# Renormalisation scheme dependence of the trilinear Higgs coupling in extended scalar sectors

Herbstschule of High-Energy Phyiscs, Bad Honnef 2025

Marc Hannig, Elina Fuchs, Marco Menen, Henning Bahl,
Georg Weiglein, Johannes Braathen

Leibniz Universität Hannover

Herbstschule HEP
formerly Maria Laach

DESY.

# Motivation: Trilinear Coupling

[Bahl, Braathen, Gabelmann, Weiglein 2305.03015]

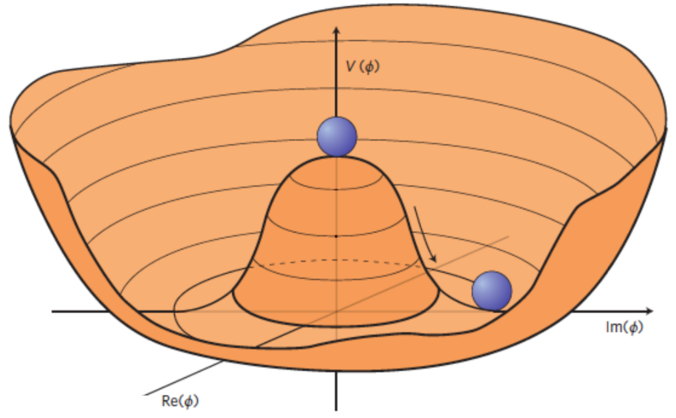
- Trilinear Higgs coupling determines the shape of the Higgs potential:
  - crucial for understanding dynamics of electroweak phase transitions

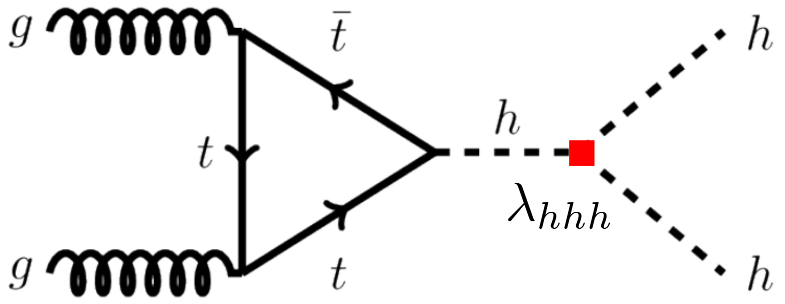
$$V_{\mathrm{SM}} \supset \frac{m_h^2}{2} h^2 + \underbrace{\frac{3m_h^2}{v}}_{\lambda_{hhh}^{\mathrm{SM},0}/6} h^3 + \frac{3m_h^2}{v^2} h^4$$

  - sensitive to BSM physics
  - exp. limit: $-1.2 < \frac{\lambda_{hhh}}{\lambda_{hhh}^{\mathrm{SM},0}} < 7.2$ at 95% C.L.

    [ATLAS, 2406.09971]

    [CMS, 2407.13554]



# Singlet Extension of the Standard Model (SSM)

[Bahl, Braathen, Gabelmann, Weiglein 2305.03015]

- we consider the real singlet extension of the SM

$$V(\Phi, S) = \mu^2 |\Phi| + \frac{\lambda}{2} |\Phi|^2 + \kappa_{SH} \left(\Phi^\dagger \Phi\right) S + \frac{\lambda_{SH}}{2} \left(\Phi^\dagger \Phi\right) S^2 + \frac{m_S^2}{2} S^2 + \frac{\kappa_S}{3} S^3 + \frac{\lambda_S}{2} S^4$$

# Singlet Extension of the Standard Model (SSM)

[Bahl, Braathen, Gabelmann, Weiglein 2305.03015]

- we consider the real singlet extension of the SM

$$V(\Phi, S) = \mu^2 |\Phi| + \frac{\lambda}{2} |\Phi|^2 + \kappa_{SH} \left(\Phi^\dagger \Phi\right) S + \frac{\lambda_{SH}}{2} \left(\Phi^\dagger \Phi\right) S^2 + \frac{m_S^2}{2} S^2 + \frac{\kappa_S}{3} S^3 + \frac{\lambda_S}{2} S^4$$

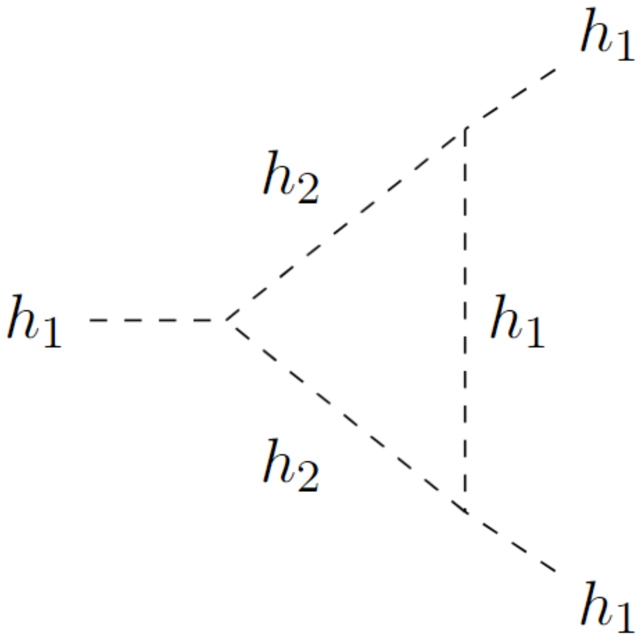
- after symmetry breaking: $\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2} G^+ \\ v+h+iG \end{pmatrix}$ $\quad S = s + v_S$
- the fields mix to 2 CP-even eigenstates $h_{1,2}$
- the Lagrangian parameters are traded for $m_{h_1}, m_{h_2}, \alpha, v, v_S, \kappa_S, \kappa_{SH}$

5 new independent parameters

- trilinear coupling at tree level: $\lambda_{h_1 h_1 h_1} = 3 m_{h_1}^2 \left( \frac{\cos^3 \alpha}{v} + \frac{\sin^3 \alpha}{v_S} \right)$

# Motivation: Precision Calculations

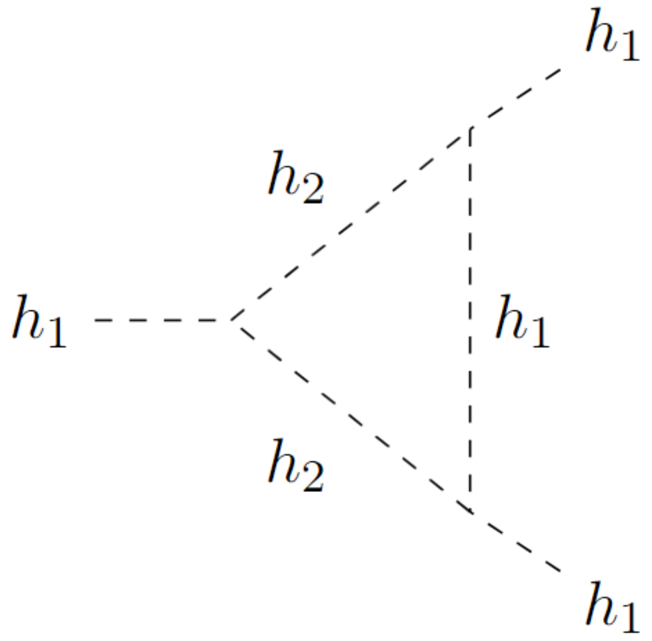
- reliable and numerically stable theoretical predictions of physical observables are imperative
  - important for comparison with experimental data
  - needed for consistent constraints on parameter space of BSM models



# Problem: Precision Calculations

- integrals over loop momenta diverge

  extracted by dimensional regularisation
  $D = 4 - 2\epsilon$



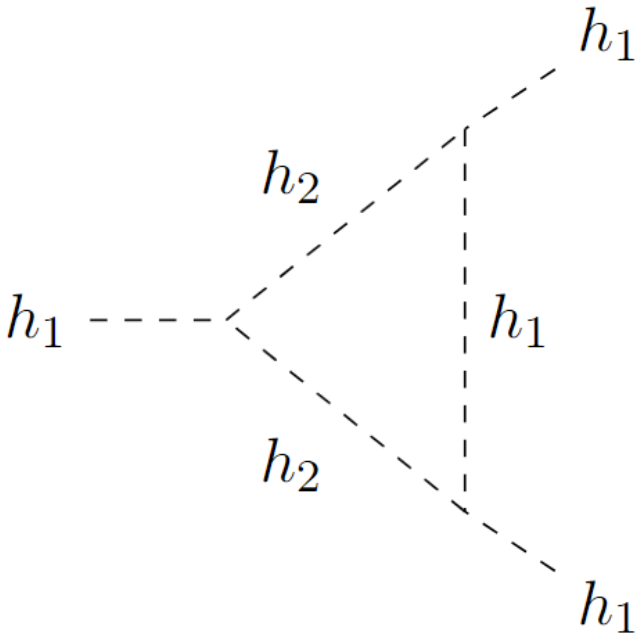
# Problem: Precision Calculations

- integrals over loop momenta diverge

  extracted by dimensional regularisation ✓
  $D = 4 - 2\epsilon$

- final expression contains divergence for D→4

  absorbed in renormalised parameters ✓




understanding of renormalisation scheme dependence needed

# Choice of Renormalization scheme

| On shell renormalisation | |
|---|---|
| • renormalised parameters correspond to measurable quantities<br>• impose a renormalisation condition $\rightarrow$ this subtracts the divergence | |

# Choice of Renormalization scheme

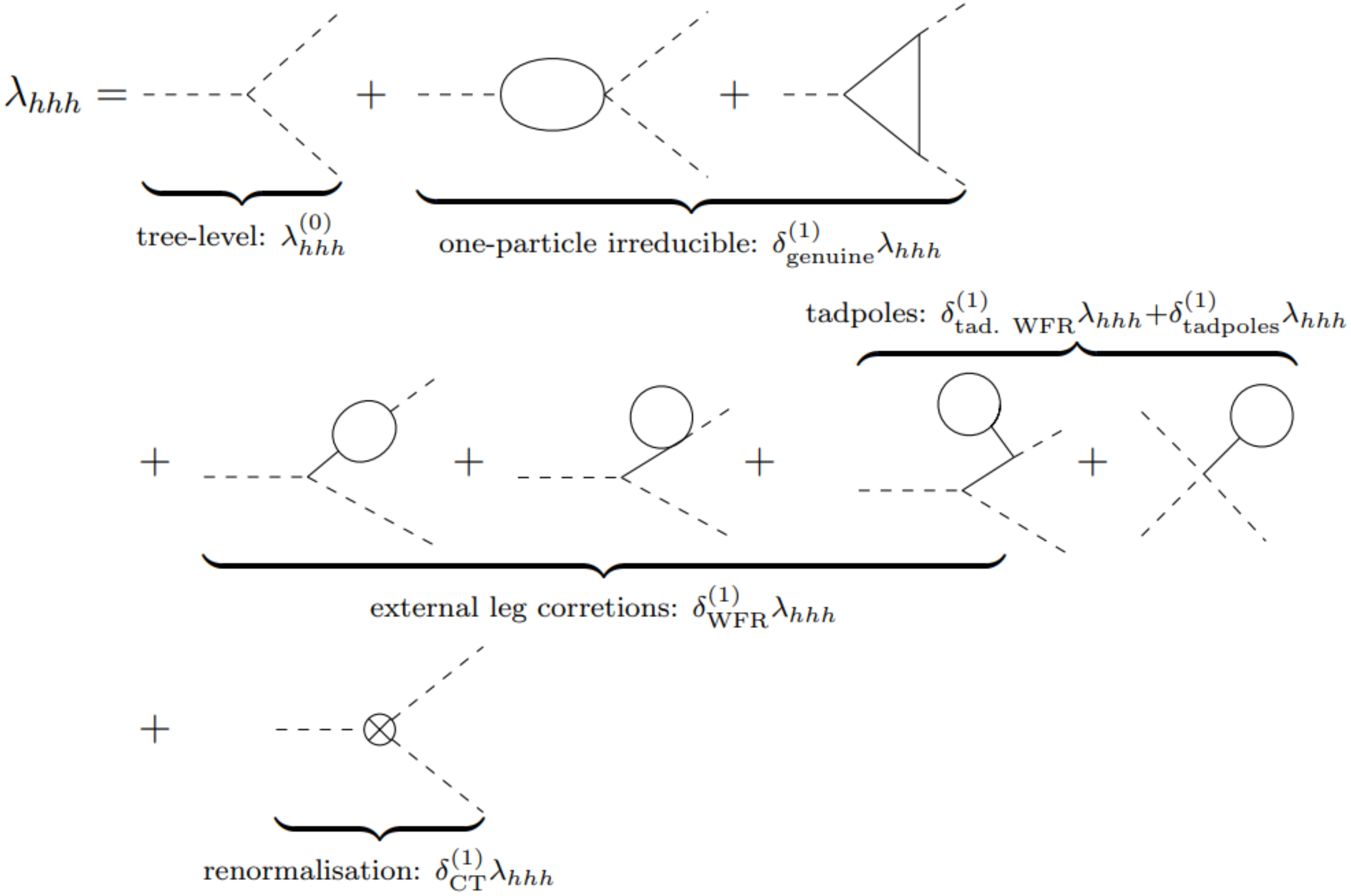
| On shell renormalisation | Minimal subtraction renormalisation |
|---|---|
| • renormalised parameters correspond to measurable quantities<br>• impose a renormalisation condition $\rightarrow$ this subtracts the divergence | • no conditions $\longrightarrow$ only divergent part is subtracted<br>$\delta m^2 = a\frac{1}{\epsilon} + \cancel{b\epsilon^0 + c\epsilon}$<br>• $\mu$ dependence $\longrightarrow$ running of parameters |

$$m_B = m^{\overline{\text{MS}}}(\mu) + \delta m^{\overline{\text{MS}}}(\mu) = m^{\text{OS}} + \delta m^{\text{OS}}$$

# Predicting the Trilinear Coupling: anyH3

[Bahl, Braathen, Gabelmann, Weiglein; 2305.03015]

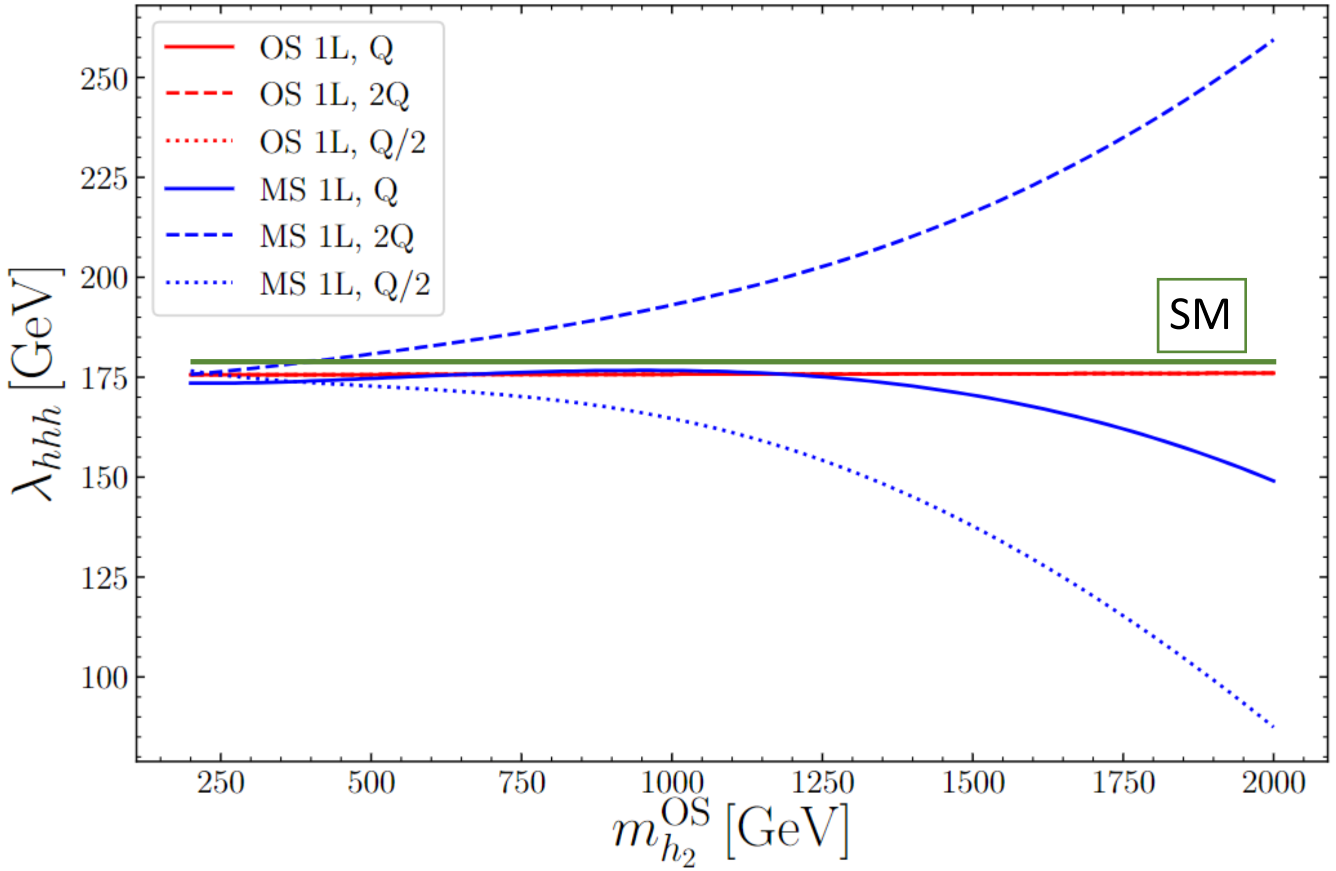
- Python code anyH3 allows fast computation of $\lambda_{hhh}$ at full one-loop level
- customize:
  - parameters/couplings
  - renormalisation
  - tadpole treatment



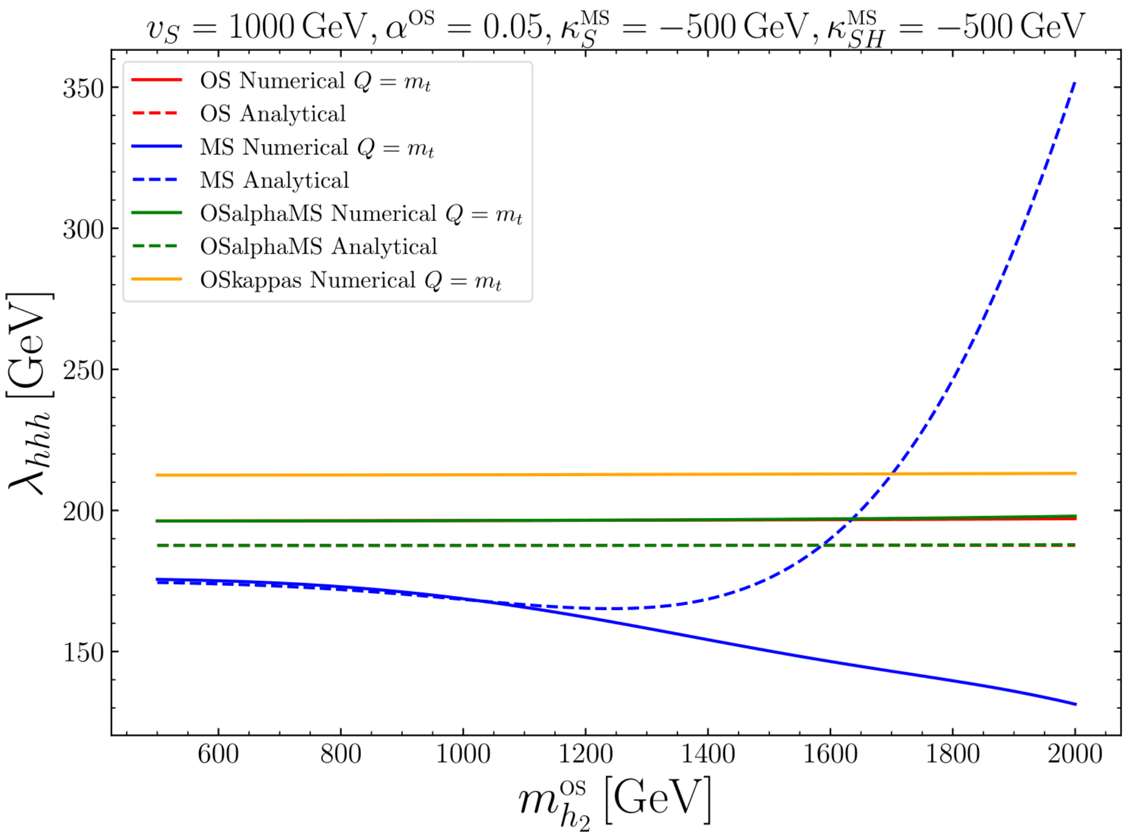
# Scheme Dependence of the Trilinear Coupling

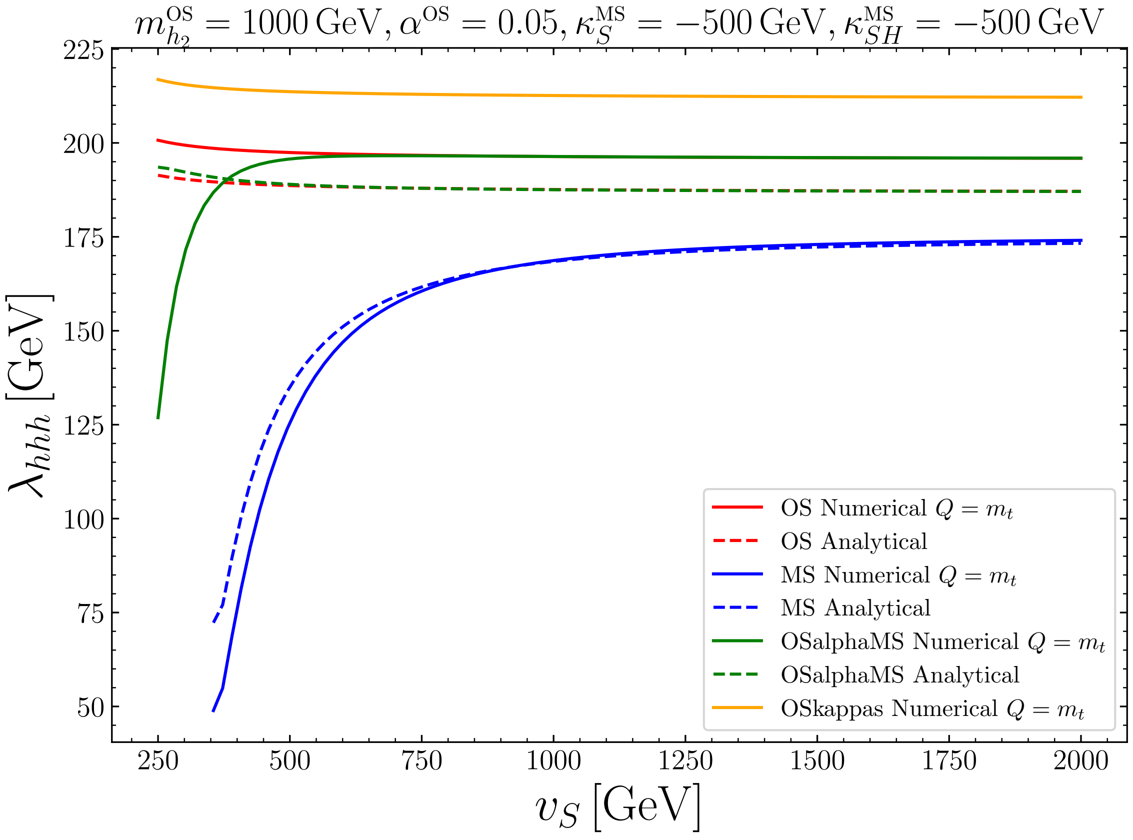
- implemented conversion
- varying $Q$ provides an estimate of size of higher order effects
- MS band contains OS values

$v_S = 1000$ GeV, $\alpha^{\text{OS}} = 0.05$, $\kappa_S^{\overline{\text{MS}}} = -500$ GeV, $\kappa_{SH}^{\overline{\text{MS}}} = -500$ GeV



# Comparison of different schemes



# Summary

- trilinear Higgs coupling as an important observable
- first tests of renormalisation scheme dependence in SSM
- further steps: investigate full parameter space, criteria definition for the schemes, extension to other BSM models

Goal: find algorithmic criteria for which scheme to use depending on region in parameter space $\longrightarrow$ deliver a robust, consistent prediction for $\lambda_{hhh}$

# Backup Slides

# Tadpole Treatment

- Tadpole-free $\overline{\text{MS}}$ scheme:

$$T_h = t_h + \delta^{(1)} t_h = 0 \Rightarrow t_h = -\delta^{(1)} t_h = - \left. \frac{\partial V^{(1)}}{\partial h} \right|_{\min}$$

- OS tadpole renormalisation:

$$t_h = 0 \qquad \delta^{(1)}_{\text{CT}} t_h = -\delta^{(1)} t_h$$

- MS tadpole renormalisation at the tree-level minimum (FJ):

$t_h = 0$ and $\overline{\text{MS}}$ renormalised tadpoles

# Renormalisation Schemes for SSM

- full OS scheme:

$$\delta\alpha = \frac{1}{2}\frac{\Sigma_{h_1 h_2}(p^2 = m_{h_1}^2) + \Sigma_{h_1 h_2}(p^2 = m_{h_2}^2)}{m_{h_2}^2 - m_{h_1}^2} \qquad \delta v_S = \cos\alpha \frac{\delta^{(1)} t_{h_2}}{m_{h_2}^2} + \sin\alpha \frac{\delta^{(1)} t_{h_1}}{m_{h_1}^2}$$

- MS scheme: all parameters renormalised $\overline{\text{MS}}$

- hybrid scheme: OS masses and VEV but $\overline{\text{MS}}$ mixing angle

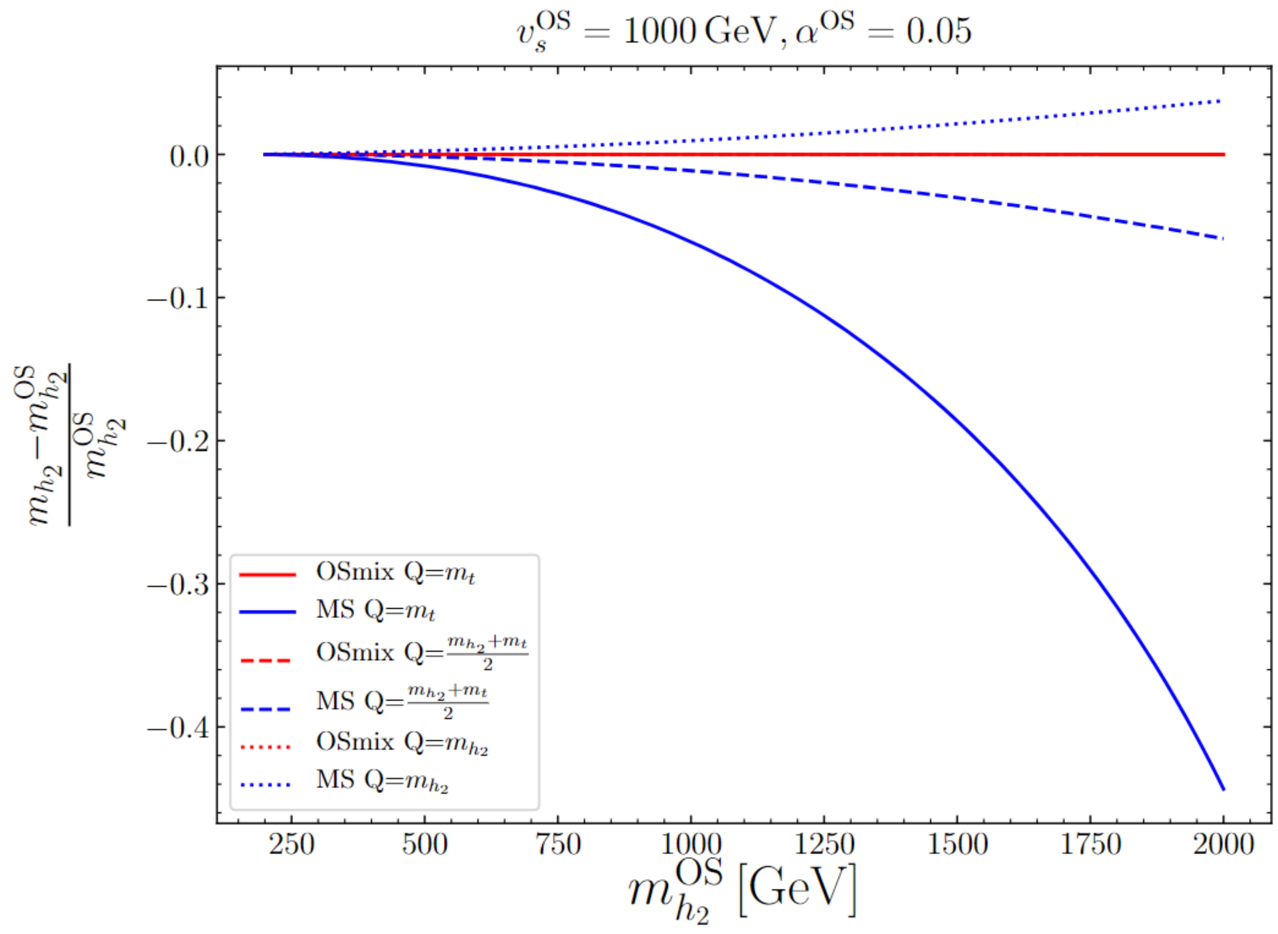
$$\delta v_S = \cos\alpha \frac{\delta^{(1)} t_{h_2}}{m_{h_2}^2} + \sin\alpha \frac{\delta^{(1)} t_{h_1}}{m_{h_1}^2}$$

# Analytic Formulas

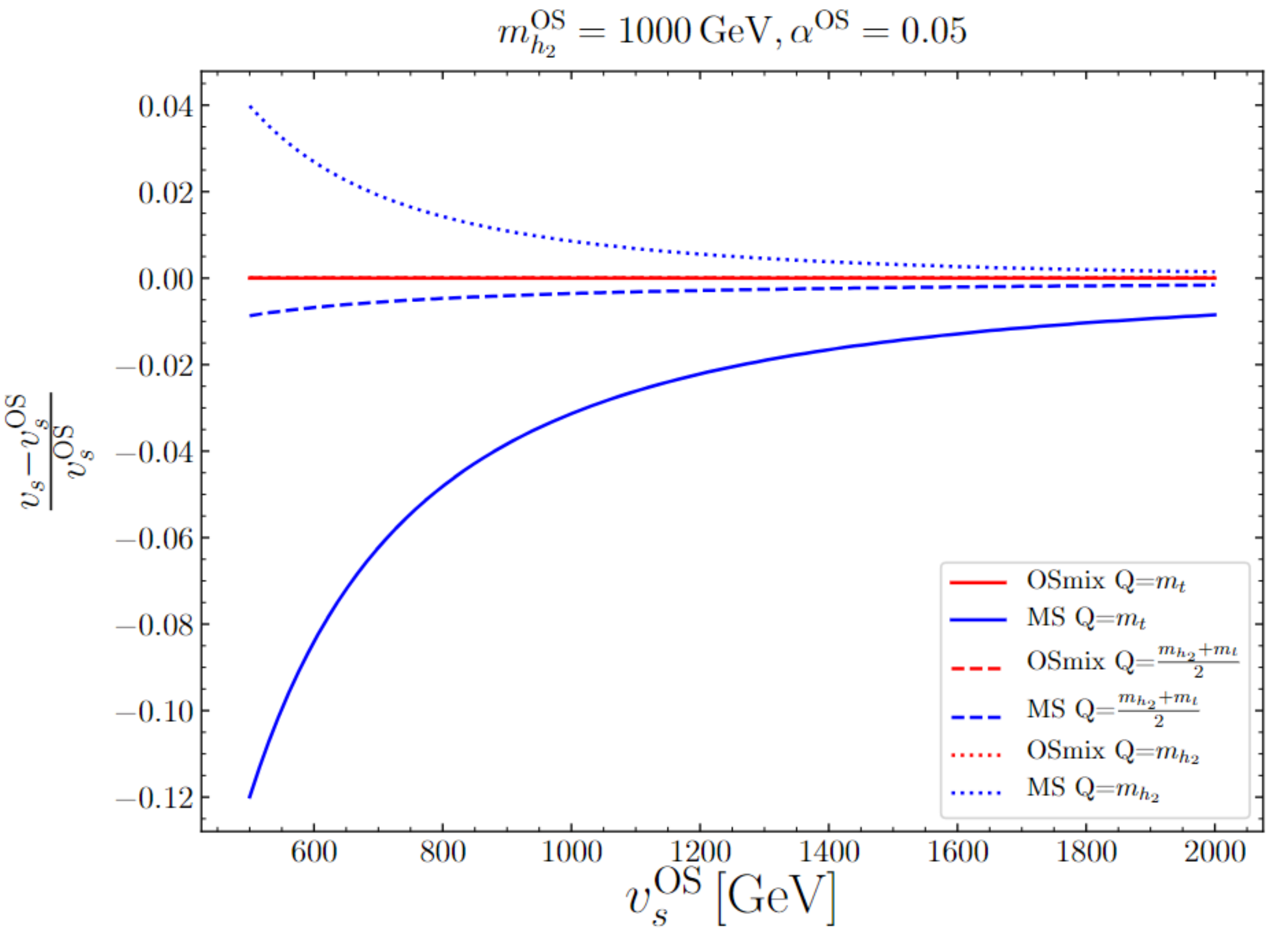
$$\lambda_{hhh}^{OS,(1)} = \frac{\kappa_{SH}^2 v^2}{64 m_{h_2}^2 \pi^2 v_S^4}\Big[6\alpha m_{h_2}^2(-4+\sqrt{3}\pi)v_S - 2\kappa_{SH} v_S v - \alpha(-3+\sqrt{3}\pi)\big(2\kappa_S v_S^2 - 3\kappa_{SH} v^2\big)\Big]$$

$$\lambda_{hhh}^{MS,(1)} = \frac{1}{64 m_{h_2}^2 \pi^2 v_S^4 v^2}\Big\{6\alpha m_{h_2}^6 v_S^3 + 6\kappa_{SH} m_{h_2}^4 v_S^3 v - 6\alpha\kappa_{SH}^2 v_S(-2m_{h_2}^2 + \kappa_S v_S)v^4 - 2\kappa_{SH}^3 v_S v^5 + 9\alpha\kappa_{SH}^3 v^6 - 6\Big[\kappa_{SH} m_{h_2}^2 v_S^2 v(m_{h_2}^2 v_S - \kappa_{SH} v^2) + \alpha\big(m_{h_2}^6 v_S^3 + 2\kappa_{SH} m_{h_2}^2 v_S^2(2m_{h_2}^2 - \kappa_S v_S)v^2 + \kappa_{SH}^2 v_S(9m_{h_2}^2 - 2\kappa_S v_S)v^4 + 3\kappa_{SH}^3 v^6\big)\Big]\cdot \ln\left(\frac{m_{h_2}^2}{\mu^2}\right)\Big\}$$

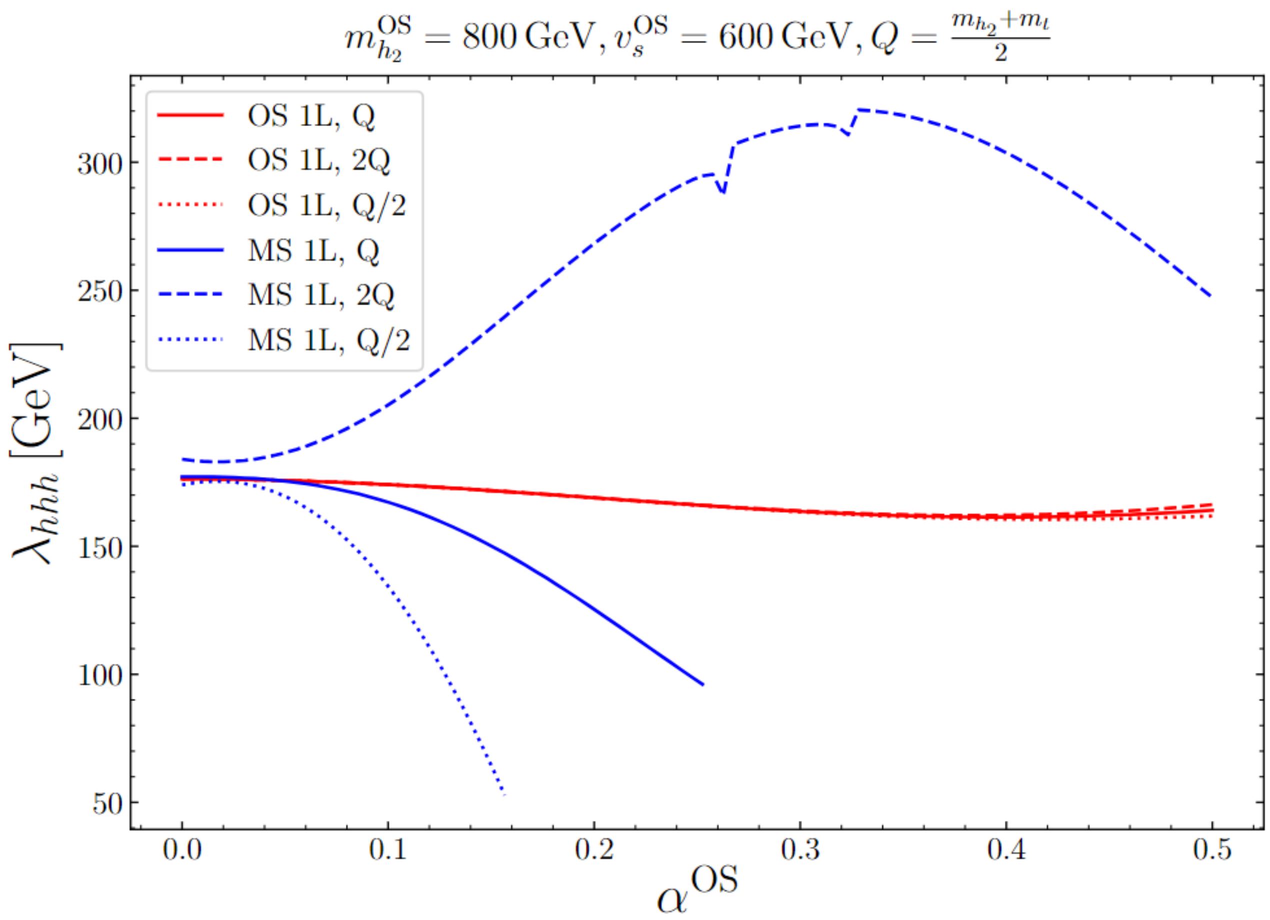
# Change of BSM mass



# Change of singlet VEV



# Dependence on the mixing angle



# Reparametrisation of Lagrangian

$$\lambda = \frac{c_\alpha^2 m_{h_1}^2 + s_\alpha^2 m_{h_2}^2}{v^2} - \frac{t_\phi}{v^3}$$

$$\lambda_{SH} = \frac{\left(m_{h_1}^2 - m_{h_2}^2\right) c_\alpha s_\alpha}{v v_S} - \frac{\kappa_{SH}}{v_S}$$

$$\lambda_S = \frac{\left(m_{h_1}^2 + m_{h_2}^2\right) v_S + \left(m_{h_2}^2 - m_{h_1}^2\right) v_S c_{2\alpha} - 2t_S - 2\kappa_S v_S^2 + \kappa_{SH} v^2}{8 v_S^3}$$

$$M_S^2 = \frac{6t_S - \kappa_{SH} v^2 - 2\kappa_S v_S^2 - \left(m_{h_1}^2 + m_{h_2}^2\right) v_S + \left(m_{h_2}^2 - m_{h_1}^2\right) \left(v s_{2\alpha} - v_S c_{2\alpha}\right)}{4 v_S}$$

$$\mu^2 = \frac{3}{2}\frac{t_\phi}{v} - \frac{1}{2}\kappa_{SH} v_S - \frac{1}{2}\left(m_{h_1}^2 c_\alpha^2 + m_{h_2}^2 s_\alpha^2\right) - \frac{v_S}{2v}\left(m_{h_1}^2 - m_{h_2}^2\right) c_\alpha s_\alpha .$$