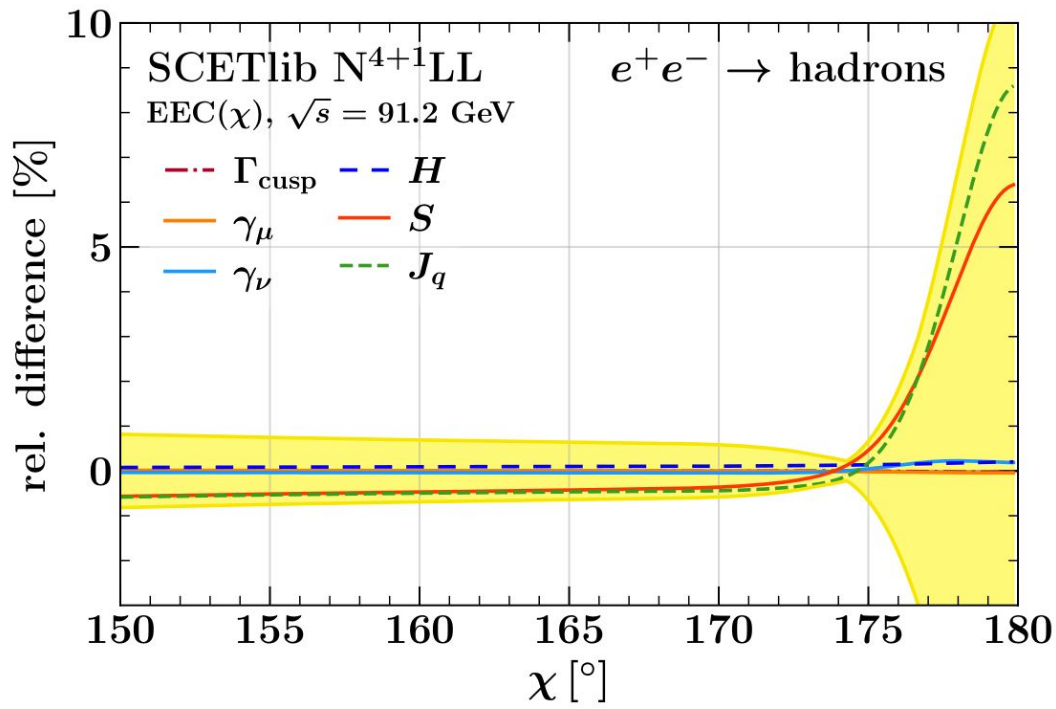
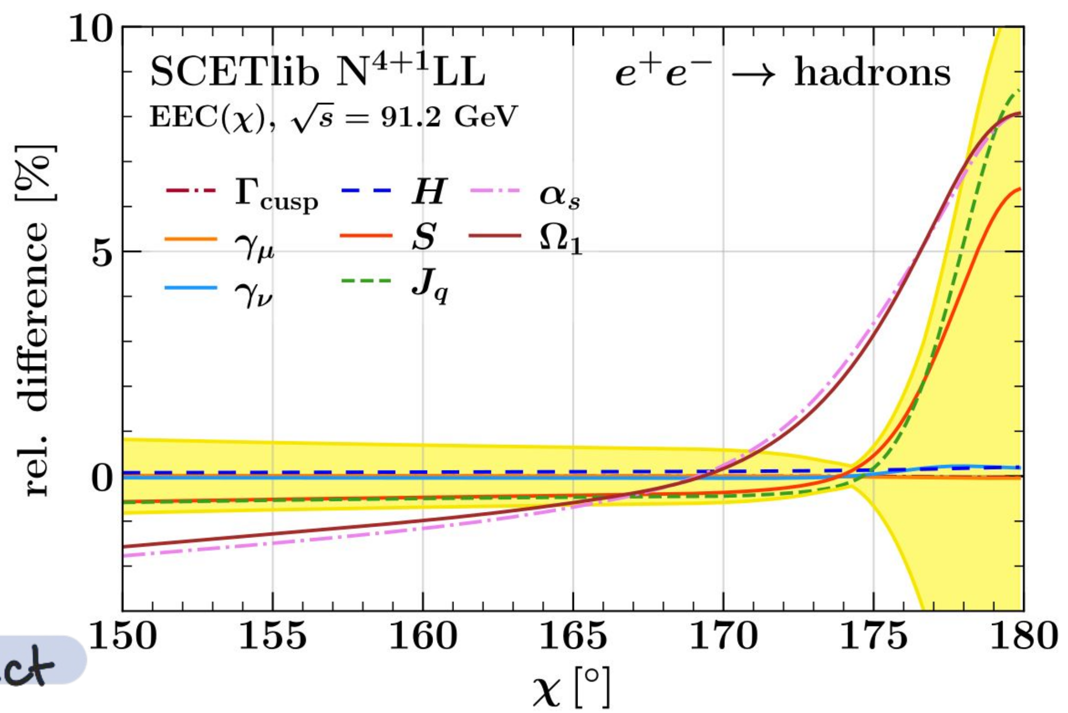
- Point-by-point correlations are important



RESULTS: α_s SENSITIVITY

- 68% CL → $\theta_i = 0 \pm 1$
- Changes in shape
  ↳ Point-by-point correlations are important!
- α_s is a %-level shape effect
  ↳ highly precise!



APPLICATION: FITTING α_s

[Cridge, Marinelli, Tackmann; 2506.13874]
[OPAL collaboration; doi: 10.1007/BF01555834]

- Current state: using Asimov data
  - pseudodata, constructed from a model of (usually) higher order + experimental uncertainties from OPAL data
    ↳ useful to test fitting setup, no unaccounted for fluctuations
- Use χ^2 minimization; Minuit
  - Scan and profile TNPs → Profiling gives smaller bias + smaller uncertainties

Scan: vary TNPs one-by-one, fit α_s; add uncs. in quadrature

profile: set fit constraints; fit TNPs + α_s at the same time

SUMMARY & OUTLOOK

- Applied theory nuisance parameters to back-to-back EEC
  - ↳ obtained resummation uncertainties up to $N^{4+1}LL$
  - ↳ implemented N^5LL structure
- Additional: perform Asimov fits for α_s, examine effect of nonperturbative Ω_{1q} parameter [Marinelli, NO, Tackmann, 26xx.xxxx (hopefully)]
- Improvements: more sophisticated nonperturbative models, nonsingular contributions, ...

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REFERENCES

- Moult, Zhu; 2506.09119 [arXiv]
- Tackmann; 2411.18606 [arXiv]
- Cridge, Marinelli, Tackmann; 2506.13874 [arXiv]
- OPAL collaboration; doi: 10.1007/BF01555834 [CDS]

BACKUP SLIDES

ABOUT ME AND MY PROJECT

- Nicole Oberth, (end of) 1st year PhD student working in DESY theory/pheno group under advisor Frank Tackmann

- General project: Apply theory nuisance parameters to color singlet processes ($e^+e^- \to$ hadrons, Drell-Yan)

  ↳ End goals: improve theory uncertainty analysis and fit α_s

- Current project with Giulia Marinelli and Frank Tackmann

NONPERTURBATIVE MODELS

- linear Ω_1 parameter:

$$\boxed{J(b_T) = J^{\text{pert.}}(b_T) - \Omega_1 b_T}$$

with $\Omega_1 = 0.305$ GeV

- Collins-Soper kernel:

$$\tilde{\gamma}_\nu^{np}(b_T) = -\lambda_\infty \tanh\left(\frac{\lambda_2}{\lambda_\infty} b_T^2 + \frac{\lambda_4}{\lambda_\infty} b_T^4\right) \text{ with }$$

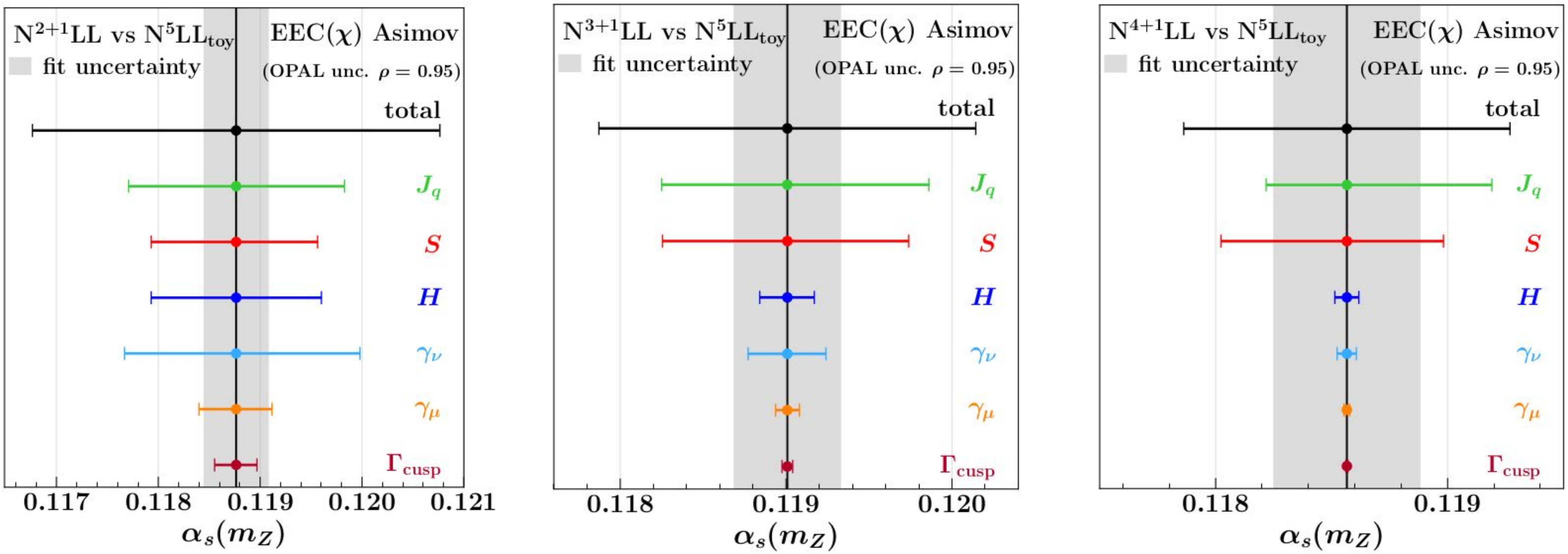
$\lambda_\infty = 1.68529$
$\lambda_2 = 0.087011$
$\lambda_4 = 0.00739108$

- TMD PDF boundary condition:

$$\tilde{f}^{np}(b_T) = \exp\left\{-2 b_T^2 (\lambda_2 + \lambda_4 b_T^2)\right\} \text{ with }$$

$\lambda_2 = 0.106$
$\lambda_4 = 0$

BACKUP: ASIMOV FIT RESULTS



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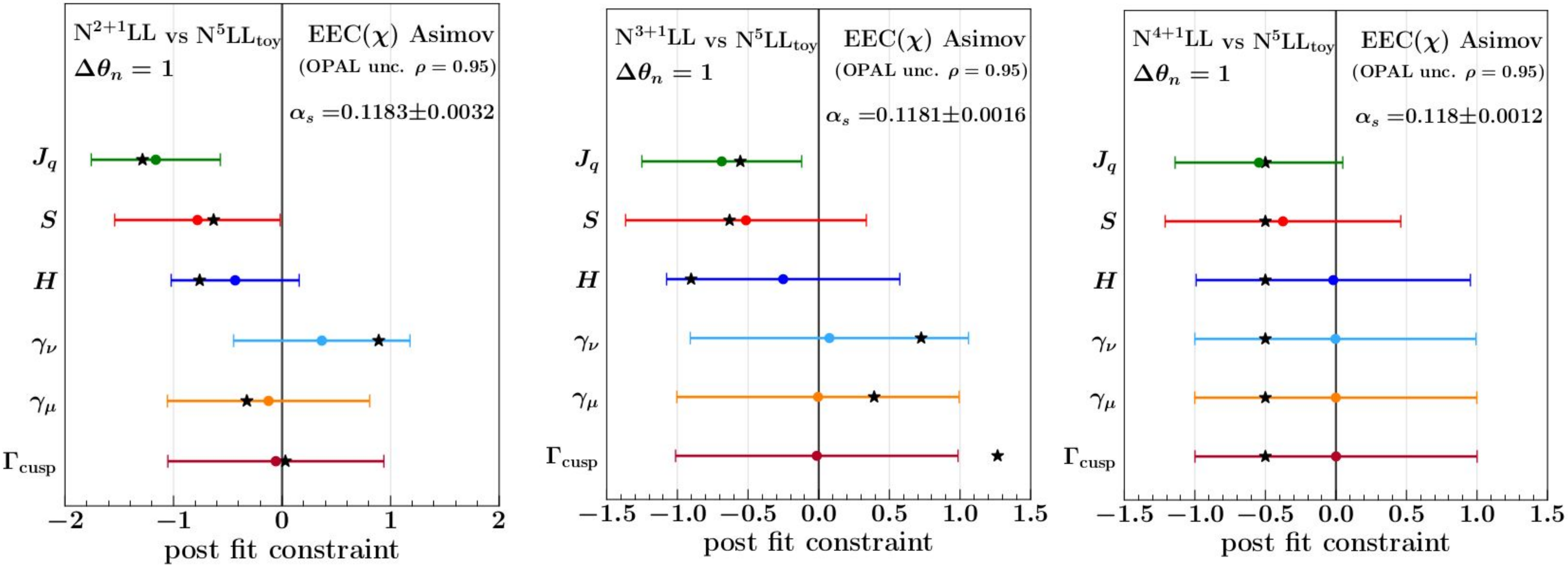
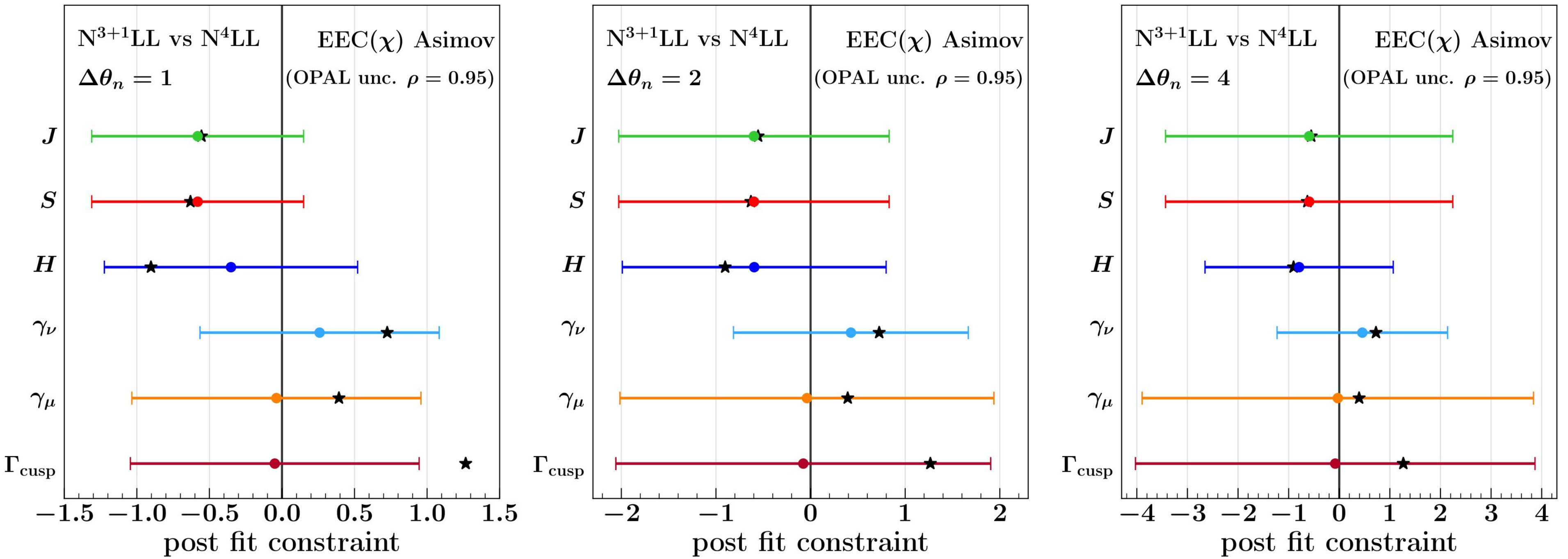


RESULTS: PROFILING TNPS

[Backup]

95% correlation : different $\Delta\theta_n = \{1., 2., 4.\}$



note: old plots without linear \Omega_{1q} parameter! Just for illustration purposes

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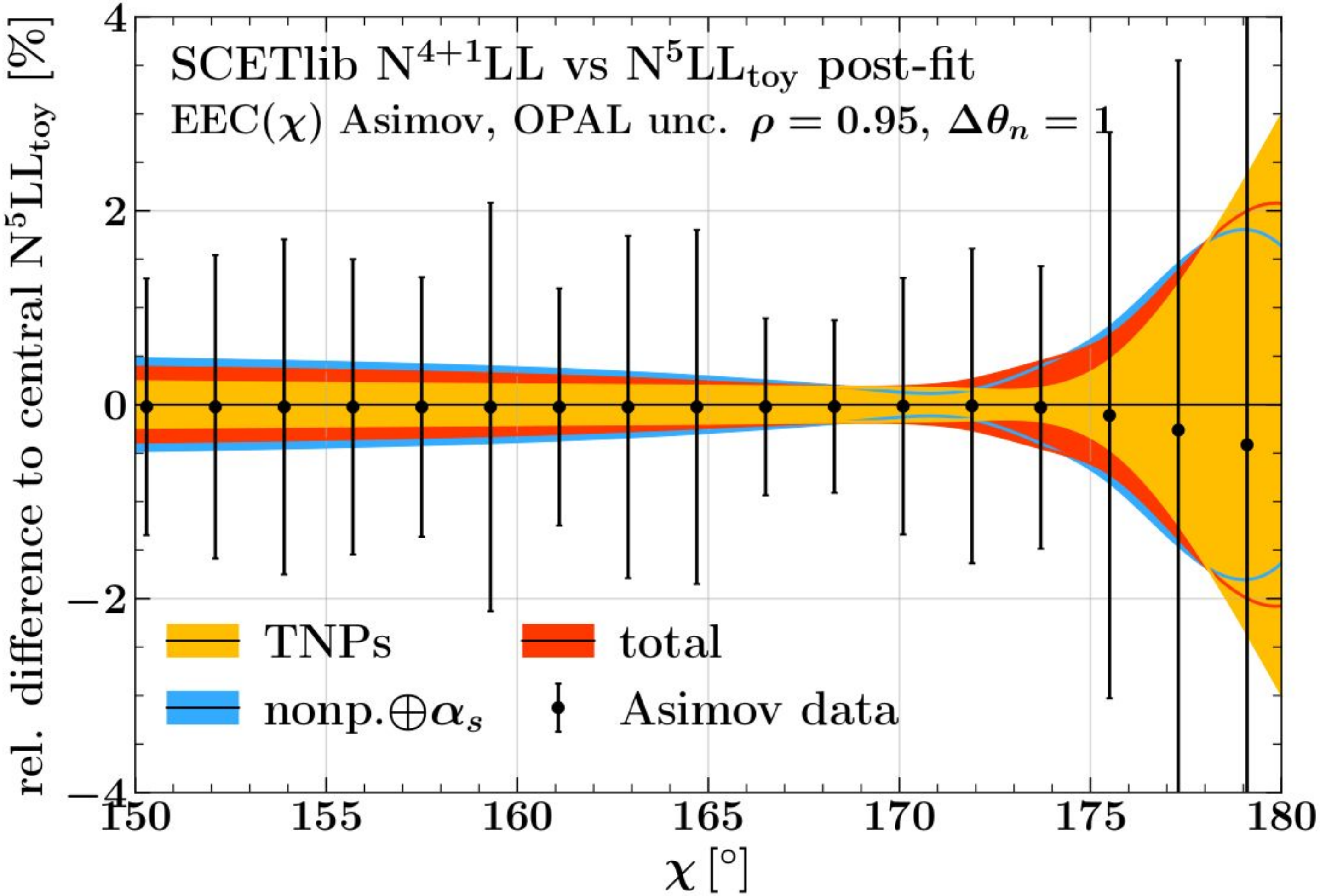

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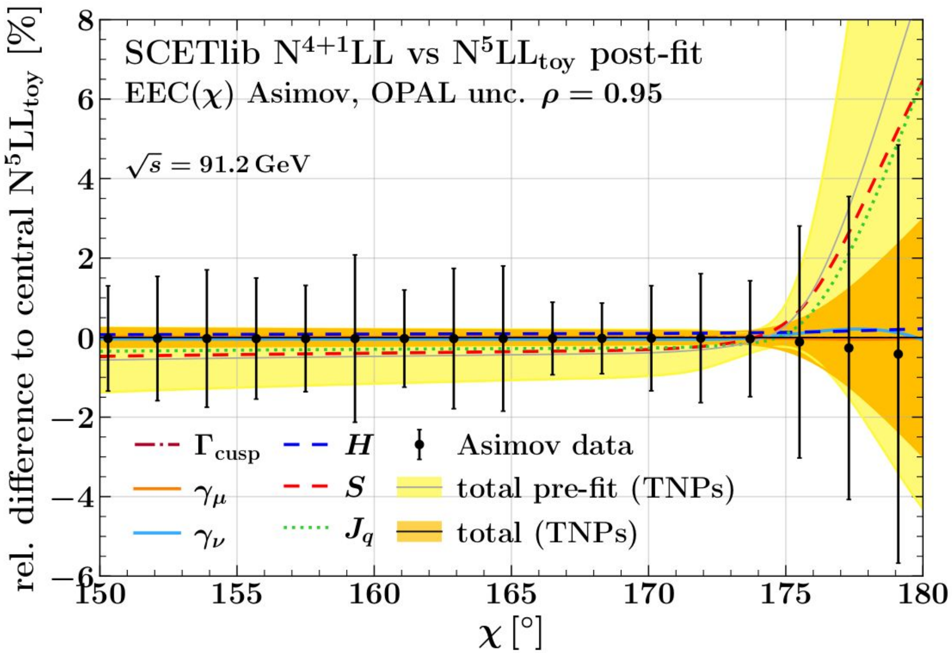

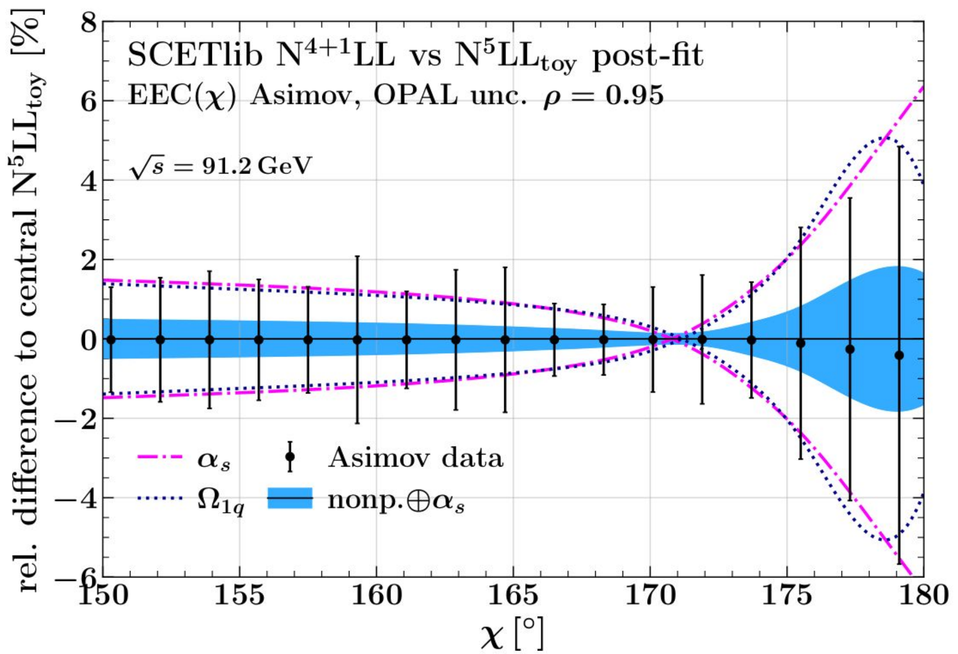

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