# Perturbative uncertainties in the back-to-back energy-energy-correlator

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Theory uncertainties provide valuable insight into the spectrum of a scattering process and a correct treatment of them is crucial to the extraction of physical quantities such as the strong coupling $\alpha_s$. The aim of my current project (in collaboration with G. Marinelli and F. Tackmann) is to improve the theoretical uncertainty analysis of color-singlet processes using the recently developed theory nuisance parameter (TNP) approach. Specifically, it applies TNPs to calculate the perturbative resummation uncertainty of the energy-energy correlator in the back-to-back limit, for the process $e^+e^- \to$ hadrons. Building upon this, Asimov fits to extract $\alpha_s$ are perfomed.

The philosophy of the TNP approach is that the source of uncertainty of a perturbative series of a function $f(x, \alpha)$ expanded around $\alpha$ lies in its unknown coefficients after truncation. As an example, writing a series with coefficients known up to order $\alpha^2$ would yield

$$f(x, \alpha) = \hat{f}_0(x) + \hat{f}_1(x)\alpha + \hat{f}_2(x)\alpha^2 + f_3(x)\alpha^3 + \dots, \tag{1}$$

split into the known coefficients $\hat{f}_n(x)$ and unknown coefficients $f_n(x)$. This would correspond to a NNLO truncation of a fixed-order perturbative series. Despite their true numerical values being unknown, the coefficients $f_n$ generally have a known structure, which are parametrized as a set of TNPs $\{\theta_n\}_i \equiv \theta_n$, such that $f_n(x) = f_n(x, \theta_n)$. These parameters have true (but not necessarily known) values $\hat{\theta}_n$, such that $\hat{f}_n(x) = f_n(x, \hat{\theta}_n)$, and have a central value and uncertainty [1].

TNPs present an alternative to scale variations, which have been the go-to method for theorists to approximate uncertainties. In scale variations, the resummation scales are varied and the envelope of the resulting cross section variations is taken in order to get an approximation for the theory uncertainty. This approach requires a choice of variations, which can be arbitrary and are not necessarily representative of the actual uncertainty sources. Using TNPs, the total uncertainty can be traced back and broken down into individual physical sources, to which the sensitivity of quantities such as $\alpha_s$ can be observed [2].

Energy correlators have enjoyed much interest in recent years, providing connections between formal, phenomenological and experimental groups. They are based on an abstraction of a detector in form of operators measuring the energy flux. This project examines the energy-energy correlator (EEC), which is the weighted correlation function of two detector operators in dependence on their angle $\chi$ from each other. In the usual literature, it is expressed in terms of the variable $z = \frac{1}{2}(1 - \cos\chi)$. The limits of the angle dictate the kinematical behavior of the EEC as well as its resummation requirements. In the back-to-back limit $\chi \to 180^\circ$ ($z \to 1$), large logarithms of $(1-z)$ are resummed [3]. TNPs can be applied well to the back-to-back EEC because it has a factorization in which the perturbative components have known parametrizations, but are truncated at different orders. The factorization components are the hard function $H(Q,\mu)$, the soft function $S(b_\perp,\mu,\nu)$ and the two jet functions $J_i(b_\perp,Q,\mu,\nu)$. They depend on the impact parameter $b_\perp$ (Fourier transform of the transverse momentum) and the renormalization scales $\mu$ and $\nu$, corresponding to the virtuality and rapidity, respectively. From the renormalization group equations of the factorization components, one can define six TNPs: those for the boundary terms $\theta_i^F$ : $\{\theta_H, \theta_S, \theta_{J_i}\}$ and those for the anomalous dimensions $\theta_i^\gamma$ : $\{\theta_{\Gamma_{\mathrm{cusp}}}, \theta_{\gamma_\mu}, \theta_{\gamma_\nu}\}$. The cross section also takes into account nonperturbative effects in form of simplified nonperturbative models, specifically describing the nonperturbative contributions to the Collins-Soper kernel and TMD boundary terms as well as the linear parameter $\Omega_1$ which flows into the jet function boundary terms [4].

So far, several interesting applications have presented themselves. One is to extend the $\mathrm{N}^4$LL results by using TNPs for the unknown components of the $\mathrm{N}^5$LL parametrization, allowing the incorporation of uncertainties from one order higher. Another application, which will be the main focus of the upcoming publication, is the sensitivity of $\alpha_s$ and its extraction from the EEC spectrum. For now, Asimov fits are used, which fit against pseudodata constructed from a theory model and are a useful tool to test the fitting environment. Eventually, the aim is to refine the current model by using more sophisticated nonperturbative models and including further corrections (such as nonsingular contributions) not currently accounted for to then perform a fit to experimental data.

## References


[1] Frank J. Tackmann. *Beyond Scale Variations: Perturbative Theory Uncertainties from Nuisance Parameters*. 2025. arXiv: `2411.18606 [hep-ph]`. URL: `https://arxiv.org/abs/2411.18606`.

[2] Thomas Cridge, Giulia Marinelli and Frank J. Tackmann. *Theory Uncertainties in the Extraction of $\alpha_s$ from Drell-Yan at Small Transverse Momentum*. 2025. arXiv: `2506.13874 [hep-ph]`. URL: `https://arxiv.org/abs/2506.13874`.

[3] Ian Moult and Hua Xing Zhu. *Energy Correlators: A Journey From Theory to Experiment*. 2025. arXiv: `2506.09119 [hep-ph]`. URL: `https://arxiv.org/abs/2506.09119`.

[4] Max Jaarsma **andothers**. *From DGLAP to Sudakov: Precision Predictions for Energy-Energy Correlators*. 2025. arXiv: `2512.11950 [hep-ph]`. URL: `https://arxiv.org/abs/2512.11950`.