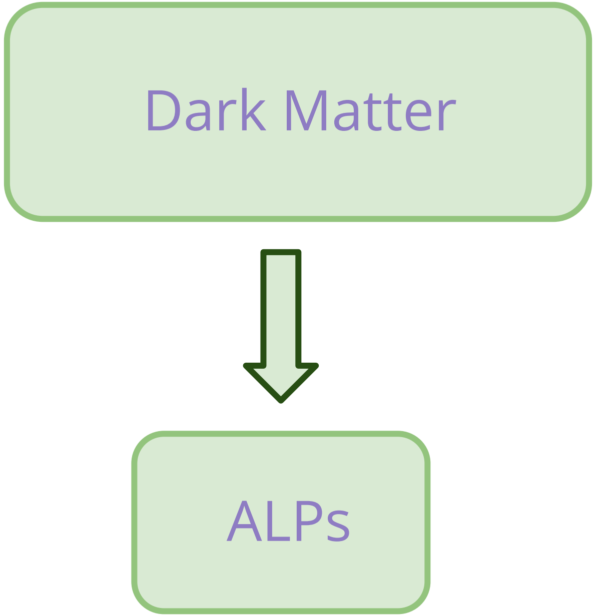
# Axion-Like-Particles in External Magnetic Fields

Ozan Semin
University of Tuebingen
56. Herbstschule Bad Honnef

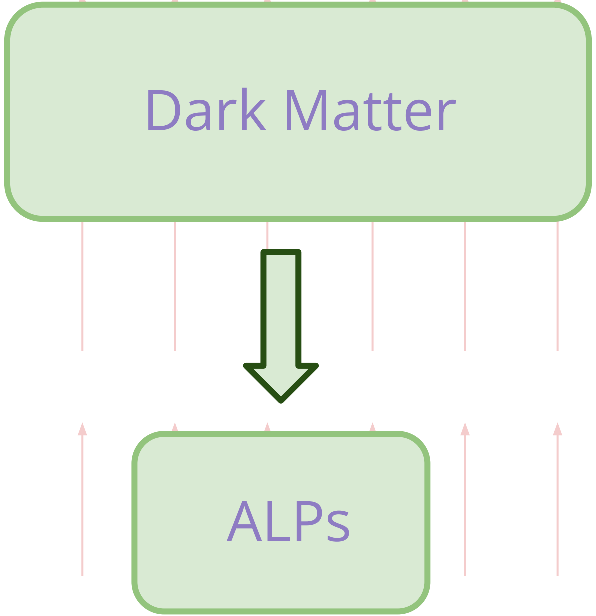
# Today's Topics

Dark Matter

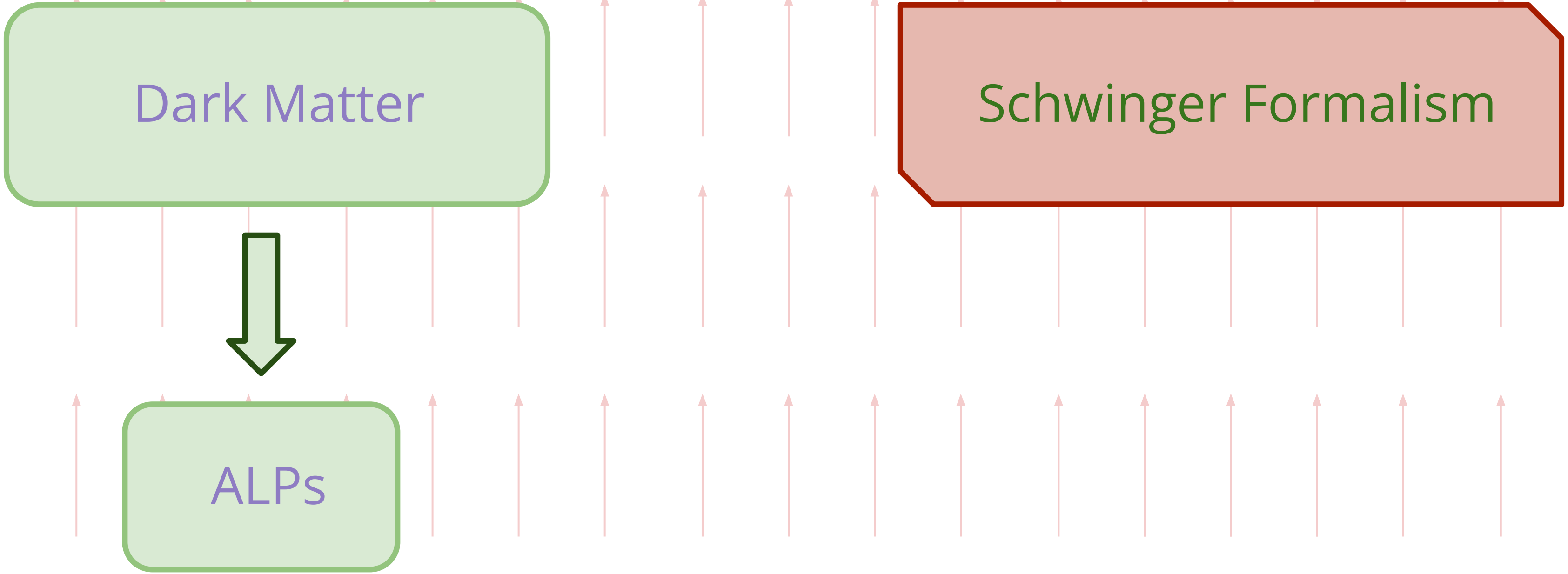
# Today's Topics



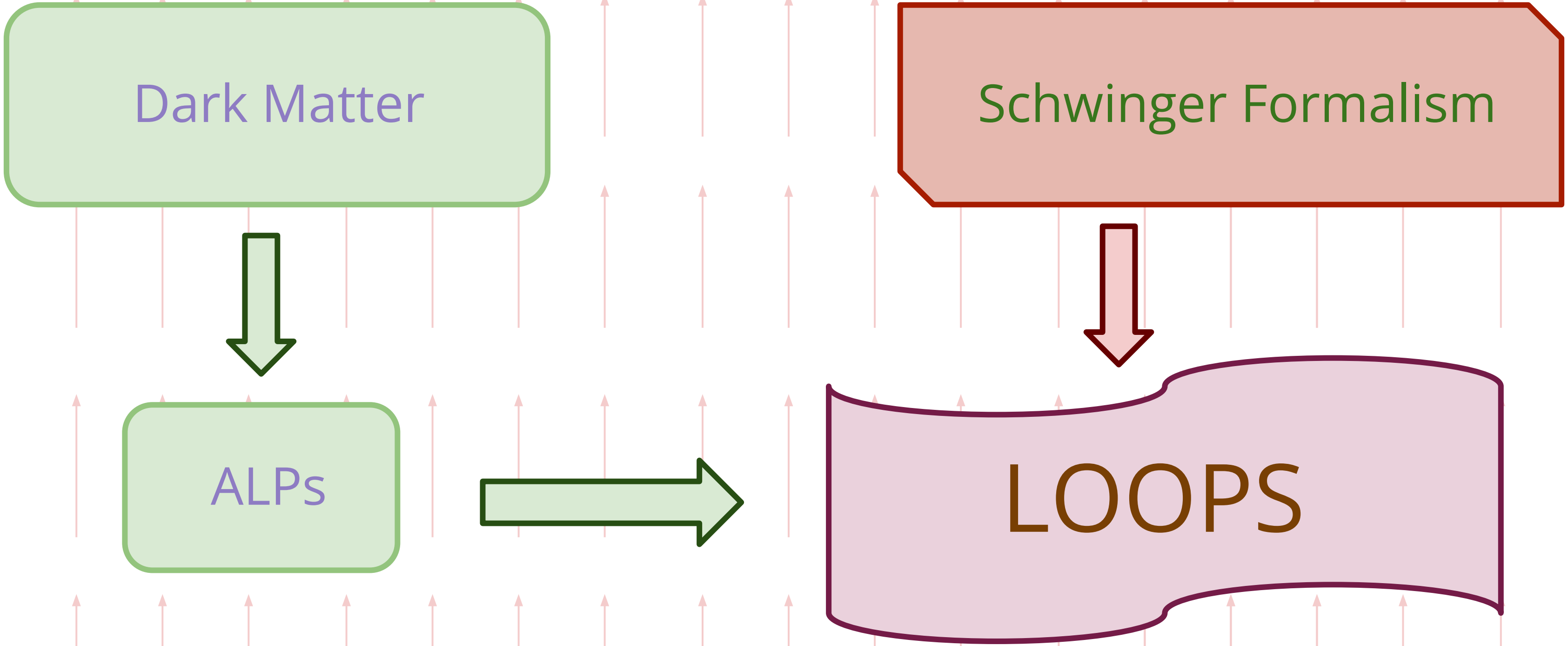
# Today's Topics



# Today's Topics



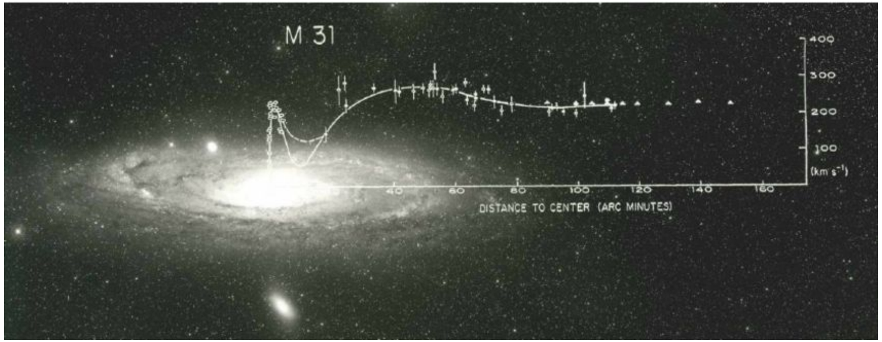
# Today's Topics



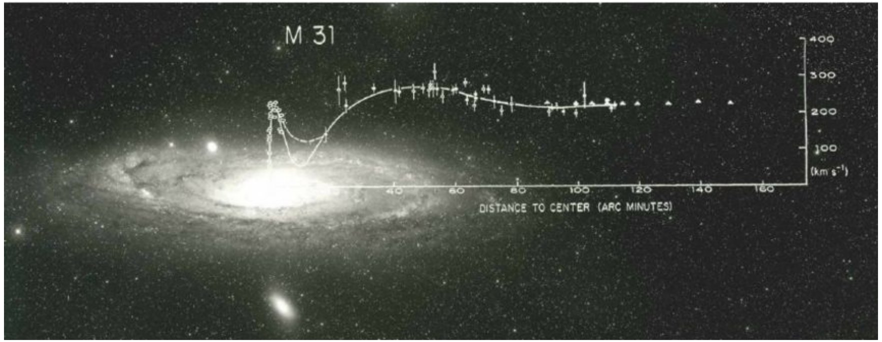
# Mysteries

- Neutrino Mass and oscillations
- Muon g-2
- Hierarchy problem
- Matter-Antimatter asymmetry
- QCD-CP problem
- ……

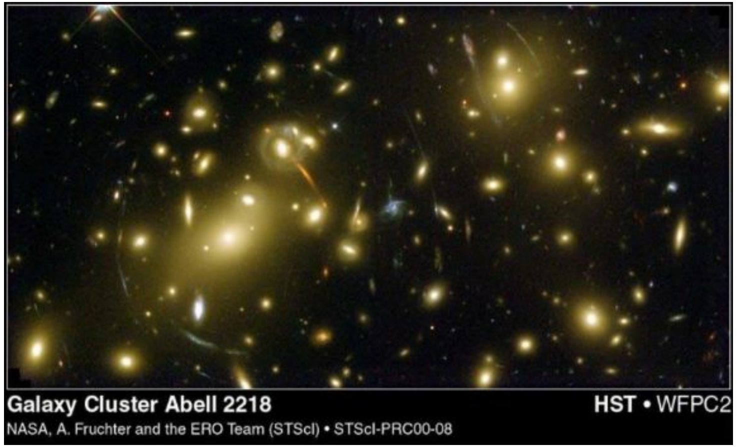
BUT THERE IS MORE!



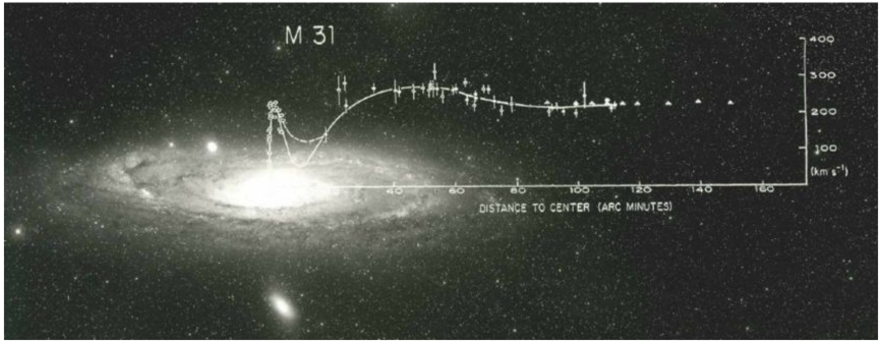

Arms of galaxies rotate faster than out expectations!



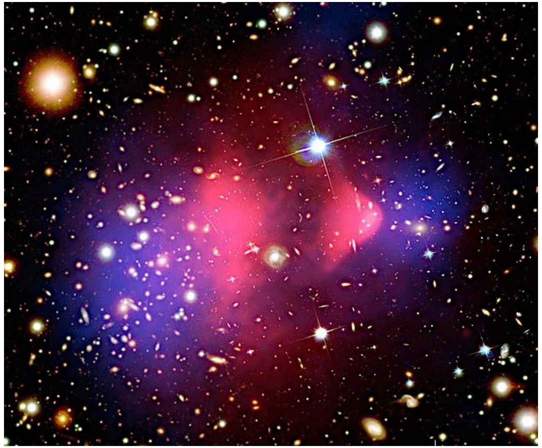

Arms of galaxies rotate faster than out expectations!



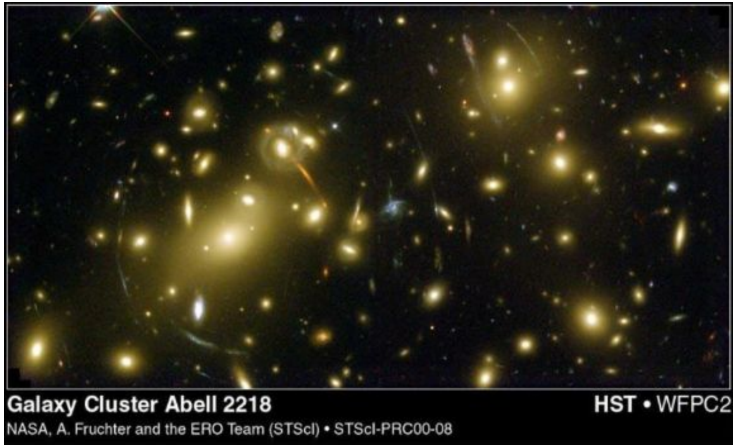

Clusters bend light too much!




Arms of galaxies rotate faster than out expectations!



Bullet Cluster is very weird




Clusters bend light too much!

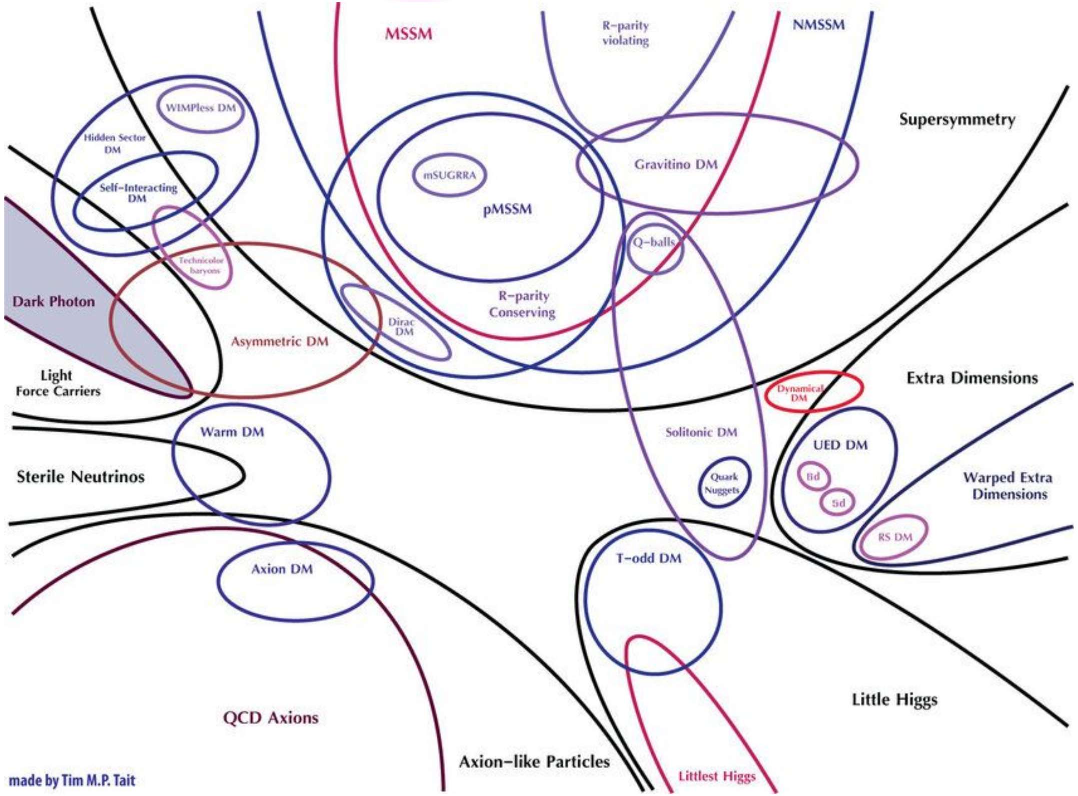
# Solution:

# New Particles

Solution:

Solution:

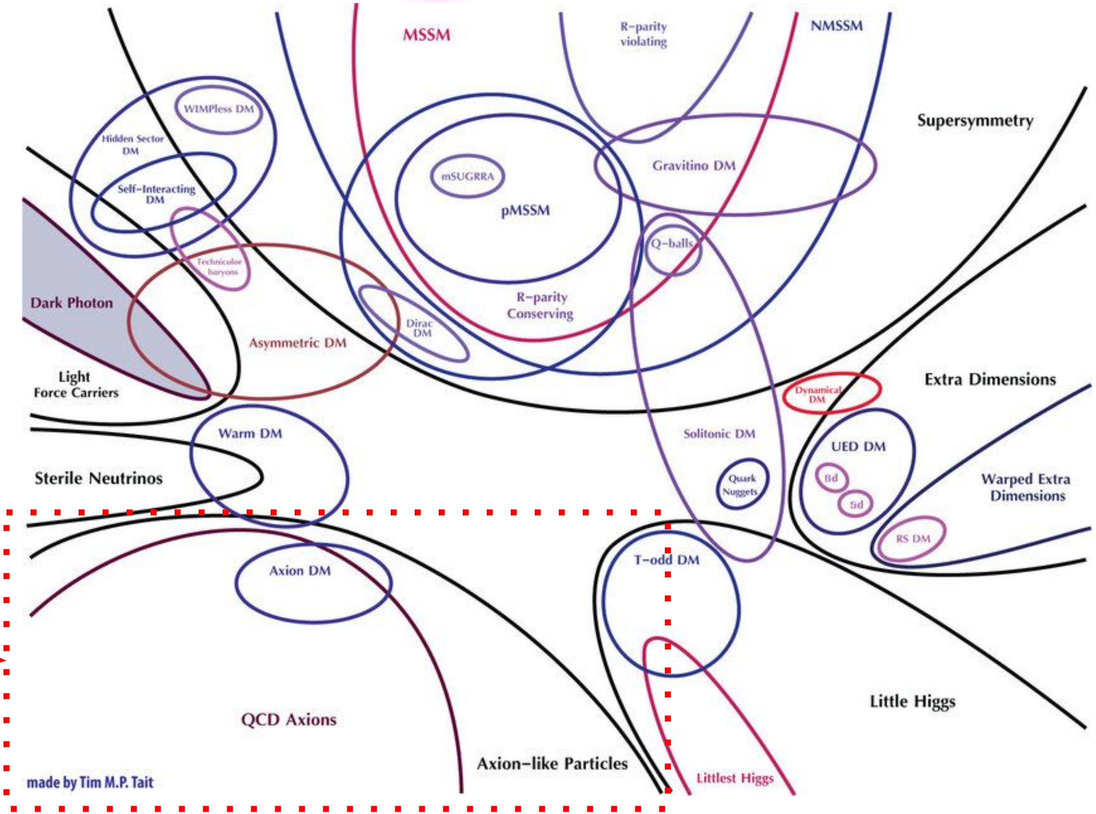
# New Particles



Solution: **New Particles**

We pick a very general category of particles

ALPs



# Axion-Like-Particles

- Pseudoscalar
- Shift-symmetric $a \to a + x$
- Mass is independent of the symmetry breaking scale!
- Modeled by EFTs

# Axion-Like-Particles

- Pseudoscalar
- Shift-symmetric $a \rightarrow a + x$
- Mass is independent of the symmetry breaking scale!
- Modeled by EFTs

$$\mathcal{L}^5_{\text{eff}} = \frac{\partial^\mu a}{\Lambda} \sum_f \bar{\psi}_f C_{ff} \gamma^5 \gamma_\mu \psi_f + \frac{a}{\Lambda} \left( g_s^2 C_{GG} G^A_{\mu\nu} \tilde{G}^{\mu\nu,A} + g^2 C_{WW} W^A_{\mu\nu} \tilde{W}^{\mu\nu,A} + g'^2 C_{BB} B_{\mu\nu} \tilde{B}^{\mu\nu} \right)$$

# Axion-Like-Particles

- Pseudoscalar
- Shift-symmetric $a \rightarrow a + x$
- Mass is independent of the symmetry breaking scale!
- Modeled by EFTs

$$\mathcal{L}_{\text{eff}}^{5} = \frac{\partial^\mu a}{\Lambda} \sum_f \bar{\psi}_f C_{ff} \gamma^5 \gamma_\mu \psi_f + \frac{a}{\Lambda} \left( g_s^2 C_{GG} G^A_{\mu\nu} \tilde{G}^{\mu\nu,A} + g^2 C_{WW} W^A_{\mu\nu} \tilde{W}^{\mu\nu,A} + g'^2 C_{BB} B_{\mu\nu} \tilde{B}^{\mu\nu} \right)$$

EWSB

$$\mathcal{L}_{a\gamma\gamma} = \frac{g_{a\gamma\gamma}}{4} a F_{\mu\nu} \tilde{F}^{\mu\nu}, \quad \text{with} \quad g_{a\gamma\gamma} = 4\frac{e^2}{\Lambda}(C_{BB} + C_{WW}) \equiv 4\frac{e^2}{\Lambda} C_{\gamma\gamma}$$

The most probed part:

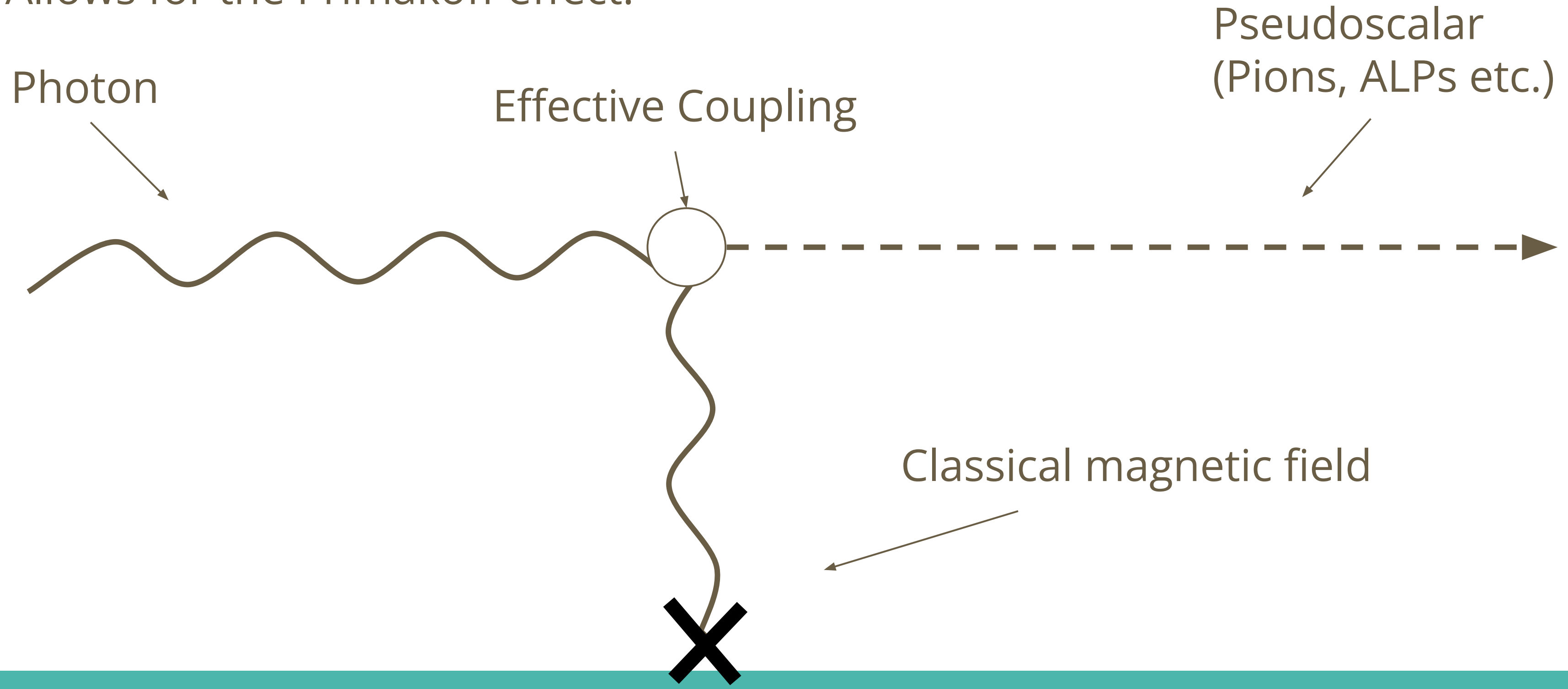
$$\mathcal{L}_{a\gamma\gamma} = \frac{g_{a\gamma\gamma}}{4} a F_{\mu\nu} \tilde{F}^{\mu\nu}$$

The most probed part: $$\mathcal{L}_{a\gamma\gamma} = \frac{g_{a\gamma\gamma}}{4} a F_{\mu\nu}\tilde{F}^{\mu\nu}$$

Allows for the Primakoff effect:

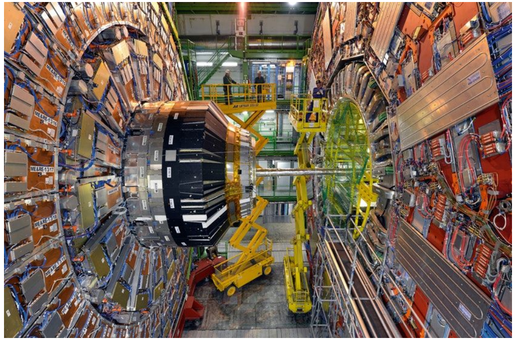
The most probed part: $$\mathcal{L}_{a\gamma\gamma} = \frac{g_{a\gamma\gamma}}{4} a F_{\mu\nu} \tilde{F}^{\mu\nu}$$

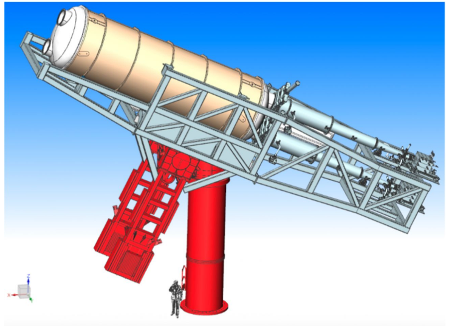
Allows for the Primakoff effect:



CERN Axion Solar Telescope (CAST)

IAXO

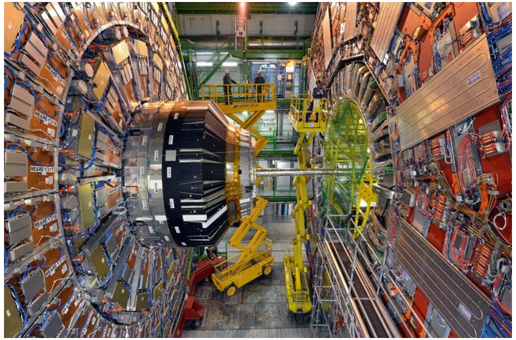


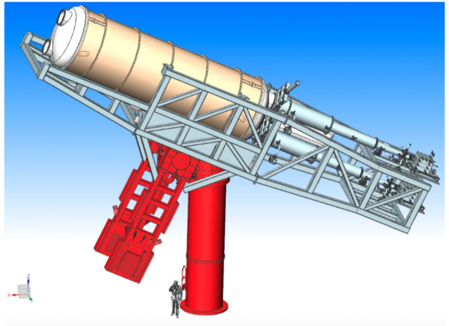


These, and many more, aim to constraint the ALP-Photon-Photon coupling using the Primakoff effect!

CERN Axion Solar Telescope (CAST)

IAXO

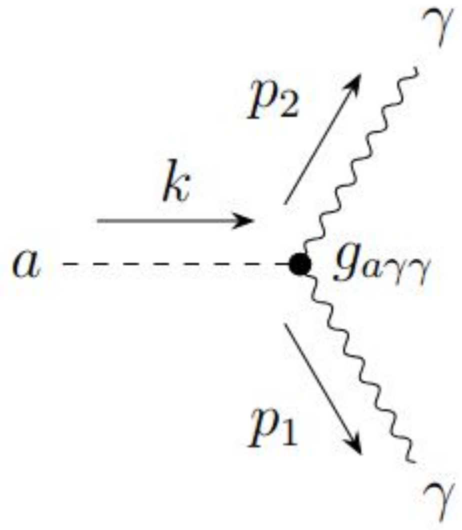




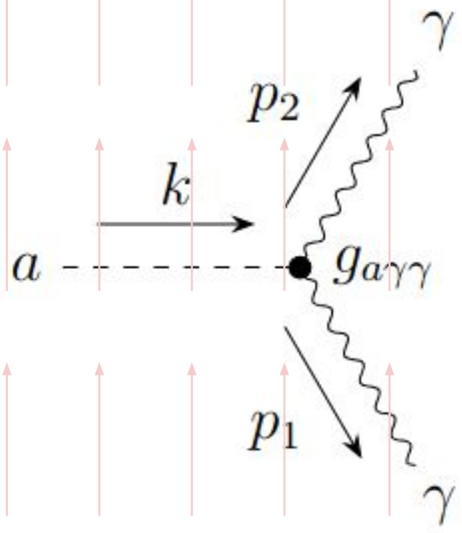
These, and many more, aim to constraint the ALP-Photon-Photon coupling using the Primakoff effect!

**.... or basically any process where magnetic fields are present: Ion collisions, magnetars....**

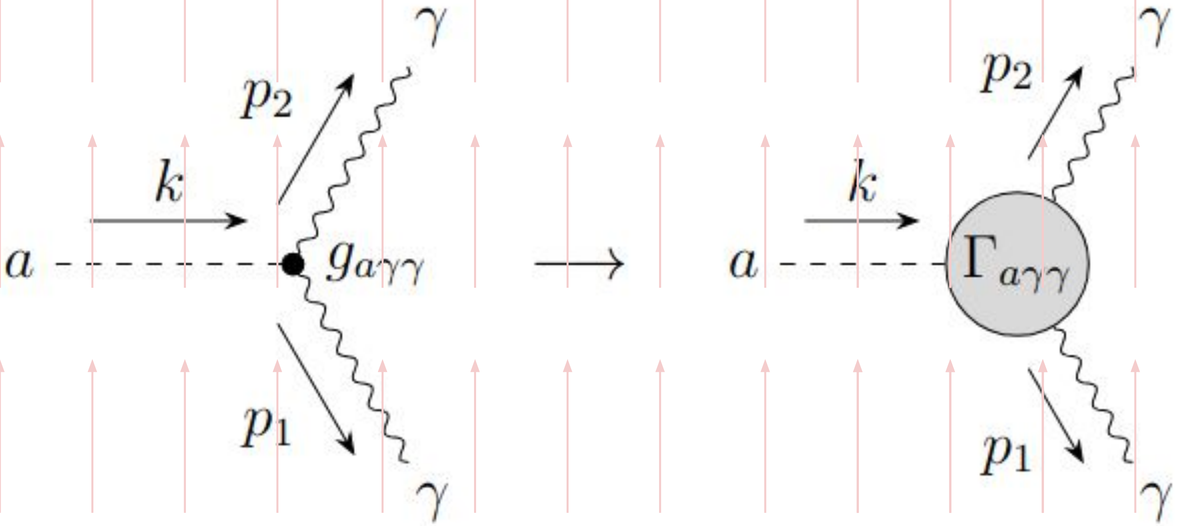
But we are interested in something different:
How does the ALP-Photon-Photon coupling term even behave under external fields?



But we are interested in something different:
How does the ALP-Photon-Photon coupling term even behave under external fields?



But we are interested in something different:
How does the ALP-Photon-Photon coupling term even behave under external fields?



# Schwinger Formalism

Typically we work with “free” particle states.

If there is an external magnetic field, this can be treated as a classical potential, instead of dynamic photon fields!

# Schwinger Formalism

Typically we work with “free” particle states.

If there is an external magnetic field, this can be treated as a classical potential, instead of dynamic photon fields!

But we also have the usual, quantized, dynamic photons.

# Schwinger Formalism

Typically we work with “free” particle states.

If there is an external magnetic field, this can be treated as a classical potential, instead of dynamic photon fields!

But we also have the usual, quantized, dynamic photons.

$$A_\mu = A_\mu^{\mathrm{c}} + A_\mu^{\mathrm{q}}$$

Classical Quantum

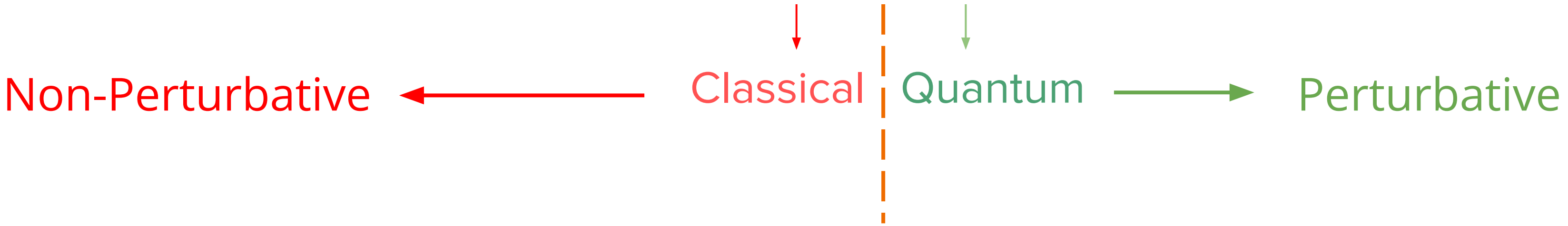
# Schwinger Formalism

Typically we work with “free” particle states.

If there is an external magnetic field, this can be treated as a classical potential, instead of dynamic photon fields!

But we also have the usual, quantized, dynamic photons.

$$A_\mu = A_\mu^{\mathrm{c}} + A_\mu^{\mathrm{q}}$$



QED Lagrangian becomes:

$$\mathcal{L}_{\text{EM}} = -\frac{1}{4} F_{\mu\nu} \tilde{F}^{\mu\nu} + \bar{\psi} \Bigg( \underbrace{i\partial\!\!\!/ - e_f A\!\!\!/_{\text{c}}}_{=:iD\!\!\!\!/} - m \Bigg) \psi - e_f \bar{\psi} A\!\!\!/_{\text{q}} \psi$$

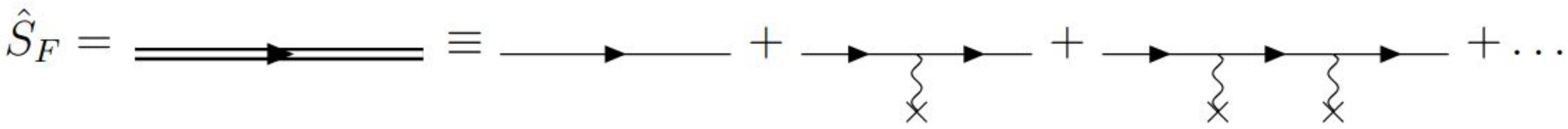
Dirac wave functions do not work anymore!

We do not have to compute the solution to the modified Dirac Eq., only the propagators are needed.

$$(i\not{D} - m)S_F(x, x') = \delta^4(x - x')$$

We do not have to compute the solution to the modified Dirac Eq., only the propagators are needed.

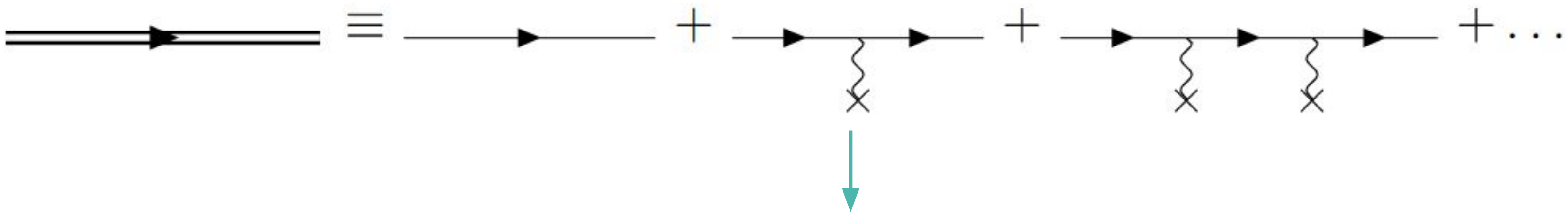
$$(i\not{D} - m)S_F(x, x') = \delta^4(x - x')$$



We do not have to compute the solution to the modified Dirac Eq., only the propagators are needed.

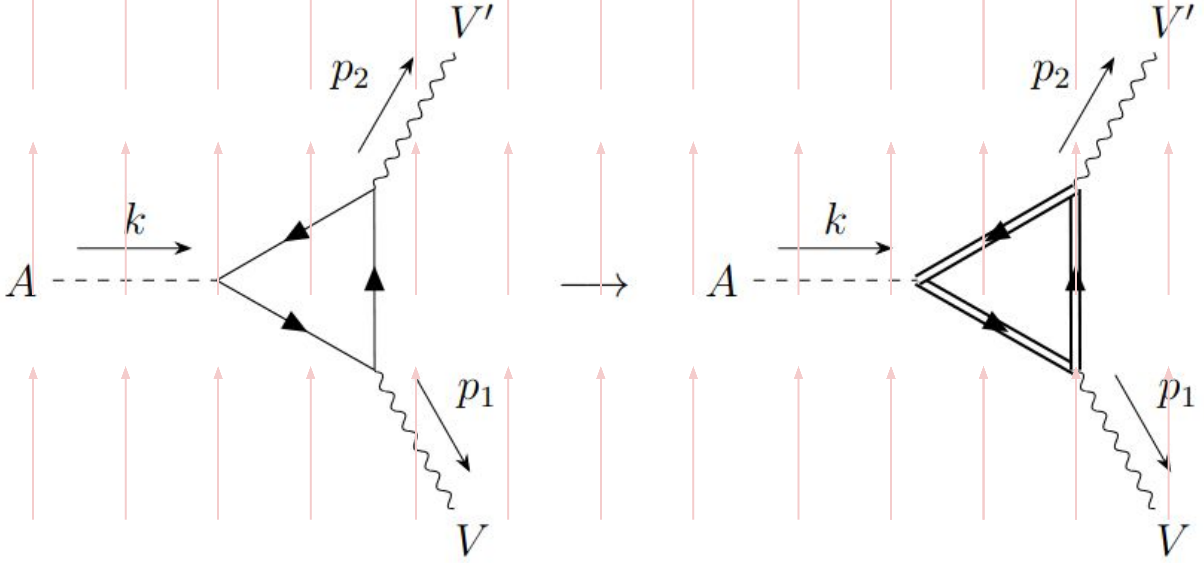
$$(i\not{D} - m)S_F(x, x') = \delta^4(x - x')$$

$\hat{S}_F =$



Primakoff

But we are interested in something different:
How does the ALP-Photon-Photon coupling term even behave under external fields?



Lorentz-invariance is lost, and the resulting propagator is kinda complicated.

$$S(p) = \frac{i}{\not{p} - m}$$

Lorentz-invariance is lost, and the resulting propagator is kinda complicated.

$$S(p) = \frac{i}{\not{p} - m}$$

becomes....

Lorentz-invariance is lost, and the resulting propagator is kinda complicated.

$$S_f(X) = e^{i\Phi(x,x')} \frac{i\beta}{2(4\pi)^2} \int_0^\infty \frac{ds}{s\sin(\beta s)} \Bigg\{ \frac{1}{s} \Big[ \cos(\beta s) X_\mu \tilde{\Lambda}^{\mu\nu} \gamma_\nu + i \sin(\beta s) X_\mu \hat{\tilde{F}}^{\mu\nu} \gamma_\nu \gamma^5 \Big]$$

$$- \frac{\beta}{\sin(\beta s)} X_\mu \tilde{\Lambda}^{\mu\nu} \gamma_\nu + m_f \left[ 2\cos(\beta s) + \sin(\beta s) \gamma_\mu \hat{F}^{\mu\nu} \gamma_\nu \right] \Bigg\}$$

$$\times\, e^{-i\left(m^2 s + \frac{X_\mu \tilde{\Lambda}^{\mu\nu} X_\nu}{4s} - \frac{\beta}{4\tan(\beta s)} X_\mu \tilde{\Lambda}^{\mu\nu} X_\nu\right)}.$$

with $$\Phi(x,x') = -e_f \int_{x'}^{x} \mathrm{d}\xi^\mu \left[ A_\mu^{\mathrm{c}} + \frac{F_{\mu\nu}^{\mathrm{c}} (\xi - x')^\nu}{2} \right]$$

Lorentz-invariance is lost, and the resulting propagator is kinda complicated.

$$S_f(X) = e^{i\Phi(x,x')} \frac{i\beta}{2(4\pi)^2} \int_0^\infty \frac{ds}{s\sin(\beta s)} \left\{ \frac{1}{s} \left[ \cos(\beta s) X_\mu \tilde{\Lambda}^{\mu\nu} \gamma_\nu + i \sin(\beta s) X_\mu \hat{\tilde{F}}^{\mu\nu} \gamma_\nu \gamma^5 \right] \right.$$
$$\left. - \frac{\beta}{\sin(\beta s)} X_\mu \tilde{\Lambda}^{\mu\nu} \gamma_\nu + m_f \left[ 2\cos(\beta s) + \sin(\beta s) \gamma_\mu \hat{F}^{\mu\nu} \gamma_\nu \right] \right\}$$
$$\times e^{-i\left(m^2 s + \frac{X_\mu \tilde{\Lambda}^{\mu\nu} X_\nu}{4s} - \frac{\beta}{4\tan(\beta s)} X_\mu \tilde{\Lambda}^{\mu\nu} X_\nu\right)}.$$

with $$\Phi(x, x') = -e_f \int_{x'}^{x} d\xi^\mu \left[ A^c_\mu + \frac{F^c_{\mu\nu} (\xi - x')^\nu}{2} \right]$$

Lorentz-invariance is lost, and the resulting propagator is kinda complicated.

$$S_f(X) = e^{i\Phi(x,x')} \frac{i\beta}{2(4\pi)^2} \int_0^\infty \frac{ds}{s\sin(\beta s)} \left\{ \frac{1}{s} \left[ \cos(\beta s) X_\mu \tilde{\Lambda}^{\mu\nu} \gamma_\nu + i \sin(\beta s) X_\mu \hat{\tilde{F}}^{\mu\nu} \gamma_\nu \gamma^5 \right] - \frac{\beta}{\sin(\beta s)} X_\mu \tilde{\Lambda}^{\mu\nu} \gamma_\nu + m_f \left[ 2\cos(\beta s) + \sin(\beta s) \gamma_\mu \hat{F}^{\mu\nu} \gamma_\nu \right] \right\}$$

$$\times\, e^{-i\left(m^2 s + \frac{X_\mu \tilde{\Lambda}^{\mu\nu} X_\nu}{4s} - \frac{\beta}{4\tan(\beta s)} X_\mu \tilde{\Lambda}^{\mu\nu} X_\nu\right)}.$$

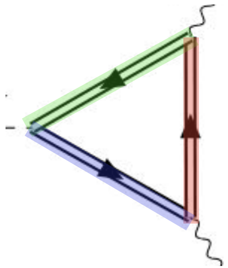
Source of the problem

with $$\Phi(x,x') = -e_f \int_{x'}^{x} d\xi^\mu \left[ A^c_\mu + \frac{F^c_{\mu\nu}(\xi - x')^\nu}{2} \right]$$

Explicit form of the integral:

$$\Gamma^{\mu\nu\rho}_{a\gamma\gamma}(k,p_1,p_2) = -\frac{i}{2}\sum_f \frac{C_{ff}}{\Lambda}\iiint \frac{\mathrm{d}^4q}{(2\pi)^4}\frac{\mathrm{d}^4\ell}{(2\pi)^4}\frac{\mathrm{d}^4w}{(2\pi)^4}\iiint \mathrm{d}^4x\,\mathrm{d}^4y\,\mathrm{d}^4z$$
$$\times\, e^{i\Phi_\Sigma(x,y,z)}\mathrm{Tr}\left[\gamma^5\gamma^\mu S_f(q)\gamma^\nu S_f(\ell)\gamma^\rho S_f(w)\right]$$
$$\times\, e^{-ix(k+q-w)-iy(-p_1+w-\ell)-iz(-p_2+\ell-q)}.$$

$$\Phi_\Sigma(x,y,z) := -\frac{\beta}{2}\left(x_\mu \hat{F}^{\mu\nu}_{\mathrm{c}} y_\nu + y_\mu \hat{F}^{\mu\nu}_{\mathrm{c}} z_\nu + z_\mu \hat{F}^{\mu\nu}_{\mathrm{c}} x_\nu\right)$$

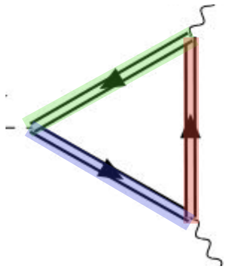


Explicit form of the integral:

$$\Gamma^{\mu\nu\rho}_{a\gamma\gamma}(k,p_1,p_2) = -\frac{i}{2}\sum_f \frac{C_{ff}}{\Lambda}\iiint \frac{\mathrm{d}^4q}{(2\pi)^4}\frac{\mathrm{d}^4\ell}{(2\pi)^4}\frac{\mathrm{d}^4w}{(2\pi)^4}\iiint \mathrm{d}^4x\,\mathrm{d}^4y\,\mathrm{d}^4z$$
$$\times\, e^{i\Phi_\Sigma(x,y,z)}\mathrm{Tr}\left[\gamma^5\gamma^\mu S_f(q)\gamma^\nu S_f(\ell)\gamma^\rho S_f(w)\right]$$
$$\times\, e^{-ix(k+q-w)-iy(-p_1+w-\ell)-iz(-p_2+\ell-q)}.$$

$\mathrm{d}^2x_\parallel \mathrm{d}^2x_\perp$ $\mathrm{d}^2y_\parallel \mathrm{d}^2y_\perp$ $\mathrm{d}^2z_\parallel \mathrm{d}^2z_\perp$

$$\Phi_\Sigma(x,y,z) := -\frac{\beta}{2}\left(x_\mu \hat{F}^{\mu\nu}_{\mathrm{c}} y_\nu + y_\mu \hat{F}^{\mu\nu}_{\mathrm{c}} z_\nu + z_\mu \hat{F}^{\mu\nu}_{\mathrm{c}} x_\nu\right)$$



These variables live on the perpendicular plane!

Evaluate the delta distributions:

$$\Gamma^{\mu\nu\rho}_{a\gamma\gamma}(k,p_1,p_2) = -2\frac{i}{(2\pi\beta)^2}\sum_f \frac{C_{ff}}{\Lambda}\iiint \frac{\mathrm{d}s_1\ \mathrm{d}s_2\ \mathrm{d}s_3}{\mathfrak{c}_1\mathfrak{c}_2\mathfrak{c}_3}\delta^{(4)}(k-p_2-p_1)$$
$$\times\, e^{-im_f^2(s_1+s_2+s_3)}\iiint \mathrm{d}^2q_\perp\ \mathrm{d}^2\ell_\perp\ \mathrm{d}^4w\ \underline{I^{\mu\nu\rho}},$$

All the nasty stuff is now hidden here.

$$\mathfrak{t}_j := \tan(\beta s_j)$$
$$\mathfrak{c}_j := \cos(\beta s_j)$$
$$\mathfrak{s}_j := \sin(\beta s_j)$$
$$\mathfrak{e}_j := e^{i\beta s_j\Sigma_3} = \mathfrak{c}_j + i\mathfrak{s}_j\Sigma_3$$

$$\iiint \mathrm{d}^4w\ \mathrm{d}^2\ell_\perp\ \mathrm{d}^2q_\perp I^{\mu\nu\rho} = \frac{i\pi^4\beta^3}{2}\frac{e^{\frac{i}{\beta}\frac{p_{1\perp}^2\mathfrak{t}_2\mathfrak{t}_3+p_{2\perp}^2\mathfrak{t}_1\mathfrak{t}_2+k_\perp^2\mathfrak{t}_1\mathfrak{t}_3+2\mathfrak{t}_1\mathfrak{t}_2\mathfrak{t}_3k_\perp\hat{F}_\mathrm{c}p_{1\perp}}{\mathfrak{t}_1+\mathfrak{t}_2+\mathfrak{t}_3-\mathfrak{t}_1\mathfrak{t}_2\mathfrak{t}_3}}}{\mathfrak{t}_1+\mathfrak{t}_2+\mathfrak{t}_3-\mathfrak{t}_1\mathfrak{t}_2\mathfrak{t}_3}$$
$$\times\frac{1}{s_1+s_2+s_3}e^{\frac{i}{s_1+s_2+s_3}\left(s_1s_3k_\parallel^2+s_2s_3p_{1\parallel}^2+s_1s_2p_{2\parallel}^2\right)}$$
$$\times\left(\frac{i\beta\mathrm{Tr}\left[\gamma^5\gamma^\mu\mathfrak{T}_1^{\nu\rho}\right]}{\mathfrak{t}_1+\mathfrak{t}_2+\mathfrak{t}_3-\mathfrak{t}_1\mathfrak{t}_2\mathfrak{t}_3}-\frac{i\mathrm{Tr}\left[\gamma^5\gamma^\mu\mathfrak{T}_2^{\nu\rho}\right]}{s_1+s_2+s_3}+2\mathrm{Tr}\left[\gamma^5\gamma^\mu\mathfrak{T}_3^{\nu\rho}\right]\right) \quad (44$$

with the operators

$$\mathfrak{T}_1^{\nu\rho} := \frac{\eta_\perp^{\alpha\beta}+\mathfrak{t}_1\hat{F}_\mathrm{c}^{\alpha\beta}}{\mathfrak{c}_2\mathfrak{c}_3}\mathfrak{P}_1\gamma^\nu\gamma_\alpha^\perp\gamma^\rho\gamma_\beta^\perp+\frac{\eta_\perp^{\alpha\beta}+\mathfrak{t}_3\hat{F}_\mathrm{c}^{\alpha\beta}}{\mathfrak{c}_1\mathfrak{c}_2}\gamma_\alpha^\perp\gamma^\nu\gamma_\beta^\perp\gamma^\rho\mathfrak{P}_3+\frac{\eta_\perp^{\alpha\beta}+\mathfrak{t}_2\hat{F}_\mathrm{c}^{\alpha\beta}}{\mathfrak{c}_1\mathfrak{c}_3}\gamma_\alpha^\perp\gamma^\nu\mathfrak{P}_2\gamma^\rho\gamma_\beta^\perp,$$
$$\mathfrak{T}_2^{\nu\rho} := \mathfrak{P}_1\gamma^\nu\gamma_\perp^\alpha\mathfrak{e}_2\gamma^\rho\gamma_\alpha^\perp\mathfrak{e}_3+\gamma_\perp^\alpha\mathfrak{e}_1\gamma^\nu\gamma_\alpha^\perp\mathfrak{e}_2\gamma^\rho\mathfrak{P}_3+\gamma_\perp^\alpha\mathfrak{e}_1\gamma^\nu\mathfrak{P}_2\gamma^\rho\gamma_\alpha^\perp\mathfrak{e}_3,$$
$$\mathfrak{T}_3^{\nu\rho} := \mathfrak{P}_1\gamma^\nu\mathfrak{P}_2\gamma^\rho\mathfrak{P}_3 \quad (45$$

where

$$\mathfrak{P}_1 := \Pi_1\Bigg(\underbrace{-\frac{s_3k_\parallel+s_2p_{2\parallel}}{s_1+s_2+s_3}}_{a_1}+\underbrace{\frac{\mathfrak{t}_2\mathfrak{t}_3\hat{F}_\mathrm{c}p_{1\perp}-\mathfrak{t}_2p_{2\perp}-\mathfrak{t}_3k_\perp}{\mathfrak{t}_1+\mathfrak{t}_2+\mathfrak{t}_3-\mathfrak{t}_1\mathfrak{t}_2\mathfrak{t}_3}}_{b_1}\Bigg)$$
$$\mathfrak{P}_2 := \Pi_2\Bigg(\underbrace{\frac{s_1p_{2\parallel}-s_3p_{1\parallel}}{s_1+s_2+s_3}}_{a_2}+\underbrace{\frac{-\mathfrak{t}_1\mathfrak{t}_3\hat{F}_\mathrm{c}k_\perp-\mathfrak{t}_3p_{1\perp}+\mathfrak{t}_1p_{2\perp}}{\mathfrak{t}_1+\mathfrak{t}_2+\mathfrak{t}_3-\mathfrak{t}_1\mathfrak{t}_2\mathfrak{t}_3}}_{b_2}\Bigg)$$
$$\mathfrak{P}_3 := \Pi_3\Bigg(\underbrace{\frac{s_2p_{1\parallel}+s_1k_\parallel}{s_1+s_2+s_3}}_{a_3}+\underbrace{\frac{\mathfrak{t}_1\mathfrak{t}_2\hat{F}_\mathrm{c}p_{2\perp}+\mathfrak{t}_1k_\perp+\mathfrak{t}_2p_{1\perp}}{\mathfrak{t}_1+\mathfrak{t}_2+\mathfrak{t}_3-\mathfrak{t}_1\mathfrak{t}_2\mathfrak{t}_3}}_{b_3}\Bigg).$$

$$\Pi_j(p) := (m_f+\not{p}_\parallel)\mathfrak{e}_j+\frac{\not{p}_\perp}{\mathfrak{c}_j}$$

$$\iiint \mathrm{d}^4 w \; \mathrm{d}^2 \ell_\perp \; \mathrm{d}^2 q_\perp I^{\mu\nu\rho} = \frac{i\pi^4 \beta^3}{2} \frac{e^{\frac{i}{\beta}\frac{p_{1\perp}^2 \mathfrak{t}_2\mathfrak{t}_3 + p_{2\perp}^2 \mathfrak{t}_1\mathfrak{t}_2 + k_\perp^2 \mathfrak{t}_1\mathfrak{t}_3 + 2\mathfrak{t}_1\mathfrak{t}_2\mathfrak{t}_3 k_\perp \hat{F}_c p_{1\perp}}{\mathfrak{t}_1+\mathfrak{t}_2+\mathfrak{t}_3-\mathfrak{t}_1\mathfrak{t}_2\mathfrak{t}_3}}}{\mathfrak{t}_1 + \mathfrak{t}_2 + \mathfrak{t}_3 - \mathfrak{t}_1\mathfrak{t}_2\mathfrak{t}_3}$$

$$\times \frac{1}{s_1+s_2+s_3} e^{\frac{i}{s_1+s_2+s_3}\left(s_1 s_3 k_\parallel^2 + s_2 s_3 p_{1\parallel}^2 + s_1 s_2 p_{2\parallel}^2\right)}$$

$$\times \left( \frac{i\beta \mathrm{Tr}\left[\gamma^5\gamma^\mu \mathfrak{T}_1^{\nu\rho}\right]}{\mathfrak{t}_1 + \mathfrak{t}_2 + \mathfrak{t}_3 - \mathfrak{t}_1\mathfrak{t}_2\mathfrak{t}_3} - \frac{i \mathrm{Tr}\left[\gamma^5\gamma^\mu \mathfrak{T}_2^{\nu\rho}\right]}{s_1+s_2+s_3} + 2\mathrm{Tr}\left[\gamma^5\gamma^\mu \mathfrak{T}_3^{\nu\rho}\right] \right) \qquad (44)$$

114 terms of products of trig. functions

with the operators

$$\mathfrak{T}_1^{\nu\rho} := \frac{\eta_\perp^{\alpha\beta} + \mathfrak{t}_1 \hat{F}_c^{\alpha\beta}}{\mathfrak{c}_2\mathfrak{c}_3} \mathfrak{P}_1 \gamma^\nu \gamma_\alpha^\perp \gamma^\rho \gamma_\beta^\perp + \frac{\eta_\perp^{\alpha\beta} + \mathfrak{t}_3 \hat{F}_c^{\alpha\beta}}{\mathfrak{c}_1\mathfrak{c}_2} \gamma_\alpha^\perp \gamma^\nu \gamma_\beta^\perp \gamma^\rho \mathfrak{P}_3 + \frac{\eta_\perp^{\alpha\beta} + \mathfrak{t}_2 \hat{F}_c^{\alpha\beta}}{\mathfrak{c}_1\mathfrak{c}_3} \gamma_\alpha^\perp \gamma^\nu \mathfrak{P}_2 \gamma^\rho \gamma_\beta^\perp,$$

$$\mathfrak{T}_2^{\nu\rho} := \mathfrak{P}_1 \gamma^\nu \gamma_\perp^\alpha \mathfrak{e}_2 \gamma^\rho \gamma_\alpha^\perp \mathfrak{e}_3 + \gamma_\perp^\alpha \mathfrak{e}_1 \gamma^\nu \gamma_\alpha^\perp \mathfrak{e}_2 \gamma^\rho \mathfrak{P}_3 + \gamma_\perp^\alpha \mathfrak{e}_1 \gamma^\nu \mathfrak{P}_2 \gamma^\rho \gamma_\alpha^\perp \mathfrak{e}_3,$$

$$\mathfrak{T}_3^{\nu\rho} := \mathfrak{P}_1 \gamma^\nu \mathfrak{P}_2 \gamma^\rho \mathfrak{P}_3 \qquad (45)$$

where

$$\mathfrak{P}_1 := \Pi_1 \left( \underbrace{-\frac{s_3 k_\parallel + s_2 p_{2\parallel}}{s_1+s_2+s_3}}_{a_1} + \underbrace{\frac{\mathfrak{t}_2\mathfrak{t}_3 \hat{F}_c p_{1\perp} - \mathfrak{t}_2 p_{2\perp} - \mathfrak{t}_3 k_\perp}{\mathfrak{t}_1 + \mathfrak{t}_2 + \mathfrak{t}_3 - \mathfrak{t}_1\mathfrak{t}_2\mathfrak{t}_3}}_{b_1} \right)$$

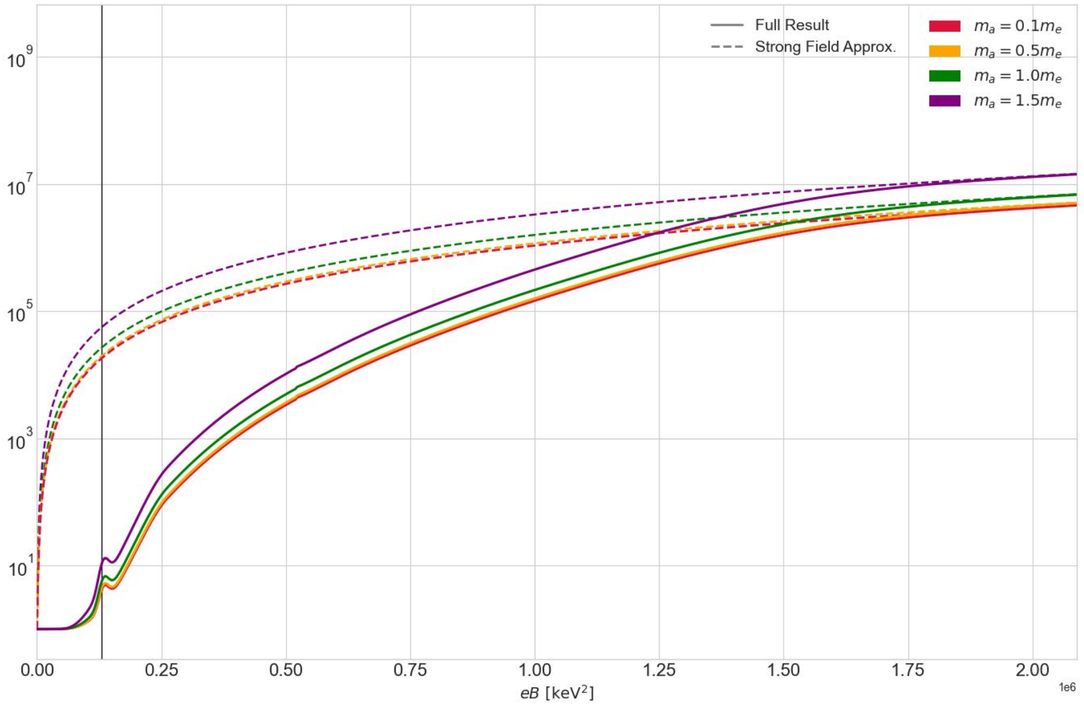
$$\mathfrak{P}_2 := \Pi_2 \left( \underbrace{\frac{s_1 p_{2\parallel} - s_3 p_{1\parallel}}{s_1+s_2+s_3}}_{a_2} + \underbrace{\frac{-\mathfrak{t}_1\mathfrak{t}_3 \hat{F}_c k_\perp - \mathfrak{t}_3 p_{1\perp} + \mathfrak{t}_1 p_{2\perp}}{\mathfrak{t}_1 + \mathfrak{t}_2 + \mathfrak{t}_3 - \mathfrak{t}_1\mathfrak{t}_2\mathfrak{t}_3}}_{b_2} \right)$$

$$\mathfrak{P}_3 := \Pi_3 \left( \underbrace{\frac{s_2 p_{1\parallel} + s_1 k_\parallel}{s_1+s_2+s_3}}_{a_3} + \underbrace{\frac{\mathfrak{t}_1\mathfrak{t}_2 \hat{F}_c p_{2\perp} + \mathfrak{t}_1 k_\perp + \mathfrak{t}_2 p_{1\perp}}{\mathfrak{t}_1 + \mathfrak{t}_2 + \mathfrak{t}_3 - \mathfrak{t}_1\mathfrak{t}_2\mathfrak{t}_3}}_{b_3} \right).$$

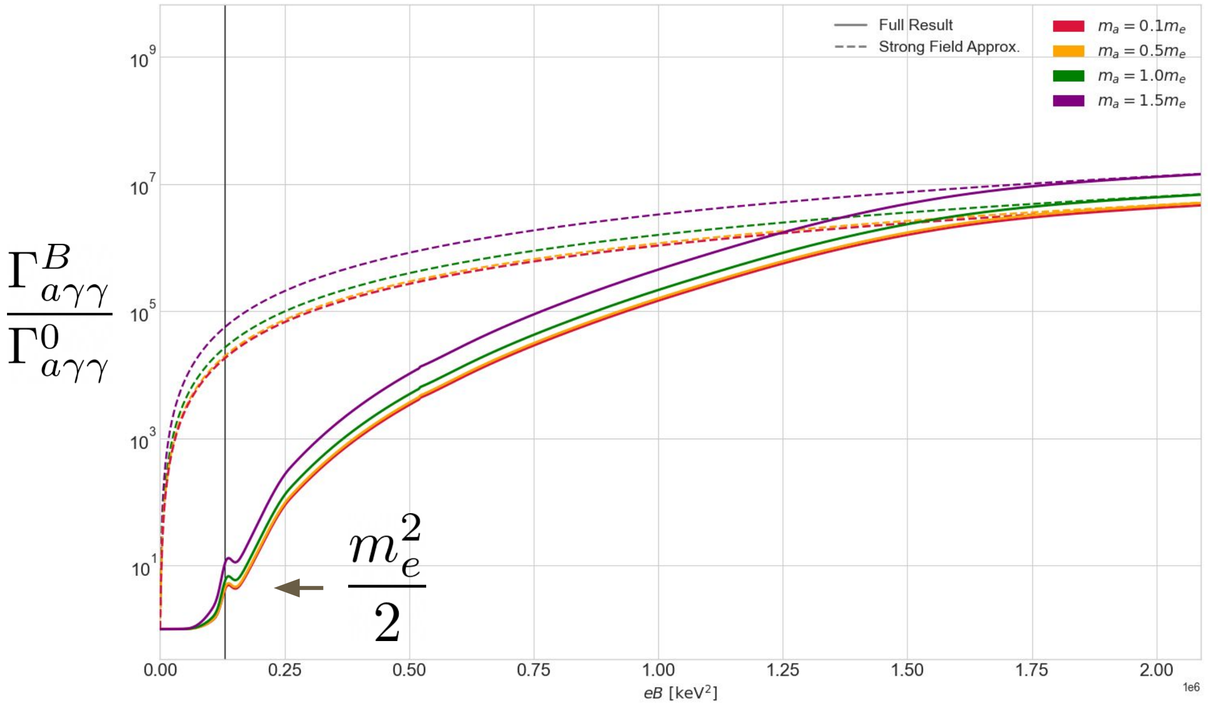
$$\Pi_j(p) := (m_f + \not{p}_\parallel)\mathfrak{e}_j + \frac{\not{p}_\perp}{\mathfrak{c}_j}$$

Closed form when the external momenta are parallel to the magnetic field:

$$\frac{\Gamma^B_{a\gamma\gamma}}{\Gamma^0_{a\gamma\gamma}}$$



Closed form when the external momenta are parallel to the magnetic field:



## Summary

- For external fields, Schwinger formalism can be used
- **Effective** couplings have non-trivial external field dependance
- Closed form for specific kinetic configurations

## Outlook

- Particles with different mathematical structures can be used
- Bosonic corrections are a possibility
- *Phenomenology work*
- *Other BSM candidates can be considered*