Universität Münster

Entanglement in Top-Quark Pair Production at the LHC

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Motivation

- Quantum entanglement is a fundamental aspect of quantum mechanics.
- Top-quark pair production provides a unique testing ground to understand entanglement in high-energy physics.
- Understanding entanglement with [(arXiv:2003.02280)] and comparing with data [(arXiv:2406.03976v2)], [(arXiv:2311.07288v3)]

The Top Quark

- Heaviest Quark: $m_t \approx 173,3\,\text{GeV}$
- Charge: $2/3$
- Spin: $\frac{1}{2}$
- lifetime: $\propto 10^{-25}$ s
- timescales:
  - hadronisation: $\propto 10^{-23}$ s
  - spin decorrelation $\propto 10^{-21}$ s

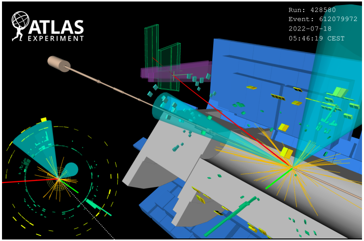

Figure: Modulation of $t\bar{t}$ production at LHC [(4)]

Cross Section

- Hadronic Cross Section:

$$\sigma = \sum_{ij} \int_0^1 dx_1 dx_2\, f_i(x_1, \mu_f) f_j(x_2, \mu_f)\, \sigma_{ij}$$

Cross Section

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PDFs

Cross Section

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PDFs

partonic cross section

Density Matrix

$$\langle\hat{O}\rangle = \mathrm{Tr}(\rho\hat{O})$$

$$\rho = \frac{\mathcal{M}(\lambda_1,\lambda_2)\mathcal{M}^\dagger(\lambda_1',\lambda_2')}{|\overline{\mathcal{M}}|^2}$$

- $\mathcal{M}$: scattering amplitude
- λ_i : Spin eigenvalues
- $\mathrm{Tr}(\rho) = 1$
- $|\overline{\mathcal{M}}|^2 = 4\tilde{A}$

connection to relative cross section

$$\beta = \sqrt{1 - \frac{4m_t^2}{M_{t\bar{t}}^2}}$$

$$\frac{d\sigma_I}{d\Omega dM_{t\bar{t}}} = \frac{\alpha_s^2\beta}{2M_{t\bar{t}}^2}\tilde{A}^I$$

Spin density matrix ρ

$\sigma^i \equiv$ Pauli matrix

Density matrix ρ:

$$\rho = \frac{1}{4}\left(I_4 + \sum_i (B_i^+ \sigma^i \otimes I_2 + B_i^- I_2 \otimes \sigma^i) + \sum_{i,j} C_{ij} \sigma^i \otimes \sigma^j \right)$$

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non-polarized component

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non-polarized component

polarized component

spin-correlation component

Entanglement criteria

- spin density matrix

$$\rho = \frac{I_4 + \sum_i (B_i^+ \sigma^i \otimes I_2 + B_i^- I_2 \otimes \sigma^i) + \sum_{i,j} C_{ij} \sigma^i \otimes \sigma^j}{4}$$

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- separable state: $\rho = \sum_n p_n \rho_n^a \otimes \rho_n^b$
- negation of above statement is sufficient condition for entanglement

Peres-Horodecki criterion

- spin density matrix
  $\rho = \frac{I_4 + \sum_i (B_i^+ \sigma^i \otimes I_2 + B_i^- I_2 \otimes \sigma^i) + \sum_{i,j} C_{ij} \sigma^i \otimes \sigma^j}{4}$
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- state is separable if

$$\rho^{T_2} = \sum_n p_n \rho_n^a \otimes \left(\rho_n^b\right)^T \quad \text{is non-negative}$$

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- Skipping mathematical details leads to:

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- negation of above statement is sufficient condition for entanglement
- Skipping mathematical details leads to:

$$\begin{aligned} -C_{33} + \left|C_{11} + C_{22}\right| - 1 > 0, \quad C_{33} < 0, \\ C_{33} + \left|C_{11} - C_{22}\right| - 1 > 0, \quad C_{33} > 0. \end{aligned}$$

Peres-Horodecki criterion

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$$-C_{33} + |C_{11} + C_{22}| - 1 > 0, \quad C_{33} < 0,$$
$$C_{33} + |C_{11} - C_{22}| - 1 > 0, \quad C_{33} > 0.$$

- Define : $\delta = -C_{33} + |C_{11} + C_{22}| - 1 \quad \Rightarrow$ entanglement with $\delta > 0$

Entanglement in gluon-fusion

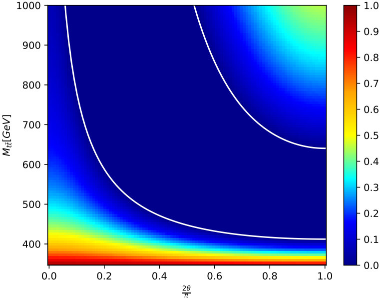
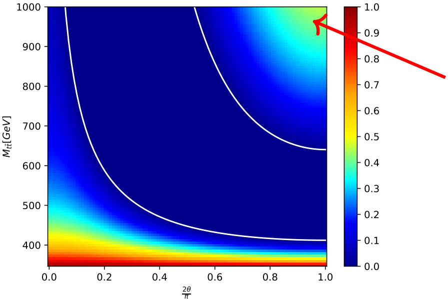


Figure: Concurrence $\frac{\max(\delta,0)}{2}$

- high energy and large angle: triplet state: $|1,0\rangle$
- Threshold: singlet state: $|0,0\rangle$

Entanglement in gluon-fusion



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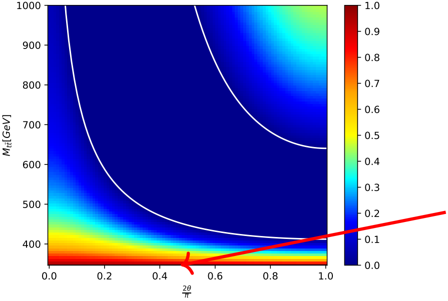

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Entanglement in quark-antiquark annihilation

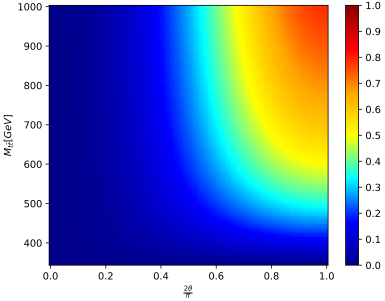

Figure: Concurrence $\frac{\max(\delta,0)}{2}$

- high energy and large angle: triplet state: $|1,0\rangle$
- Threshold or small angle: triplet states $|1,1\rangle$ and $|1,-1\rangle$

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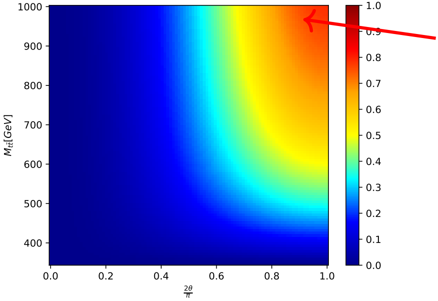

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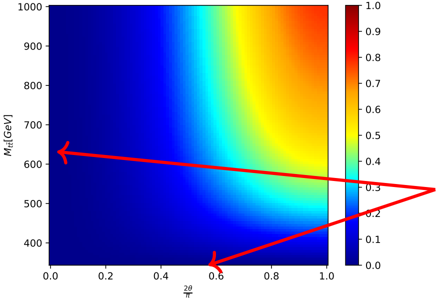

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- high energy and large angle: triplet state: $|1,0\rangle$
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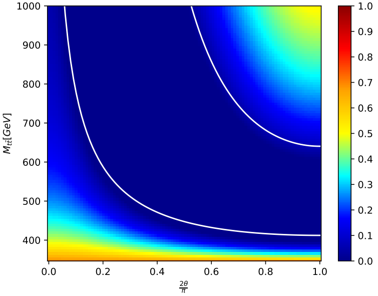
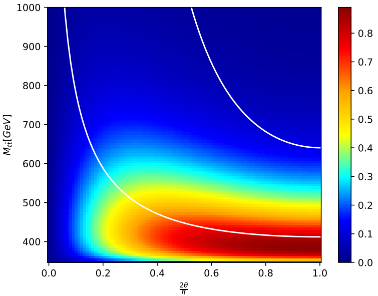
Entanglement from proton-proton collision



(a) Concurrence $\frac{\max(\delta,0)}{2}$



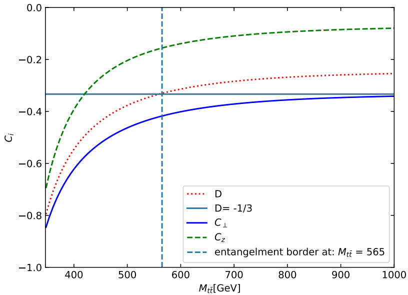
(b) Differential cross section

- Gluon fusion dominant counterpart
- Large cross section close to threshold

Angle Averaging and Mass Integration

- Introduce observable: $D = -\frac{1-\delta}{3} = \frac{1}{3}\mathrm{Tr(C)}$
- **Peres-Horodecki** criterion for entanglement: $D < -\frac{1}{3}$

  - angle averaged and mass integrated over bins $[m_t, M_{t\bar{t}}]$
  - $C_{11} = C_{22} = C_{\perp}$
  - $C_{33} = C_z$
  - Border at $M_{t\bar{t}} = 565$ GeV.



Results are in agreement with the paper [(arXiv:2003.02280)]

Comparison with Atlas/CMS

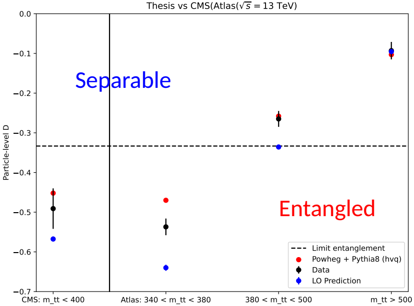


Figure: Atlas: [(arXiv:2311.07288v3)],
CMS: [(arXiv:2406.03976v2)]

Comparison with Atlas/CMS

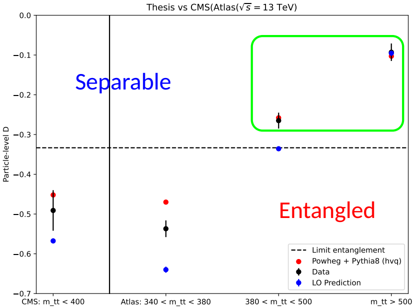


Figure: Atlas: [(arXiv:2311.07288v3)], CMS: [(arXiv:2406.03976v2)]

- High Energies
  $\Rightarrow$ agreement

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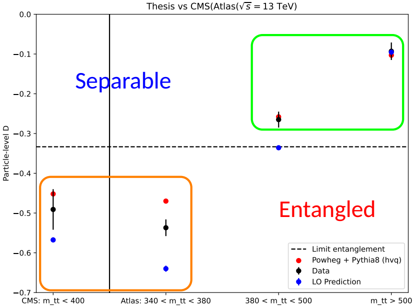
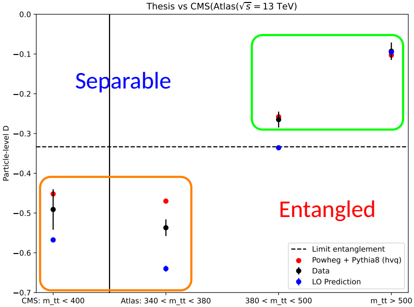


Figure: Atlas: [(arXiv:2311.07288v3)], CMS: [(arXiv:2406.03976v2)]

- High Energies
  ⇒ agreement
- Low Energies
  ⇒ mismatch

Comparison with Atlas/CMS

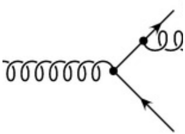


- High Energies
  - ⇒ agreement
- Low Energies
  - ⇒ mismatch

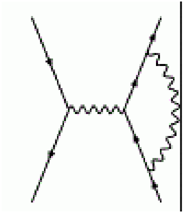
Figure: Atlas: [(arXiv:2311.07288v3)],
CMS: [(arXiv:2406.03976v2)]

⇒ non-relativistic QCD processes not accounted

Outlook: Soft Limit Correction



real emission



virtual correction

- consider only soft limit

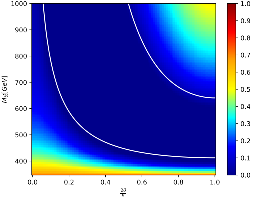
$\Rightarrow$ NLL $\rightarrow$ Momentum space: $\mathcal{O}\left(\alpha^k \left[\frac{\ln^{2k-2}(1-z)}{1-z}\right]_+\right)$

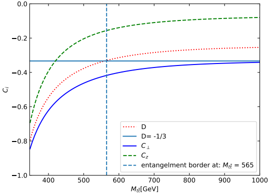
Thank You!

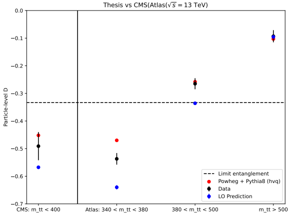
$$\rho = \frac{1}{4}\left[I_4 + \sum_i (B_i^+ \sigma^i \otimes I_2 + B_i^- I_2 \otimes \sigma^i) + \sum_{i,j} C_{ij}\sigma^i \otimes \sigma^j\right]$$

Entanglement criteria: $\delta = -C_{33} + |C_{11} + C_{22}| - 1$

Observable: $D = -\frac{1}{3}\mathrm{Tr}\, C < -\frac{1}{3}$







- ✓ Agreement at high energies
- × Mismatch at low energies (threshold)
- ⇒ NLO, soft limit resummation

Sources I

[(arXiv:2003.02280)] Yoav Afik and Juan Ramón Muñoz de Nova. "Entanglement and quantum tomography with top quarks at the LHC". In: *European Physical Journal Plus* 136.11 (Sept. 2021), pp. 1–10. DOI: 10.1140/epjp/s13360-021-01902-1. eprint: 2003.02280.

Sources II

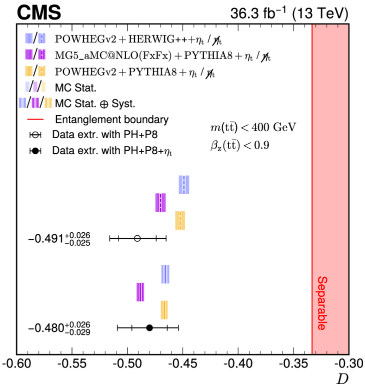
[(arXiv:2406.03976v2)] The CMS Collaboration. "Observation of quantum entanglement in top quark pair production in proton proton collisions". In: *Reports on Progress in Physics* 87.11 (Oct. 2024), p. 117801. ISSN: 1361-6633. DOI: 10.1088/1361-6633/ad7e4d. eprint: 2406.03976v2. URL: http://dx.doi.org/10.1088/1361-6633/ad7e4d.

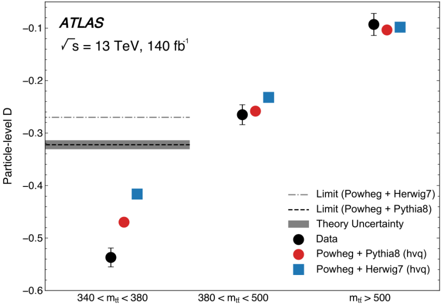
Sources III

[(arXiv:2311.07288v3)] G. Aad et al. “Observation of quantum entanglement with top quarks at the ATLAS detector”. In: *Nature* 633.8030 (Sept. 2024), pp. 542–547. ISSN: 1476-4687. DOI: 10.1038/s41586-024-07824-z. eprint: 2311.07288v3. URL: http://dx.doi.org/10.1038/s41586-024-07824-z.

[(4)] *ATLAS Event Display of Top-pair production in 13.6 TeV collisions during Run 3.* URL: https://cds.cern.ch/record/2842591.

Backup: Plots





Backup: Separability Example

- state is separable if

$$\rho = \sum_n p_n \rho_n^a \otimes \rho_n^b, \quad \text{with} \sum_n p_n = 1, \quad p_n \geq 0$$

$$\begin{aligned}\rho &= |\uparrow\uparrow\rangle\langle\uparrow\uparrow| \\ &= \begin{pmatrix}1\\0\\0\\0\end{pmatrix} \otimes \begin{pmatrix}1 & 0 & 0 & 0\end{pmatrix} \\ &= \begin{pmatrix}1&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix}\end{aligned}$$

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Backup: Matrix Elements

- $q\bar{q}$–Channel

$$\tilde{A}^{q\bar{q}} = F_q(2 - \beta^2 \sin^2\theta)$$
$$\tilde{C}_{rr}^{q\bar{q}} = F_q(2 - \beta^2)\sin^2\theta$$
$$\tilde{C}_{nn}^{q\bar{q}} = -F_q\beta^2 \sin^2\theta$$
$$\tilde{C}_{kk}^{q\bar{q}} = F_q(2\cos^2\theta + \beta^2 \sin^2\theta)$$

- gg–Channel

$$\tilde{A}^{gg} = 2F_g(\theta)\left(1 + 2\beta^2\sin^2\theta - \beta^4(1+\sin^4\theta)\right)$$
$$\tilde{C}_{rr}^{gg} = -2F_g(\theta)\left(1 - \beta^2(2-\beta^2)(1+\sin^4\theta)\right)$$
$$\tilde{C}_{nn}^{gg} = -2F_g(\theta)\left(1 - 2\beta^2 + \beta^4(1+\sin^4\theta)\right)$$
$$\tilde{C}_{kk}^{gg} = -2F_g(\theta)\left[1 - \beta^2\frac{\sin^2 2\theta}{2} - \beta^4(1+\sin^4\theta)\right]$$