Λ_b baryon LCDAs in the short distance expansion

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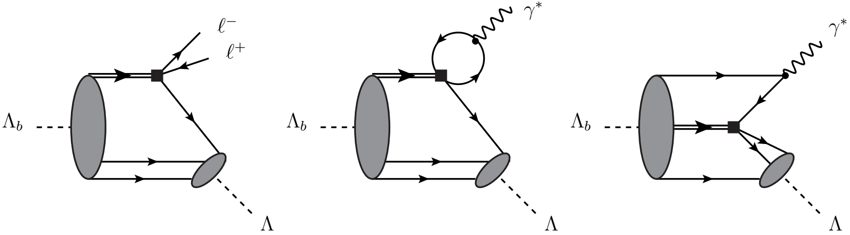
Outline

- What are Light-Cone Distribution Amplitudes (LCDAs) ?
- Short-distance expansion
- Results
- Modelling the LCDAs
- Conclusion & Outlook



(Fig. taken from arXiv:2312.14146)

- Light-Cone Distribution Amplitudes (LCDAs) appear in the context of QCD factorization or light-cone sum rules, based on an expansion in $\Lambda_{\mathrm{QCD}}/m_b$
- LCDAs provide the relevant information about the hadronic bound state: momentum distribution of light quarks (coupled to static heavy quark in HQET)
- Universal quantities, but not directly accessible in experiment

Definition of the Λ_b LCDA ϕ_2

The “leading-twist” LCDA is defined as a *non-local* hadronic matrix element:

$$\epsilon^{abc} \langle 0| \left(u^a(\tau_1 n)\, C\gamma_5 \not{n}\, d^b(\tau_2 n)\right) h_v^c(0)|\Lambda_b(v,s)\rangle = f_{\Lambda_b}^{(2)}\, \tilde{\phi}_2(\tau_1, \tau_2)\, u_{\Lambda_b}(v,s)$$

[Ball, Braun, Gardi, 0804.2424]

- Colour-neutral baryon current $\to \epsilon^{abc}$
- Charge-conjugation matrix C (Lorentz-invariant definition of diquark current)
- Normalization constant $f_{\Lambda_b}^{(2)}$
- On-shell spinor $u_{\Lambda_b}(v,s)$ for Λ_b moving with velocity v^μ in Heavy Quark Effective Theory (HQET) $(v^2 = 1)$

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[Ball, Braun, Gardi, 0804.2424]

- Light-cone direction: n^μ with $n^2 = 0$ and $v \cdot n = 1$
- Implicit: Gauge-links (“Wilson lines”), make the defintion gauge-invariant

$$u(\tau_1 n) \to [0,\tau_1 n]\, u(\tau_1 n)\,, \qquad d(\tau_2 n) \to [0,\tau_2 n]\, d(\tau_2 n)$$

- Momentum distribution from Fourier transform $(\tau_1,\tau_2) \to (\omega_1,\omega_2)$
- Dirac structure between light quarks depends on the process

Hadronic matrix elements cannot be computed perturbatively

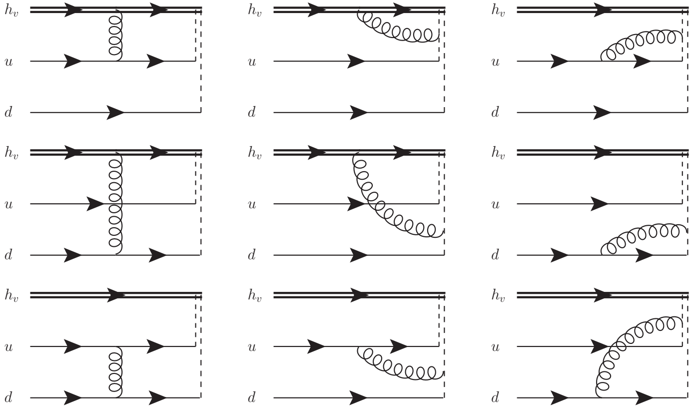
⇒Operator product expansion (OPE) allows us to express the non-local operator in terms of local structures:

$$\epsilon^{abc}\left(u^a(\tau_1 n)\, C\gamma_5\, \not{n}\, d^b(\tau_2 n)\right) h_v^c(0)$$
$$= \sum_{i=0}^{\infty}\sum_{k=1}^{K_i} c_k^{(i+9/2)}(\tau_1, \tau_2)\, \mathcal{O}_k^{(i+9/2)}(0)$$

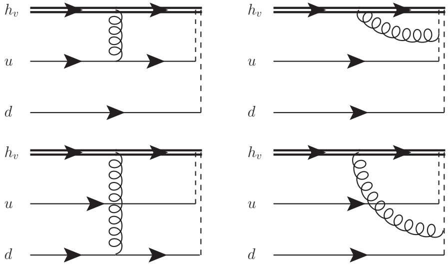
(for mesonic case, see [Kawamura, Tanaka, 0810.5628], [Feldmann, Lüghausen, Seitz, 2306.14686])

- Here: Truncate the expansion including order τ_1, τ_2 and α_s
- Determine coefficients c_k from matching calculation with on-shell *partonic* states

All possible diagrams at order α_s:



Only diagrams where gluons couple to heavy quark contribute to the matching:



Reason: Light quark dynamics is reproduced in local matrix elements

- The definition of the LCDA requires hadronic matrix elements
- We can use equations of motion, together with Lorentz and isospin symmetry,

$$iv \cdot \partial \langle 0|\mathcal{O}_1^{(9/2)}|\Lambda_b(v,s)\rangle = \bar{\Lambda}\,\langle 0|\mathcal{O}_1^{(9/2)}|\Lambda_b(v,s)\rangle\,,$$

to express the local matrix elements in terms of $\bar{\Lambda} = M_{\Lambda_b} - m_b$ and $f_{\Lambda_b}^{(2)}$

- Putting everything together results in short-distance expansion for the LCDA:

$$\tilde{\phi}_2(\tau_1,\tau_2) = \left(1 - i(\tau_1+\tau_2)\frac{2\bar{\Lambda}}{3}\right)\left(1 - \frac{\alpha_s C_F}{4\pi}\left(L_1^2 + L_2^2 + L_1 + L_2 + \frac{5\pi^2}{12}\right)\right) + i\bar{\Lambda}\frac{\alpha_s C_F}{4\pi}\left(\tau_1\left(\frac{2L_1}{3} - \frac{3}{4}\right) + \tau_2\left(\frac{2L_2}{3} - \frac{3}{4}\right)\right) + \mathcal{O}(\tau_i^2, \alpha_s^2)\,,$$

- Notice the logarithmic dependence on the light-cone separations, entering via

$$L_j = \ln(i\tau_j\,\mu e^{\gamma_E})$$

These give rise to a non-trivial “radiative tail” in momentum space (!)

- Reason: Heavy quark can radiate an infinite amount of energy to the light quark

(Similar expressions for sub-leading LCDAs ϕ_3^s, ϕ_3^σ and ϕ_4 can be found in the paper)

Idea:

- "radiative tail" to describe behaviour for small $|\tau_i| \sim \mathcal{O}(1/\mu)$
- hadronic model to describe behavour for large $|\tau_i| \sim \mathcal{O}(1/\Lambda_{\text{QCD}})$

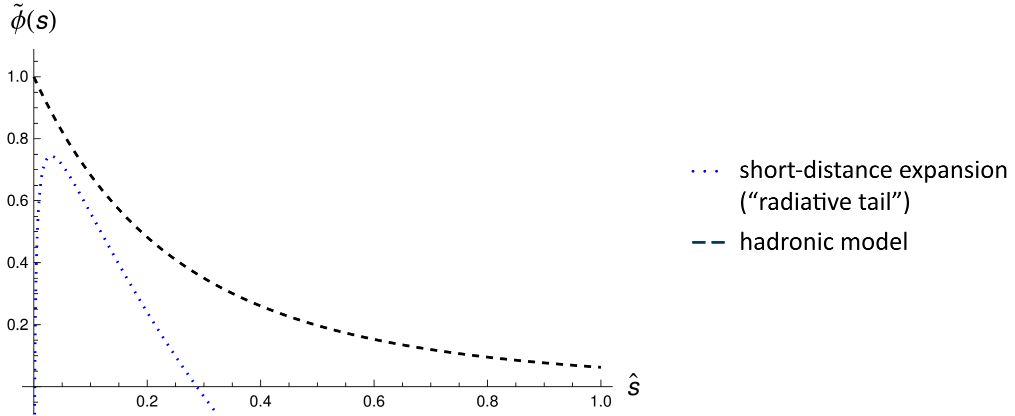
$$\tilde{\phi}_2(\tau_1,\tau_2) = \frac{1}{(1+i\omega_0\tau_1)^2(1+i\omega_0\tau_2)^2} \quad \Leftrightarrow \quad \phi_2(\omega_1,\omega_2) = \frac{\omega_1\omega_2}{\omega_0^4}\, e^{-(\omega_1+\omega_2)/\omega_0}$$

here, $\omega_0 \sim$ few 100 MeV is to be fixed in terms of $\bar{\Lambda} = M_{\Lambda_b} - m_b$

- interpolate between the two regions, using the 1-loop RGE
(see paper for details)

Plot

Result of the interpolation at equal light-cone times $\hat{s} = i\tau_1\omega_0 = i\tau_2\omega_0$



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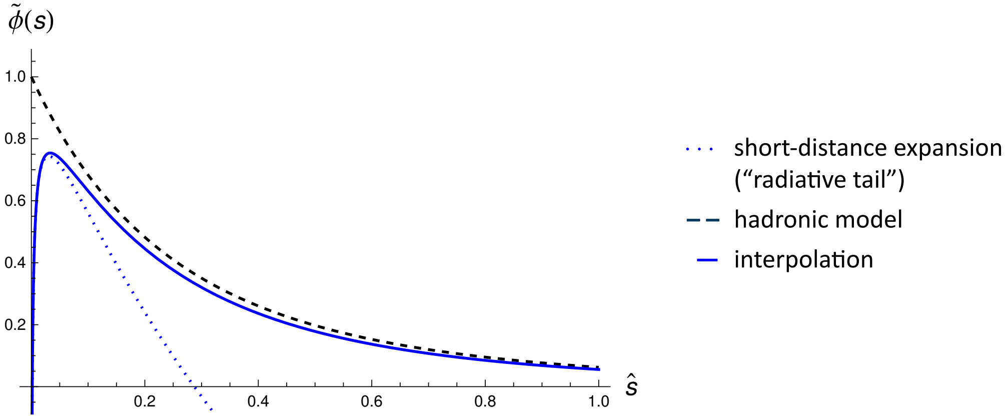

- We calculated the 3-particle Λ_b baryon LCDAs in the short-distance expansion
- Our result for the “radiative tail” provides perturbative constraints, on an otherwise poorly known hadronic quantity
- Allows for QCD-improved modelling of Λ_b LCDAs
- Possible applications:

  $\Rightarrow$ $|V_{ub}|$ extraction from $\Lambda_b \to p\ell\nu$

  $\Rightarrow$ New physics studies in rare decays $\Lambda_b \to \Lambda\ell^+\ell^-$

Kinematic properties

- Light-like vectors:

$$n^2 = \bar{n}^2 = 0 \quad \text{and} \quad n \cdot \bar{n} = 2$$

- Heavy-quark velocity:

$$v^\mu = \frac{n^\mu + \bar{n}^\mu}{2}$$

- HQET field:

$$\not{v}\, h_v = h_v \,, \qquad (iv \cdot D)\, h_v = 0 \qquad \text{(on-shell)}$$

LCDA definitions

- Chiral-odd:

$$\epsilon^{abc}\,\langle 0|\left(u^a(\tau_1 n)\,C\gamma_5 \not{n}\, d^b(\tau_2 n)\right) h_v^c(0)|\Lambda_b(v,s)\rangle = f^{(2)}_{\Lambda_b}\,\tilde{\phi}_2(\tau_1,\tau_2)\,u_{\Lambda_b}(v,s)$$

$$\epsilon^{abc}\,\langle 0|\left(u^a(\tau_1 n)\,C\gamma_5 \not{\bar{n}}\, d^b(\tau_2 n)\right) h_v^c(0)|\Lambda_b(v,s)\rangle = f^{(2)}_{\Lambda_b}\,\tilde{\phi}_4(\tau_1,\tau_2)\,u_{\Lambda_b}(v,s)$$

- Chiral-even:

$$\epsilon^{abc}\,\langle 0|\left(u^a(\tau_1 n)\,C\gamma_5\, d^b(\tau_2 n)\right) h_v^c(0)|\Lambda_b(v,s)\rangle = f^{(1)}_{\Lambda_b}\,\tilde{\phi}^s_3(\tau_1,\tau_2)\,u_{\Lambda_b}(v,s)$$

$$\epsilon^{abc}\,\langle 0|\left(u^a(\tau_1 n)\,C\gamma_5\, i\sigma_{\mu\nu}\bar{n}^\mu n^\nu\, d^b(\tau_2 n)\right) h_v^c(0)|\Lambda_b(v,s)\rangle = 2f^{(1)}_{\Lambda_b}\,\tilde{\phi}^\sigma_3(\tau_1,\tau_2)\,u_{\Lambda_b}(v,s)$$

Further definitions

$$\epsilon^{abc} \langle 0| \Big(u^a(0)\, C\gamma_5\gamma^\mu\, d^b(0) \Big)\, h_v^c(0) |\Lambda_b(v,s)\rangle = f_{\Lambda_b}^{(2)}\, v^\mu\, u_{\Lambda_b}(v,s)$$

$$\epsilon^{abc} \langle 0| \Big(u^a(0)\, C\gamma_5\, d^b(0) \Big)\, h_v^c(0) |\Lambda_b(v,s)\rangle = f_{\Lambda_b}^{(1)}\, u_{\Lambda_b}(v,s)$$

$$\phi_i(\omega_1,\omega_2) = \int\limits_{-\infty-i\epsilon}^{\infty-i\epsilon} \frac{d\tau_1}{2\pi} \int\limits_{-\infty-i\epsilon}^{\infty-i\epsilon} \frac{d\tau_2}{2\pi}\, e^{i\omega_1\tau_1 + i\omega_2\tau_2}\, \tilde{\phi}_i(\tau_1,\tau_2)$$

$$[x,y] \equiv \mathcal{P}\ \exp -ig_s n^\mu \int_x^y \mathrm{d}\lambda\, \mathcal{A}_\mu^{\mathrm{A}}(n\lambda)\, T^{\mathrm{A}}$$

Short distance expansion

$$
\begin{aligned}
\mathcal{O}_\Gamma(\tau_1,\tau_2) &\equiv \epsilon^{abc}\Big(u^a(\tau_1 n)\, C\gamma_5\, \Gamma\, d^b(\tau_2 n)\Big)\, h_v^c(0) \\
&= \sum_{i=0}^{\infty}\sum_{P}\sum_{k=1}^{K_i} c_{P,k}^{(i+9/2)}(\tau_1,\tau_2)\, \mathcal{O}_{P\Gamma,k}^{(i+9/2)}(0)
\end{aligned}
$$

$$
\begin{aligned}
P\Gamma\ :\Gamma_{++} &= \frac{\not{n}\not{\bar{n}}}{4}\,\Gamma\,\frac{\not{\bar{n}}\not{n}}{4} \qquad \Gamma_{+-} = \frac{\not{n}\not{\bar{n}}}{4}\,\Gamma\,\frac{\not{n}\not{\bar{n}}}{4} \\
\Gamma_{-+} &= \frac{\not{\bar{n}}\not{n}}{4}\,\Gamma\,\frac{\not{\bar{n}}\not{n}}{4}\,, \qquad \Gamma_{--} = \frac{\not{\bar{n}}\not{n}}{4}\,\Gamma\,\frac{\not{n}\not{\bar{n}}}{4}
\end{aligned}
$$

Tree-level coefficients and subtractions

$$
\begin{aligned}
c_P^{(9/2)}(\tau_1, \tau_2) &= 1 + \mathcal{O}(\alpha_s) \\
c_{P,1}^{(11/2)}(\tau_1, \tau_2) = c_{P,2}^{(11/2)}(\tau_2, \tau_1) &= -i\tau_1 + \mathcal{O}(\alpha_s) \\
c_{P,3}^{(11/2)}(\tau_1, \tau_2) = c_{P,4}^{(11/2)}(\tau_2, \tau_1) &= \mathcal{O}(\alpha_s)
\end{aligned}
$$

$$
\tilde{I}_{\text{subtr.}}(\tau_1, \tau_2) = \int_0^\infty d\omega_1 \int_0^\infty d\omega_2 \left(e^{-i\omega_1\tau_1 - i\omega_2\tau_2} - 1 + i\omega_1\tau_1 + i\omega_2\tau_2 - \ldots\right) I(\omega_1, \omega_2)\,.
$$

Local hadronic matrix elements

$$\epsilon^{abc} \langle 0| \left((u^a C\gamma_5)_\alpha d^b_\beta\right) h^c_{v,\gamma} |\Lambda_b(v,s)\rangle = \frac{1}{4} \left[M^{(9/2)}(v)\right]_{\beta\alpha} u_\gamma(v,s)$$

$$M^{(9/2)}(v) = f^{(1)}_{\Lambda_b} + f^{(2)}_{\Lambda_b} \not{v}$$

$$\epsilon^{abc} \langle 0| \left((u^a C\gamma_5)_\alpha (i\overleftarrow{D}_\mu)\, d^b_\beta\right) h^c_{v,\gamma} |\Lambda_b(v,s)\rangle = \frac{1}{4} \left[M^{(11/2)}_\mu(v)\right]_{\beta\alpha} u_\gamma(v,s)$$

$$M^{(11/2)}_\mu(v) = f^{(2)}_{\Lambda_b} \left(C\,\gamma_\mu + D\,\not{v}\,v_\mu\right) + f^{(1)}_{\Lambda_b} \left(E\,\not{v}\gamma_\mu + F\,\gamma_\mu\not{v}\right)$$

$$C = -\frac{\bar\Lambda}{6} \quad D = \frac{2\bar\Lambda}{3} \qquad E = \frac{\bar\Lambda}{6} \quad F = \frac{\bar\Lambda}{3}$$

LCDAs ϕ_2 and ϕ_4

$$\tilde{\phi}_2(\tau_1,\tau_2) = \left(1 - i(\tau_1+\tau_2)\,\frac{2\bar{\Lambda}}{3}\right)\left(1 - \frac{\alpha_s C_F}{4\pi}\left(L_1^2 + L_2^2 + L_1 + L_2 + \frac{5\pi^2}{12}\right)\right) + i\bar{\Lambda}\,\frac{\alpha_s C_F}{4\pi}\left(\tau_1\left(\frac{2L_1}{3} - \frac{3}{4}\right) + \tau_2\left(\frac{2L_2}{3} - \frac{3}{4}\right)\right) + \mathcal{O}(\tau_i^2)$$

$$\tilde{\phi}_4(\tau_1,\tau_2) = \left(1 - i(\tau_1+\tau_2)\,\frac{\bar{\Lambda}}{3}\right)\left(1 - \frac{\alpha_s C_F}{4\pi}\left(L_1^2 + L_2^2 - L_1 - L_2 + \frac{5\pi^2}{12}\right)\right) + i\bar{\Lambda}\,\frac{\alpha_s C_F}{4\pi}\left(\tau_1\left(\frac{2L_1}{3} - \frac{3}{4}\right) + \tau_2\left(\frac{2L_2}{3} - \frac{3}{4}\right)\right) + \mathcal{O}(\tau_i^2)$$

LCDAs ϕ_3^s and ϕ_3^σ

$$\tilde{\phi}_3^s(\tau_1,\tau_2) = \left(1 - i(\tau_1+\tau_2)\frac{\bar{\Lambda}}{2}\right)\left(1 - \frac{\alpha_s C_F}{4\pi}\left(L_1^2 + L_2^2 + \frac{5\pi^2}{12}\right)\right) + i\bar{\Lambda}\frac{\alpha_s C_F}{4\pi}\left(\tau_1\left(\frac{5}{6}L_1 - \frac{1}{6}L_2 - \frac{3}{4}\right) + \tau_2\left(\frac{5}{6}L_2 - \frac{1}{6}L_1 - \frac{3}{4}\right)\right) + \mathcal{O}(\tau_i^2)$$

$$\tilde{\phi}_3^\sigma(\tau_1,\tau_2) = \left(-i(\tau_1-\tau_2)\frac{\bar{\Lambda}}{6}\right)\left(1 - \frac{\alpha_s C_F}{4\pi}\left(L_1^2 + L_2^2 + \frac{5\pi^2}{12}\right)\right) + i\bar{\Lambda}\frac{\alpha_s C_F}{4\pi}\left((\tau_1+\tau_2)\frac{1}{2}\left(L_1 - L_2\right)\right) - \frac{\alpha_s C_F}{4\pi}\left(L_1 - L_2\right) + \mathcal{O}(\tau_i^2)$$

Modeling LCDA ϕ_2

$$\frac{d\tilde{\phi}(s;\mu)}{d\ln\mu} = -\frac{\alpha_s C_F}{4\pi}\left(\Gamma_c^{(1)}\,L + \gamma^{(1)}\right)\tilde{\phi}(s;\mu) + \frac{\alpha_s C_F}{4\pi}\,\Gamma_c^{(1)}\int_0^1 dz\,\frac{z}{1-z}\left(\tilde{\phi}_2(-izs,-is;\mu) - \tilde{\phi}_2(-is,-is;\mu)\right)$$

with $\Gamma_c^{(1)} = 4$, $\gamma^{(1)} = 2$, and $L = \ln(s\mu e^{\gamma_E})$

$$\tilde{\phi}(s,\mu) = \left(1 - \frac{\alpha_s C_F}{4\pi}\left(\left(4L - 2\ln\frac{\mu}{\mu_0(s)}\right)\ln\frac{\mu}{\mu_0(s)} + 2\,\ln\frac{\mu}{\mu_0(s)} + \mathrm{const}_1(s)\right)\right)\tilde{f}(s) + \frac{\alpha_s C_F}{4\pi}\left(4\ln\frac{\mu}{\mu_0(s)} + 4\,\mathrm{const}_2(s)\right)\int_0^1 dz\,\frac{z}{1-z}\left(\tilde{f}_2(zs,s) - \tilde{f}_2(s,s)\right)$$

$$\hat{\mu}_0(s) = \frac{1+\hat{\mu}_F s}{s}\,,\quad \mathrm{const}_1 = \frac{5\pi^2}{12}\,,\quad \mathrm{const}_2 = -\frac{9}{8}\,\frac{1}{1+\hat{\mu}_F s}$$

$\alpha_s = 0.3$ GeV, $\bar{\Lambda} = 0.6$ GeV, $\mu e^{\gamma_E} = \mu_f e^{\gamma_E} = 2$ GeV, $\omega_0 \simeq 0.23$ GeV