From Helicity Amplitudes to Entanglement at Colliders

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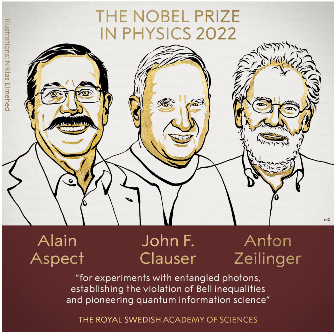
Motivation

Spin correlations have been a powerful tool collider physics

- > Constraints on anomalous couplings
- > Study CP properties

Recent growing interest in quantum information for HEP

- > Nobel prize in physics 2022 for experiments with entangled photons
- > Measurements of entanglement in $t\bar{t}$-production at LHC



For nice summaries on current research status, see e.g.

"Quantum Entanglement and Bell inequality violation at colliders", Barr et al. (2024)
"Quantum Information meets High-Energy Physics: Input to the update of the European Strategy for Particle Physics", Afik et al. (2025)

First: Little bit of theory

Quantum mechanics toolbox - The density matrix

Pure states

- A quantum system that is fully known is described by a vector in Hilbert space $|\psi\rangle$ (up to phase and normalization)
- The density matrix for a pure state is given by

$$\rho_{pure} = |\psi\rangle \langle\psi|$$

Mixed states

- More generally, a mixed quantum state is described by the density matrix ρ

$$\rho = \sum_i p_i |\psi_i\rangle \langle\psi_i|$$

p_i: probability to find system in pure state ψ_i

Properties: ρ is non-negative ($\rho \geq 0$), hermitian ($\rho^\dagger = \rho$) and normalized ($\mathrm{Tr}(\rho) = 1$)

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$$|\psi\rangle = \alpha |0\rangle + \beta |1\rangle , \qquad \text{with} \qquad |\alpha|^2 + |\beta|^2 = 1$$

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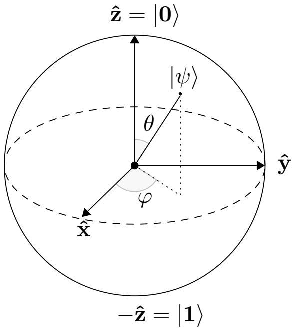

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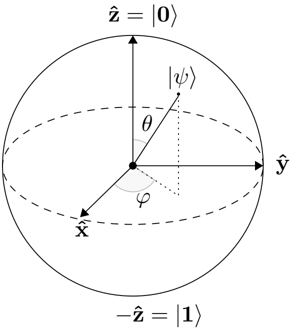
- **Why important for us?**
  - A spin-$\frac{1}{2}$ particle (like an electron, quark, or muon) naturally behaves as a qubit
  - Entanglement = correlations between multiple qubits beyond classical physics.

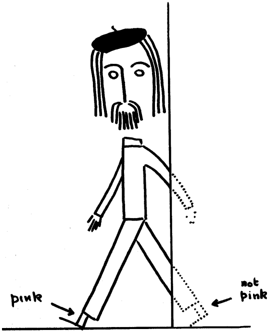


Quantum correlations vs classical correlations - Bertlmann's socks

"Bertlmann's socks and the nature of reality", John Bell (1980)

> Classical correlation (Bertlmann's socks)

- If you see Dr. Bertlmann wears one sock that is pink, you can already be sure that the second sock will not be pink
- The outcome of one foot **pre-determines** the other



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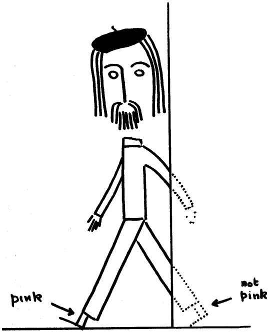
> Quantum entanglement

- Consider a pair of entangled particles, like two electrons singlet state
- If you measure the spin of one particle, the spin of the other one is suddenly fixed (regardless how far)
- Spin is **not pre-determined**, the individual outcomes are undefined until measurement!



Quantum entanglement

Definition

Consider two quantum systems A and B described by a joint density matrix ρ_{AB}. Any *mixed state* that can be written as

$$\rho_{AB} = \sum_i p_i \, \rho_i^A \otimes \rho_i^B, \qquad p_i \geq 0, \sum_i p_i = 1$$

is called **separable**. Any state that is not separable is called **entangled**.
For *pure states* $|\psi\rangle_{AB}$ a separable state can be written as $|\psi\rangle_{AB} = |\varphi_A\rangle \otimes |\varphi_B\rangle$.

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- $|\varphi\rangle = \frac{1}{2}\left(|0\rangle_A \otimes |0\rangle_B + |0\rangle_A \otimes |1\rangle_B + |1\rangle_A \otimes |0\rangle_B + |1\rangle_A \otimes |1\rangle_B\right)$

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> $|\varphi\rangle = \frac{1}{2}\left(|0\rangle_A \otimes |0\rangle_B + |0\rangle_A \otimes |1\rangle_B + |1\rangle_A \otimes |0\rangle_B + |1\rangle_A \otimes |1\rangle_B\right)$

> $= \frac{1}{2}(|0\rangle_A + |1\rangle_A) \otimes (|0\rangle_B + |1\rangle_B)$, $\Rightarrow$ separable

Quantifying Entanglement: Concurrence

"Entanglement of Formation of an Arbitrary State if Two Qubits", Wootters (1997)
"Entanglement of a pair of quantum bits", Hill & Wooters (1997)

Definition

For a bipartite qubit state with density matrix ρ, the concurrence is

$$C(\rho) \equiv \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\},$$

where λ_i are the square roots of the eigenvalues of

$$R = \rho\,(\sigma_y \otimes \sigma_y)\,\rho^*\,(\sigma_y \otimes \sigma_y),$$

ordered decreasingly, and σ_y a Pauli matrix.

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Properties

- $0 \leq C[\rho] \leq 1$ quantifies the “degree” of entanglement
- $C[\rho] = 0$ $\Rightarrow$ **separable** (no entanglement)
- $C[\rho] = 1$ $\Rightarrow$ **maximally entangled**

How is this related to High-Energy Physics?

Parametrization of the density matrix

At colliders: spin 1/2 $\widehat{=}$ qubit (e.g. lepton, quarks), spin 1 $\widehat{=}$ qutrit (e.g. W, Z boson)

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> Two qubits (bipartite system) described by (4×4) matrix

$$\rho = \frac{1}{4}\left(\mathbb{1}_2 \otimes \mathbb{1}_2 + \sum_i a_i \sigma_i \otimes \mathbb{1}_2 + \sum_j b_j \mathbb{1}_2 \otimes \sigma_j + \sum_{ij} C_{ij} \sigma_i \otimes \sigma_j\right)$$

where a_i, b_j are the polarizations, C_{ij} the spin correlation matrix

> In total 15 real parameters $\Rightarrow$ **Quantum Tomography**

Simple QED Example - Electron positron annihilation

Simple Computation: Consider $e^+e^- \to \mu^+\mu^-$- scattering

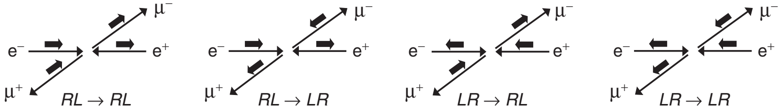
- Spin density matrix of the muon-pair:

$$\rho' = \mathcal{M}\rho_i\mathcal{M}^\dagger, \qquad \rho = \frac{\rho'}{\mathrm{Tr}(\rho')}$$

  where ρ_i is the initial state density matrix, $\mathcal{M}$ contains helicity amplitudes
- Collision of unpolarized particles: $\rho_i = \frac{1}{4}\mathbb{1}_4$
- In total 16 helicity amplitudes $\Rightarrow 4 \times 4$ matrix, e.g.

$$M_{RL\to RL} = e^2(1 + \cos\theta)$$

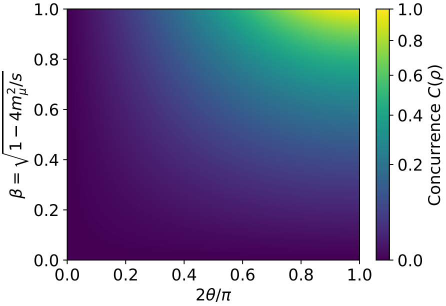
- In high-energy limit: only four non-zero amplitudes



Simple QED Example - Concurrence

> For $e^+e^- \to \mu^+\mu^-$ via photon (neglecting electron mass):

$$C[\rho] = \frac{(s - 4m_\mu^2)\sin^2\theta}{4m_\mu^2 \sin^2\theta + s(1+\cos^2\theta)}$$



Experimental Status - Recent measurements at LHC

"Observation of quantum entanglement with top quarks at the ATLAS detector", ATLAS Collaboration (2023)

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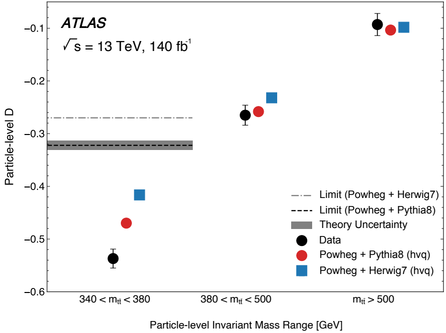
Entanglement marker D:

$$D = \frac{\mathrm{Tr}(\mathbf{C})}{3} = -3 \langle \cos \varphi \rangle$$

where $D < -1/3$ means entanglement and φ is the angle between the lepton directions.



Quantum Information for HEP - Opportunities at Future Colliders

At lepton colliders, you have the advantage of

- Clean initial state
- Vanishing backgrounds
- Full knowledge of collision kinematics
- Polarized beams

Relevant for BSM searches:

- Sensitivity of entanglement to couplings of produced particles ⇒ Observable for NP?
- In SMEFT, corrections would alter helicity amplitudes ⇒ spin density matrix ⇒ entanglement

Many processes under study:

Qubit system

- $pp \to t\bar{t}$
- $e^+e^- \to \tau^-\tau^+$
- $H \to \gamma\gamma$
- $H \to \tau^+\tau^-$

Qutrit system

- $H \to WW$
- $H \to ZZ$

...

Conclusions and Outlook

Conclusion

- Entanglement provides a new perspective on collider physics beyond traditional spin correlations
- For bipartite qubit system, analytic expressions for entanglement measures can be derived
- Experimental evidence: Observation of entanglement in top quark pairs $\Rightarrow$ collider physics is a real quantum information laboratory

Outlook

- Extend entanglement studies to more complex processes (Higgs, dibosons, tripartite systems)
- Study polarized initial states, simulations with WHIZARD
- Explore sensitivity of entanglement observables to BSM effects via SMEFT corrections

Thank you!
Any questions?

PPT criterion - Necessary condition for entanglement

"Separability of Mixed States: Necessary and Sufficient Conditions", Horodeckis (1994)

Peres-Horodecki criterion

We have a bipartite density matrix ρ_{AB}, with matrix elements

$$\rho = \sum_{ijkl} \rho_{ij,kl} \left|i\right\rangle_A \left\langle j\right| \otimes \left|k\right\rangle_B \left\langle l\right| \quad \Rightarrow \quad \rho^{T_B} = \sum_{ijkl} \rho_{ij,kl} \left|i\right\rangle_A \left\langle j\right| \otimes \left|l\right\rangle_B \left\langle k\right|$$

If ρ is separable, then its partial transpose remains positive semi-definite. If ρ^{T_B} has at least one negative eigenvalue, then ρ is **entangled**.

- **Sufficient** criteria for entanglement in lower dimension (2×2) and (2×3)
- **Example**: Density matrix for Bell state $|\phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$

$$\rho = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix} \quad \Rightarrow \quad \rho^{T_B} = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Backup - Density matrix for electron-positron annihilation

Neglecting the electron mass, we find

$$\rho = \frac{1}{\rho_0} \begin{pmatrix} 4M^2 \sin^2\theta & M\sqrt{s}\sin 2\theta & M\sqrt{s}\sin 2\theta & -4M^2\sin^2\theta \\ M\sqrt{s}\sin 2\theta & s(1+\cos^2\theta) & -s\sin^2\theta & -M\sqrt{s}\sin 2\theta \\ M\sqrt{s}\sin 2\theta & -s\sin^2\theta & s(1+\cos^2\theta) & -M\sqrt{s}\sin 2\theta \\ -4M^2\sin^2\theta & -M\sqrt{s}\sin 2\theta & -M\sqrt{s}\sin 2\theta & 4M^2\sin^2\theta \end{pmatrix}$$

with

$$\rho_0 = 2s(1+\cos^2\theta) + 8M^2\sin^2\theta$$

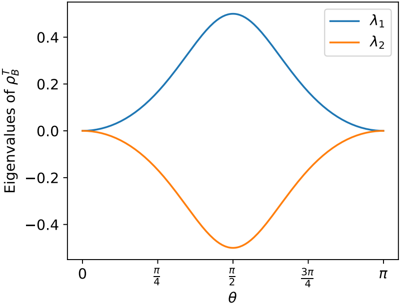
Simple QED Example - PPT criterion and eigenvalues

After applying partial transposition to the density matrix ρ, we get the eigenvalues

$$\lambda_1 = \frac{(s - 4m_\mu^2)\sin^2\theta}{2(s + s\cos^2\theta) + 4m_\mu^2\sin^2\theta}$$

$$\lambda_2 = -\frac{(s - 4m_\mu^2)\sin^2\theta}{2(s + s\cos^2\theta) + 4m_\mu^2\sin^2\theta}$$

$$\lambda_{3/4} = 1/2$$



$\Rightarrow$ Entangled for all scattering angles θ

Backup - Qutrit Parametrization

For massive spin 1-particles ($W^{\pm}, Z^0$), the situation is more complex:

- Single qutrit, density matrix (3×3) is given by

$$\rho = \frac{1}{3}\left(\mathbb{1}_3 + \sum_{a=1}^{8} f_a T_a\right)$$

  where T_i are the *Gell-Mann matrices* and f_a real coefficients

- Two qutrits (bipartite system) - (9×9-matrix)

$$\rho = \frac{1}{9}\left(\mathbb{1}_3 \otimes \mathbb{1}_3 + \sum_i f_a (T^a \otimes \mathbb{1}_3) + \sum_b g_b (\mathbb{1}_3 \otimes T^b) + \sum_{ab} h_{ab} (T^a \otimes T^b)\right)$$

Backup - Hierarchy of quantum correlations

Hierarchy of quantum correlations

Spin correlations $\supseteq$ Discord $\supseteq$ Entanglement $\supseteq$ Steering $\supseteq$ Bell inequalities

- Spin correlations: statistical correlation between spins, classical
- Discord: Quantum correlations yet in separable states
- Entanglement: Subsystems are not separable
- Steering: Measurement in one subsystem influences the other
- Bell inequalities: Correlations cannot be described by local hidden variables

Backup - Bell's inequality

"On the Einstein Podolsky Rosen paradox", Bell (1964)

Starting point: EPR paper (1935)

1.) Predictions of QM are correct
2.) Criterion of **reality** (measurement outcomes determined by pre-existing properties)
3.) Physics is **local** (no faster-than-light influences)
$\Rightarrow$ Conclusion: QM is *incomplete*, there are "hidden variables"

Reply: John Bell (1964)
Setup - Pair of spin 1/2-particles prepared in singlet state

- Measurement results of Alice and Bob: $A(\vec{a}, \lambda) = \pm 1$, $B(\vec{b}, \lambda) = \pm 1$ with additional hidden variable λ
- Expectation value of joint spin-measurement

$$E(\vec{a}, \vec{b}) = \int d\lambda \rho(\lambda) A(\vec{a}, \lambda) B(\vec{b}, \lambda)$$

- **Bell's inequality**

$$|E(\vec{a}, \vec{b}) - E(\vec{a}, \vec{c})| \geq E(\vec{b}, \vec{c}) + 1$$

Backup - CHSH inequality

"Proposed experiment to test local hidden variable theories", Clauser, Horne, Shimony & Holt (1969)

For a bipartite qubit system, Bell's inequality is the Clauser-Horne-Shimonu-Holt (CHSH) inequality:

$$|\langle \vec{a}_1 \cdot \vec{\sigma} \otimes \vec{b}_1 \cdot \vec{\sigma} \rangle - \langle \vec{a}_1 \cdot \vec{\sigma} \otimes \vec{b}_1 \cdot \vec{\sigma} \rangle + \langle \vec{a}_2 \cdot \vec{\sigma} \otimes \vec{b}_1 \cdot \vec{\sigma} \rangle + \langle \vec{a}_2 \cdot \vec{\sigma} \otimes \vec{b}_2 \cdot \vec{\sigma} \rangle| \leq 2$$

- $\vec{a}_1, \vec{a}_2, \vec{b}_1, \vec{b}_2$ are the measurement axes
- At colliders, each term corresponds to a spin measurement with

Backup - Concurrence in $t\bar{t}$−production

"Entanglement and quantum tomography with top quarks at the LHC", Afik & Nova (2021)

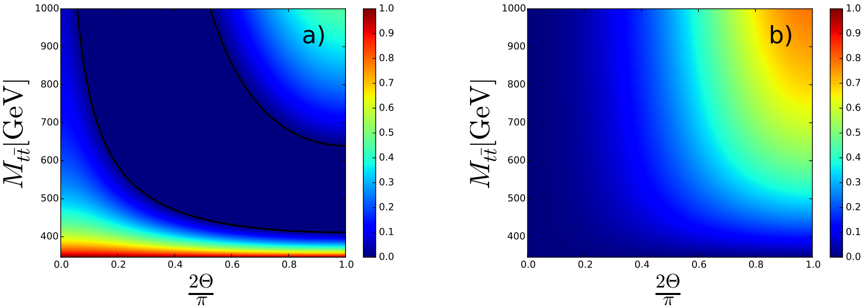


Figure: a) Concurrence for $gg \to t\bar{t}$ (gluon fusion) as a function of invariant mass $m_{t\bar{t}}$ and scattering angle θ in the $t\bar{t}$ CM frame. The black lines represent the boundaries between separability and entanglement. b) Concurrence for $q\bar{q} \to t\bar{t}$