Herbstschule HEP

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An effective hadronic field theory for B-meson decays at high recoil

[in preparation]

Thorsten Feldmann, Jack Jenkins, JdPL

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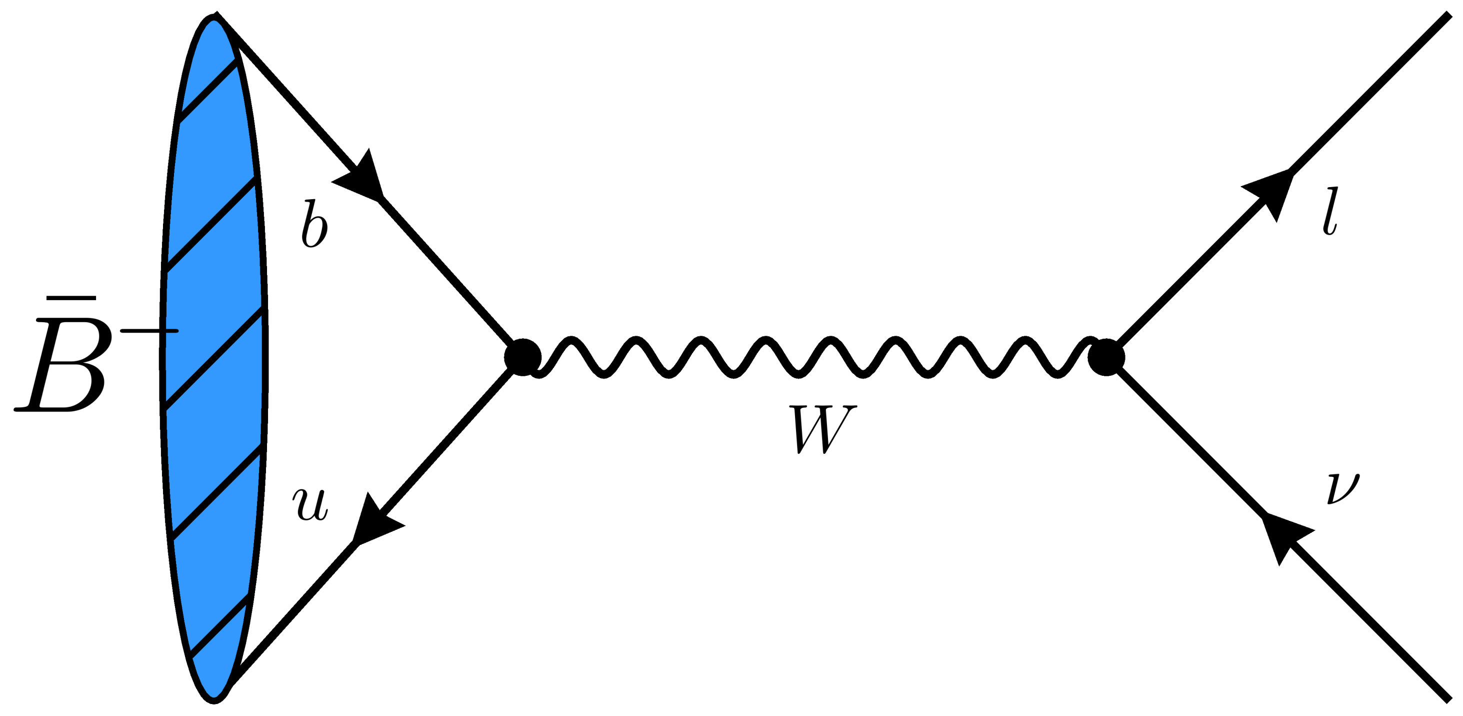
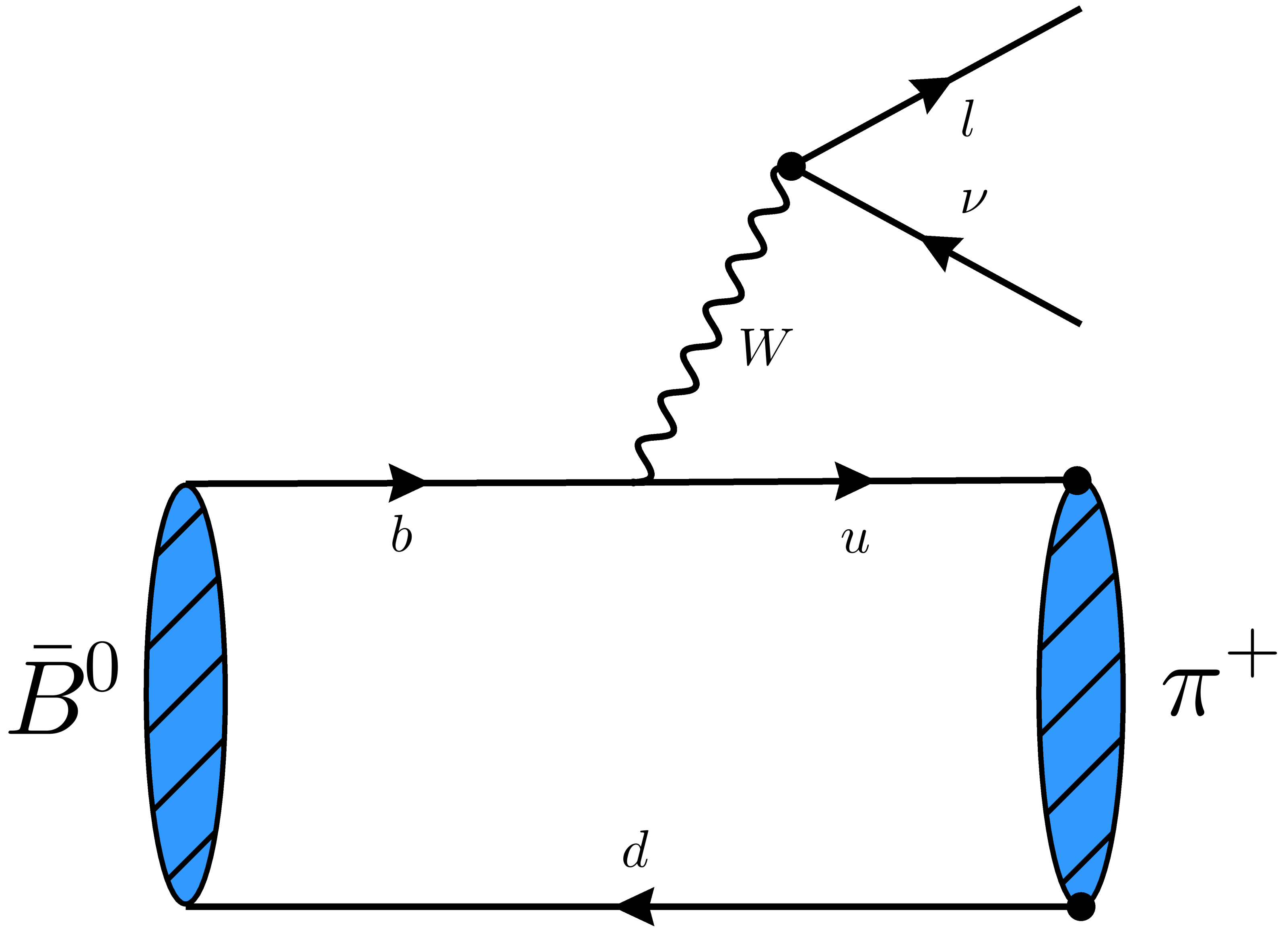
Universität Siegen

TP1 Theoretical Particle Physics

QCD is “easy” (but non-perturbative)

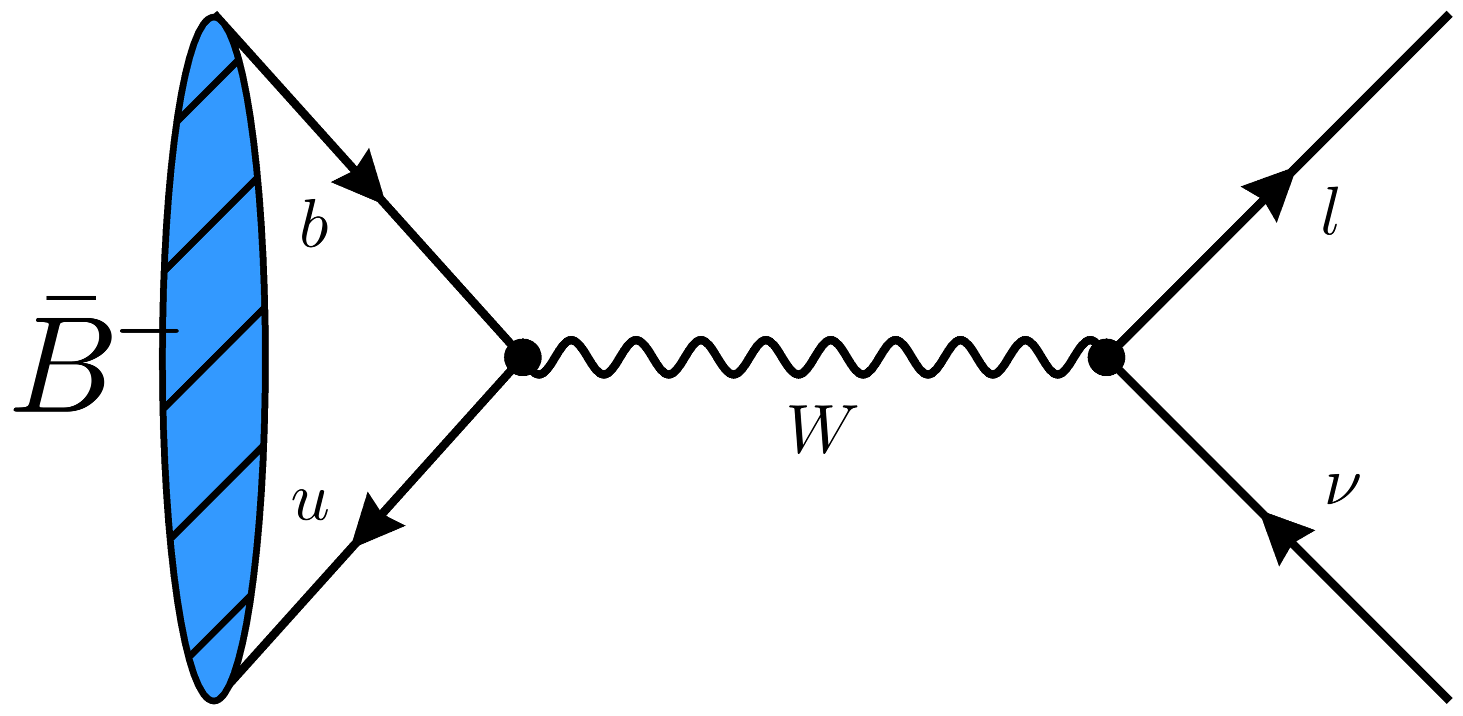


$$\langle 0 \,|\, \bar{u}\gamma_\mu(1-\gamma_5)b \,|\, B^-(p)\rangle$$

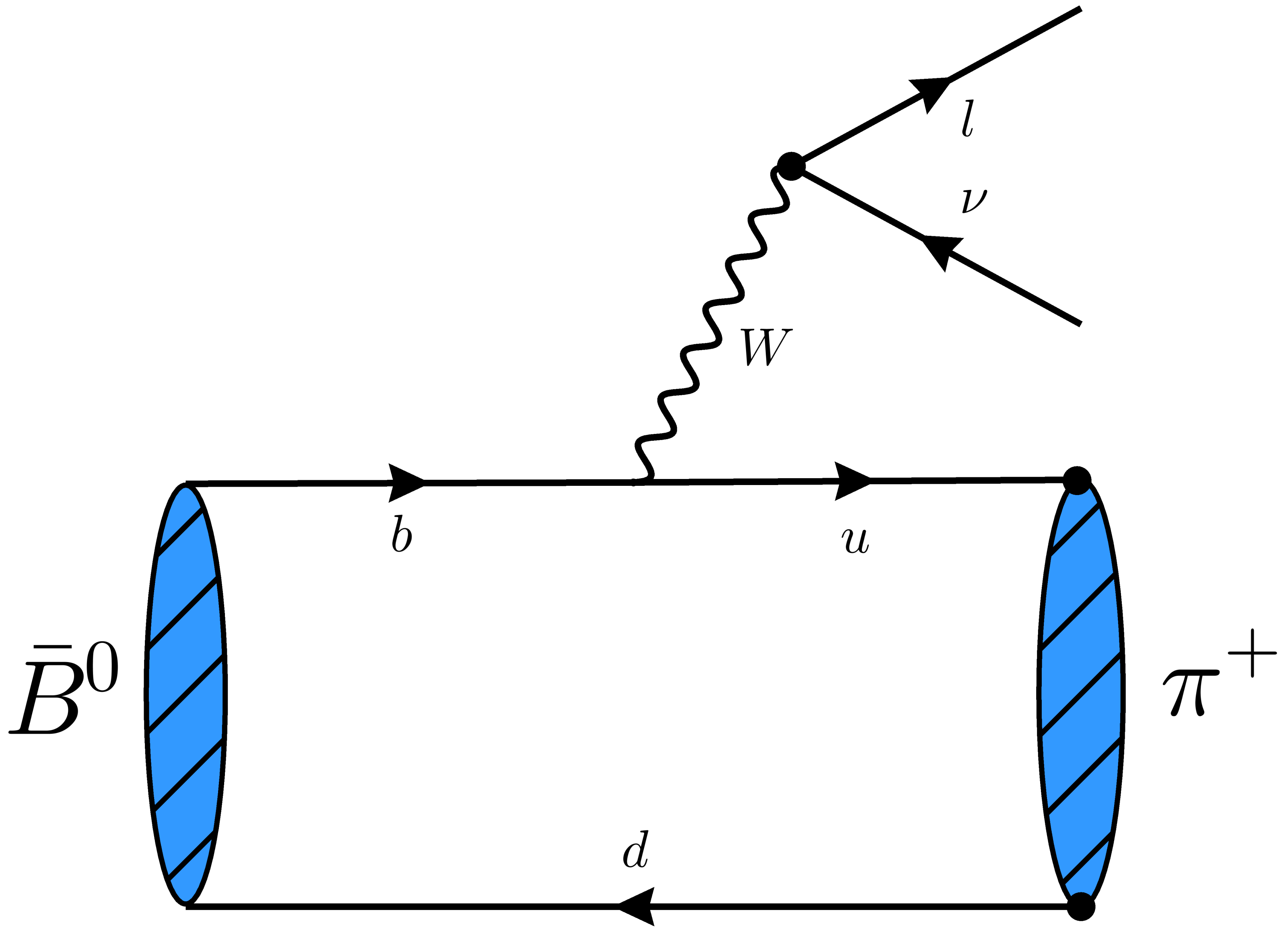


$$\langle \pi^+(k) \,|\, \bar{u}\gamma_\mu(1-\gamma_5)b \,|\, \bar{B}^0(p)\rangle$$

QCD is "easy" (but non-perturbative)



$$\langle 0 | \bar{u}\gamma_\mu(1-\gamma_5)b | B^-(p)\rangle = i f_B p_\mu$$



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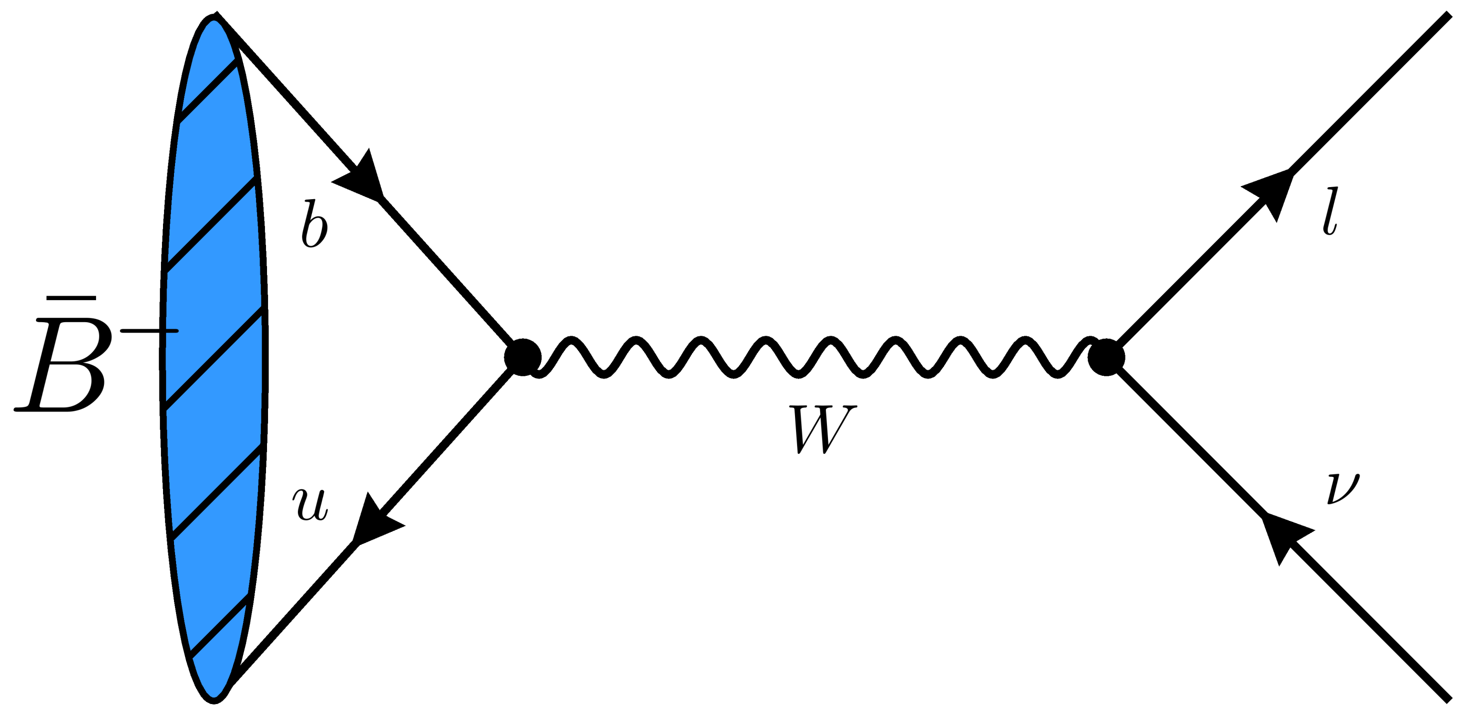

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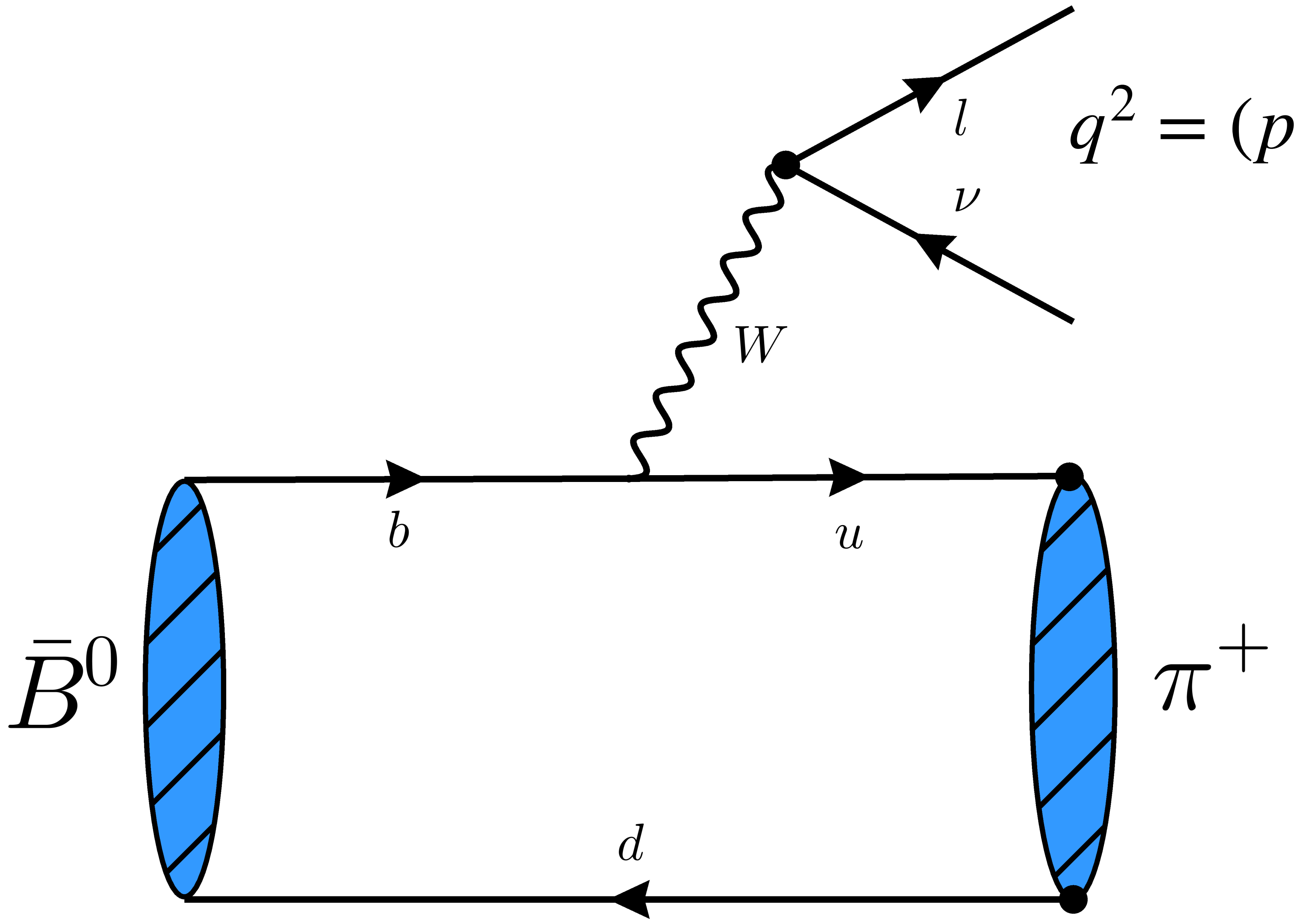

$$q^2 = (p-k)^2 = M_B^2 + m_\pi^2 - 2M_B E_\pi$$

$$\langle \pi^+(k) | \bar{u}\gamma_\mu(1-\gamma_5)b | \bar{B}^0(p)\rangle$$

$$= f_+^{B\to\pi}(q^2)(p_\mu + k_\mu) + f_-^{B\to\pi}(q^2)(p_\mu - k_\mu)$$

QCD is difficult

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$m_Q \gg \Lambda_{QCD}$

→*symmetries!*

Heavy Quark Effective Theory

$B \to X$ $\quad$ $B \to X_u \ell \nu$

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Heavy Quark Effective Theory

$B \to X \qquad B \to X_u \ell \nu$

$m_q \ll 4\pi f_\pi$

→*symmetries!*

Chiral Perturbation Theory

$\pi \to \ell \nu \qquad \pi\pi \to \pi\pi$

QCD is difficult

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Heavy Hadron Chiral Perturbation Theory

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QCD is difficult

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Heavy Quark Effective Theory

$B \to X \qquad B \to X_u \ell \nu$

$m_q \ll 4\pi f_\pi$

→*symmetries!*

Chiral Perturbation Theory

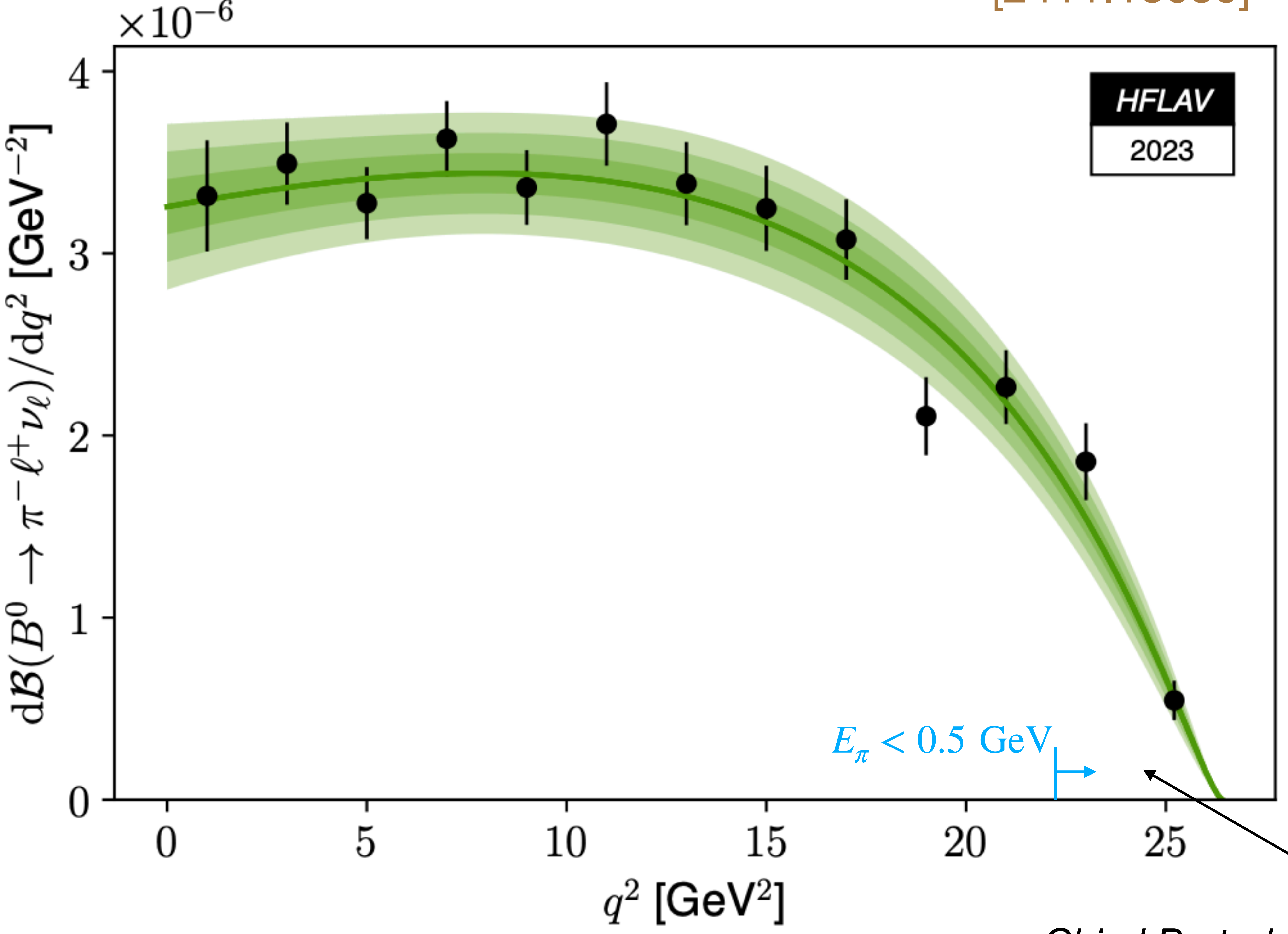
$\pi \to \ell \nu \qquad \pi_{\text{soft}}\pi_{\text{soft}} \to \pi_{\text{soft}}\pi_{\text{soft}}$

Heavy Hadron Chiral Perturbation Theory

$B \to \ell \nu \qquad B \to \pi_{\text{soft}} \ell \nu$

High / low recoil regions

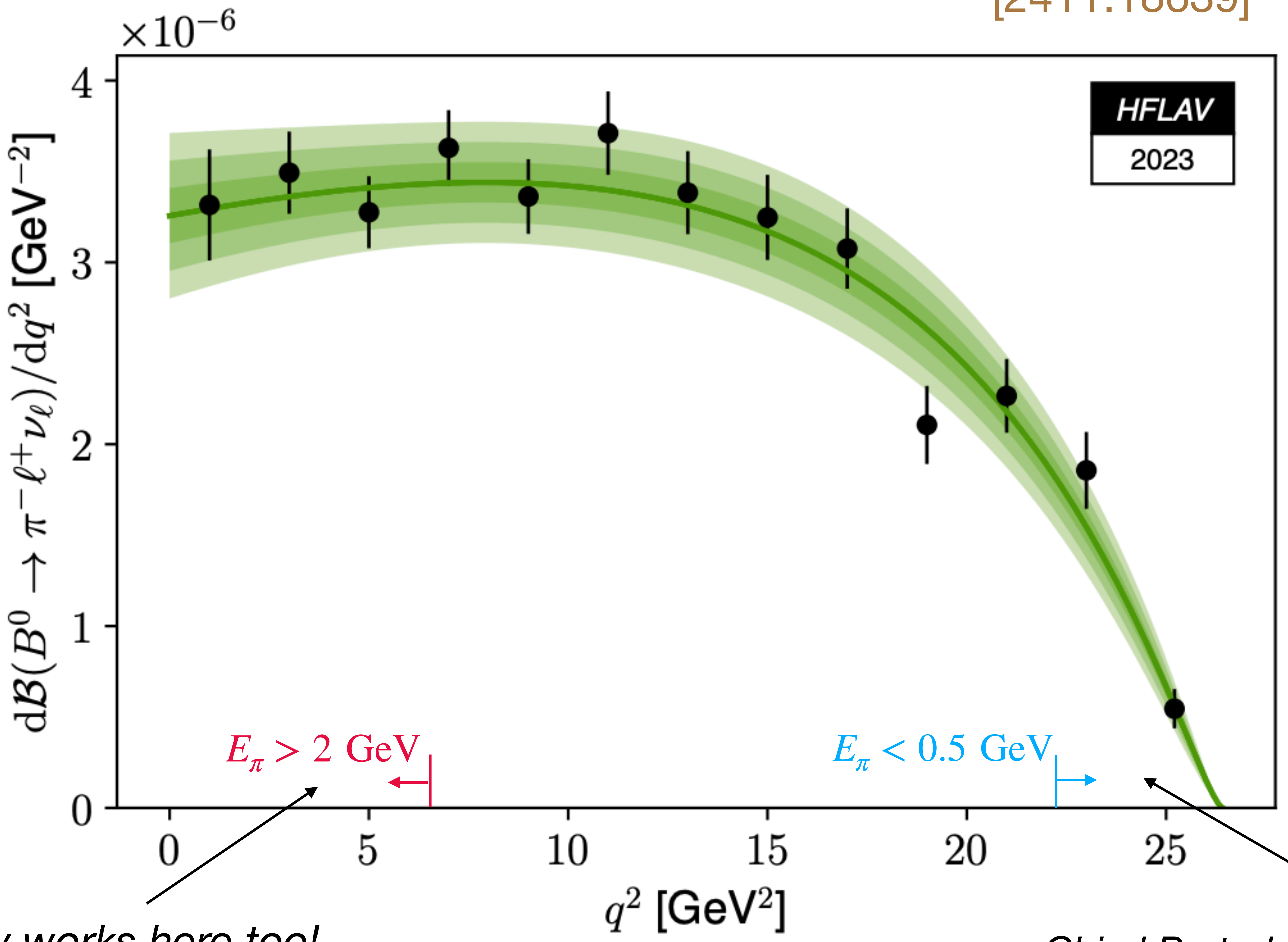
[2411.18639]



Chiral Perturbation Theory only works here!

High / low recoil regions

[2411.18639]



New Theory works here too!

Chiral Perturbation Theory only works here!

QCD is difficult

QCD is difficult

Heavy Hadron Chiral Perturbation Theory

QCD is difficult

Heavy Hadron Chiral Perturbation Theory

Soft-Collinear Effective Theory

- Based on the method of regions: $q \to q_s \, , q_c$

QCD is difficult

Heavy Hadron Chiral Perturbation Theory

Soft-Collinear Effective Theory

- Based on the method of regions: $q \to q_s \, , q_c$

SCET for χPT

$$\mathscr{L}^{(0)} = -i\mathrm{Tr}\,\bar{H}_a v_\mu \left(\partial^\mu \delta_{ab} + iV^\mu_{s_{ba}}\right) H_b + g_\pi \mathrm{Tr}\,\bar{H}_a H_b \gamma_\nu \gamma_5 A^\nu_{s_{ba}} + \mathscr{L}^{(0)}_{\pi_s} + \mathscr{L}^{(0)}_{\pi_c} +$$

External Currents: Non-local operators, energy dependent couplings

$$\mathscr{L}^{(0)}_{\pi_c} \to \frac{f_\pi^2}{4}\mathrm{Tr}\left[\partial_\mu \Sigma_c^\dagger \partial^\mu \Sigma_c\right]$$

$$H_v = \frac{1+\not{v}}{2}[\bar{B}^*_v - \bar{B}_v \gamma_5]$$

External currents: Matching QCD $\to$ HHChPT

$$QCD \to HQET \to HHChPT$$

External currents: Matching QCD $\rightarrow$ HHChPT

$$QCD \rightarrow HQET \rightarrow HHChPT$$

QCD

$$\langle 0 \,|\, \bar{u}\gamma_\mu(1-\gamma_5)b \,|\, \bar{B}(p)\rangle \equiv i f_B p_\mu$$

External currents: Matching QCD $\rightarrow$ HHChPT

$$QCD \rightarrow HQET \rightarrow HHChPT$$

QCD

$$\langle 0 | \bar{u}\gamma_\mu(1-\gamma_5)b | \bar{B}(p)\rangle \equiv i f_B p_\mu$$

HQET

$$\bar{u}\gamma_\mu(1-\gamma_5)b \stackrel{!}{=} C_{V-A}(\mu)\bar{u}\gamma_\mu(1-\gamma_5)h_v$$

External currents: Matching QCD → HHChPT

$QCD \to HQET \to HHChPT$

QCD

$$\langle 0 | \bar{u}\gamma_\mu(1-\gamma_5)b | \bar{B}(p)\rangle \equiv i f_B p_\mu$$

HQET

$$\bar{u}\gamma_\mu(1-\gamma_5)b \stackrel{!}{=} C_{V-A}(\mu)\bar{u}\gamma_\mu(1-\gamma_5)h_v$$

HHChPT

$$\bar{q}_L \Gamma h_v \stackrel{!}{=} C(\mu)\mathrm{Tr}\left[H_v \Gamma P_R\right]\xi^\dagger$$

$$C(\mu = m_b) = \sqrt{M_B} f_B + \mathcal{O}(1/m_b, \alpha_s, 1/f_\pi)$$

External currents: Matching SCET $\to$ HHChPT

External currents: Matching SCET $\rightarrow$ HHChPT

SCET II

$$\mathcal{O} = [\bar{q}_c(0)\ldots q_c(s\bar{n})] \times [\bar{q}_s(tn)\ldots h_v(0)]$$

External currents: Matching SCET → HHChPT

SCET II

$$\mathcal{O} = [\bar{q}_c(0)\ldots q_c(s\bar{n})] \times [\bar{q}_s(tn)\ldots h_v(0)]$$

HHChPT

$$\mathcal{O} \stackrel{!}{=} c(t,s)\ \mathrm{Tr}[H(0)\gamma^\mu(1-\gamma_5)][0,tn]\xi_s^\dagger(tn) \times \xi_c(s\bar{n})[s\bar{n},0]\xi_c^\dagger(0)$$

External currents: Matching SCET → HHChPT

SCET II

$$\mathcal{O} = [\bar{q}_c(0)\ldots q_c(s\bar{n})] \times [\bar{q}_s(tn)\ldots h_v(0)]$$

HHChPT

$$\mathcal{O} \stackrel{!}{=} c(t,s)\ \mathrm{Tr}[H(0)\gamma^\mu(1-\gamma_5)][0,tn]\xi_s^\dagger(tn) \times \xi_c(s\bar{n})[s\bar{n},0]\xi_c^\dagger(0)$$

Related to the form factor, not the decay constant!

What can you actually calculate?

Heavy Hadron Chiral Perturbation Theory

+ SCET

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Heavy Hadron Chiral Perturbation Theory

- $SU(3)$ relations at high q^2

+ SCET

What can you actually calculate?

Heavy Hadron Chiral Perturbation Theory

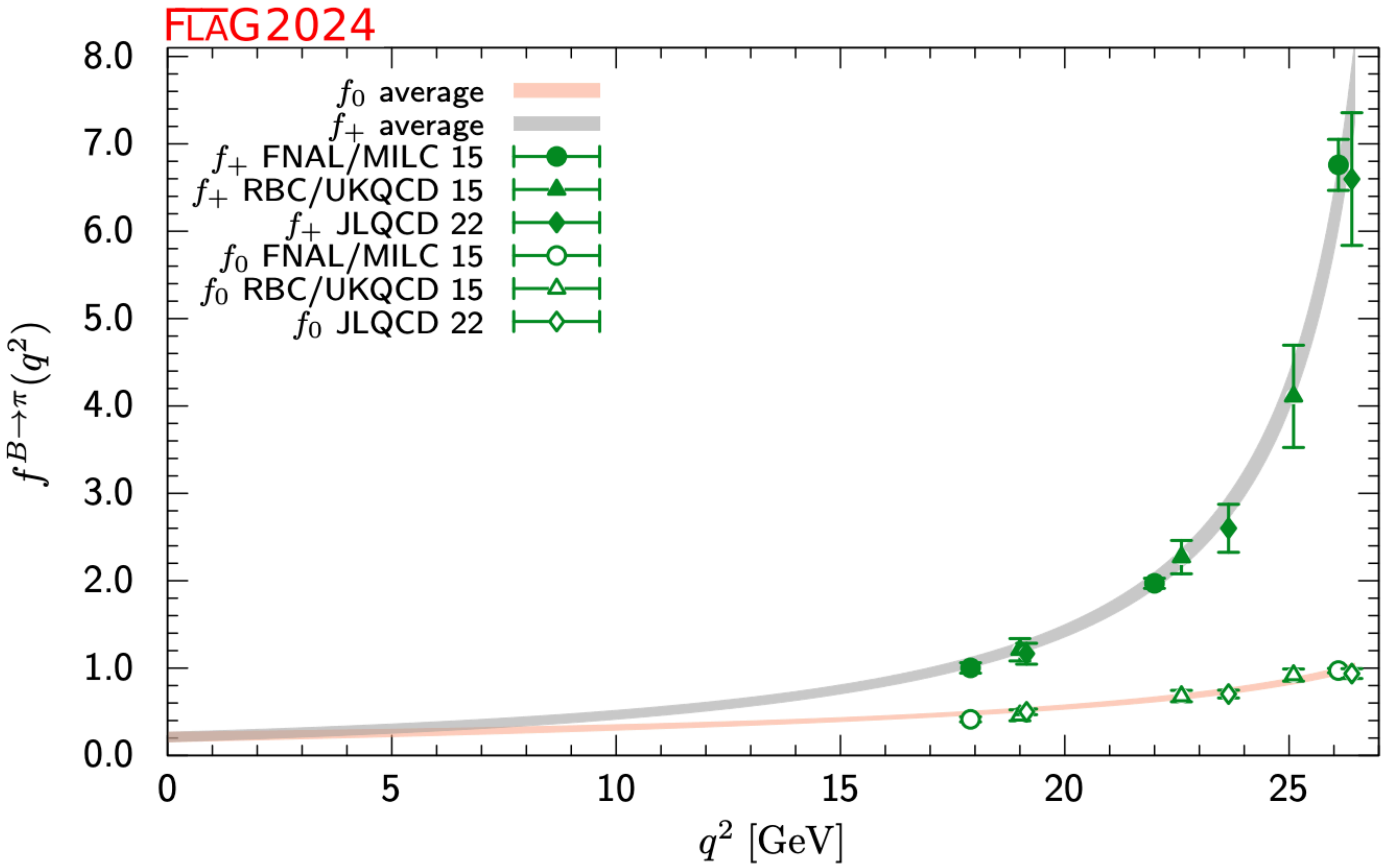
- $SU(3)$ relations at high q^2

+ SCET

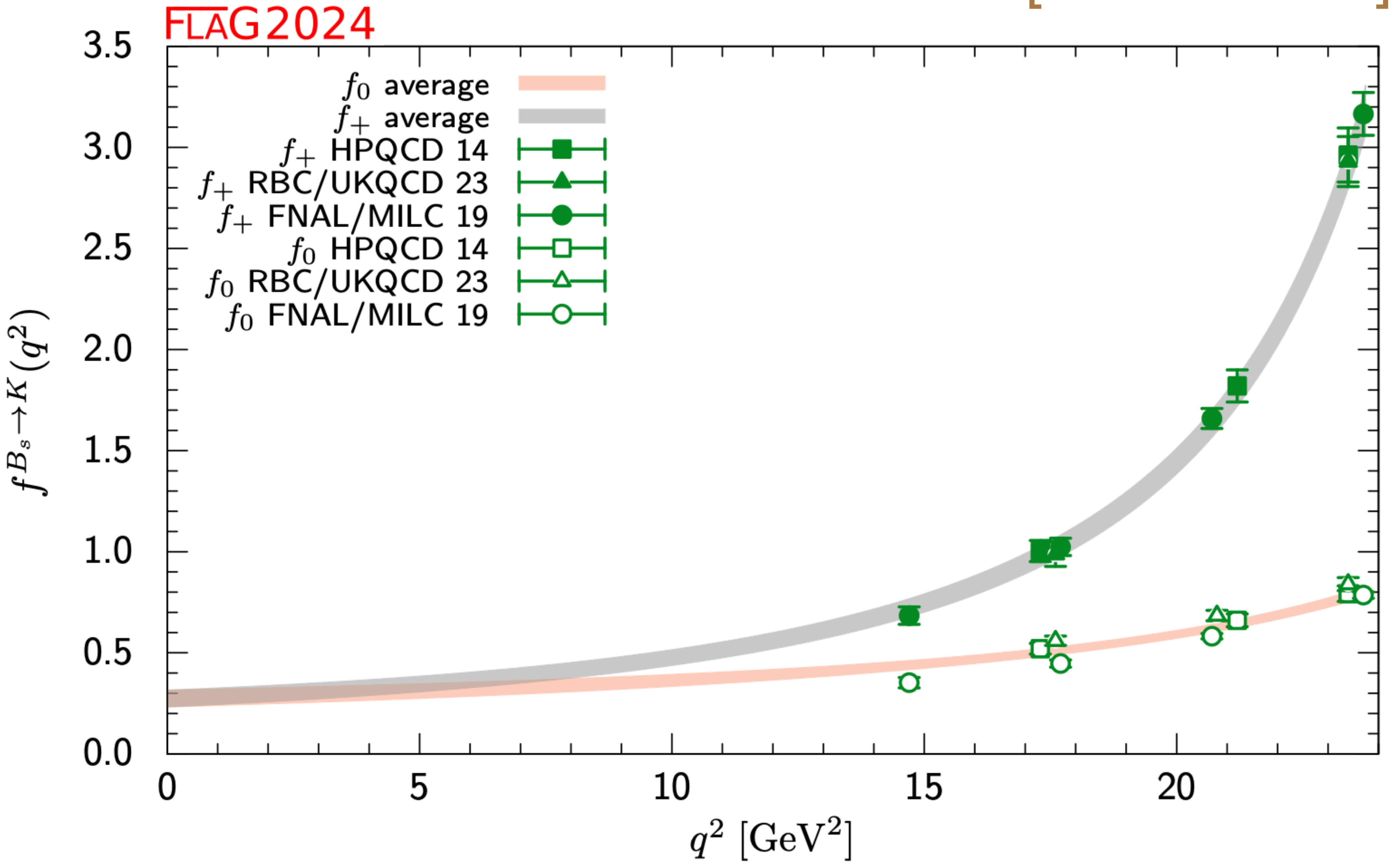
- $SU(3)$ relations at low q^2

What can you actually calculate?

$B \to \pi$ and $B_s \to K$ form factors are equivalent in the $SU(3)$ limit



[2411.04268]



$$q^2 = M_B^2 + m_\pi^2 - 2M_B E_\pi$$

What can you actually calculate?

Heavy Hadron Chiral Perturbation Theory

- $SU(3)$ relations at high q^2

+ SCET

- $SU(3)$ relations at low q^2

What can you actually calculate?

Heavy Hadron Chiral Perturbation Theory

- $SU(3)$ relations at high q^2
- Chiral extrapolation at high q^2

+ SCET

- $SU(3)$ relations at low q^2

What can you actually calculate?

Heavy Hadron Chiral Perturbation Theory

- $SU(3)$ relations at high q^2
- Chiral extrapolation at high q^2

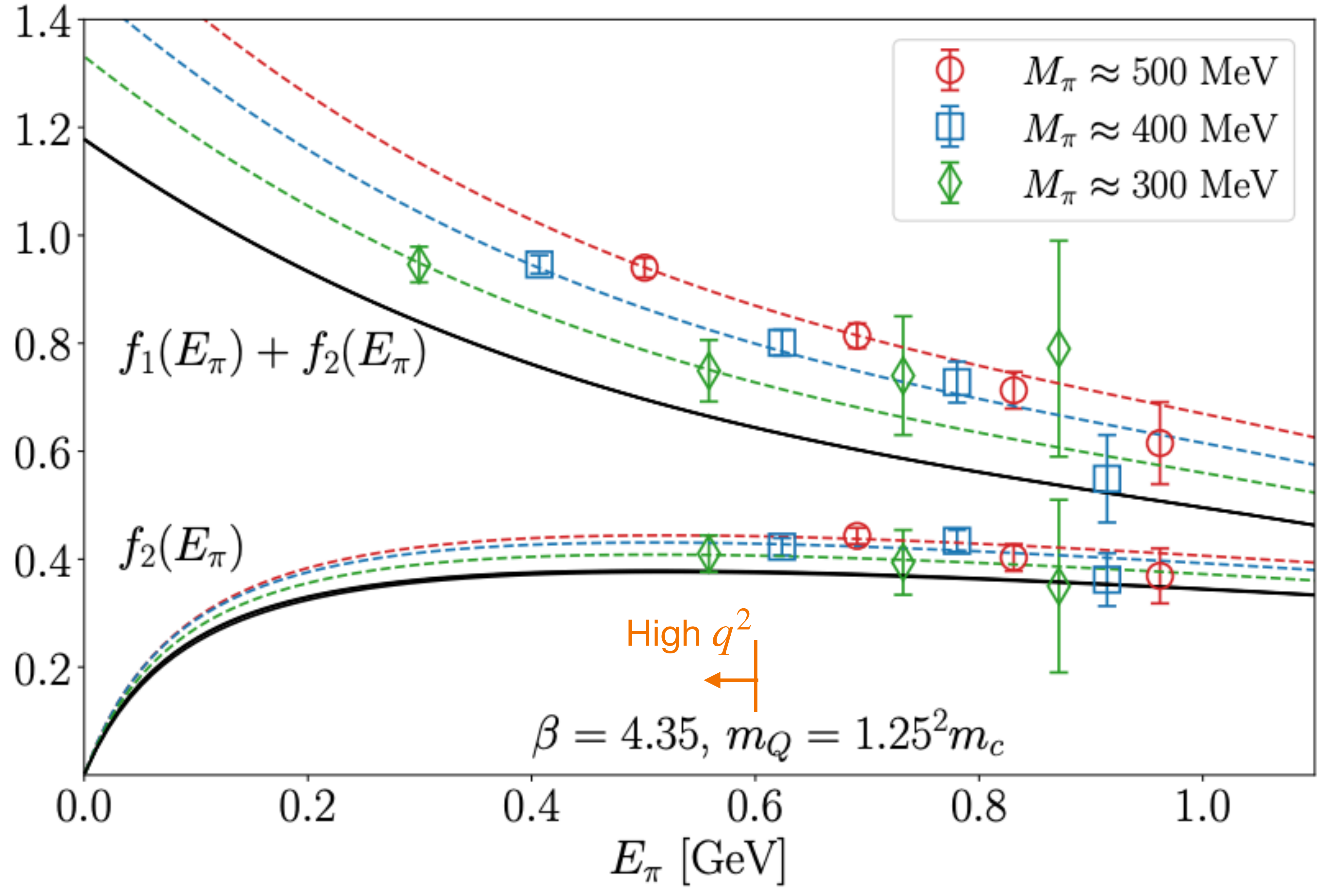
+ SCET

- $SU(3)$ relations at low q^2
- Chiral extrapolation at low q^2

What can you actually calculate?

Lattice at high q^2 with chiral extrapolation (HHChPT)

JLQCD [2203.04938]



$$q^2 = M_B^2 + m_\pi^2 - 2M_B E_\pi$$

Conclusions

- We are building SCET for a hadronic theory: SCHHChPT (placeholder name)
- Matching based on symmetries (and more that was not shown)
- Allows us to calculate:
  - $SU(3)$ relations at low q^2: Relate form factors of different decays
  - Chiral extrapolation at low q^2: Allow lattice to work in more energetic regimes

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Thank you!

Jaime del Palacio-Lirola

P H

Universität Siegen

TP1 Theoretical Particle Physics

Backup slides

Hard Pion Chiral Perturbation Theory for $B \to \pi$ and $D \to \pi$ Formfactors

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We should thus be able to describe the hard part of any diagram by an effective Lagrangian. This effective Lagrangian should include the most general terms allowed consistent with all the symmetries and have coefficients that depend on the hard kinematical quantities and can even be complex. A two-loop example will be given in [15]. We expect that a proof along the lines of SCET [20] should be possible. Once it is accepted that one can do this, a second step is to prove that the effective Lagrangian one uses is sufficient to describe the neighbourhood of the hard process and calculate chiral logarithms.

HHChPT: Matching

$$QCD \to HQET \to HHChPT$$

QCD

$$\langle 0 | \bar{u}\gamma_\mu(1-\gamma_5)b | \bar{B}(p)\rangle = if_B p_\mu$$

$$\langle \pi(k) | \bar{u}\gamma_\mu(1-\gamma_5)b | \bar{B}(p)\rangle$$

$$= f_+^{B\to\pi}(q^2)(p_\mu + k_\mu) + f_-^{B\to\pi}(q^2)(p_\mu - k_\mu)$$

HQET

$$\bar{u}\gamma_\mu(1-\gamma_5)b = C_{V-A}(\mu)\bar{u}\gamma_\mu(1-\gamma_5)h_v$$

HHChPT

$$\bar{q}_L\Gamma h_v = C(\mu)\mathrm{Tr}\left[H_v\Gamma P_R\right]\xi^\dagger$$

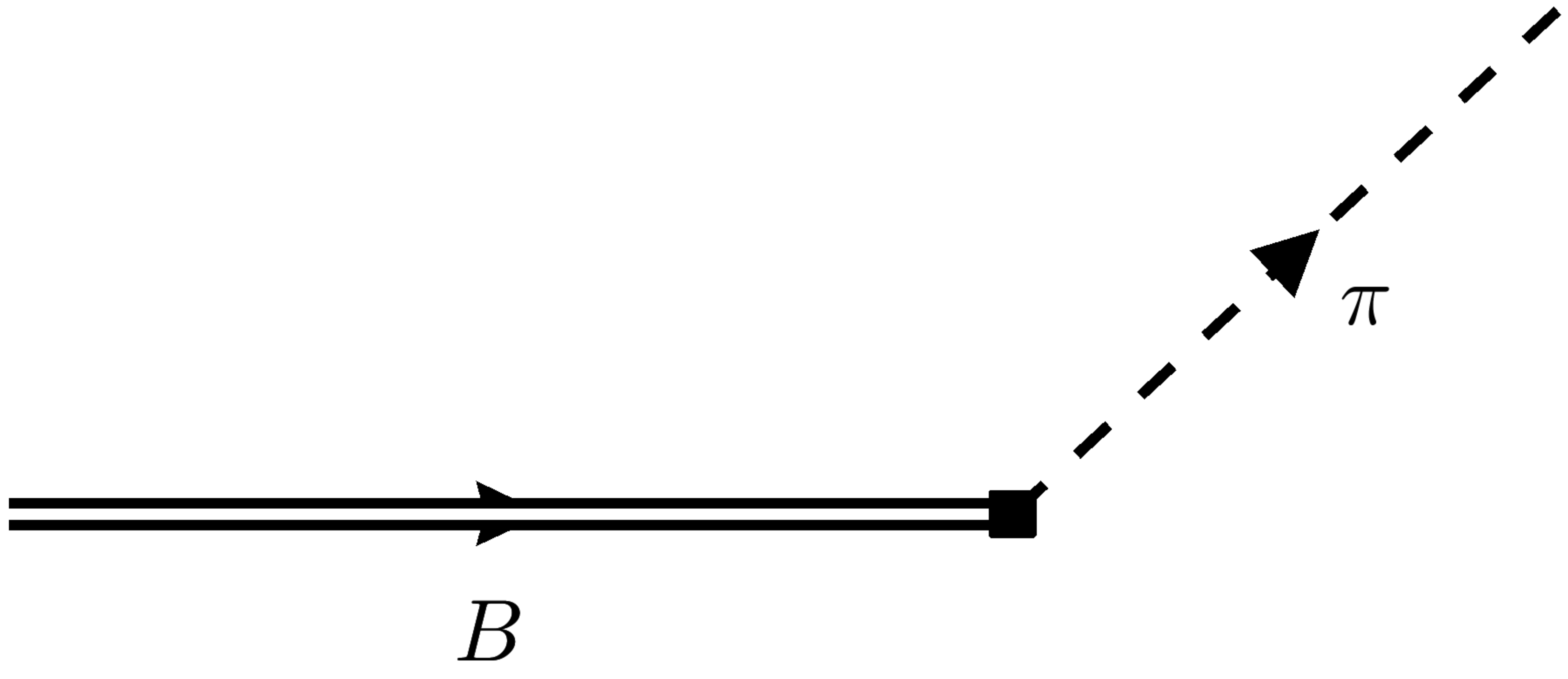
$$\bar{q}_R\Gamma h_v = C(\mu)\mathrm{Tr}\left[H_v\Gamma P_L\right]\xi$$

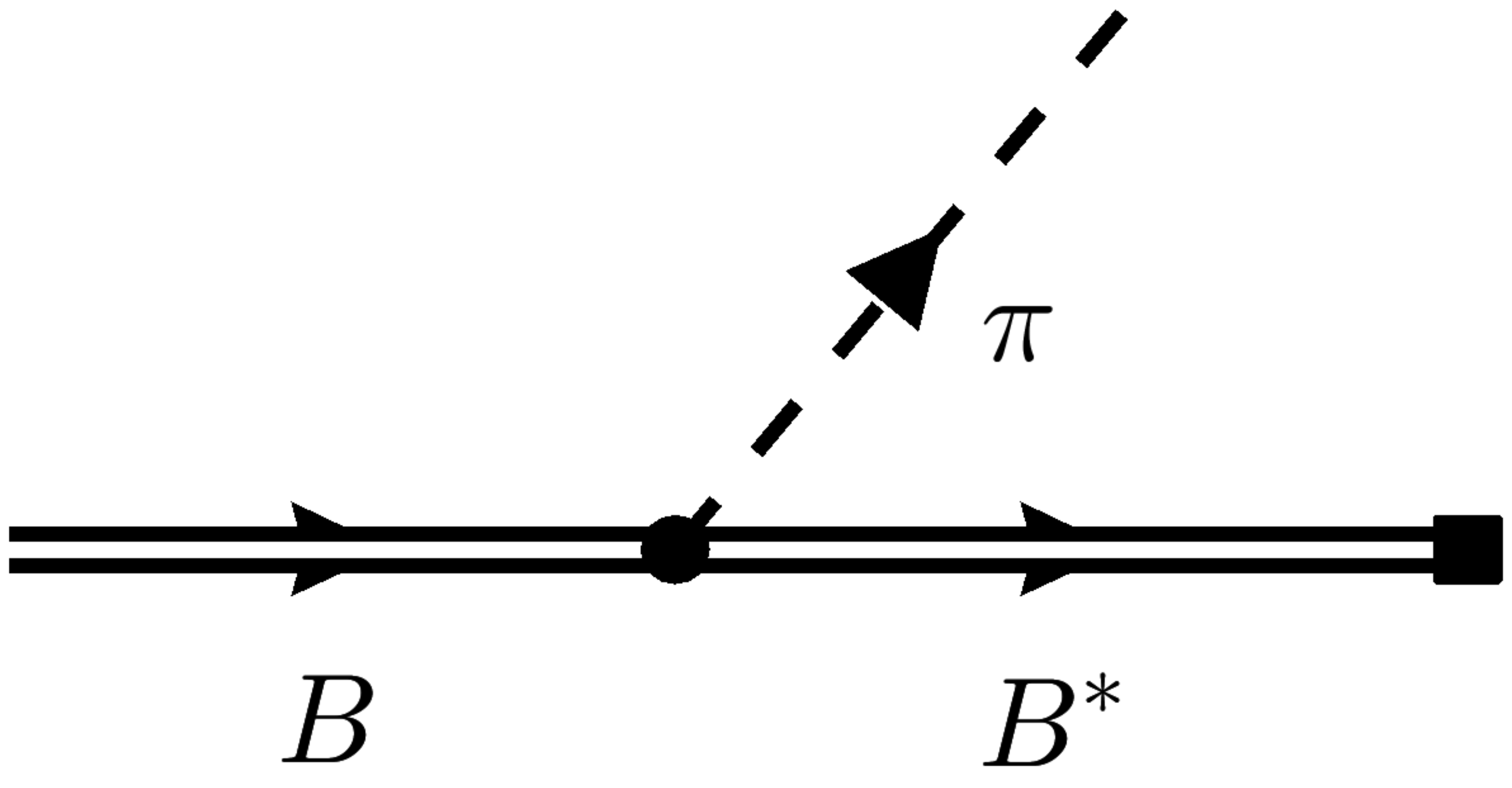
$$\langle 0 | \bar{q}\not{v}(1-\gamma_5)h_v | \bar{B}\rangle = -2\langle 0 | \bar{q}_L h_v | \bar{B}\rangle$$

$$= -2C(\mu)\langle 0 | \mathrm{Tr}\left[H_v P_R\right]\xi^\dagger | \bar{B}\rangle$$

$$C(\mu = m_b) = \sqrt{M_B}f_B + \mathcal{O}(1/m_b, \alpha_s, 1/f_\pi)$$

HHChPT: Form factors





$$f_+^{B\to\pi} + f_-^{B\to\pi} = \left[\frac{f_B}{f_\pi}\right]\left[1 - \frac{g_\pi v\cdot p_\pi}{v\cdot p_\pi + \Delta^B}\right] \qquad f_+^{B\to\pi} - f_-^{B\to\pi} = \frac{g_\pi f_B m_B}{f_\pi\left[v\cdot p_\pi + \Delta^B\right]}$$

$g_\pi = 0.492(29)$ [ALPHA, 1404.6951] $\qquad \Delta^B = M_{B*} - M_B \approx 45\ \text{MeV}$

Heavy to light form factors

$|V_{ub}|$ Determination from $B_{(s)} \to \pi(K)\ell\nu$, need the form factors over full kinematic range

$$|V_{ub}| = (3.75 \pm 0.06_{exp} \pm 0.19_{theo}) \times 10^{-3}$$

HFLAV[2411.18639]

External currents: Matching SCET → HHChPT

$$O_1^{(P)}\big|_L = g^2 \left[\bar{Q}_c^a(0)\,\bar{P}_L\,\frac{\bar{\not{n}}}{2}\,\gamma_5\,P_L\,Q_c^b(s\bar{n})\right]\left[\bar{Q}_s^b(tn)\,\bar{P}_L\,\frac{\bar{\not{n}}\not{n}}{4}\,\gamma_5\,h_v(0)\right]$$

$$= -g^2 \left[\bar{Q}_c^a(0)\,\bar{P}_L\,\frac{\bar{\not{n}}}{2}\,Q_c^b(s\bar{n})\right]\left[\bar{Q}_s^b(tn)\,\bar{P}_L\,\frac{\bar{\not{n}}\not{n}}{4}\,h_v(0)\right] \qquad (37)$$

Let us now redefine the chiral quark fields by rotation with the Goldstone matrices in the non-linear representation of ChSB,

$$P_L Q_c(x) = \xi_c(x)\,\tilde{Q}_{c,L}(x)\,, \qquad \tilde{Q}_{c,L}(x) \to U_c(x)\,\tilde{Q}_{c,L}(x)\,,$$
$$P_L Q_s(x) = \xi_s(x)\,\tilde{Q}_{s,L}(x)\,, \qquad \tilde{Q}_{s,L}(x) \to U_s(x)\,\tilde{Q}_{s,L}(x)\,, \qquad (39)$$

The quark fields in the last expression still have leading interactions with the pion fields arising from the analogous field redefinitions in the kinetic terms,

$$\bar{Q}_c \bar{P}_L\, i\not{\partial} P_L Q_c = \bar{\tilde{Q}}_{c,L}\gamma_\mu \left(i\partial^\mu + i\,\xi_c^\dagger \partial^\mu \xi_c\right)\tilde{Q}_{c,L}\,, \qquad (42)$$

$$\tilde{Q}_{c,L}(s\bar{n}) = Y_{c,L}(s\bar{n})\chi_{c,L}(s\bar{n}) \qquad (43)$$

where $Y_{c,L}(t\bar{n})$ is a straight Wilson line connecting the point $t\bar{n}$ to some reference point at infinity, satisfying

$$\left(\bar{n}\cdot\partial + \xi_c^\dagger\,(\bar{n}\cdot\partial)\,\xi_c\right) Y_{c,L} = 0\,. \qquad (44)$$

External currents: Matching SCET → HHChPT

HQET

SCET II

$$\bar{q}_L \Gamma h_v \stackrel{!}{=} C(\mu) \mathrm{Tr}\left[H_v \Gamma P_R\right] \xi^\dagger$$

$$C(\mu = m_b) = \sqrt{M_B} f_B + \mathcal{O}(1/m_b, \alpha_s, 1/f_\pi)$$

Wilson coefficient is non-perturbative,
related to form factors in chiral and HQ limits

External currents: Matching SCET → HHChPT

HQET

$$\bar{q}_L \Gamma h_v \overset{!}{=} C(\mu)\mathrm{Tr}\left[H_v \Gamma P_R\right]\xi^\dagger$$

$$C(\mu = m_b) = \sqrt{M_B} f_B + \mathcal{O}(1/m_b, \alpha_s, 1/f_\pi)$$

Wilson coefficient is non-perturbative, related to form factors in chiral and HQ limits

SCET II

Soft and collinear *chiral* Wilson lines:

$$S_n^{(\pi)}(x) = P\ \exp\left[i\int_{-\infty}^{0} ds\,(\bar{n}\cdot V_s(x+sn))\right]$$

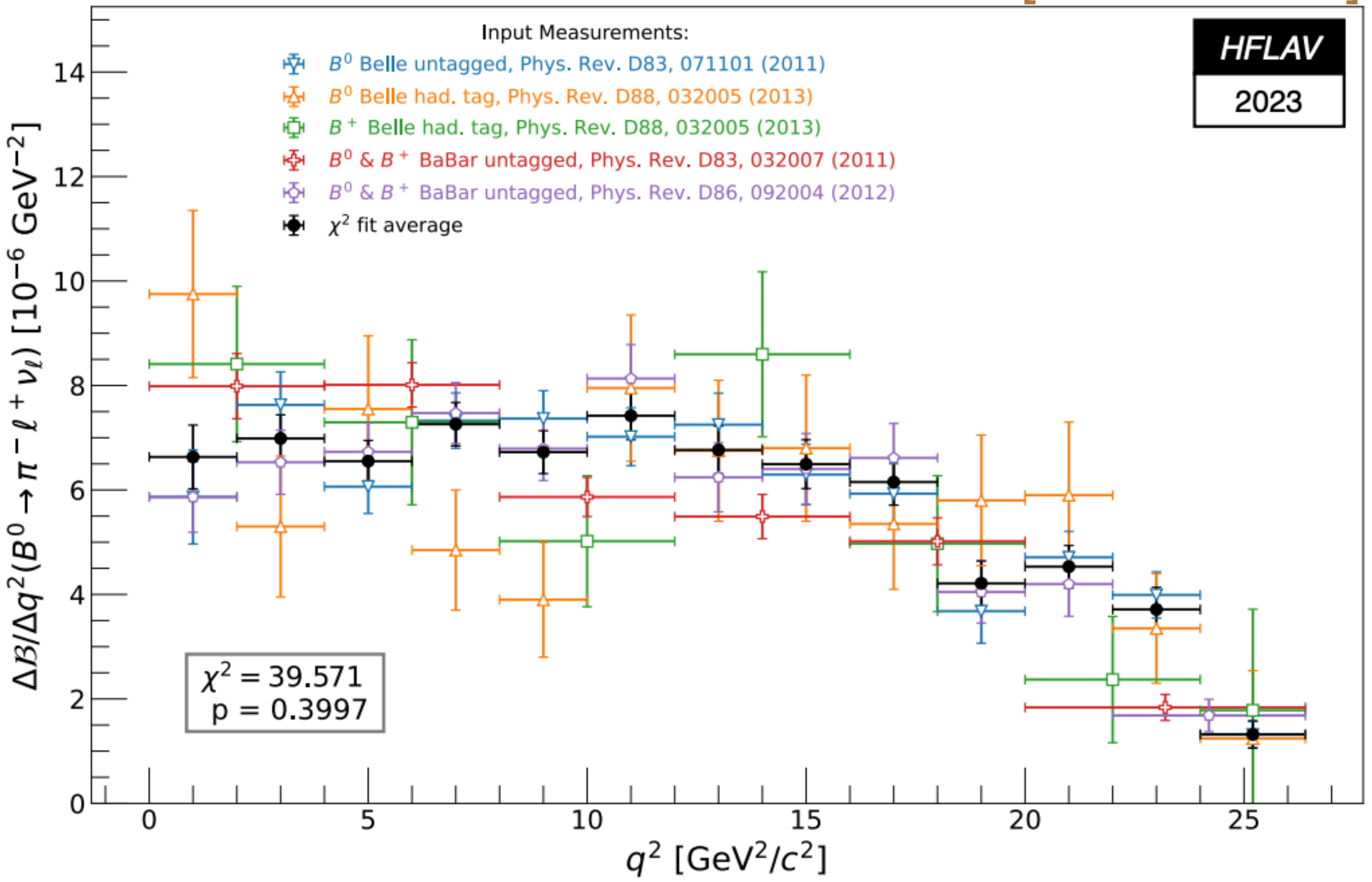
$$W_n^{(\pi)}(x) = P\ \exp\left[i\int_{-\infty}^{0} ds\,(\bar{n}\cdot V_c(x+sn))\right]$$

$$S_n^{(\pi)}(x) \to U_s(x) Y_n^{(\pi)}(x)$$

$$W_n^{(\pi)}(x) \to U_c(x) W_n^{(\pi)}(x)$$

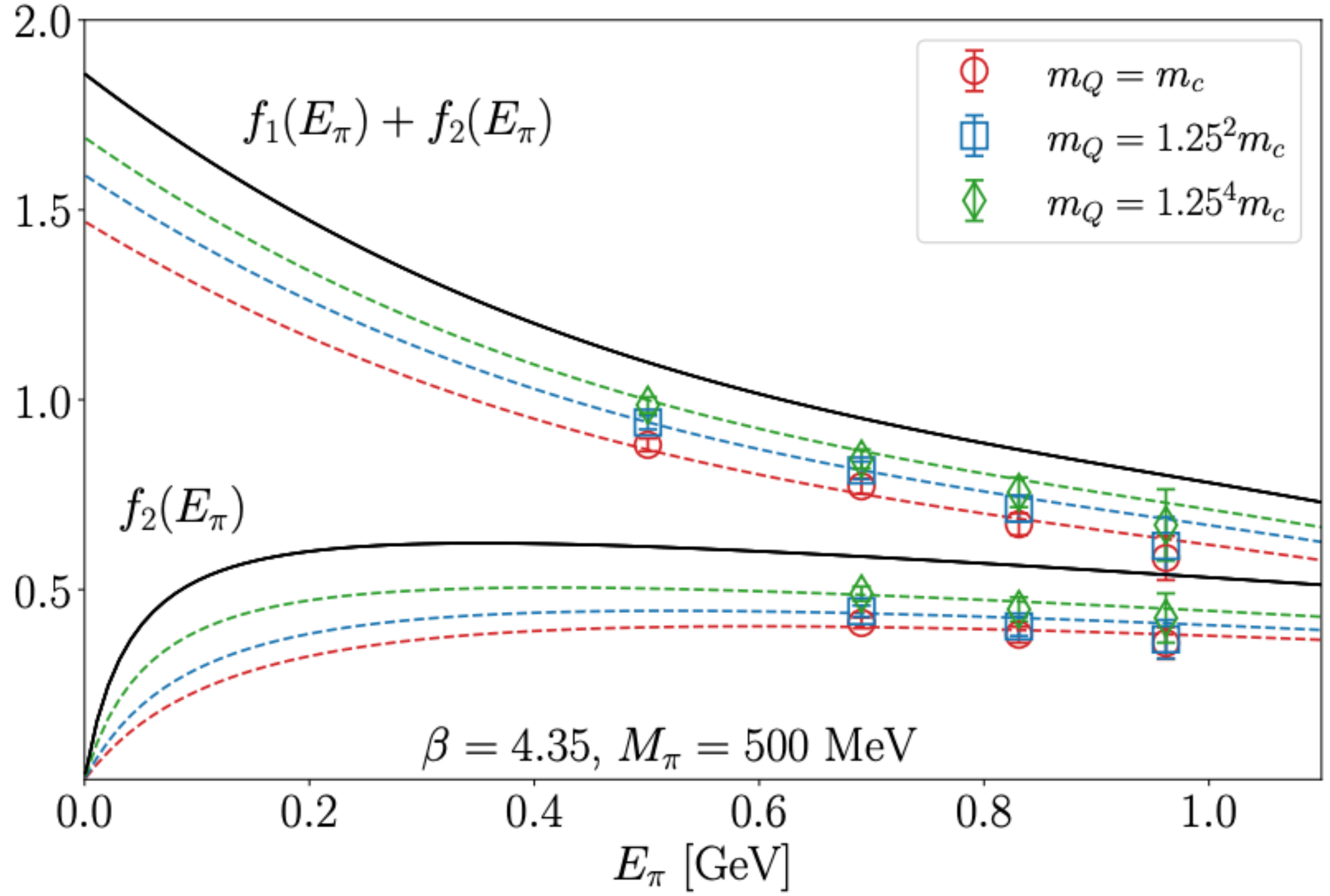
Various $B \to \pi \ell \nu$ measurements

[2411.18639]



HQ extrapolation

JLQCD [2203.04938]



Hard pion ChPT diagrammatic analysis

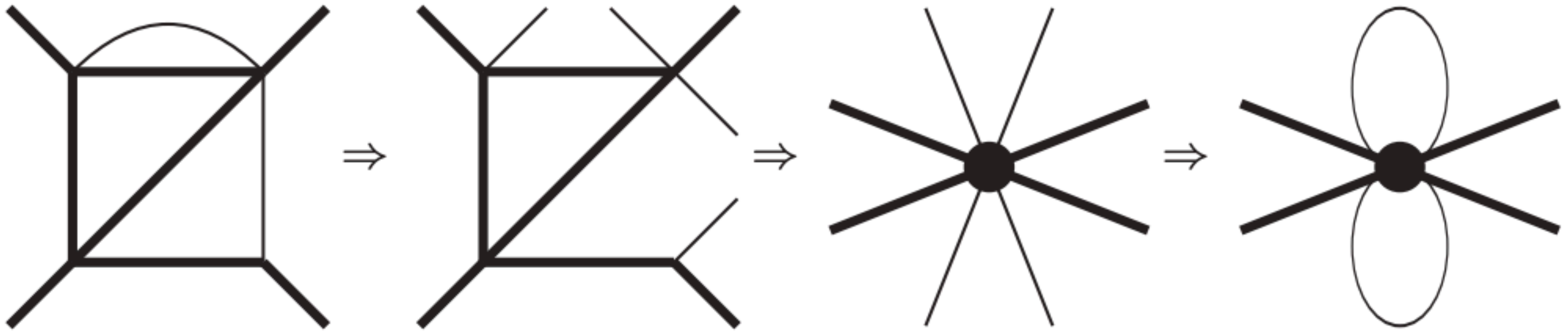


Figure 1: An example of the argument used. The thick lines contain a large momentum, the thin lines a soft momentum. Left: a general Feynman diagram with hard and soft lines. Middle-left: we cut the soft lines to remove the soft singularity. Middle-right: The contracted version where the hard part is assumed to be correctly described by a "vertex" of an effective Lagrangian. Right: the contracted version as a loop diagram. This is expected to reproduce the chiral logarithm of the left diagram. Figure from [14].

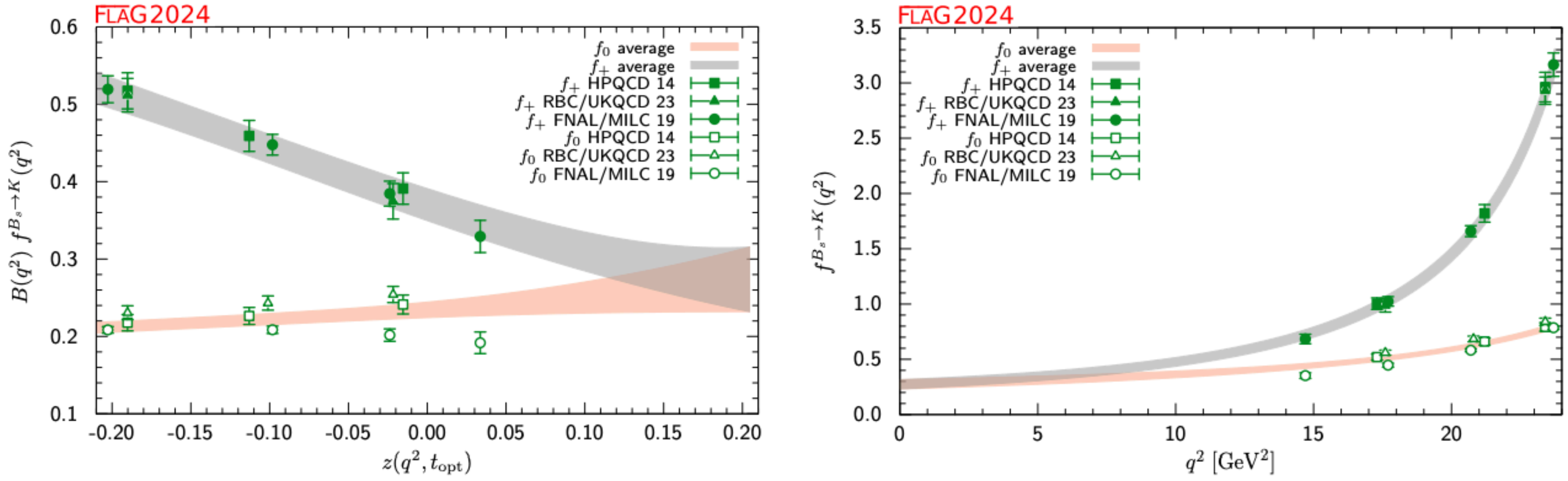


Figure 26: The form factors $f_+(q^2)$ and $f_0(q^2)$ for $B_s \to K\ell\nu$ plotted versus z (left panel) and q^2 (right panel). In the left plot, we remove the Blaschke factors. See text for a discussion of the data sets. The grey and salmon bands display our preferred $N^+ = N^0 = 4$ BCL fit (seven parameters).

$B_s \to K$ form factors

[2411.04268]

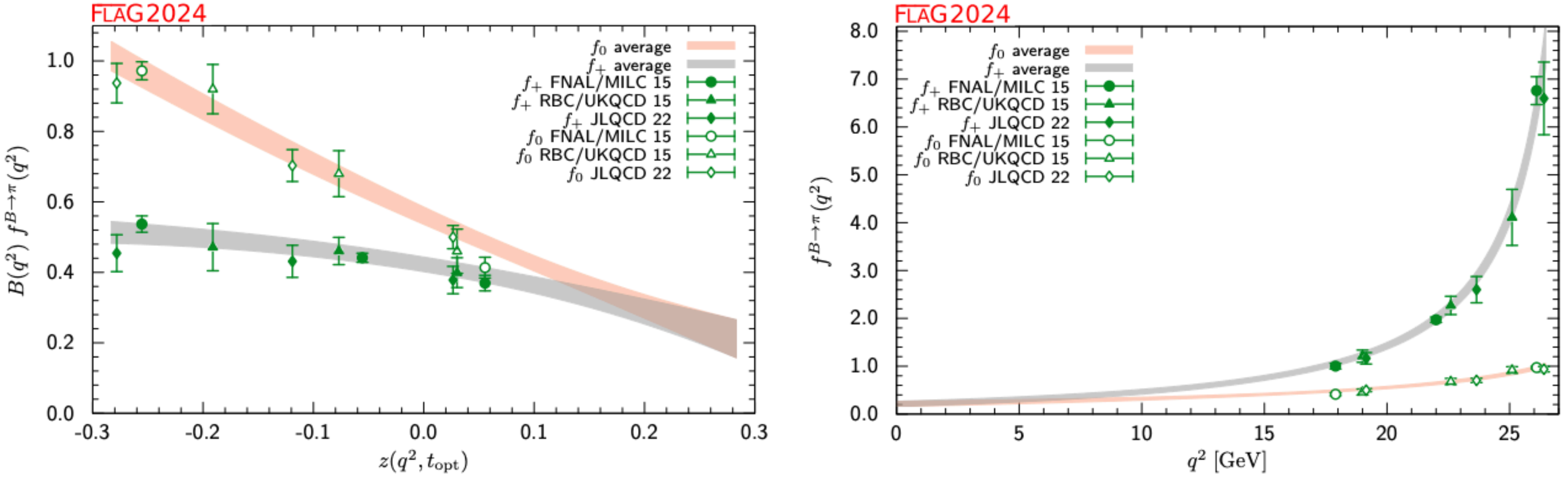


Figure 25: The form factors $f_+(q^2)$ and $f_0(q^2)$ for $B \to \pi\ell\nu$ plotted versus z (left panel) and q^2 (right panel). In the left plot, we removed the Blaschke factors. See text for a discussion of the data set. The grey and salmon bands display our preferred $N^+ = N^0 = 3$ BCL fit (five parameters).

$B \to \pi$ form factors

[2411.04268]

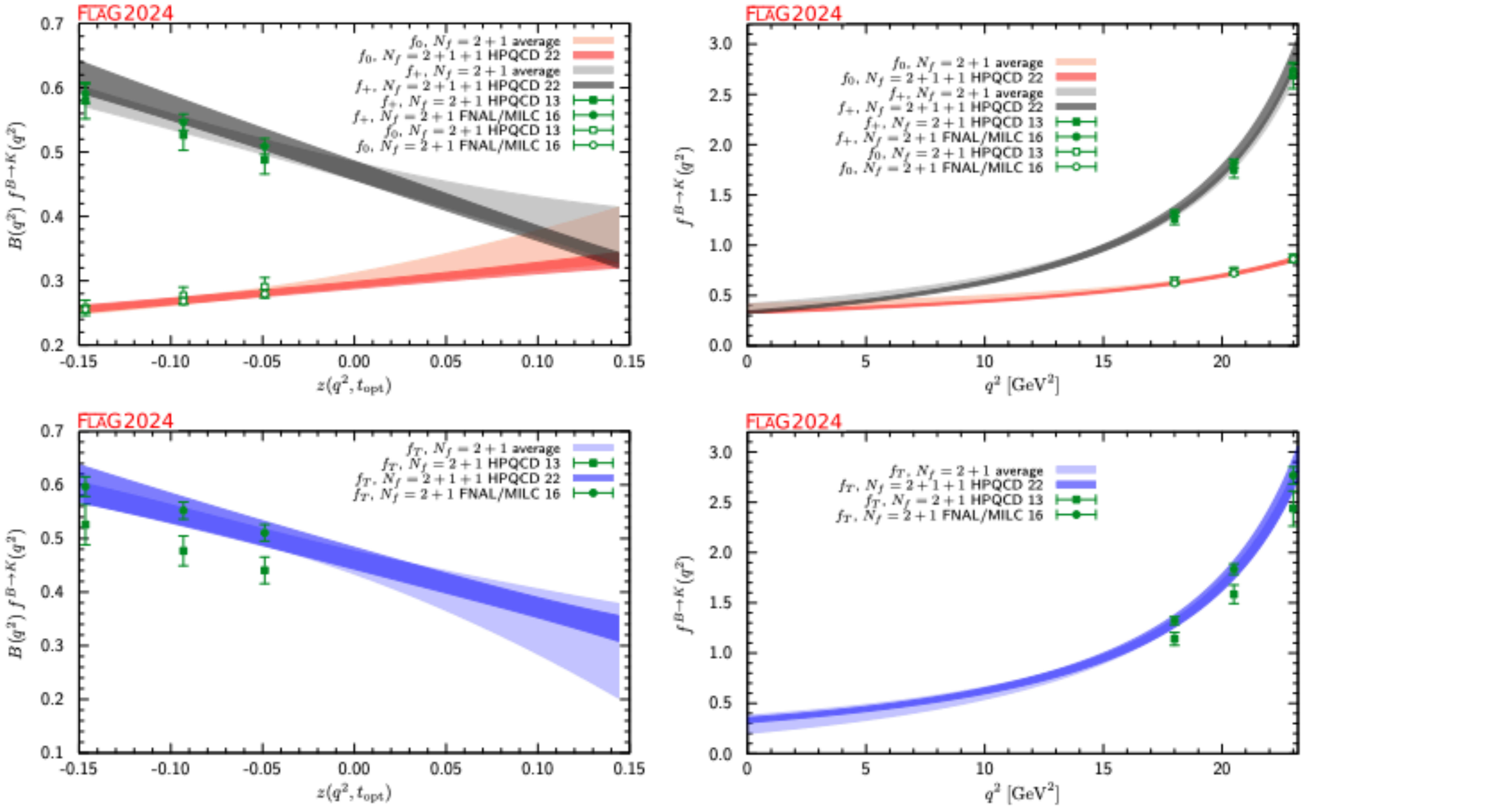


Figure 27: The $B \to K$ form factors $f_+(q^2)$, $f_0(q^2)$ and $f_T(q^2)$ plotted versus z (left panels) and q^2 (right panels). In the plots as a function of z, we remove the Blaschke factors. See text for a discussion of the data sets. The light-shaded grey, salmon and blue bands display our preferred $N^+ = N^0 = N^T = 3$ BCL fit (eight parameters) to the $N_f = 2+1$ lattice results. The dark-shaded grey, salmon and blue bands display the $N_f = 2+1+1$ HPQCD 22 results [487].

$B \to K$ form factors

[2411.04268]

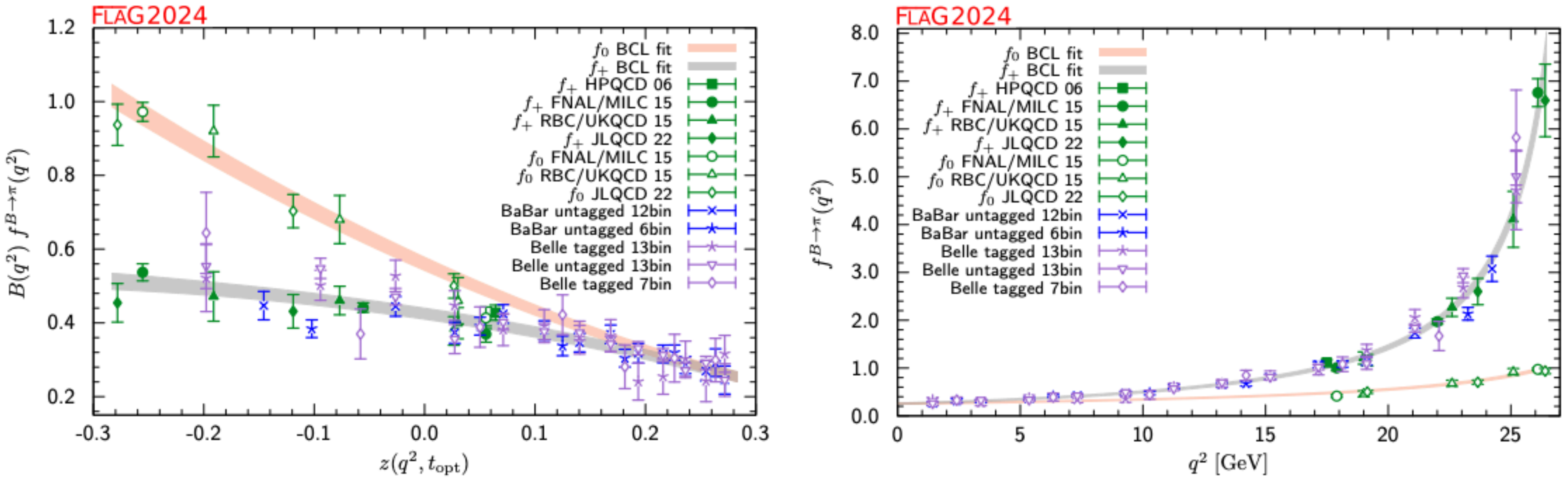


Figure 31: Lattice and experimental data for $f_+^{B\to\pi}(q^2)$ and $f_0^{B\to\pi}(q^2)$ versus z (left panel) and q^2 (right panel). Experimental data has been rescaled by the value for $|V_{ub}|$ found from the joint fit. Green symbols denote lattice-QCD points included in the fit, while blue and indigo points show experimental data divided by the value of $|V_{ub}|$ obtained from the fit. The grey and orange bands display the preferred $N^+ = N^0 = 3$ BCL fit (five z-parameters and $|V_{ub}|$).

$B \to \pi$ form factors

[2411.04268]

$D \to K$ form factors

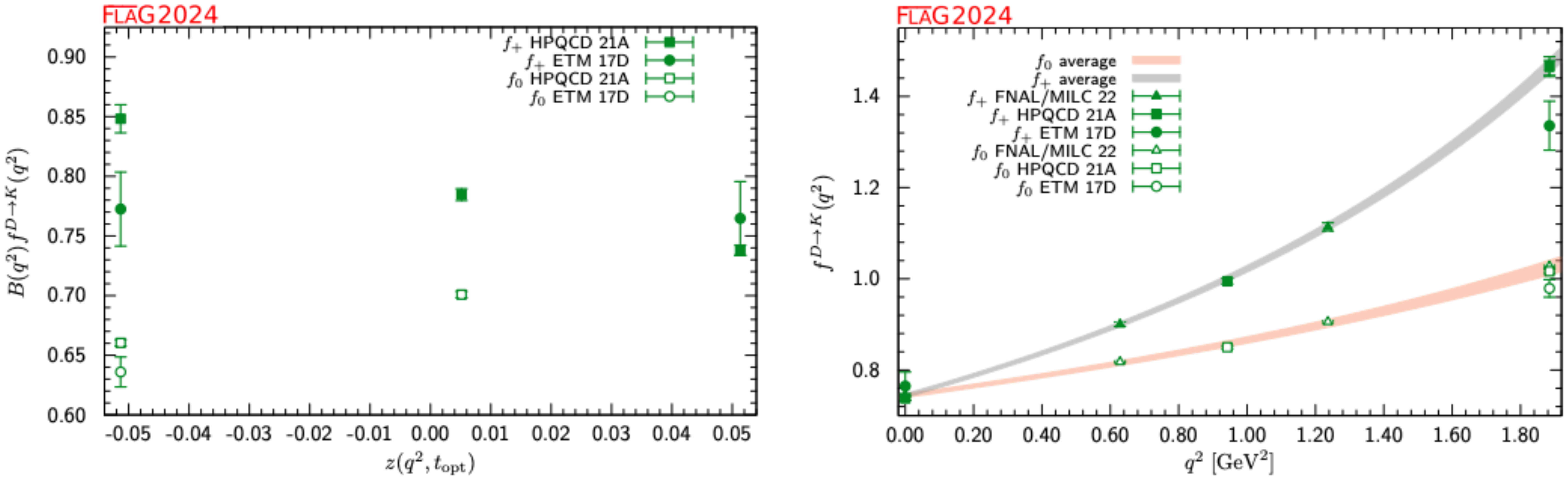


Figure 18: The form factors $f_+(q^2)$ and $f_0(q^2)$ for $D \to K\ell\nu$ plotted versus z (left panel) and q^2 (right panel). In the left plot, we removed the Blaschke factors. See text for a discussion of the data set. The grey and salmon bands display our preferred $N^+ = N^0 = 4$ BCL fit (seven parameters).

[2411.04268]

SCET for ChPT

Covariant formulation of ChPT

$$\xi(x) \equiv \exp\left[\frac{i\pi^a(x)t^a}{f_\pi}\right]$$

$$\Sigma(x) = \xi^2(x) \to L\xi^2(x)R^\dagger$$

$$\xi(x) \to L\xi(x)U^\dagger(x) = U(x)\xi(x)R^\dagger$$

$$V^\mu = \frac{1}{2}\left[\xi^\dagger iD_L^\mu \xi + \xi iD_R^\mu \xi^\dagger\right] \qquad V^\mu \to UV^\mu U^\dagger + U[iD^\mu, U^\dagger]$$

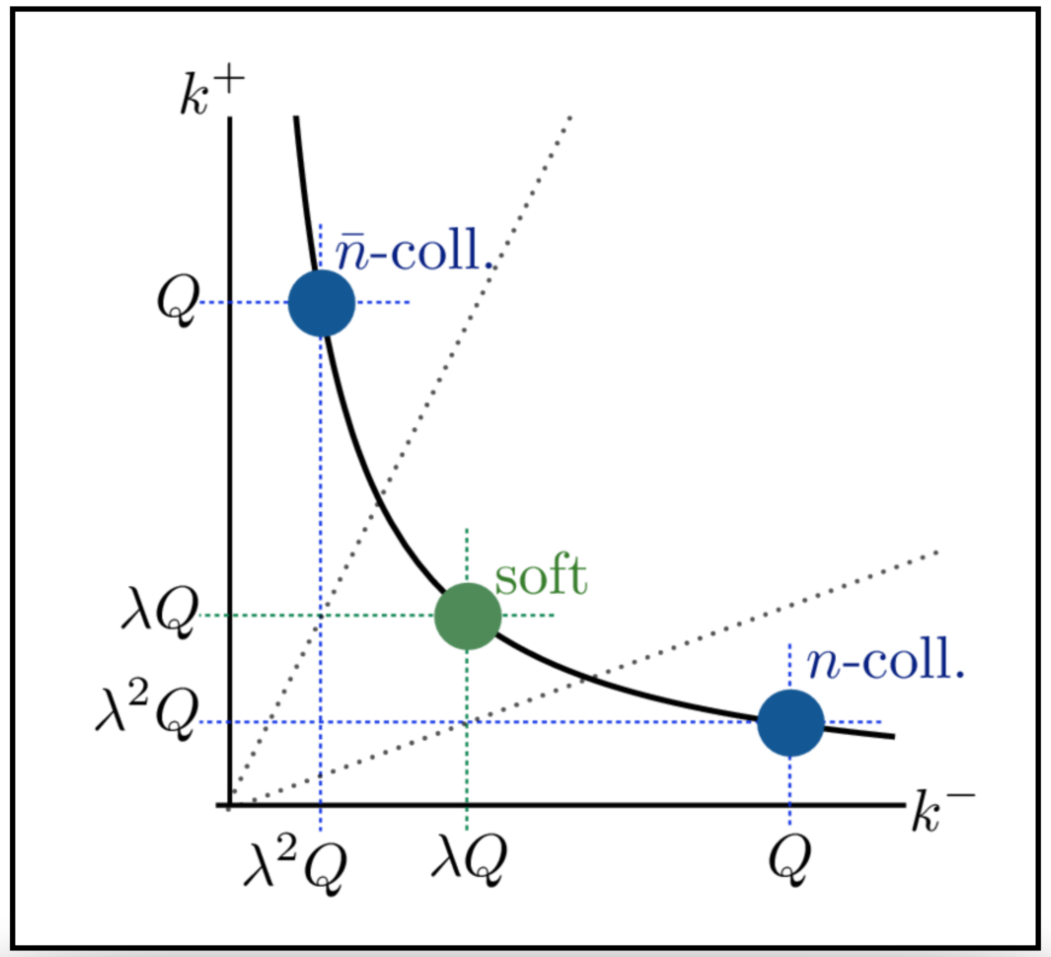
$$A^\mu = \frac{1}{2}\left[\xi^\dagger iD_L^\mu \xi - \xi iD_R^\mu \xi^\dagger\right] \qquad A^\mu \to UA^\mu U^\dagger$$

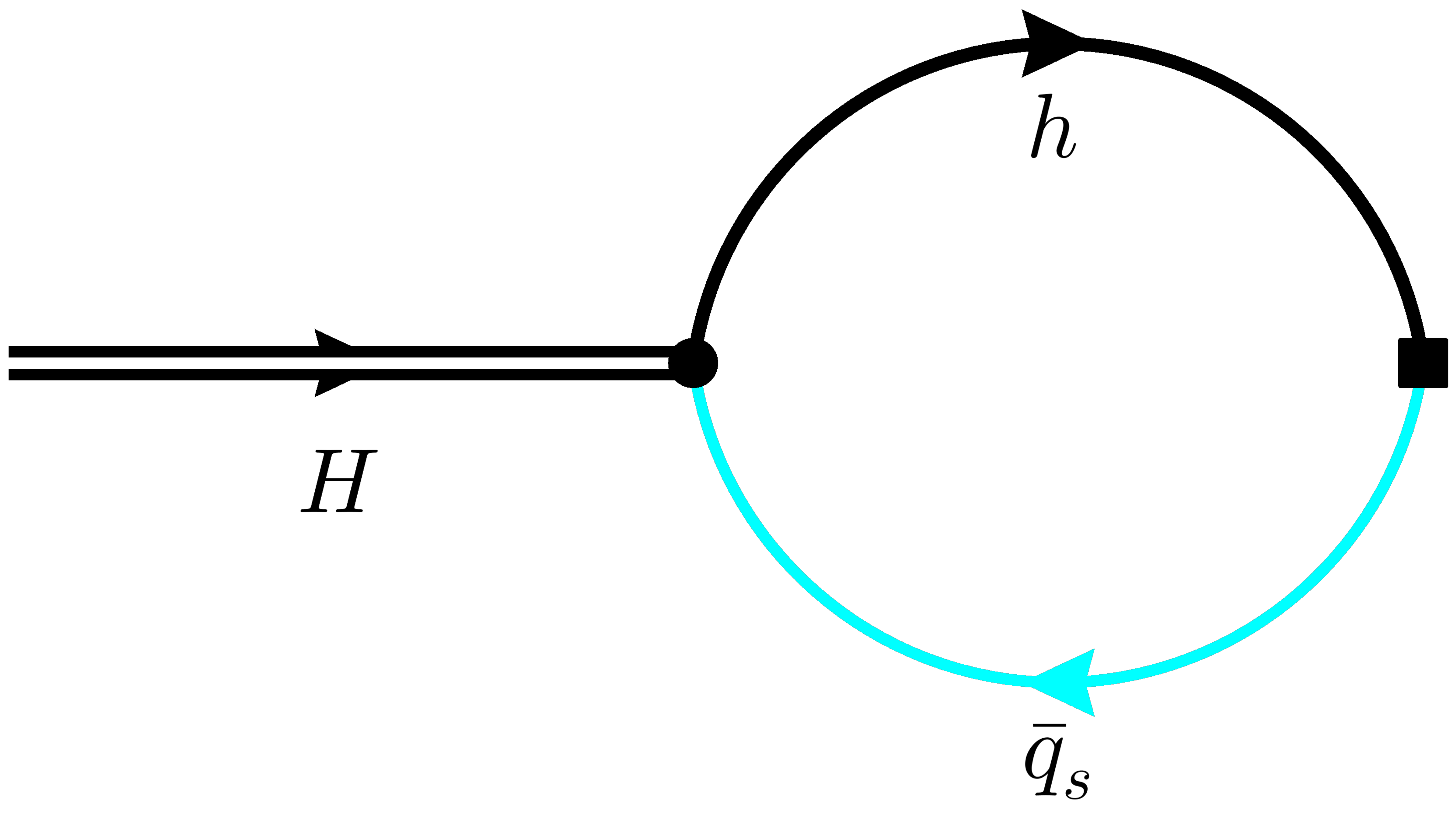
$$D^\mu_{L(R)} = \partial^\mu - i(V_{\rm ext} \mp A_{\rm ext})^\mu$$

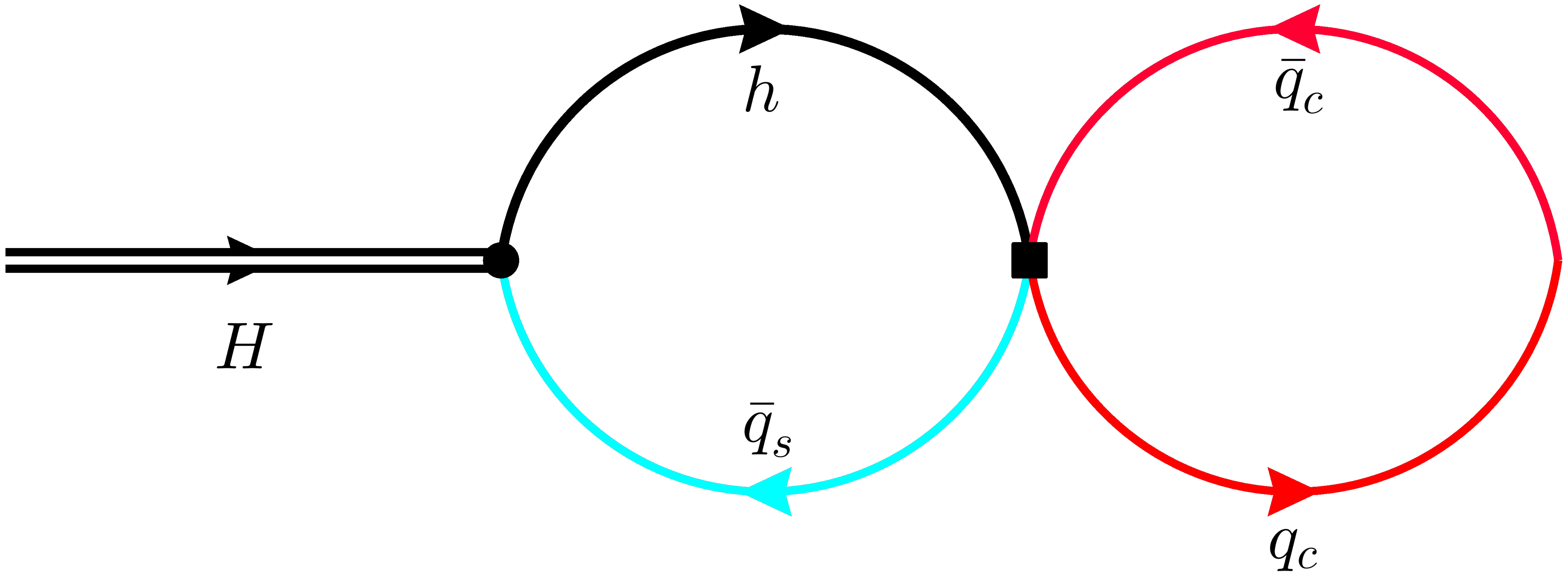
$$D^\mu = \partial^\mu - iV^\mu_{\rm ext}$$

Introducing the soft, collinear, and soft-collinear (messenger) sectors

$$V^\mu \to V^\mu_s, V^\mu_c, V^\mu_{sc}$$




h
H
$\bar{q}_s$


h
$\bar{q}_c$
H
$\bar{q}_s$
q_c