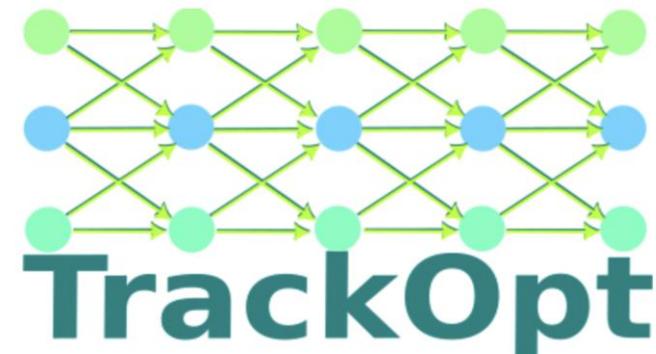


Metrics for vertex finding in particle physics

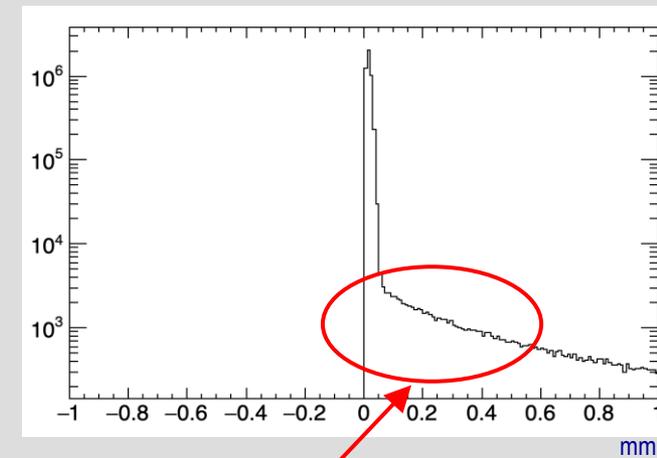
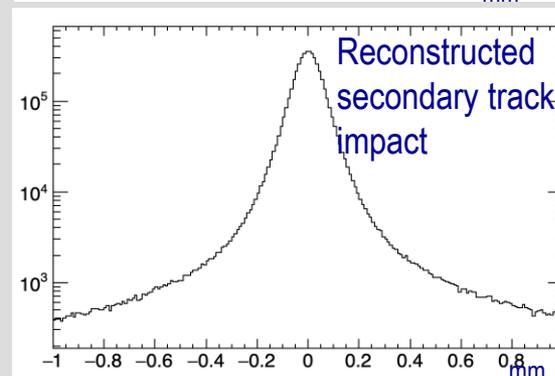
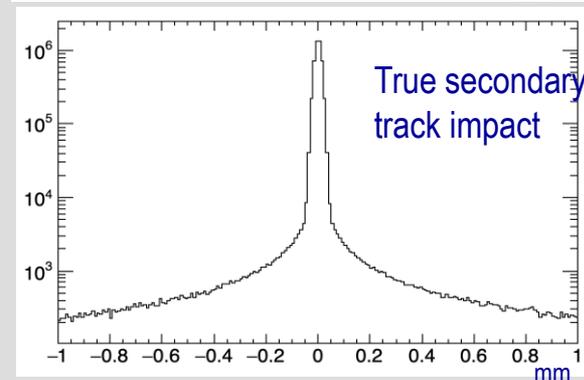
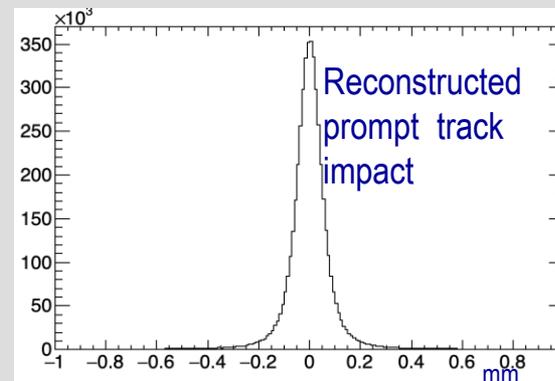
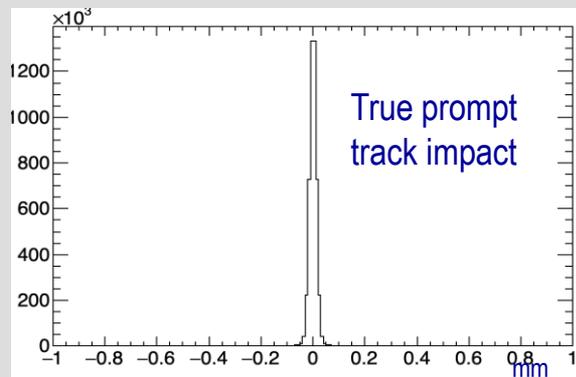
V. Kostyukhin, D.Biswas, M.Cristinziani
Siegen university



Primary+Secondary vertex challenge

Terminology:

- Prompt - tracks produced in primary proton-proton interactions. Production vertices are distributed along a (beam)line
- Secondary - tracks are produced in subsequent interaction/decay vertices scattered in 3D space, sometimes very far from the beamline

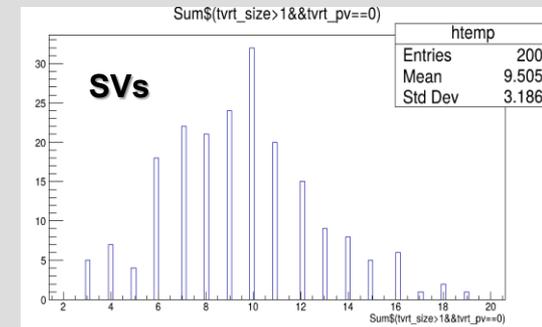
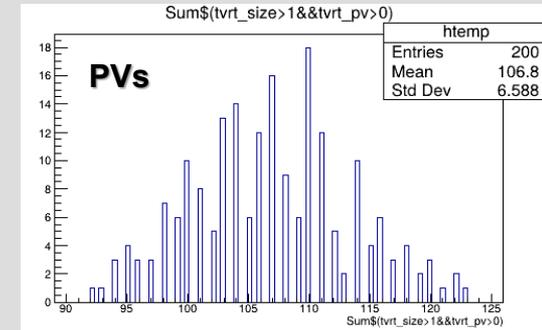


Problematic region for PV+SV reconstruction due to resolution (~equivalent to image blurring?). Hoped to resolve using ML.

Hope - efficient PV+SV vertex finding must efficiently resolve close vertices in the region around beamline with big track density and bad resolution

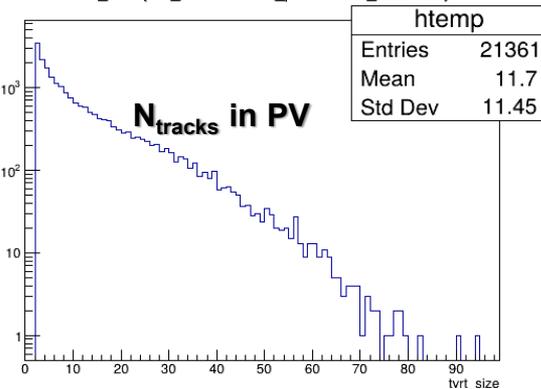
MC data features

- ~1380 tracks/event (LLP sample, reco track $p_T > 0.7 \text{ GeV}$, pileup=200)
- ~7.3% secondary track (produced in SV)
- ~98 vertices/event have only one reconstructed track
 - 68% of them are SVs (interaction in material)
- ~116 vertices/events have ≥ 2 reconstructed tracks
 - 9.5 SVs/event
 - 107 PVs/event

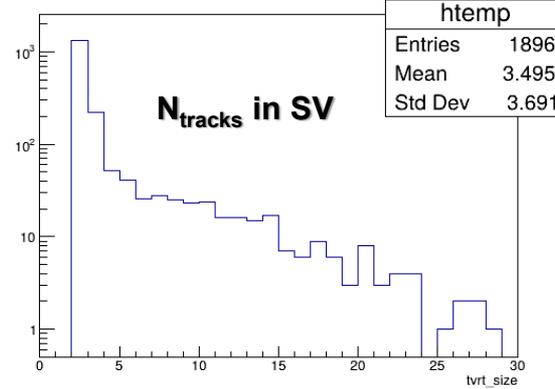


SVs and PVs categories are imbalanced (factor ~10)

tvrt_size (tvrt_size>1\&\&tvrt_pv>0\&\&tvrt_size<100)



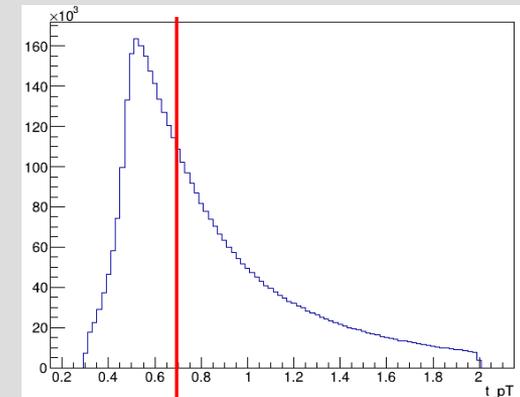
tvrt_size (tvrt_size>1\&\&tvrt_pv==0\&\&tvrt_size<30)



Track p_T spectrum is exponentially falling:

$$1380(p_T > 0.7 \text{ GeV}), 760(p_T > 1.0 \text{ GeV}), 340(p_T > 1.5 \text{ GeV})$$

Problem complexity can be easily tuned/scanned by changing the track p_T cut.
Good for the algorithm development. Physics – higher cut less physics info.



- 1) One-track clusters (C1t) - not vertices, garbage collector
- 2) Primary clusters (PVc) - made of only prompt tracks
- 3) Secondary clusters (SVc) - made of only secondary tracks
- 4) Mixed n-track primary clusters (mixPVc) - fraction of prompt tracks >50%
- 5) Mixed n-track secondary clusters (mixSVc) - fraction of secondary tracks >50%
- 6) Mixed 2-track clusters (mixC2t) - prompt+secondary tracks

Idea

After clustering select only prompt/secondary tracks and check statistical metrics for them only as these categories have different properties.

- 1) Variation of information (comparison with truth, $VI==0$ if identical.
Smaller VI \rightarrow better clustering) [[Journal of Multivariate Analysis 98 \(2007\) 873–895](#)]
- 2) Adjusted Rand Index (comparison with truth, $ARI==1$ - perfect agreement,
 $ARI==0$ - random clustering)
- 3) Possibly Silhouette for PVs only. Cluster compactness vs inter-cluster distance.
[[Journal of Computational and Applied Mathematics, v20, 1987, pp.53-65](#)]

Evident requirements

- 1) $C1t/N_{\text{tracks}} \rightarrow$ truth value
- 2) $PVc+SVc+mixPVc+mixSVc+mixC2t \rightarrow$ truth number of N-track vertices
- 3) $mixPVc \rightarrow 0$, $mixSVc \rightarrow 0$, $mixC2t \rightarrow 0$

Due to SV/PV imbalance, the PV reconstruction is not affected much by secondary track presence \rightarrow stat metrics only for the moment (metric from PV paper comes later...)

Physics metrics

- 1) SV efficiency: $(SVc+mixSVc)/nSV(nTrack>1)_{\text{true}}$
- 2) SV purity: $SVc/(SVc+mixSVc)$
- 3) Number of mixC2t vertices: N_{mixC2t}
- 4) Secondary track fraction in mixPVc: $nTsec_in_mixPVc/N_{\text{secondary_tracks}}$

(3)+(4) estimate how many secondary tracks are "lost" due to prompt track presence

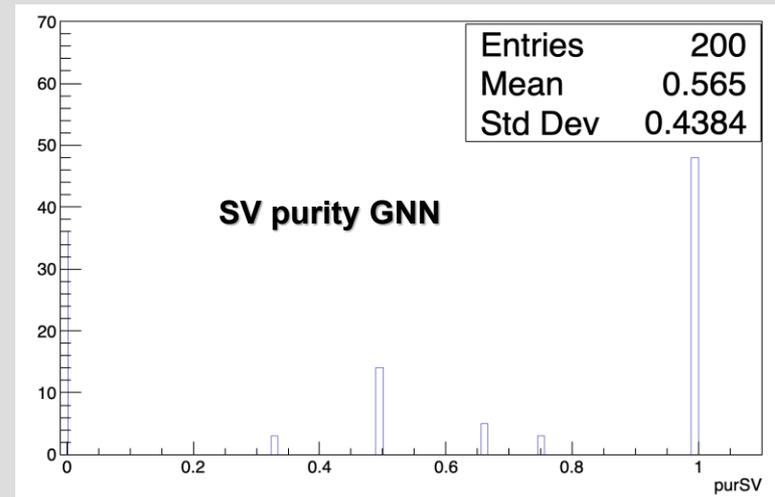
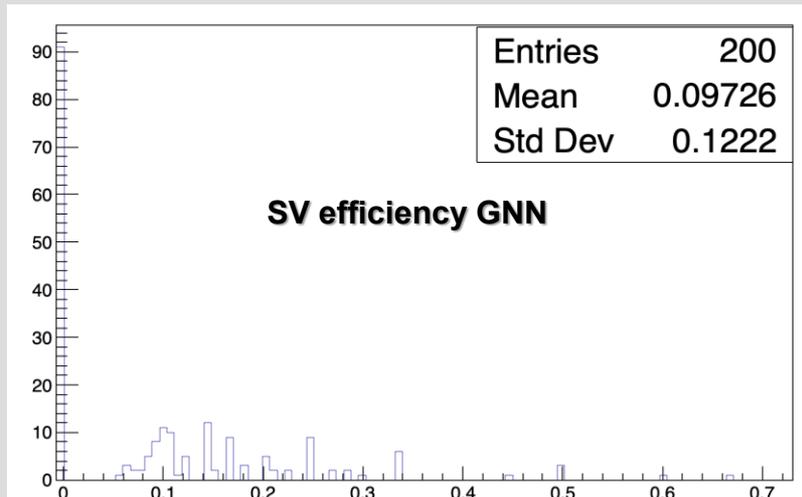
Results (LLP, $p_T > 0.7 \text{ GeV}$, $pu=200$)

Compare LMC clustering in 2 cases

- 1) Edge weights are obtained in GNN processing
- 2) Edge weights are based on simple 2-track vertex fit χ^2

LMP weight	Ncl 1-track	Ncl N-track	effic. SVc	purity SVc	mixed 2trk clst	Lost sec.track	VI PVc	ARI PVc	VI SVc	ARI SVc
GNN wgt	321	74	10%	54%	3.0	52%	3.28	0.285	1.45	0.107
Edge Chi2	197	81	6%	43%	1.6	69%	4.40	0.153	1.89	0.095
Truth	98	116	100%	100%	0	0	0	1	0	1

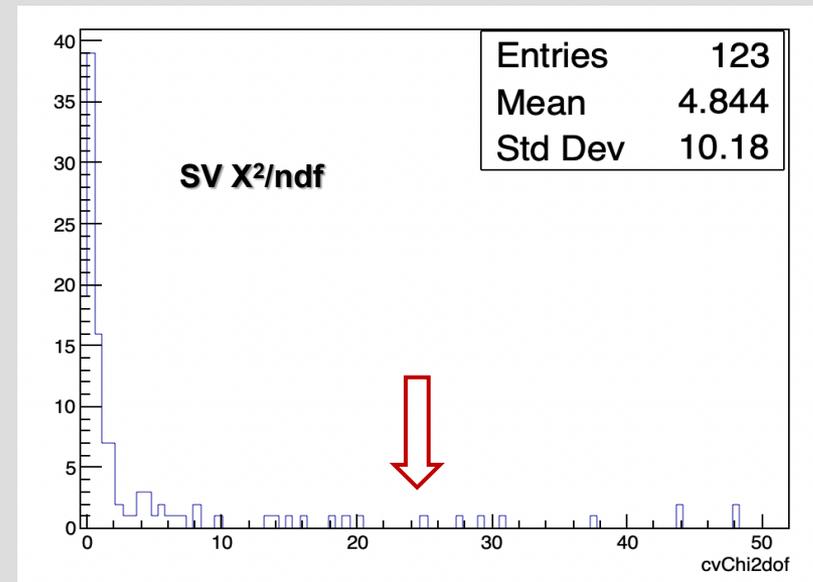
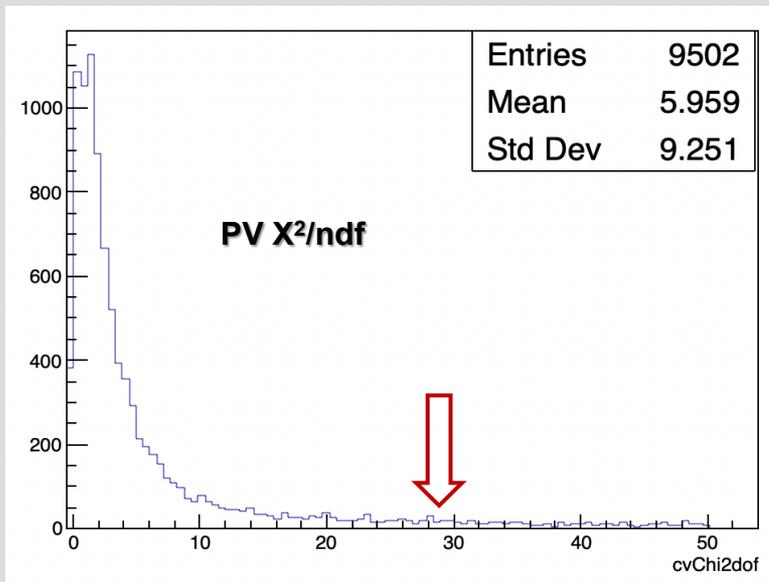
Prb=TMath::Prob(Chi2,nDoF=1)



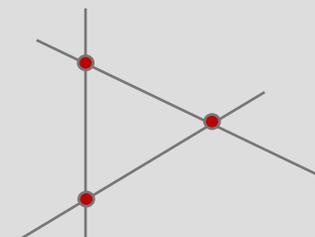
Clustering with GNN weights is significantly better

Lifted minimum cost multi-cut problem

Current LMC (no constraints, hyperedges, etc.) setup provides non-compact clusters. Can be seen in cluster vertex fit $\chi^2/n\text{DoF}$. They can't become physics vertices.



In SV case it might be a triangular (quartic, etc.) anomaly:



This is not a 3-track SV!

In PV case no such explanation (effective 1D space) - further study needed

- 1) Metric to compare/tune simultaneous SV+PV finding is proposed
- 2) Graph processing does improve the vertex finding efficiency as compared to the simple vertex χ^2 based one.
- 3) Still far from expected:
 - LMC clustering doesn't look ideal for the moment
 - 1000ev GNN training only
 - Balancing Secondary/Prompt fractions
 - Try LMC constraints
 - Better physics information in GNN input
 -