The Electroweak Standard Model

facilitating precision physics at LHC and beyond

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Recap: lecture II

The Electroweak Standard Model

- $SU(2)_L \otimes U(1)_Y$ gauge invariant Lagrangia
  - $\rightsquigarrow$ weak charged current coupling to left-chiral particles only
  - $\rightsquigarrow$ non-trivial ground state for $SU(2)$ scalar field doublet (SSB)
  - $\rightsquigarrow$ gauge symmetry hidden in the spectrum of the theory
  - $\rightsquigarrow$ finite masses for matter fermions and $W^{\pm}$, Z^0 bosons

the road ahead

- consider Standard Model at the quantum level
  - $\rightsquigarrow$ inputs and predictions
- evaluation of higher-order corrections
  - $\rightsquigarrow$ virtual and real-emission corrections, QED IR divergences
  - $\rightsquigarrow$ NLO EW corrections for scattering processes

Parameters of the Standard Model

- GWS theory describes EW interactions in terms of

$$g_2, g_1 = g_2 \tan\theta_W, v \text{ and } \lambda$$

- these inputs can be accurately determined from [PDG '25]
  - the fine structure constant

$$\alpha = \frac{e^2}{4\pi} = \frac{g_2^2 \sin^2\theta_W}{4\pi} = 1/137.035\,999\,177(21)$$

  - the Fermi weak coupling constant (derived from muon lifetime)

$$G_F = \frac{g_2^2\sqrt{2}}{8M_W^2} = \frac{1}{\sqrt{2}v^2} = 1.166\,378\,8(6) \times 10^{-5}\ \text{GeV}^{-2}$$

  - the Z^0 boson mass

$$M_Z = \frac{g_2 v}{2\cos\theta_W} = 91.1880(20)\ \text{GeV}$$

  - the Higgs boson mass, fixing the quartic coupling λ

$$M_H = v\sqrt{2\lambda} = 125.20(11)\ \text{GeV}$$

$\rightsquigarrow$ note, relations subject to higher-order corrections

$\rightsquigarrow$ different schemes to define consistent set of input parameters

- the Higgs–fermion interaction strength determined by Yukawa couplings

$$f_f = \frac{\sqrt{2} m_f}{v}$$

  $\rightsquigarrow$ 9 fermion masses m_f, neutrinos considered massless

- however, up- and down-quark states allowed to mix among generations

  $\rightsquigarrow$ gauge basis states *not* mass eigenstates

  $\rightsquigarrow$ linked by unitary Cabibbo–Kobayashi–Maskawa (CKM) matrix

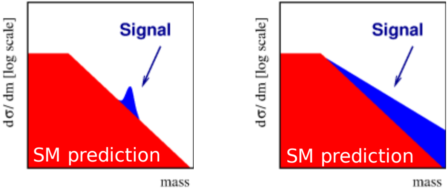
$$\begin{aligned} V_{\mathsf{CKM}} &= \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \\ &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \end{aligned}$$

  $\rightsquigarrow$ three real parameters (θ_{12},θ_{13},θ_{23}) and one complex phase (δ)

- note, quark mixing only source for CP violation in Standard Model

Precision EW physics

- historically, search for unknown elements & inputs of the SM
  - ⇝ e.g. discoveries of $W^{\pm}$, Z^0, ν_τ, top quark and Higgs boson
  - ⇝ evidence for CP violation in the K and B systems
- at present, all inputs to the Standard Model established and known
  - ⇝ within parameter uncertainties theory fully determined
- framework for precision predictions to confront with experimental data
  1. yet better determine SM input parameters
  2. detailed probes of SM predictions, e.g. Higgs self-coupling, rare decays
  3. seeking for evidence for physics Beyond the Standard Model



- to get to higher theoretical accuracy of predictions, in perturbative calculations more higher-order terms need to be included

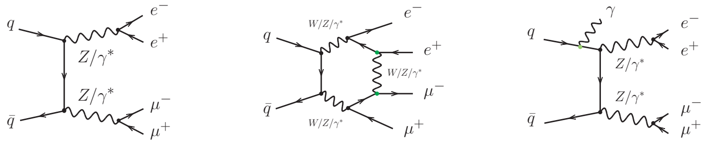
Quantum Theory of EW interactions

- EW SM – non-abelian gauge theory with SSB – is renormalizable
  ⇝ all UV divergences can be absorbed in renormalization constants
  ⇝ parameter and wave-function renormalizations, e.g.

$$M_{0,W}^2 = M_W^2 + \delta M_W^2\,, \qquad W_{0,\mu}^{\pm} = Z_W^{1/2} W_\mu^{\pm}$$
$$M_{0,h}^2 = M_h^2 + \delta M_h^2\,, \qquad h_0 = Z_h^{1/2} h$$

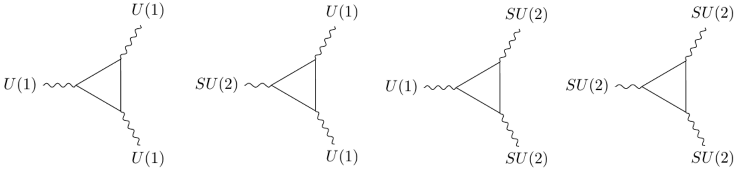
$$\boxed{\mathcal{L}_{\mathsf{EW},0} = \mathcal{L}_{\mathsf{EW}} + \mathcal{L}_{\mathsf{EW},ct}}$$

- consistent QFT with quantum effects beyond the classical theory
  ⇝ particle interpretation, interferences, $\hbar$-type corrections, etc
- means to calculate scattering matrix elements
  ⇝ inclusion of virtual & real-emission corrections



Quantum Theory of EW interactions: gauge anomalies

- classical SM Lagrangian is $SU(2)_L \otimes U(1)_Y$ symmetric (and $SU(3)_c$)
  Is this symmetry preserved at quantum level or broken by gauge anomalies?
- SM features chiral fermions giving rise to anomalous contributions
  $\rightsquigarrow$ proper cancellation needed for theory to be anomaly free
- need to consider triangle graphs with external bosons and fermion loops



- links quantum numbers of all left- and right-chiral fermions[1], e.g.

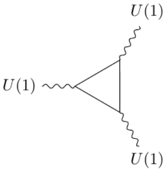
$$U(1)^3 \quad : \quad \sum_L Y^3_{\psi_L} - \sum_R Y^3_{\psi_R} \stackrel{!}{=} 0$$

$$SU(2)^2 U(1) \quad : \quad \sum_L Y_{\psi_L} \stackrel{!}{=} 0$$

[1] includes QCD, i.e. quarks come with $N_c = 3$

Quantum Theory of EW interactions: gauge anomalies

- consider the $U(1)_Y$ abelian axial current for single fermion generation



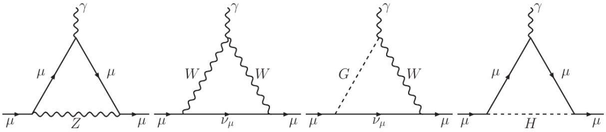
$$
\begin{aligned}
0 &\stackrel{!}{=} \sum_L Y^3_{\psi_L} - \sum_R Y^3_{\psi_R} \\
&= Y^3_{\nu_{eL}} + Y^3_{e_L} + (Y^3_{u_L} + Y^3_{d_L})N_c - Y^3_{e_R} - (Y^3_{u_R} + Y^3_{d_R})N_c \\
&= (-1)^3 + (-1)^3 + \left(\frac{1}{27} + \frac{1}{27}\right)3 - (-2)^3 - \left(\frac{64}{27} - \frac{8}{27}\right)3 \\
&= -2 + \frac{2}{9} + 8 - \frac{56}{9} = 0 \quad \checkmark
\end{aligned}
$$

- $U(1)^3_Y$ gauge anomaly cancels, symmetry preserved
  - $\rightsquigarrow$ fermions need to come in complete lepton–quark generations
  - $\rightsquigarrow$ SM quantum number assignments strongly constrained
- all SM gauge anomalies cancel, gauge symmetry preserved at quantum level

An example: EW corrections to the muon magnetic moment

[Aoyama et al. Phys Rept 887 (2020) 1]

- besides QED contributions $a_\mu = \frac{g_\mu - 2}{2}$ receives EW corrections
  $\rightsquigarrow$ loop diagrams with exchange of at least one EW boson Z, W, h



- physical result gauge independent, diagrams depend on gauge
  $\rightsquigarrow$ in non-unitary gauges Goldstone bosons ($G^\pm$, G^0) appear
- EW corrections known up to two-loop order

$$a_\mu^{\mathsf{EW}} = a_\mu^{\mathsf{EW(1)}} + a_\mu^{\mathsf{EW(2)}} + \mathcal{O}(g_2^6)$$

- one-loop EW contributions yield

$$a_\mu^{\mathsf{EW(1)}} = \frac{G_F}{\sqrt{2}} \frac{m_\mu^2}{8\pi^2} \left(\frac{5}{3} + \frac{1}{3} \left(1 - 4\sin^2\theta_W\right) \right) = 194.79(1) \times 10^{-11}$$

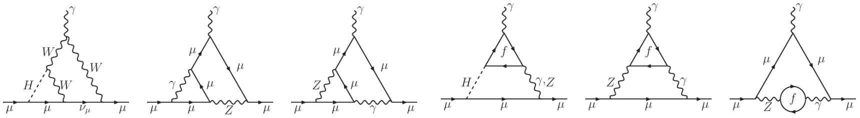
An example: EW corrections to the muon magnetic moment

- one-loop result can be expressed as

$$a_\mu^{\mathsf{EW(1)}} \propto \frac{g_2^2}{16\pi^2}\frac{m_\mu^2}{M_W^2} \sim 10^{-9}$$

$\rightsquigarrow$ besides loop factor suppressed by $m_\mu^2/M_W^2 \simeq 10^{-6}$

- there appear a large number of bosonic/fermionic two-loop contributions



- in total the EW corrections rather small [Aliberti et al. 2505.21476]

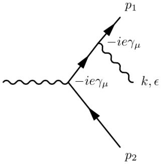
$$\begin{aligned} a_\mu^{\mathsf{QED}} &= 116\,584\,718.8(2)\times 10^{-11}\\ a_\mu^{\mathsf{EW}} &= 154.4(4)\times 10^{-11}\\ a_\mu^{\mathsf{SM}} &= 116\,592\,033(62)\times 10^{-11}\\ a_\mu^{\mathsf{exp}} &= 116\,592\,059(22)\times 10^{-11}\end{aligned}$$

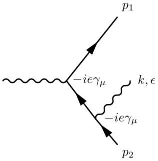
$$\Delta a_\mu \equiv a_\mu^{\mathsf{exp}} - a_\mu^{\mathsf{SM}} = 38(63)\times 10^{-11}$$

theory & experiment fully consistent

SM uncertainty dominated by QCD

consider QED Bremsstrahlung to $\gamma^* \to \mu^+\mu^-$, i.e. $\gamma^* \to \mu^+\mu^-\gamma$

$\mathcal{M}^R_{\mu^+\mu^-\gamma} =$ 

$+$ 

$$= \bar{u}(p_1)(-ie)\not{\epsilon}\frac{-i(\not{p}_1 + \not{k})}{(p_1+k)^2}(-ie)\gamma_\mu v(p_2) + \bar{u}(p_1)(-ie)\gamma_\mu\frac{i(\not{p}_2 + \not{k})}{(p_2+k)^2}(-ie)\not{\epsilon} v(p_2)$$

consider QED Bremsstrahlung to $\gamma^* \to \mu^+\mu^-$, i.e. $\gamma^* \to \mu^+\mu^-\gamma$

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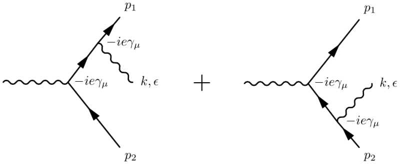


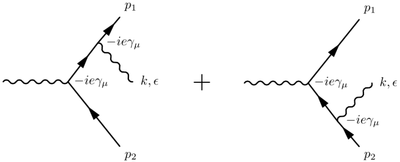
$$= \bar{u}(p_1)(-ie)\not{\epsilon}\frac{-i(\not{p}_1+\not{k})}{(p_1+k)^2}(-ie)\gamma_\mu v(p_2) + \bar{u}(p_1)(-ie)\gamma_\mu\frac{i(\not{p}_2+\not{k})}{(p_2+k)^2}(-ie)\not{\epsilon}v(p_2)$$

assume photon is soft, i.e. $k^0 \ll p_1^0, p_2^0$, ignore terms suppressed by powers of k

$$\mathcal{M}^R_{\mu^+\mu^-\gamma} \simeq \bar{u}(p_1)ie\gamma_\mu v(p_2)\, e\left(\frac{p_1\cdot\epsilon}{p_1\cdot k} - \frac{p_2\cdot\epsilon}{p_2\cdot k}\right)$$

Higher-order electroweak effects: real corrections

consider QED Bremsstrahlung to $\gamma^* \to \mu^+\mu^-$, i.e. $\gamma^* \to \mu^+\mu^-\gamma$

$\mathcal{M}^R_{\mu^+\mu^-\gamma} =$ 

$$= \bar{u}(p_1)(-ie)\not{\epsilon}\frac{-i(\not{p}_1+\not{k})}{(p_1+k)^2}(-ie)\gamma_\mu v(p_2) + \bar{u}(p_1)(-ie)\gamma_\mu\frac{i(\not{p}_2+\not{k})}{(p_2+k)^2}(-ie)\not{\epsilon}v(p_2)$$

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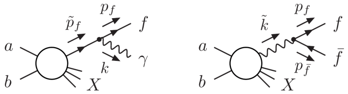
$$|\mathcal{M}^R_{\mu^+\mu^-\gamma}|^2 \simeq \sum_{\text{pol}}\left|\bar{u}(p_1)ie\gamma_\mu v(p_2)\; e\left(\frac{p_1\cdot\epsilon}{p_1\cdot k} - \frac{p_2\cdot\epsilon}{p_2\cdot k}\right)\right|^2$$

$$= |\mathcal{M}^R_{\mu^+\mu^-}|^2 e^2 \frac{2p_1\cdot p_2}{(p_1\cdot k)(p_2\cdot k)}$$ $\rightsquigarrow$ squared amplitude factorizes

- singular for soft ($k \to 0$) and/or collinear ($\angle(p_i, k) \to 0$) photon emission

Higher-order electroweak effects: real corrections

- QED with massless photon features IR singularities
  $\rightsquigarrow$ low energetic photons & collinear splittings of light particles/photons



- can be regularized, e.g. in dimensional regularization (poles in ϵ)

Higher-order electroweak effects: real corrections

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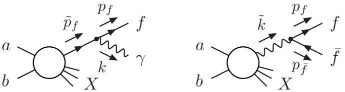
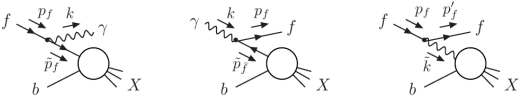
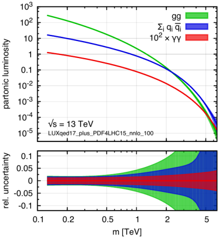


- can be regularized, e.g. in dimensional regularization (poles in ϵ)
- collinear singularities also for initial-state emissions



⇝ source for initial-state photons, need for QED corrected PDFs



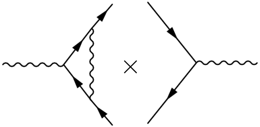
[Manohar et al. JHEP 12 (2017) 046]

- available in all modern PDF sets
  e.g. LUXQED, NNPDF, ...
- γ flux much smaller than q, g
- enhanced e.g. in $pp \to W^+W^-$ or l^+l^-

Higher-order electroweak effects: NLO corrections

- total cross sections IR finite [Bloch, Nordsieck: Phys Rev 133 (1937) 650]

  $\rightsquigarrow$ account for real and virtual contributions at given order
- very real-emission singularities cancel against virtual corrections

$2\Re\left(\mathcal{M}^{V}_{\mu^+\mu^-}\mathcal{M}^{B,\dagger}_{\mu^+\mu^-}\right) \supset$ 

$$\sigma_{\text{tot}}(\gamma^* \to \mu^+\mu^-) = \sigma^{\text{Born}}_{\mu^+\mu^-}\left(\underbrace{1}_{\text{LO}} + \underbrace{C_1\frac{\alpha}{\pi}}_{\text{NLO EW}} + \underbrace{\ldots}_{\text{higher orders}}\right)$$

- no soft divergences for emission of heavy particles W, Z, h

  $\rightsquigarrow$ however, inclusion of virtual EW corrections

  $\rightsquigarrow$ justified to classify final states by their multiplicity: W vs. WW, WZ, ...
- automation of NLO EW calculations for LHC phenomenology

  e.g. OpenLoops [Pozzorini et al.], Recola [Denner et al.], MadGraph [Frederix et al.]

Example NLO EW results for LHC from MG5_aMC@NLO

[Frederix et al. JHEP 07 (2018) 185]

Process	Cross section (in pb) LO	NLO EW	Correction (in %)
$pp \to e^+\nu_e$	$5.2498(5) \cdot 10^3$	$5.2113(6) \cdot 10^3$	$-0.73(1)$
$pp \to e^+\nu_e j$	$9.1468(2) \cdot 10^2$	$9.0449(14) \cdot 10^2$	$-1.11(2)$
$pp \to e^+\nu_e jj$	$3.1562(3) \cdot 10^2$	$3.0985(5) \cdot 10^2$	$-1.83(2)$
$pp \to e^+e^-$	$7.5367(8) \cdot 10^2$	$7.4997(10) \cdot 10^2$	$-0.49(2)$
$pp \to e^+e^- j$	$1.5059(1) \cdot 10^2$	$1.4909(2) \cdot 10^2$	$-1.00(2)$
$pp \to e^+e^- jj$	$5.1424(4) \cdot 10^1$	$5.0410(7) \cdot 10^1$	$-1.97(2)$
$pp \to e^+e^-\mu^+\mu^-$	$1.2750(1) \cdot 10^{-2}$	$1.2083(1) \cdot 10^{-2}$	$-5.23(1)$
$pp \to e^+\nu_e\mu^-\bar{\nu}_\mu$	$5.1144(7) \cdot 10^{-1}$	$5.3019(9) \cdot 10^{-1}$	$+3.67(2)$
$pp \to He^+\nu_e$	$6.7643(1) \cdot 10^{-2}$	$6.4914(12) \cdot 10^{-2}$	$-4.03(2)$
$pp \to He^+e^-$	$1.4554(1) \cdot 10^{-2}$	$1.3700(2) \cdot 10^{-2}$	$-5.87(2)$
$pp \to Hjj$	$2.8268(2) \cdot 10^0$	$2.7075(3) \cdot 10^0$	$-4.22(1)$
$pp \to W^+W^-W^+$	$8.2874(4) \cdot 10^{-2}$	$8.8017(12) \cdot 10^{-2}$	$+6.21(2)$
$pp \to ZZW^+$	$1.9874(1) \cdot 10^{-2}$	$2.0189(3) \cdot 10^{-2}$	$+1.58(2)$
$pp \to ZZZ$	$1.0761(1) \cdot 10^{-2}$	$0.9741(1) \cdot 10^{-2}$	$-9.47(2)$
$pp \to HZZ$	$2.1005(3) \cdot 10^{-3}$	$1.9155(3) \cdot 10^{-3}$	$-8.81(2)$
$pp \to HZW^+$	$2.4408(1) \cdot 10^{-3}$	$2.4809(5) \cdot 10^{-3}$	$+1.64(2)$
$pp \to HHW^+$	$2.7827(1) \cdot 10^{-4}$	$2.4259(27) \cdot 10^{-4}$	$-12.82(10)$
$pp \to HHZ$	$2.6914(3) \cdot 10^{-4}$	$2.3926(3) \cdot 10^{-4}$	$-11.10(2)$
$pp \to t\bar{t}W^+$	$2.4119(3) \cdot 10^{-1}$	$2.3025(3) \cdot 10^{-1}$	$-4.54(2)$
$pp \to t\bar{t}Z$	$5.0456(6) \cdot 10^{-1}$	$5.0033(7) \cdot 10^{-1}$	$-0.84(2)$
$pp \to t\bar{t}H$	$3.4480(4) \cdot 10^{-1}$	$3.5102(5) \cdot 10^{-1}$	$+1.81(2)$
$pp \to t\bar{t}j$	$3.0277(3) \cdot 10^2$	$2.9683(4) \cdot 10^2$	$-1.96(2)$
$pp \to jjj$	$7.9639(10) \cdot 10^6$	$7.9472(11) \cdot 10^6$	$-0.21(2)$
$pp \to tj$	$1.0613(1) \cdot 10^2$	$1.0539(1) \cdot 10^2$	$-0.70(2)$

$\rightsquigarrow$ EW corrections on total cross sections typically rather modest

Higher-order electroweak effects: NLO corrections

- at hadron colliders typically QCD corrections dominate

  $\rightsquigarrow$ evaluated to NLO, NNLO or even N^{3}LO accuracy in α_s (see L. Dixon)

  $\rightsquigarrow$ however, given $\alpha_s(M_Z^2) \approx 0.118$ & $\alpha(M_Z^2) \approx 1/128$: $\boxed{\alpha_s^2 \approx \alpha}$

- need to consider and combine both EW *and* QCD corrections

$$\sigma_{\text{QCD}}^{\text{NLO}} = \sigma^{\text{Born}}\left(1 + \delta\sigma_{\text{QCD}}^{\text{NLO}}\right) \qquad \sigma_{\text{EW}}^{\text{NLO}} = \sigma^{\text{Born}}\left(1 + \delta\sigma_{\text{EW}}^{\text{NLO}}\right)$$

- combine in additive or multiplicative fashion

$$\begin{aligned}
\sigma_{\text{QCD+EW}}^{\text{NLO}} &= \sigma^{\text{Born}}\left(1 + \delta\sigma_{\text{QCD}}^{\text{NLO}} + \delta\sigma_{\text{EW}}^{\text{NLO}}\right) \\
\sigma_{\text{QCD}\times\text{EW}}^{\text{NLO}} &= \sigma^{\text{Born}}\left(1 + \delta\sigma_{\text{QCD}}^{\text{NLO}}\right)\left(1 + \delta\sigma_{\text{EW}}^{\text{NLO}}\right) \\
&= \sigma_{\text{QCD}}^{\text{NLO}}\left(1 + \delta\sigma_{\text{EW}}^{\text{NLO}}\right) = \sigma_{\text{EW}}^{\text{NLO}}\left(1 + \delta\sigma_{\text{QCD}}^{\text{NLO}}\right)
\end{aligned}$$

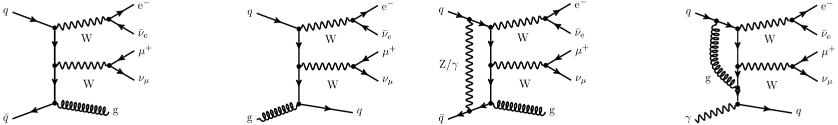
$\rightsquigarrow$ difference provides estimate for missing mixed QCD-EW corr $\mathcal{O}(\alpha\alpha_s)$

Higher-order electroweak effects: NLO corrections

An example: NLO QCD+EW results for $pp \to \mu^+\nu_\mu e^-\bar{\nu}_e j$

[Bräuer et al. JHEP 10 (2020) 159]

- consider full off-shell calculation, Born level $\mathcal{O}(\alpha_s\alpha^4)$ & $\mathcal{O}(\alpha^5)$
- NLO QCD ($\mathcal{O}(\alpha_s^2\alpha^4)$) and EW ($\mathcal{O}(\alpha_s\alpha^5)$) corrections, neglect $\mathcal{O}(\alpha^6)$
- apply veto on additional jets with $p_{T,j} > 25$ GeV
- representative LO and NLO diagrams include



$\sigma^{\rm LO}$ [fb]	$\sigma^{\rm NLO}_{\rm QCD}$ [fb]	$\sigma^{\rm NLO}_{\rm EW}$ [fb]	$\sigma^{\rm NLO}_{\rm QCD+EW}$ [fb]	$\sigma^{\rm NLO}_{\rm QCD\times EW}$ [fb]
$162.5(1)^{+11.2\%}_{-9.1\%}$	$129.5(5)^{+5.1\%}_{-8.9\%}$	$155.5(1)$	$122.5(5)$	$123.9(5)$

$\rightsquigarrow$ $\delta^{\rm NLO}_{\rm QCD} \approx -20\%$ (impact of jet veto), $\delta^{\rm NLO}_{\rm EW} \approx -4.5\%$

$\rightsquigarrow$ QCD+EW and QCD$\times$EW largely consistent for total cross section

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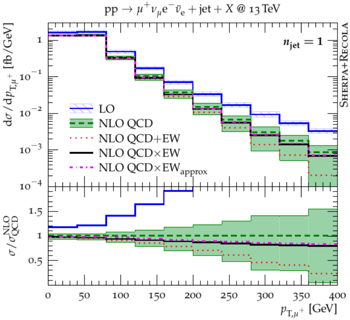
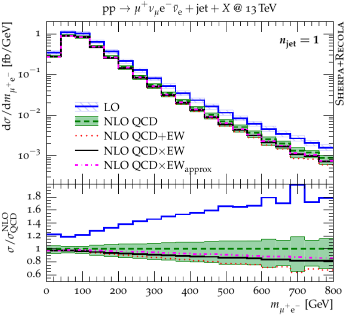
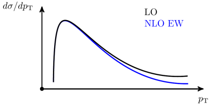
[Bräuer et al. JHEP 10 (2020) 159]

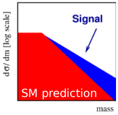




- QCD corrections clearly dominate (in particular due to jet veto)
- EW corrections also get sizable $\mathcal{O}(-20\%)$ for high-p_T/large masses

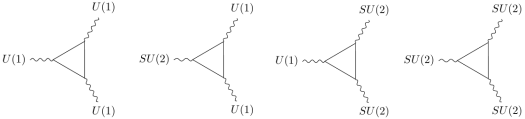
  $\rightsquigarrow$ emergence of EW Sudakov-type logarithms



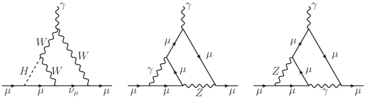


Summary lecture III

- Standard Model is renormalizable QFT, free of gauge anomalies



- EW effects from virtual and real-emission corrections
  $\rightsquigarrow$ soft & collinear singularities in QED
- contributions to precision observables, e.g. g_μ



- NLO EW corrections to LHC scattering processes

$$\sigma_{\text{tot}}^{\text{NLO}} = \sigma^{\text{Born}} \left(\underbrace{1}_{\text{LO}} + \underbrace{C_1 \frac{\alpha}{\pi}}_{\text{NLO EW}} + \underbrace{\dots}_{\text{higher orders}} \right)$$