Vacuum Stability in the Standard Model and Beyond

A guide for BSM Model Building

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Motivation: Current Situation

- Higgs boson discovered in 2012 with $m_h \approx 125$ GeV [ATLAS, CSM, 2012].
- This implies the electroweak vacuum is **metastable** [Buttazzo et al, 2012].
- Not a problem for our lifetime, but... why so close to **absolute stability**?
- Can Stability be excluded?
- Could this hint at new physics beyond the Standard Model (BSM)?

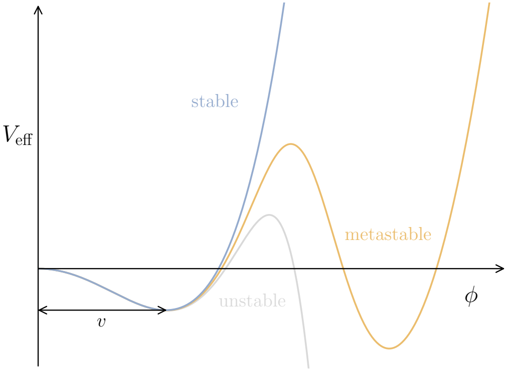
Outline:

- Stability in the SM (Updated results)
- BSM extensions to stabilise the potential

Higgs Potential: Classical vs Quantum

Classically: $V(\phi) = -\frac{1}{2}m^2\phi^2 + \frac{1}{4}\lambda\phi^4$.

- Shape determined by m^2 and λ; $m^2 > 0$ gives minimum at $\phi \neq 0$.
- stable if $\lambda > 0$.
- unstable if $\lambda < 0$.
- metastable state only possible with other couplings (e.g. ϕ^3 term) $\rightarrow$ not present in SM



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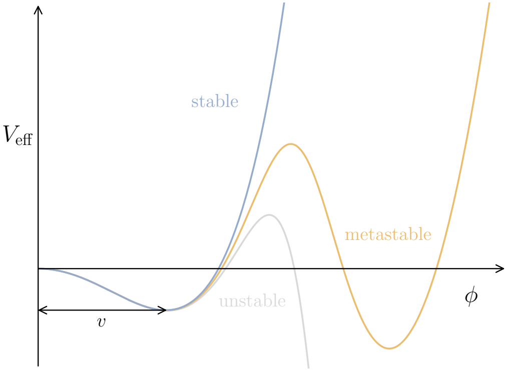


Quantum corrections: λ becomes scale-dependent $\lambda(\mu)$ via RGEs.

$\Rightarrow$ Running λ can create an effective deeper minimum at large ϕ.

Beta Functions and Running of λ

- **Definition:** Beta functions describe how couplings (here g_i) change with the renormalization scale μ:

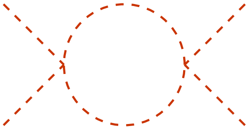
$$\beta_{g_i} \equiv \mu \frac{dg_i}{d\mu}$$

- β_{g_i} can be computed in perturbation theory
- Physical predictions are *independent* of μ, but couplings evolve with μ to ensure that independence.
- The β-functions give a set of coupled differential equations that can be solved to find how couplings evolve with the energy scale.

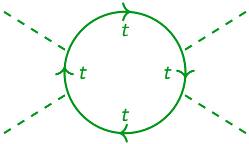
1-loop Contributions to β_λ

For the SM Higgs quartic coupling (1-loop schematic):

$$\beta_\lambda \approx \frac{1}{16\pi^2}\left(24\lambda^2 - 6y_t^4 + \ldots\right) = \mu\frac{d\lambda}{d\mu}$$



Scalar loop



Top quark loop

- Scalar self-interactions push λ up.
- Gauge interactions (e.g. W/Z-boson loops) also contribute positively.
- Yukawa interactions pull λ down (top quark is dominant).

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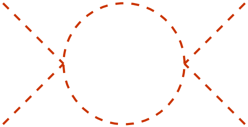

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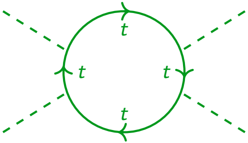

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- Scalar self-interactions push λ up.
- Gauge interactions (e.g. W/Z-boson loops) also contribute positively.
- Yukawa interactions pull λ down (top quark is dominant).

$\Rightarrow$ Large y_t can possibly turn λ negative at high scales.

How do we compute stability in the SM?

1. Experimental Inputs

- Higgs boson mass m_h
- Top-quark pole mass m_t
- QCD coupling $\alpha_s^{(5)}(m_Z)$
- Z-boson mass m_Z
- Fermi constant G_F
- Fine-structure constant α_e & hadronic shift $\Delta\alpha_e^{(5),\mathrm{had}}$
- Lepton masses $m_{e,\mu,\tau}$
- $\overline{\mathrm{MS}}$ light-quark masses: $m_b(m_b)$, $m_c(m_c)$, $m_{u,d,s}(2\,\mathrm{GeV})$

$\Rightarrow$ Newest PDG central values.

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- Matching: $\geq$ 2-loop electroweak + 3-loop QCD [Martin, Patel, 2018]
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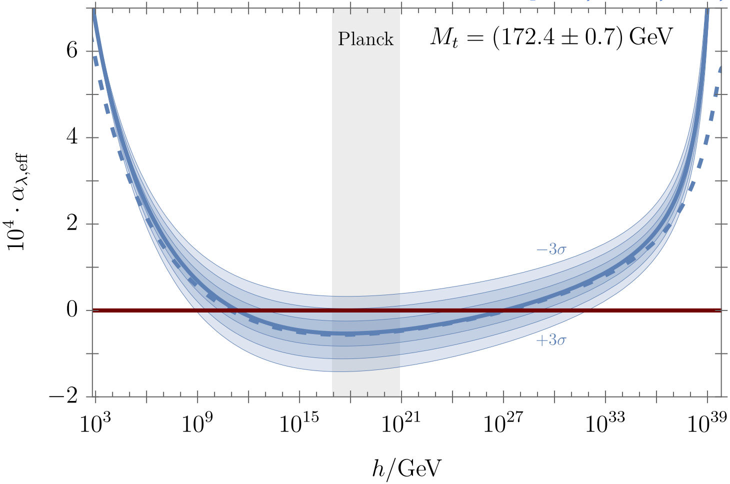
~~4. Decay Rate for Metastability~~

- Only absolute stability considered

SM Renormalization Group Evolution

So how does it look for current PDG central values?

[Hiller, Höhne, Litim, Steudtner, 2024]



Note:

$$\alpha_\lambda \equiv \frac{\lambda}{(4\pi)^2}$$

We trade:

$\lambda(\mu) \to \lambda(h)$,

with $h \equiv \phi$

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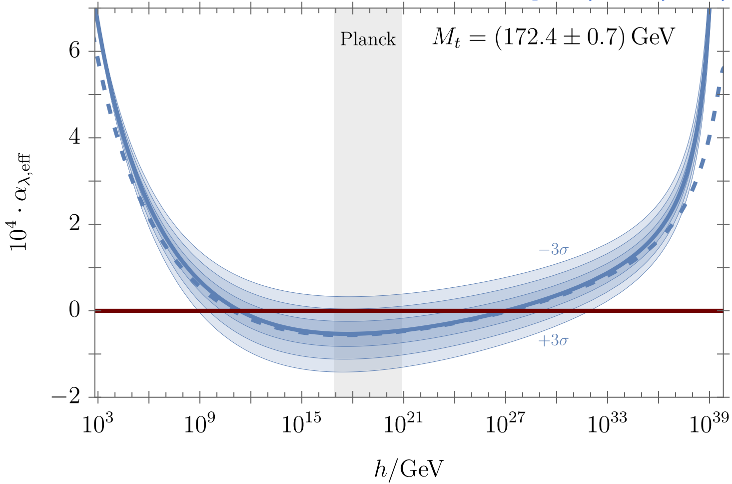

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$\Rightarrow$ The SM is metastable due to the sign-flip of $\lambda(\mu)$ at $\approx 10^{11}$ GeV.

Experimental Inputs and Their Impact

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} $m_h = 125.20(11)\,\mathrm{GeV}$

$m_t = 172.40(70)\,\mathrm{GeV}$

$\alpha_s^{(5)}(m_Z) = 0.1180(9)$

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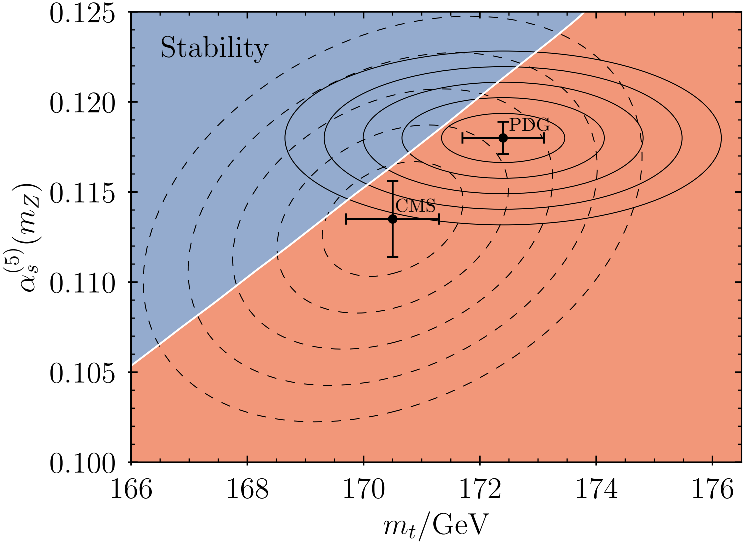
} **Impact large!**

Impact of m_t and α_s on Stability



Updated results!

We are not far off from the critical boundary!

- Stability sensitive to m_t and α_s.
- The correlation is crucial.
- Stability is roughly $1.2\,\sigma$ away from current central values.

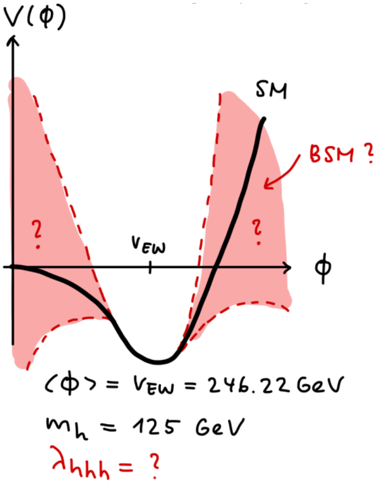
We still know fairly little about the Higgs potential!
What we do know:

- Ground state of Higgs potential with $v_{\mathrm{EW}} = 246.22$ GeV spontaneously breaks the EW symmetry:

$$SU(2)_L \times U(1)_Y \rightarrow U(1)_{\mathrm{EM}}$$

- Higgs boson mass $m_h \approx 125$ GeV.

Taken from L. Biermann



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Scalar sector is realized in nature!

⇒ Shape of the potential still unknown…

⇒ Need precise λ_{hhh} measurement.

⇒ BSM extensions of scalar sector could explain matter–antimatter asymmetry, dark matter, etc.

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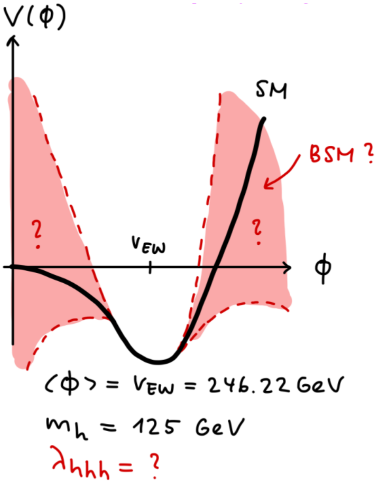

Stabilising with a scalar portal

- Minimal extension: real scalar singlet S with coupling
$$\mathscr{L} \supset -\frac{1}{2}m_S^2 S^2 - \frac{\nu}{4}S^4 - \frac{\delta}{2}\phi^2 S^2.$$
- Portal coupling δ modifies β_λ positively (at 1-loop):
$$\Delta\beta_\lambda \sim +\frac{1}{16\pi^2}\,2\delta^2 + \dots$$
- Thus λ can be kept positive up to high scales if δ is large enough.

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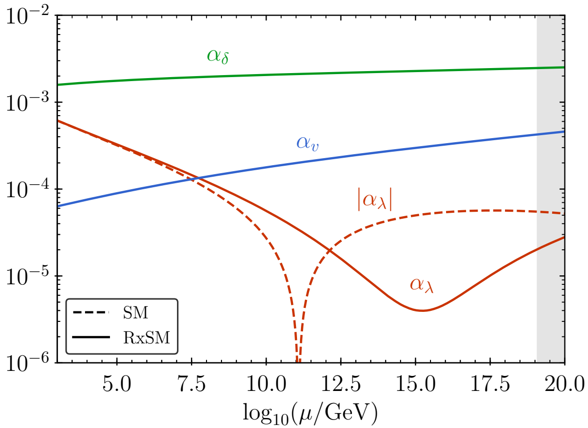
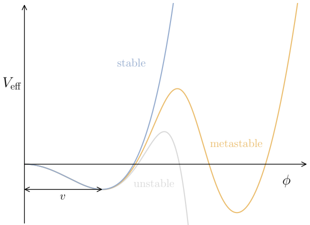
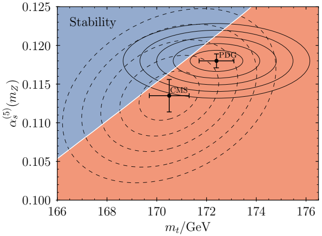
$$\mathscr{L} \supset -\frac{1}{2}m_S^2 S^2 - \frac{v}{4}S^4 - \frac{\delta}{2}H^\dagger H S^2, \quad \text{with} \quad \Delta\beta_\lambda \sim +\frac{1}{16\pi^2}2\delta^2 + \dots$$

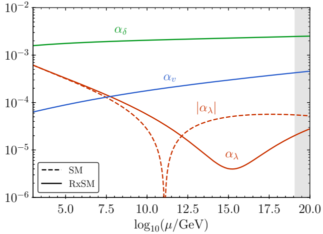


Summary / Take-home messages

- The measured Higgs mass places the SM near a critical boundary: metastability.
- The fate of the vacuum is extremely sensitive to m_t and α_s and their correlation.
- Minimal BSM physics (e.g. scalar portal) can stabilise the potential and also offer links to DM and EW baryogenesis.
- Many open questions remain: why (near-)criticality? what does it tell us about UV physics?

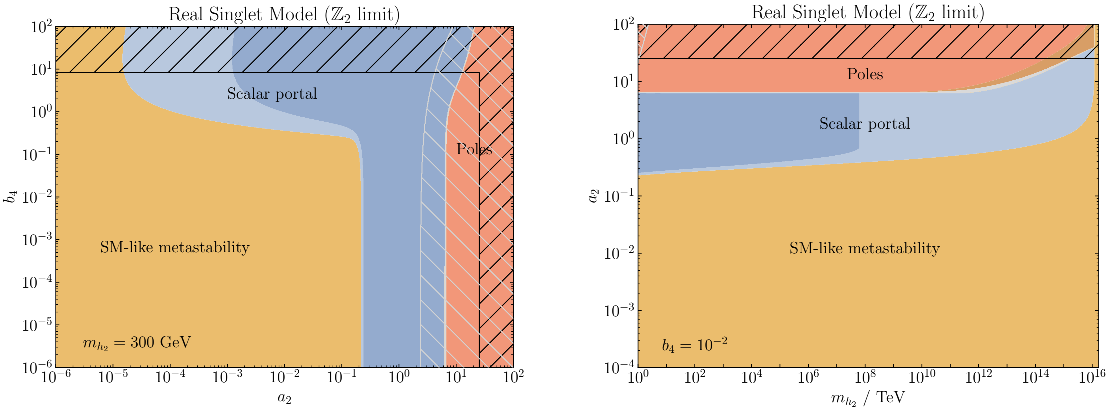






Backup

Example: parameter space where stability is restored



Cosmological connections

- Scalar singlet can be a Dark Matter candidate (i.e. in a Complex Singlet Model).
- Modified scalar potential can allow a strong first-order EW phase transition (SFOEWPT) — relevant for electroweak baryogenesis.
- Can be probed by modified hZZ-coupling measurements, Higgs self-coupling measurements, and direct searches for S.

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