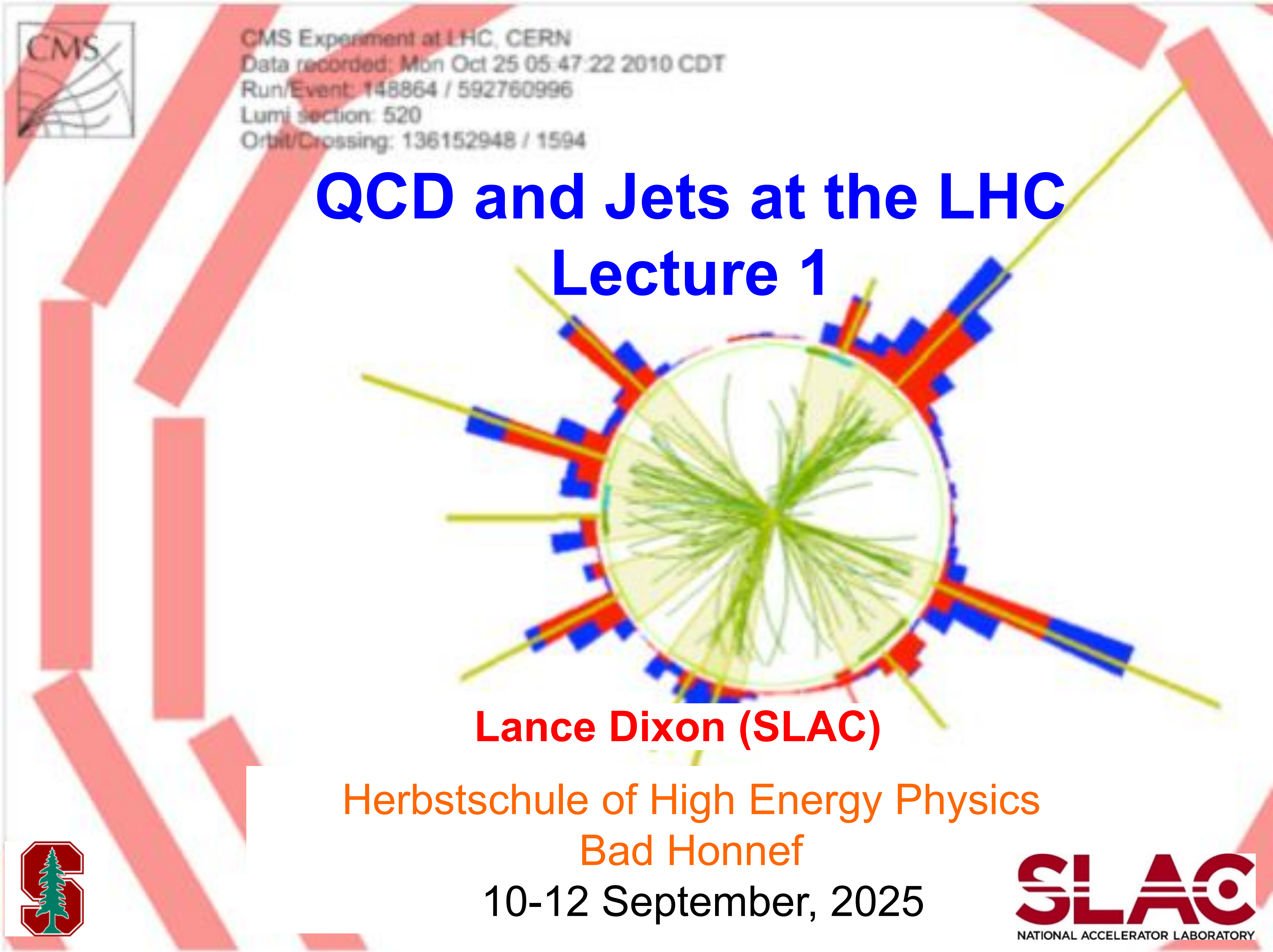


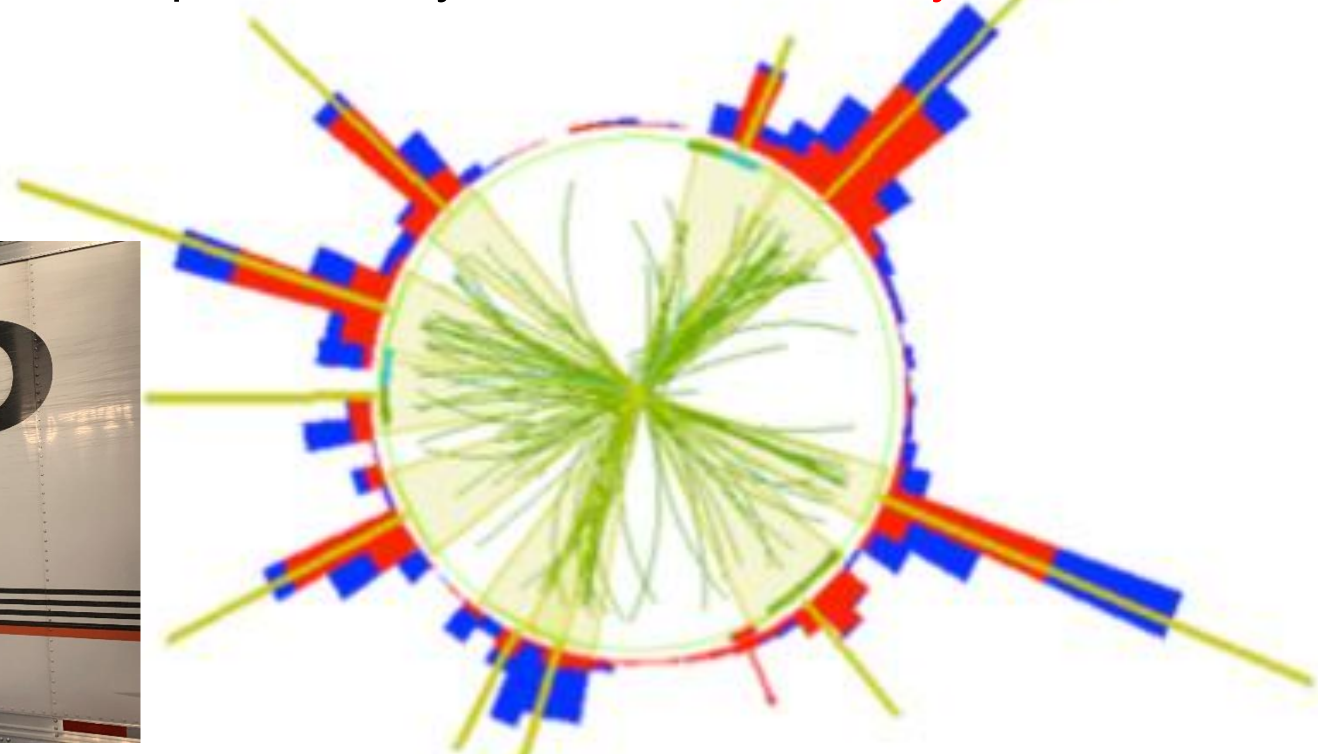
QCD and Jets at the LHC
Lecture 1

Lance Dixon (SLAC)

Herbstschule of High Energy Physics
Bad Honnef
10-12 September, 2025

LHC is QCD Machine

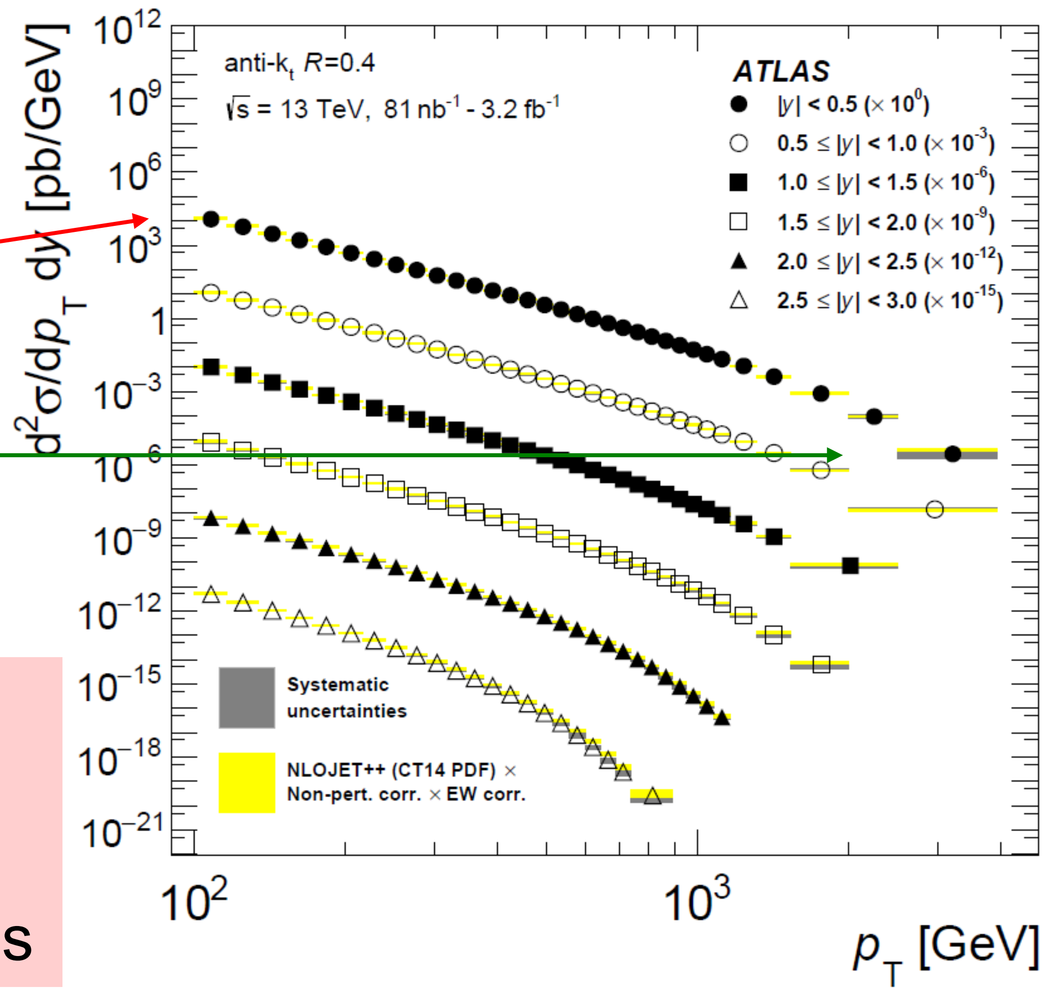
Copious production of quarks and gluons, materialize as collimated jets of hadrons, predicted by Quantum Chromodynamics



- Confrontation between experiment and theory at high precision requires higher order corrections in the strong coupling α_s

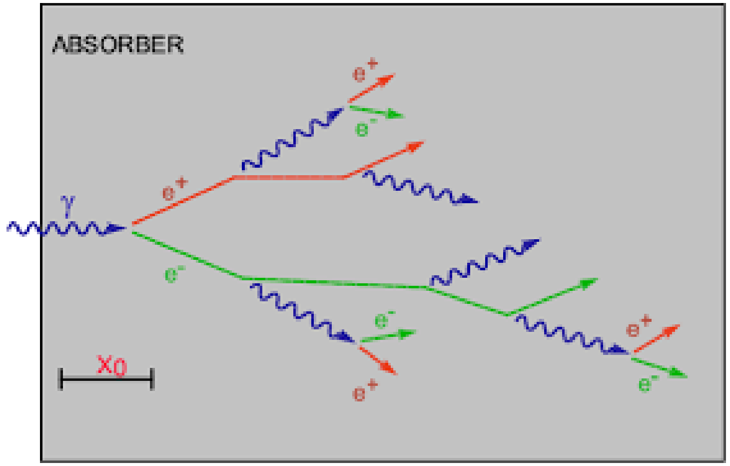
Inclusive jet cross section

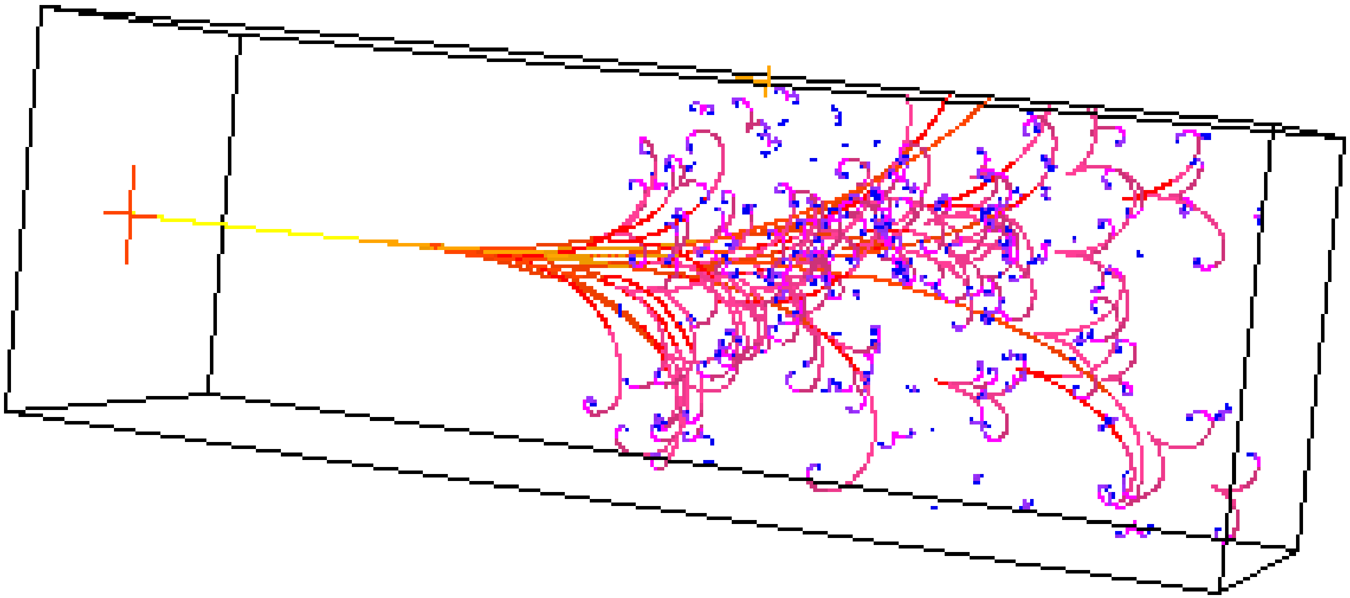
- At $\mathcal{L} = 2 \times 10^{34}\mathrm{cm}^{-2}\mathrm{s}^{-1}$
- 2000 events/second
- 5 events/day



Jets are copious – but also “messy” compared with other objects at LHC, like muons, electrons, photons

Electromagnetic shower



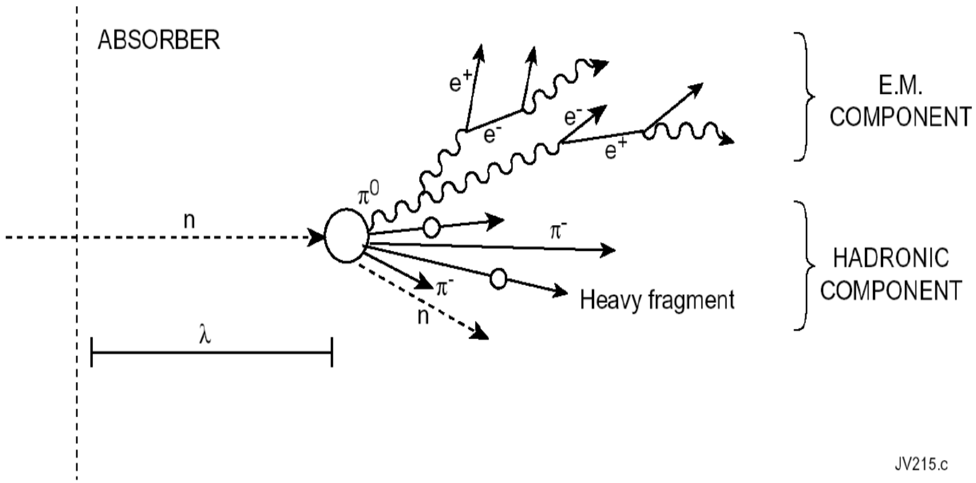


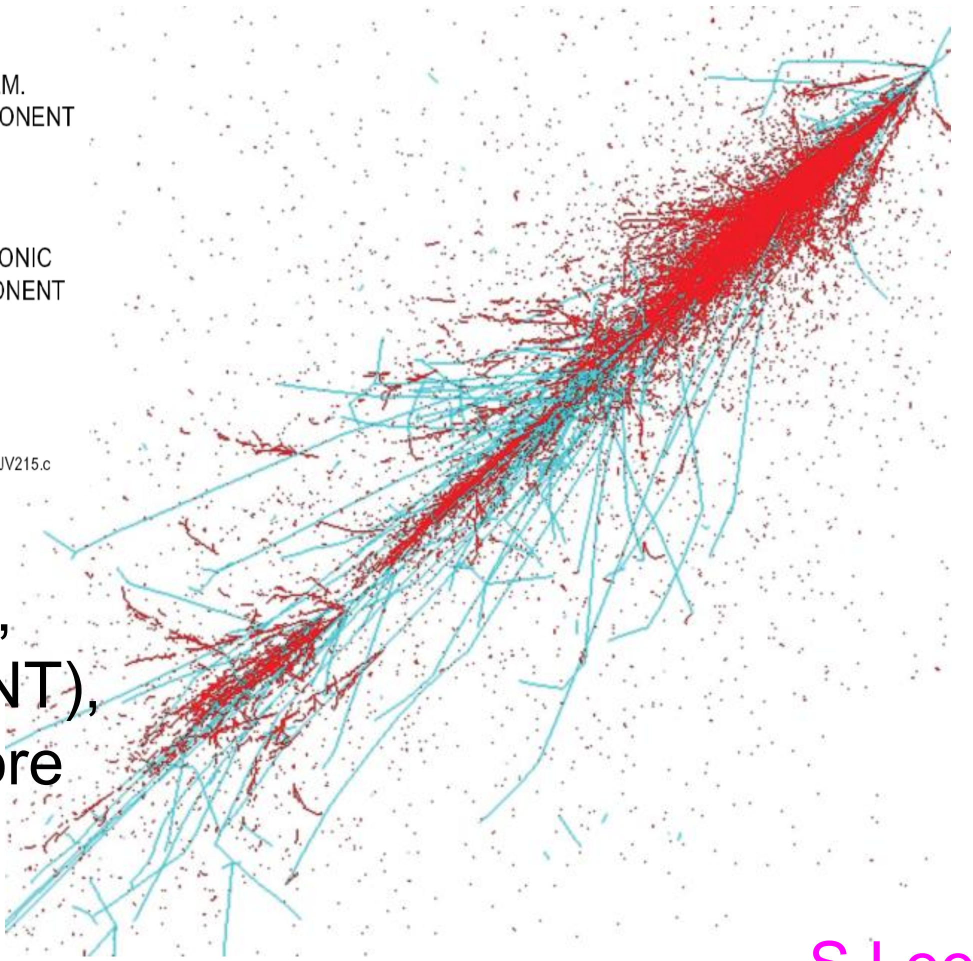
S. Menke

Hadronic interactions with matter

- Typical hadronic cross sections, e.g. πp, are of order 10s of millibarns, $\sigma \sim 10^{-26}\mathrm{cm}^2$.
- Typical nucleon density $n \sim 10^{24}\mathrm{cm}^{-3}$, so typical hadronic interaction length $\lambda = \frac{1}{n\sigma} \sim 1m$
- But most hadronic interactions are soft, so it can take many interaction lengths to remove all hadronic energy in a TeV jet
→ LHC detectors are massive!

Hadronic shower

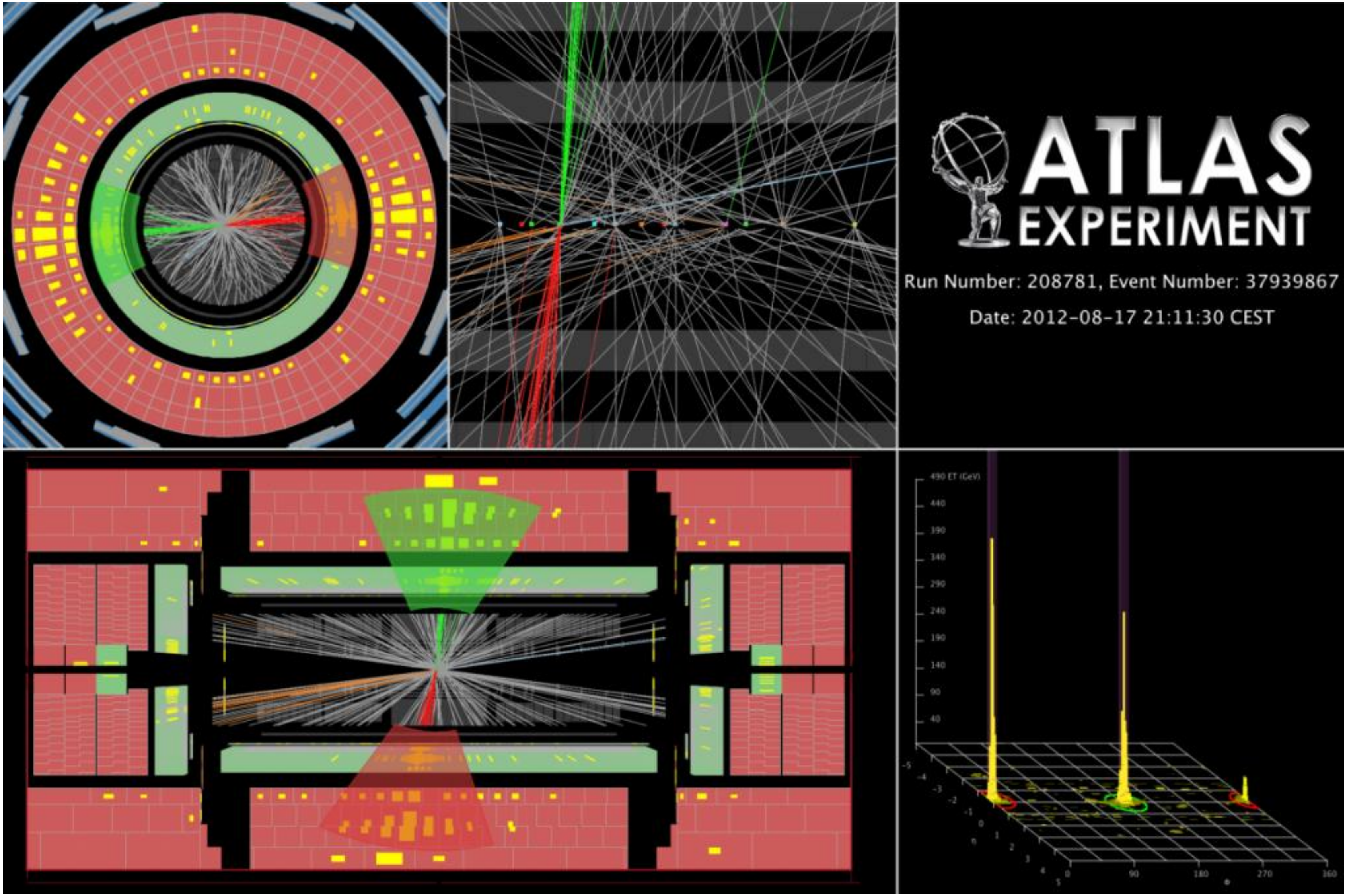




Many more processes involved, more difficult to simulate (GEANT), showers more irregular with more fluctuations than EM shower

S.Lee

At LHC outgoing hadrons → jets



“Particle flow” (PF) methods for jets

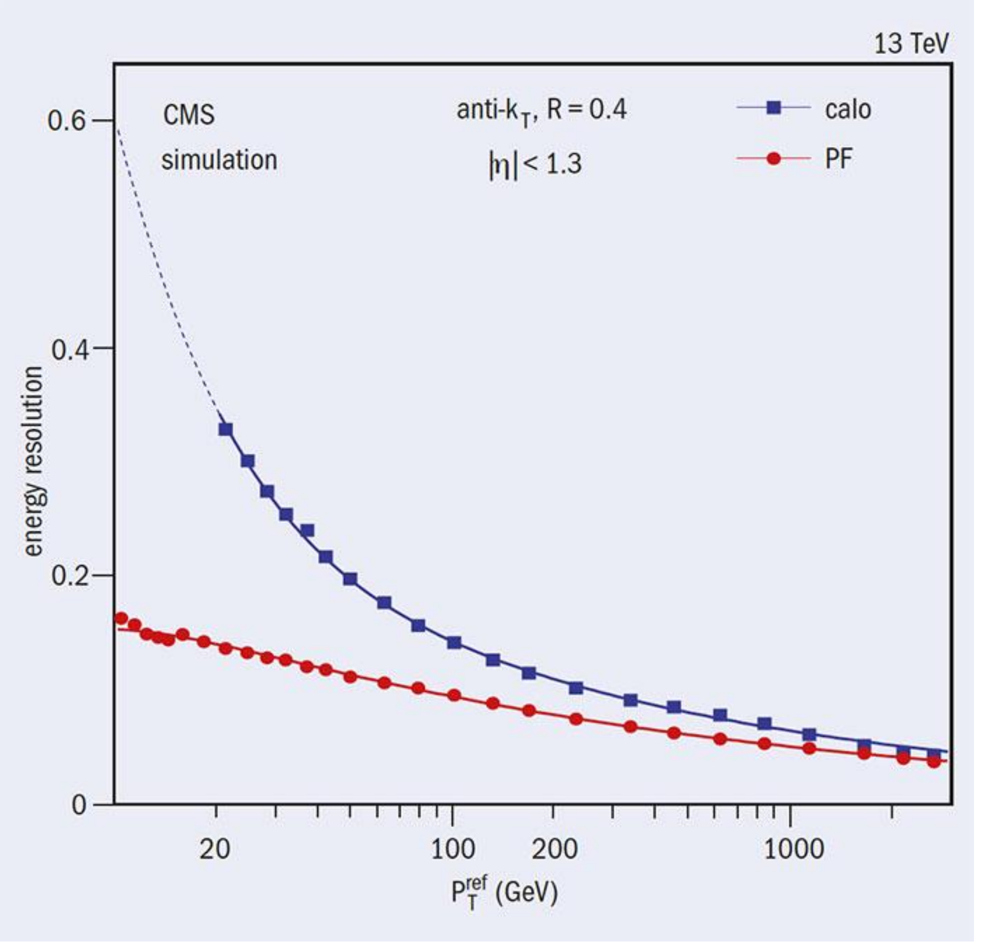
Use tracking for low momentum charged hadrons, remove calorimeter energy deposits that those tracks point to, and use calorimetry for the rest.

Dramatic improvement in jet energy resolution for low transverse momentum jets

Also improves pileup rejection

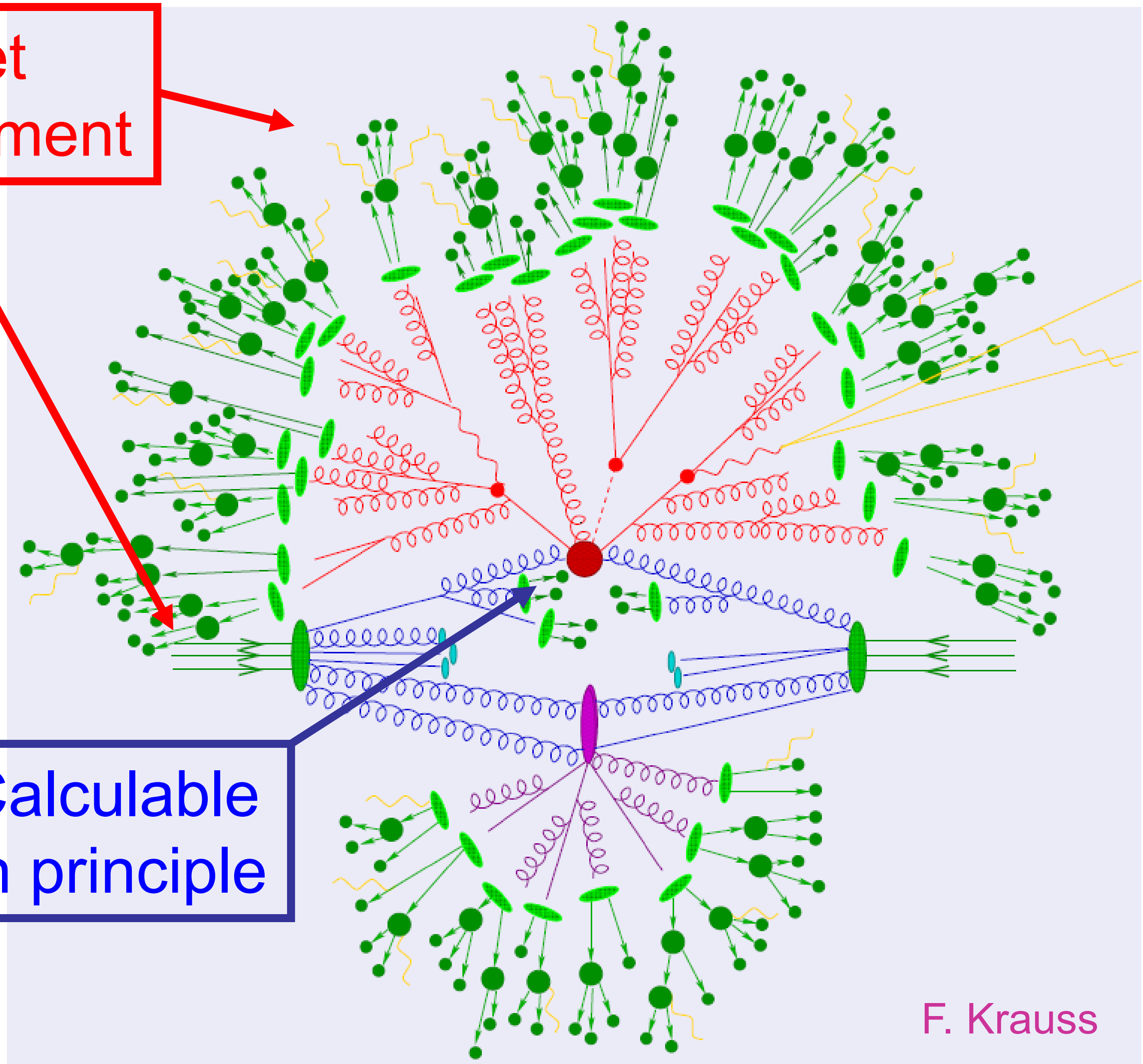
Still, resolution for 60 GeV jets is about 5-10 times worse than photons (10% vs. 1-2%)

Energy calibration also a big issue, and this limits experimental precision for (fast-falling) jet cross sections



Typical LHC Collision

Model or get from experiment



Calculable in principle

QCD as a Yang-Mills Theory

- Require local gauge invariance under a non-abelian gauge group:

$$U \in \mathrm{SU}(N_c), \quad N_c = 3, \qquad U^\dagger = U^{-1}, \ \det U = 1$$

(number of colors = components of quark field ψ):

- $\mathcal{L}_q = i\bar{q}\gamma^\mu D_\mu q \qquad\qquad D_\mu \equiv \partial_\mu - igA_\mu$
- $q(x) \to U(x)q(x), \qquad \bar{q} \to \bar{q}U^{-1}(x)$
- $\mathcal{L}_q$ invariant if $A_\mu \to UA_\mu U^{-1} + \frac{i}{g}(\partial_\mu U)U^{-1}$

so that $D_\mu q(x) \to U(x)\, D_\mu q(x)$

QCD Lagrangian (cont.)

- $A_\mu = T^a A_\mu^a, \quad a = 1,2,\ldots 8 = N_c^2 - 1$ gluons
- Field strength $F_{\mu\nu} = \frac{i}{g}\left[D_\mu, D_\nu\right]$

$$= \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu]$$

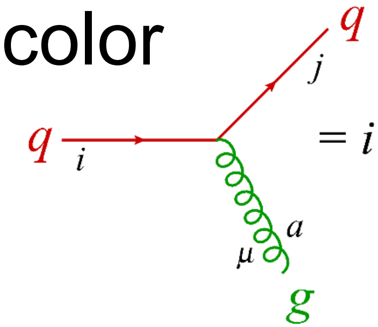
is nonlinear in A_μ, transforms covariantly, so

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4}\text{tr}\, F_{\mu\nu}^2 = -\frac{1}{4} F_{\mu\nu}^a F^{a\,\mu\nu}$$

is invariant and contains 3 and 4 gluon interactions

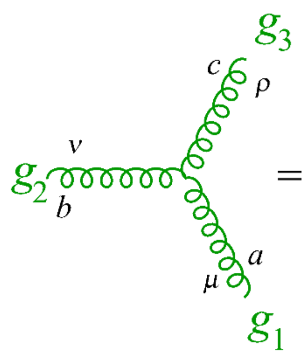
QCD interactions (cont.)

- Gauge-fermion interactions from $\bar{q}\gamma^{\mu}A_{\mu}q$ much like QED, just with a matrix color charge



$$= i\,g\,\gamma^{\mu}(T^{a})_{i}^{j}$$

- Triple gluon vertex is new, changes everything (vs QED)



$$= i\,g\,f^{abc}\left[\,\eta^{\mu\nu}\,(k_1^{\rho} - k_2^{\rho}) + \text{cyclic}\right]$$

$$\alpha_S = \frac{g^2}{4\pi}$$

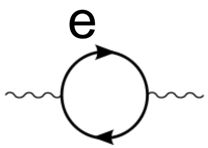
Key to calculability:

Asymptotic Freedom

Gross, Wilczek, Politzer (1973)

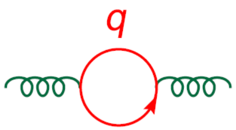
Quantum fluctuations of massless virtual particles

QED: electrons screen charge (e larger at short distances, large μ)

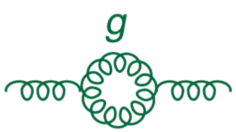


γ > 0 → $$\alpha(\mu^2) = \frac{\alpha(\mu_0^2)}{1 - \frac{1}{3\pi}\alpha(\mu_0^2)\ln(\mu^2/\mu_0^2)}$$

Non-Abelian gauge theory (Yang, Mills (1954)):
gluons anti-screen charge (g_s smaller at short distances)



> 0



< 0

→

$$\alpha_s(\mu^2) = \frac{1}{b_0 \ln(\mu^2/\Lambda^2)} = \frac{\alpha_s(\mu_0^2)}{1 + b_0\alpha_s(\mu_0^2)\ln(\mu^2/\mu_0^2)}$$

$$b_0 = \frac{11N_c - 2n_f}{12\pi}$$

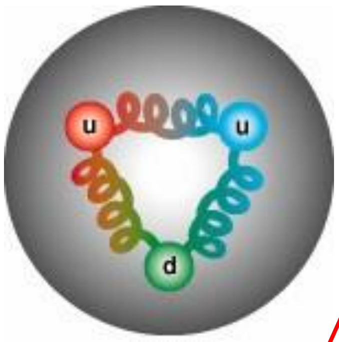
Gluon self-interactions make quarks almost free, make QCD calculable at short distances: $g_s^2/(4\pi) = \alpha_s(\mu) \to 0$ asymptotically as $\mu \to \infty$

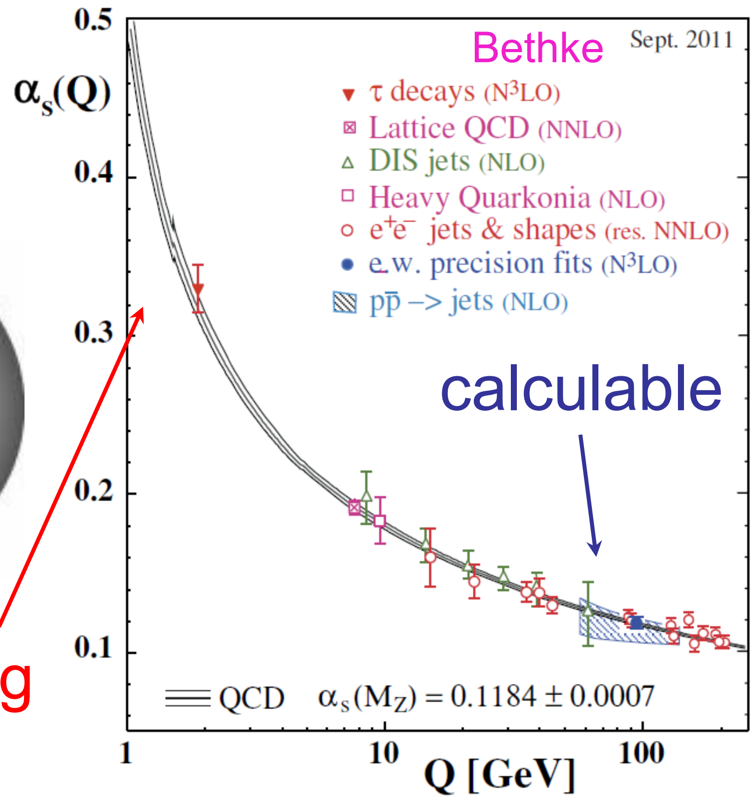
Fast forward 50 years

van Ritbergen, Vermaseren, Larin (1997); Baikov, Chetyrkin, Kühn (2016, 2017); Herzog, Ruijl, Vermaseren, Vogt (2017)

α_s , and its running with Q, now known precisely from many experiments (and high-order theory)

$$\frac{d\alpha_s(\mu^2)}{d\ln\mu^2} = \beta(\alpha_s(\mu^2)) \qquad \beta(\alpha_s) = -\frac{\alpha_s^2}{4\pi}\left[\beta_0 + \beta_1\frac{\alpha_s}{4\pi} + \beta_2\left(\frac{\alpha_s}{4\pi}\right)^2 + \beta_3\left(\frac{\alpha_s}{4\pi}\right)^3 + \cdots\right]$$





$$\beta_0 = \frac{11}{3}C_A - \frac{4}{3}T_F n_f$$

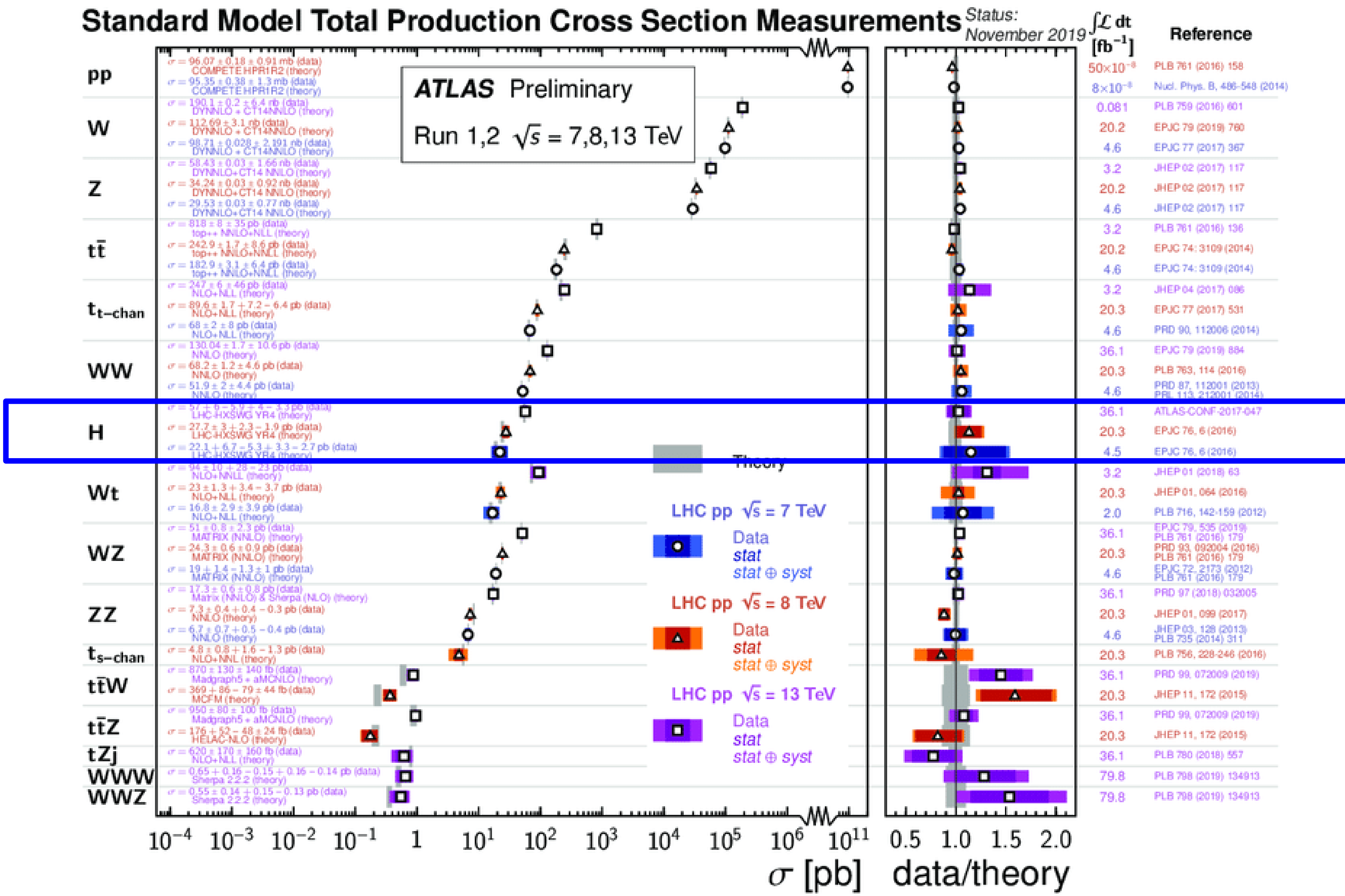
$$\beta_1 = \frac{34}{3}C_A^2 - 4C_F T_F n_f - \frac{20}{3}C_A T_F n_f$$

$$\beta_2 = \frac{2857}{54}C_A^3 + 2C_F^2 T_F n_f - \frac{205}{9}C_F C_A T_F n_f - \frac{1415}{27}C_A^2 T_F n_f + \frac{44}{9}C_F T_F^2 n_f^2 + \frac{158}{27}C_A T_F^2 n_f^2$$

$$\begin{aligned}\beta_3 = {} & C_A^4\left(\frac{150653}{486} - \frac{44}{9}\zeta_3\right) + C_A^3 T_F n_f\left(-\frac{39143}{81} + \frac{136}{3}\zeta_3\right) \\ & + C_A^2 C_F T_F n_f\left(\frac{7073}{243} - \frac{656}{9}\zeta_3\right) + C_A C_F^2 T_F n_f\left(-\frac{4204}{27} + \frac{352}{9}\zeta_3\right) \\ & + 46 C_F^3 T_F n_f + C_A^2 T_F^2 n_f^2\left(\frac{7930}{81} + \frac{224}{9}\zeta_3\right) + C_F^2 T_F^2 n_f^2\left(\frac{1352}{27} - \frac{704}{9}\zeta_3\right) \\ & + C_A C_F T_F^2 n_f^2\left(\frac{17152}{243} + \frac{448}{9}\zeta_3\right) + \frac{424}{243}C_A T_F^3 n_f^3 + \frac{1232}{243}C_F T_F^3 n_f^3 \\ & + \frac{d_A^{abcd}d_A^{abcd}}{N_A}\left(-\frac{80}{9} + \frac{704}{3}\zeta_3\right) + n_f\frac{d_F^{abcd}d_A^{abcd}}{N_A}\left(\frac{512}{9} - \frac{1664}{3}\zeta_3\right) \\ & + n_f^2\frac{d_F^{abcd}d_F^{abcd}}{N_A}\left(-\frac{704}{9} + \frac{512}{3}\zeta_3\right)\end{aligned}$$

$\beta_4 =$ ...

$C_A = 3, C_F = \frac{4}{3}, T_F = \frac{1}{2}, n_f = 5, \ldots$

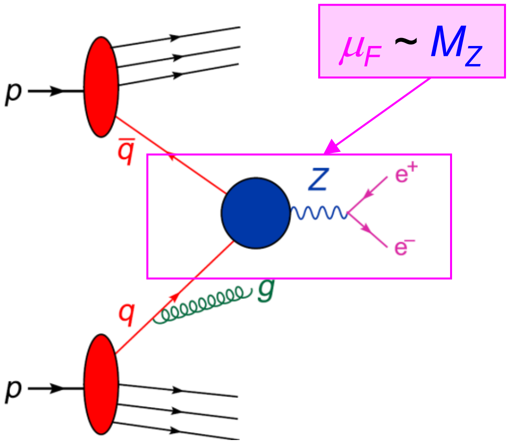

Standard Model Total Production Cross Section Measurements
Status: November 2019
ATLAS Preliminary
Run 1,2 √s = 7,8,13 TeV
Reference
pp
W
Z
tt̄
t t−chan
WW
H
Wt
WZ
ZZ
t s−chan
tt̄W
tt̄Z
tZj
WWW
WWZ
Theory
LHC pp √s = 7 TeV
Data
stat
stat ⊕ syst
LHC pp √s = 8 TeV
Data
stat
stat ⊕ syst
LHC pp √s = 13 TeV
Data
stat
stat ⊕ syst
σ [pb]
data/theory

QCD Factorization & Parton Model

Drell, Yan (1970s); …; Collins, Soper, Sterman (1980s),…

- Asymptotic freedom guarantees that at short distances (large transverse momenta), partons in the proton are almost free.
- Sampled "one at a time" in hard collisions.

→ QCD-improved parton model



suitable final state

Parton distribution function

factorization scale ("arbitrary")

$$\sigma^{pp\to X}(s;\alpha_s,\mu_R,\mu_F) = \sum_{a,b}\int_0^1 dx_1 \int_0^1 dx_2\, f_a(x_1,\alpha_s,\mu_F)\, f_b(x_2,\alpha_s,\mu_F) \times \hat{\sigma}^{ab\to X}(sx_1x_2;\alpha_s,\mu_R,\mu_F) + \frac{\Lambda}{Q}$$

Partonic cross section, computable in perturbative QCD

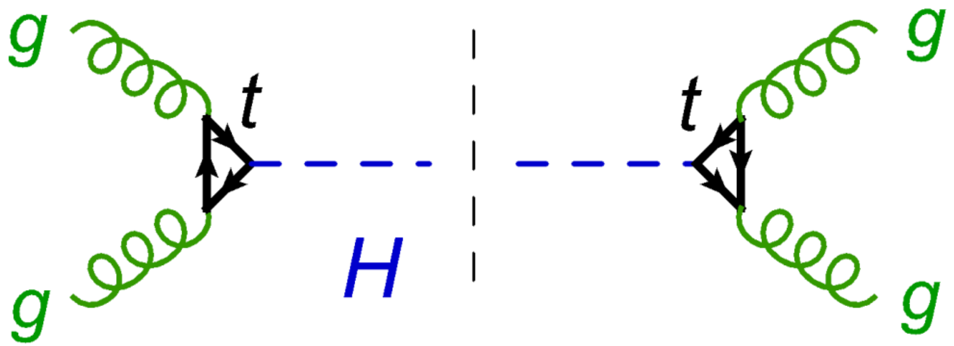
partonic CM energy^2

renormalization scale ("arbitrary")

Producing Higgs bosons at LHC

Higgs boson dominantly produced by **gluon fusion**, a quantum process at "one loop", mediated by **top quark,** because ***t*** couples strongly to both gluons and Higgs

Leading Order (LO) cross section
$= |\text{one} - \text{loop amplitude}|^2$



- Since $\boldsymbol{2m_{top}} = 350 \text{ GeV}$
  $\gg \boldsymbol{m_{Higgs}} = 125 \text{ GeV}$,

interaction between gluons and Higgs is approximately local (mediated by an operator $H G^a_{\mu\nu} G^{\mu\nu\, a}$)

Perturbative Short-Distance Cross Section

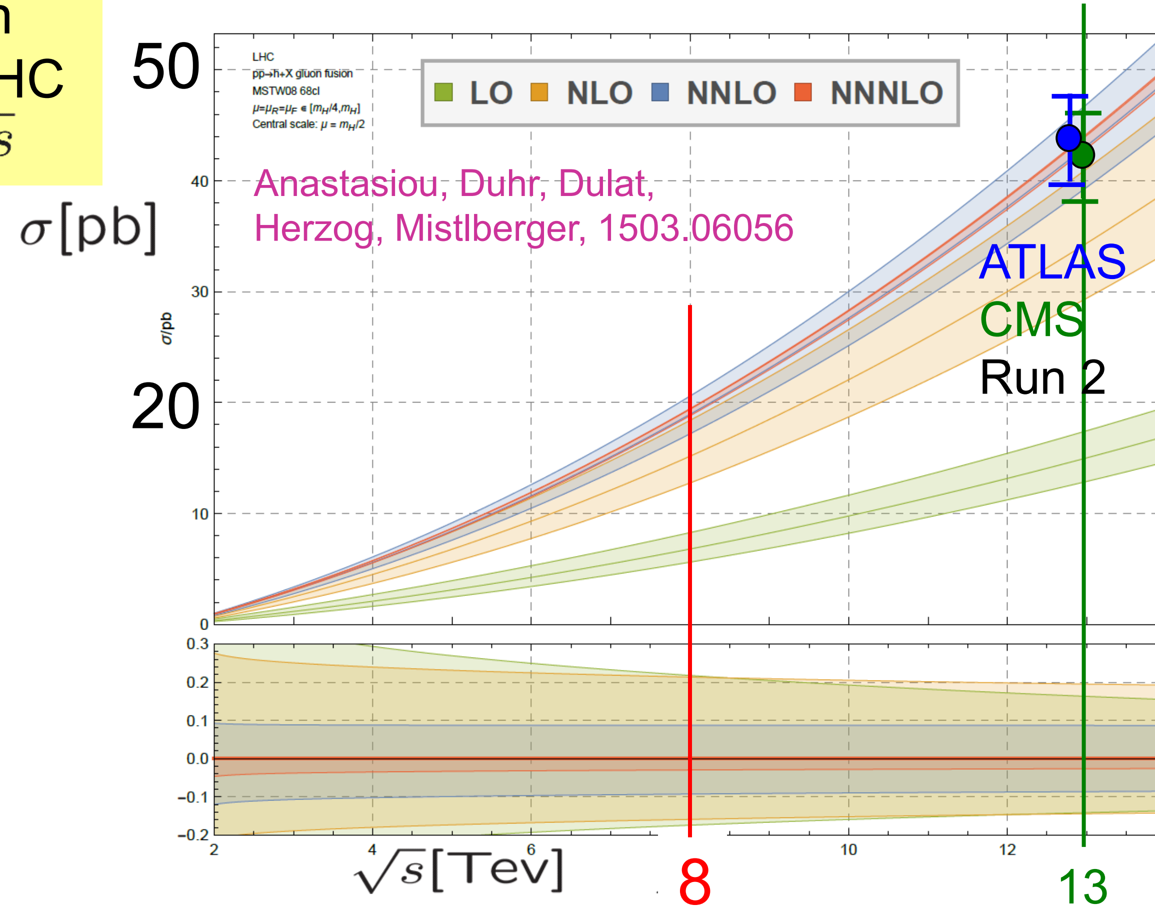
$$\hat{\sigma}(\alpha_s, \mu_F, \mu_R) \;=\; [\alpha_s(\mu_R)]^{n_\alpha}\left[\hat{\sigma}^{(0)} + \frac{\alpha_s}{2\pi}\hat{\sigma}^{(1)}(\mu_F,\mu_R) + \left(\frac{\alpha_s}{2\pi}\right)^2 \hat{\sigma}^{(2)}(\mu_F,\mu_R) + \cdots\right]$$

LO NLO NNLO

Higgs gluon fusion cross section at LHC vs. CM energy $\sqrt{s}$

LO terrible!
LO → NNNLO
→ factor of 2 or 3 increase!

Poor convergence of expansion in $\alpha_s(\mu)$ necessitates high orders!



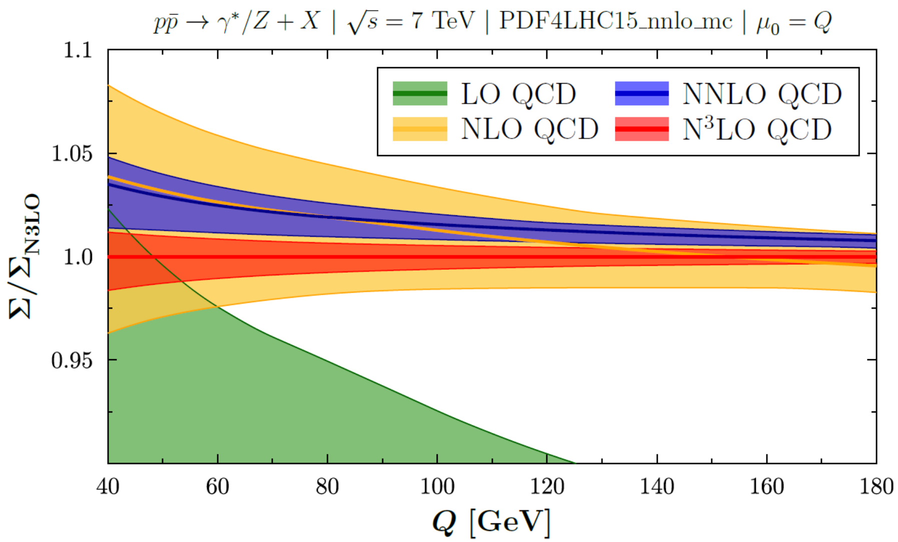
Z cross section

Much better behaved than Higgs production
→ use to constrain pdfs

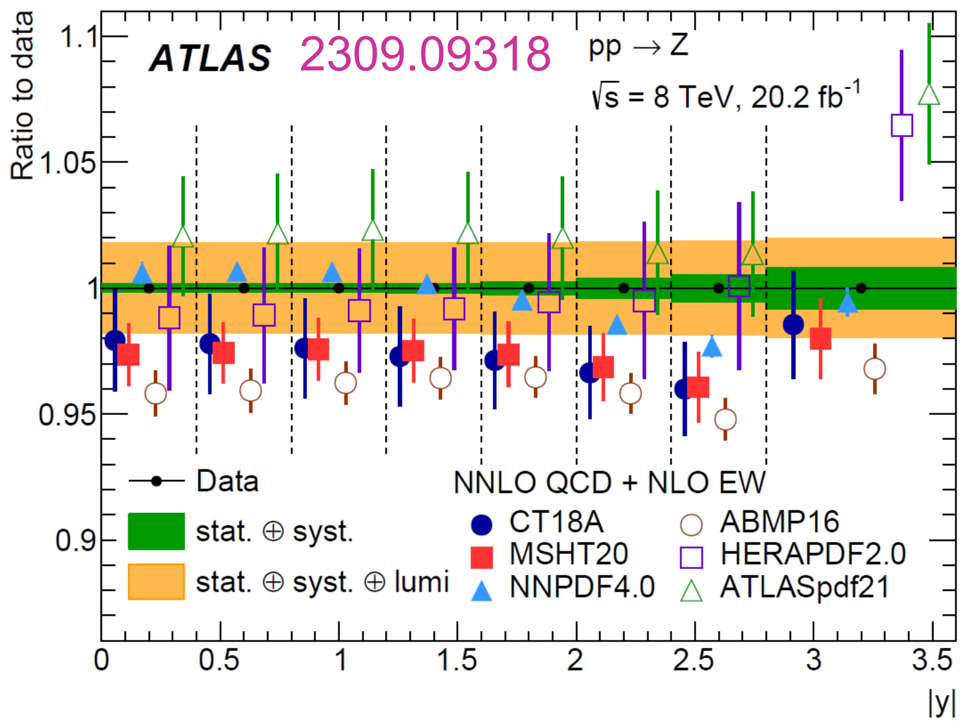
Z distribution in rapidity

$$y = \frac{1}{2}\ln\left(\frac{E+p_z}{E-p_z}\right)$$

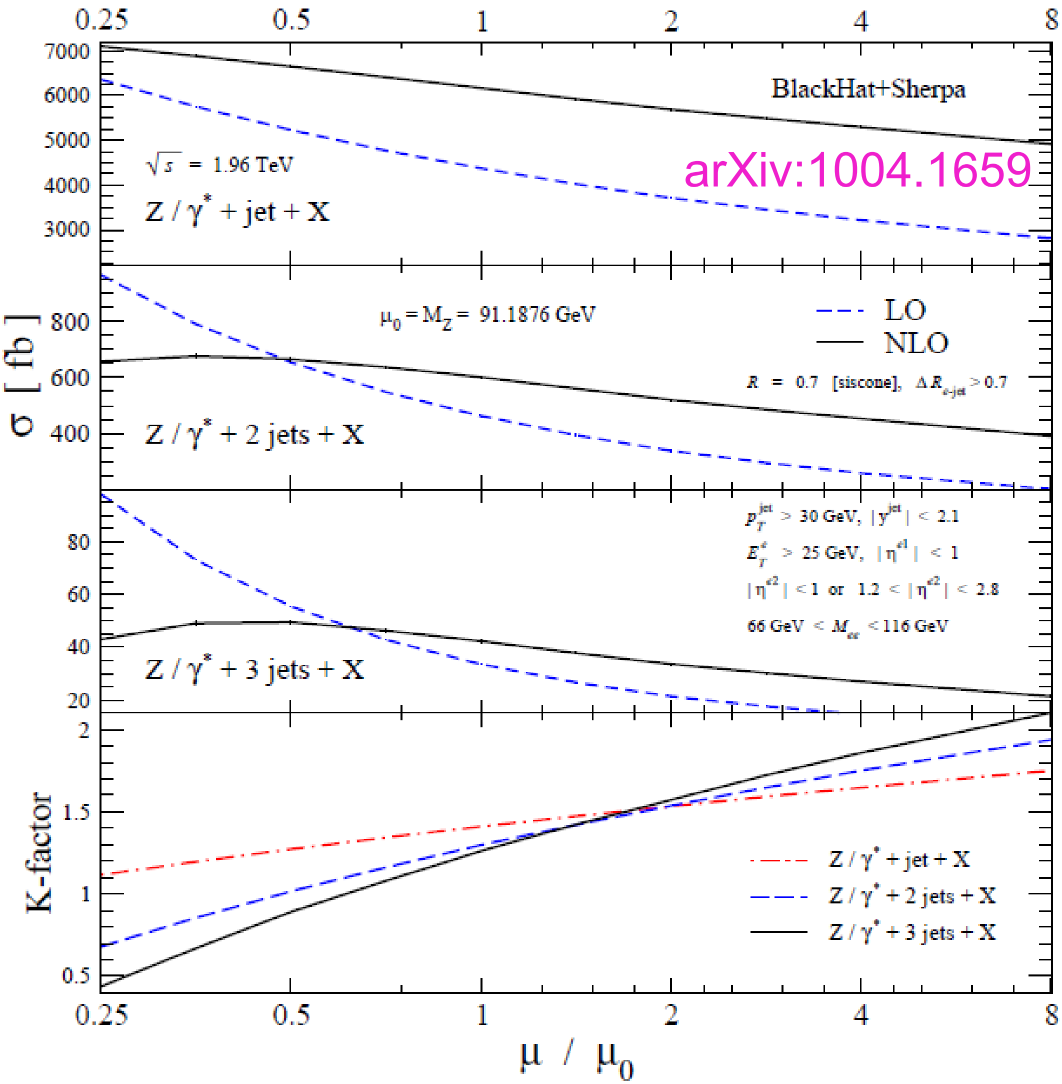
Total cross section vs. order in QCD



Baglio, Duhr, Mistlberger, Szafron, 2209.06138



Z + jets uncertainty increases with n_{jets}



Z + 1,2,3 jets at Tevatron (CDF cuts)

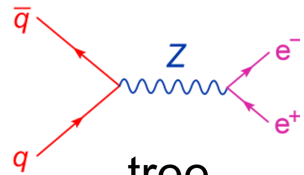
Uncertainty brought under much better control by NLO corrections: ~ 50% → ~ 15-20%

NLO really required for quantitative control of multi-jet final states

Overall structure of higher-order QCD corrections

Example of *Z* production at hadron colliders

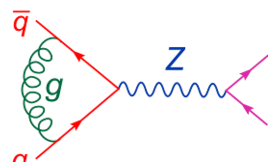
LO $\hat{\sigma}^{(0)}$



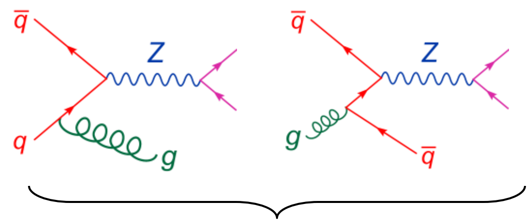
tree

convolute with pdfs
apply cuts

NLO $\hat{\sigma}^{(1)}$



1 loop

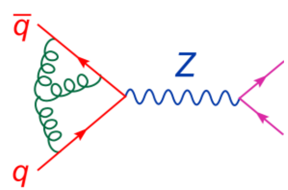


tree + 1 parton

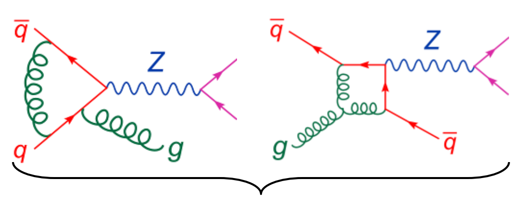
dim. reg. $D = 4 - 2\epsilon$

first, cancel infrared divergences ($1/\epsilon^2$)
between virtual & real

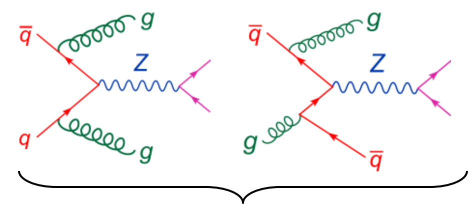
NNLO $\hat{\sigma}^{(2)}$



2 loops



1 loop + 1 parton



tree + 2 partons

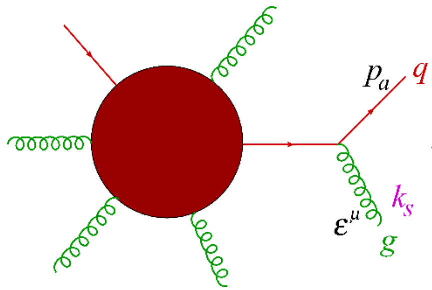
intricate ($1/\epsilon^4$) IR cancellations

Soft and Collinear Divergences

- Everywhere in perturbative QCD, due to massless gluons, quarks
- Govern the structure of jets, and what can be computed perturbatively
- Fortunately, they are quite universal
- Collectively called "infrared" divergences, because they correspond to very small virtualities and so represent interactions that can happen at very long distances from the main hard interaction
- Universality also means they can be resummed for higher accuracy in certain kinematic regions

Soft (gluon) divergences

- Emission of a soft gluon off a quark line does not depend on the quark helicity, or its spin; classical radiation from color charges.



$$\alpha \ \frac{\not p_a + \not k_s}{(p_a+k_s)^2} \not\varepsilon\, u(p_a) \ = \ \frac{\varepsilon \cdot p_a}{k_s \cdot p_a} \ u(p_a)$$

- To make gauge invariant ($\varepsilon^\mu \to \varepsilon^\mu + k_s^\mu$), combine with radiation off another colored particle (could be a gluon), with momentum p_b:
- → Eikonal factor $\dfrac{\varepsilon \cdot p_a}{k_s \cdot p_a} - \dfrac{\varepsilon \cdot p_b}{k_s \cdot p_b}$ at amplitude level
- Its square (cross section level) is $\dfrac{p_a \cdot p_b}{k_s \cdot p_a \, k_s \cdot p_b}$
- Only depends on direction of emitters a, b
- Extra collinear enhancement for $k_s \parallel p_a$ or p_a