

Framing secondary vertexing as a clustering problem: possible heuristics

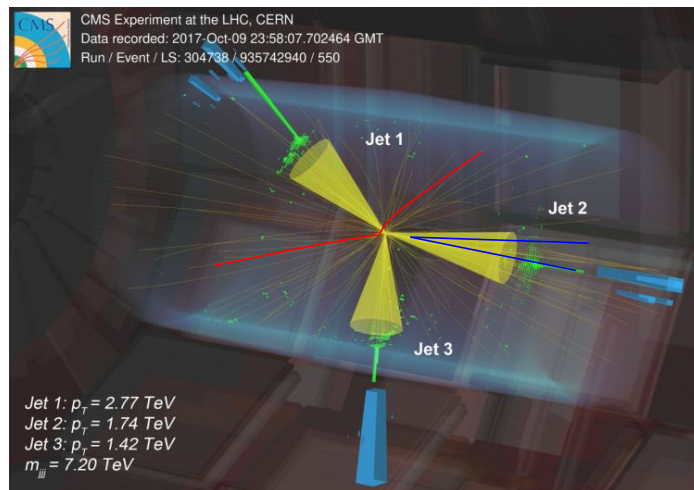
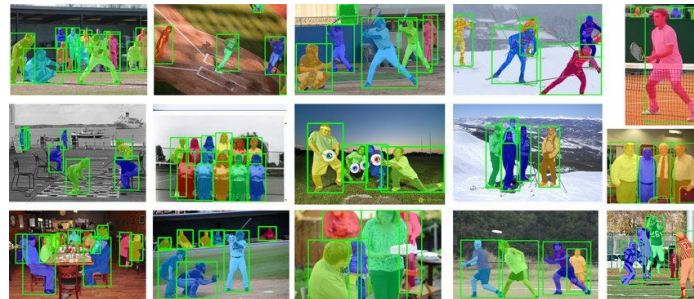
Diptaparna Biswas



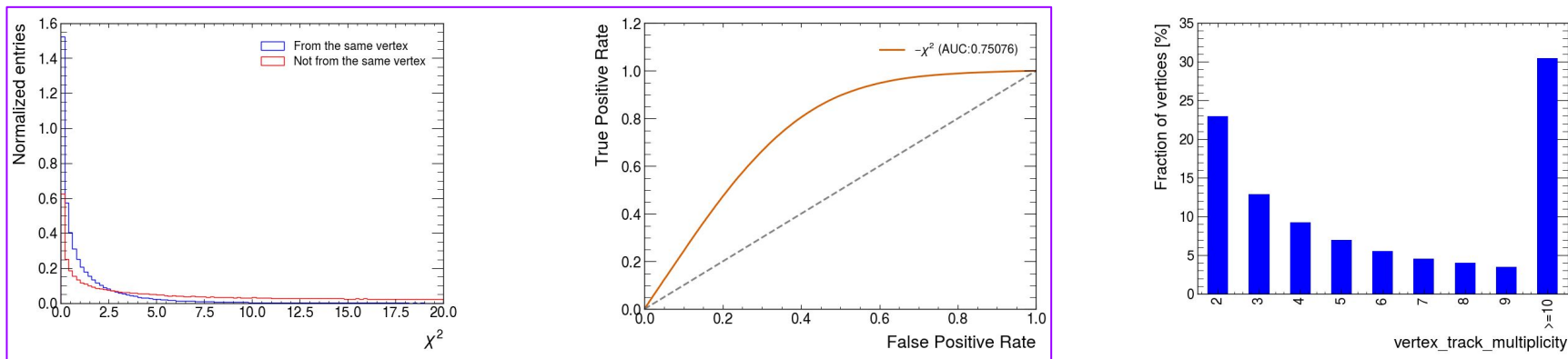
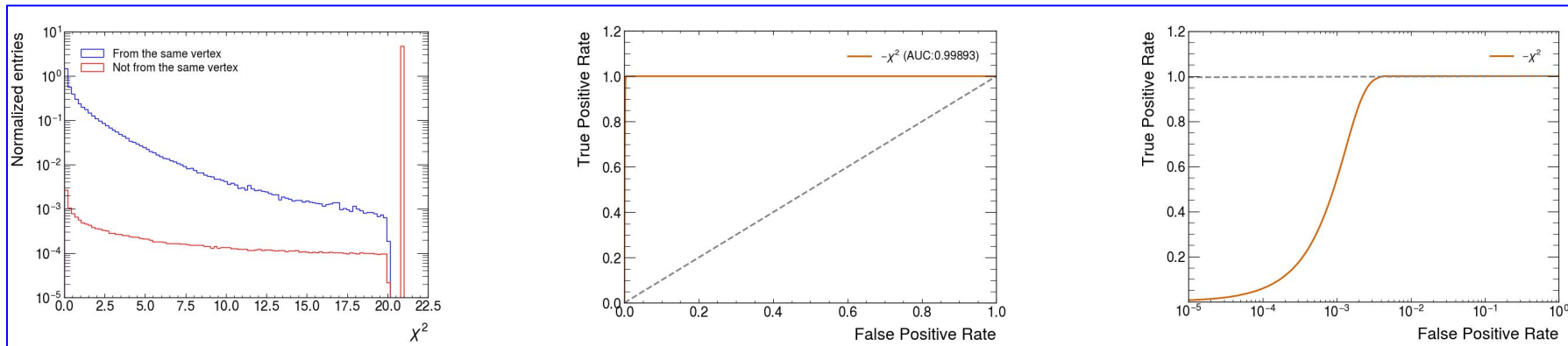
TrackOpt Meeting
7th November 2025

Looking for a heuristic

- For **vertex finding inside a jet**, the jet cone itself works as a bounding box.
 - We need to consider combinations only among the tracks (~ 20) inside the cone.
- However, for an **arbitrary secondary vertex**, tracks can go in all directions.
 - The track parameters themselves don't provide a measure of *closeness*.
 - May introduce bias.
 - How to tackle combinatorial complexity?

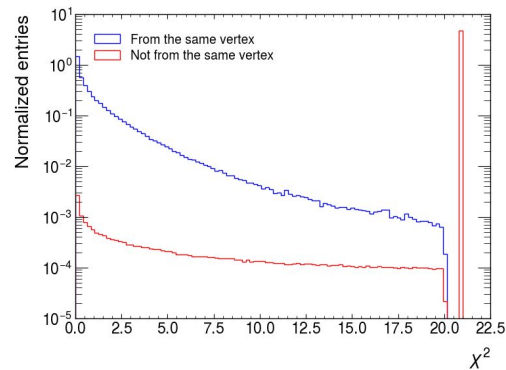


Utilizing χ^2 from Billoir fits of track-pairs

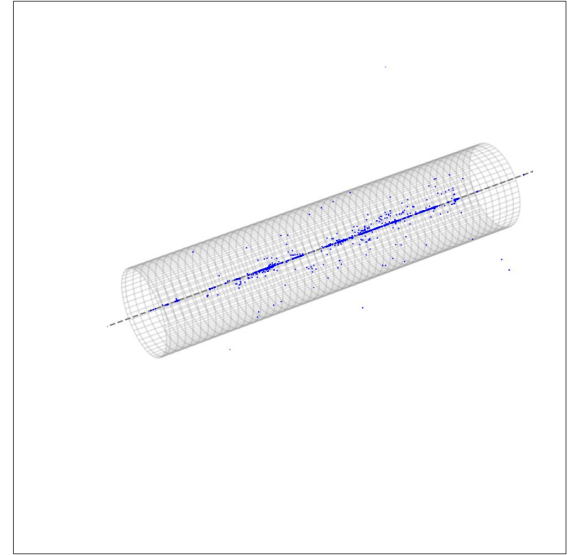
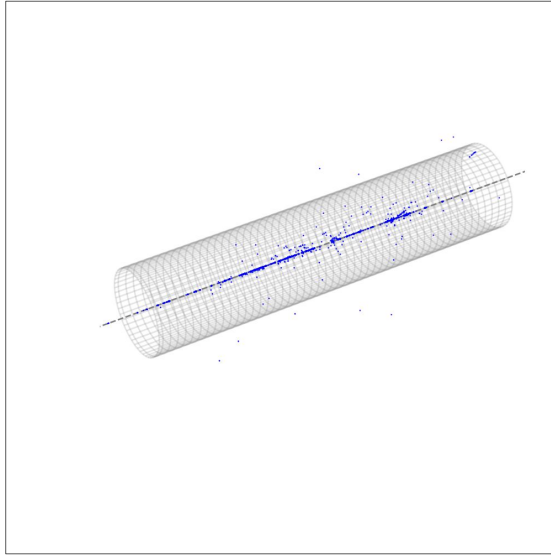
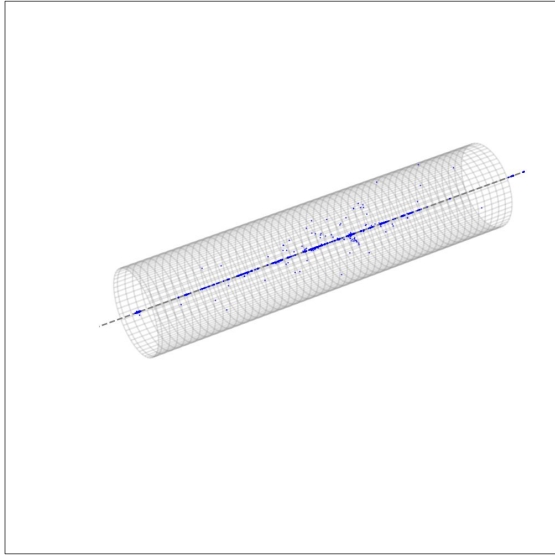


Utilizing χ^2 from Billoir fits of track-pairs

- Observation: If two tracks are from the same vertex, the fit always converges.
- Strategy: Create (overlapping) subsets of tracks.
 1. Start with an arbitrary track.
 2. Add all the tracks compatible with it to the set.
 3. Iterate over the tracks in set.
 - For each existing track, add all the compatible tracks.
 4. If any new track was added in step 3, repeat step 3.
- These subsets can be taken as “bounding boxes”, which can also be overlapped.



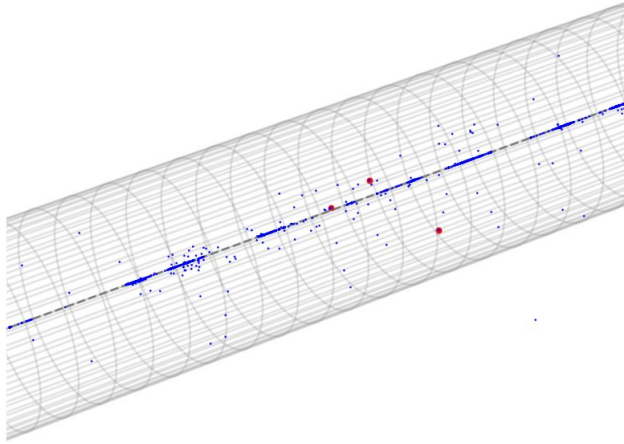
Visualizing track-pair vertices in ODD



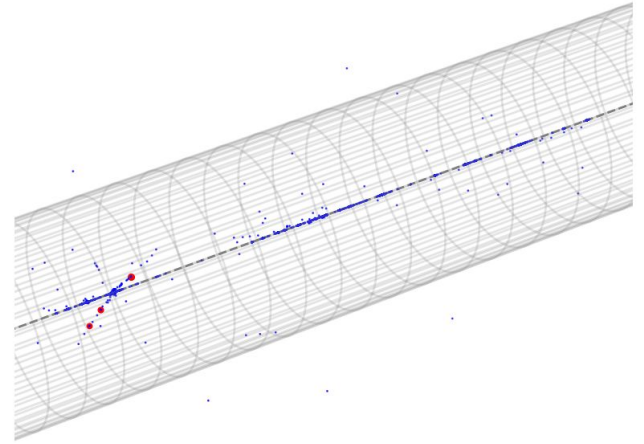
Finding multi-track vertices

- The baseline: Naive clustering solely based on position in 3D space.
 - Trivial to implement using `sklearn.cluster.DBSCAN`
 - `eps = 0.1`
 - Maximum distance between two samples to be considered as neighbours.
 - `min_samples = 1`
 - A “cluster” will be created even for a lonely two-track vertex.
 - Extremely fast.
- Visualizing the clusters:
 - Clusters with at least 2 two-track vertices has been shown in red on the next slide.
 - Transverse distance from the beam axis is required to be greater than 1 mm to eliminate the primary (including pile-up) vertices.

Visualizing vertices clusters in ODD



Event: 0



Event: 94

Performance metric for vertex finding

- Strategy for truth-matching:
 - At least 50% of the tracks from a truth vertex need to be associated with the reco vertex.
 - If multiple truth vertices match with a single reco vertex, the one with higher number of *common* tracks is chosen.
 - If ambiguity still persists, the one closest to the cluster-centre is chosen.
- For measuring the performance of the clustering method:
 - Only the truth vertices with 3 or more tracks have been considered.
 - The fraction of such vertices matched with a reco one is a metric for clustering.
- For measuring the performance of “Billoir fast vertex fit” itself:
 - Only the two-track truth vertices have been considered.
 - For truth matching, **both** tracks are required to be associated with the reco one.

Framing vertex finding as a clustering problem

- Most obvious approach: A vertex is a cluster of tracks.
 - Consider the tracks as nodes.
 - Track params are node features.
 - Different quantities from track-pair Billoir fit can be taken as edge features.
 - This is the chosen strategy for MaskFormer as well as the previous primary-vertexing work.
- My proposal: A vertex is a cluster of one or more two-track vertices.
 - Each track-pair satisfying Billoir fit is a node.
 - Different quantities from track-pair Billoir fit are node features.
 - No edge feature. The Euclidean distance might be taken as one.
 - Immediately offers us a bounding box heuristic.
 - For two-track vertices, the performance is guaranteed to be at least as good as Billoir fit.

Next steps

- Calculate the performance of naive clustering (using DBSCAN), using the truth-matching strategy described before, to establish a baseline.
- Investigate the similarities between clustering of pixels (in an image) and track-pair vertices (from Billoir fits).
 - Can we use the lifted multicut algorithm out of the box?
- Utilize hypergraphs to cluster tracks (without using any heuristics).
 - Known to be extremely parameter-efficient and fast as compared to attention-based models.
 - Example: [HyPER](#)
 - Each predicted hyperedge will correspond to a multi-track vertex.