The Electroweak Standard Model

facilitating precision physics at LHC and beyond

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QED as a role model

- QED as quantized gauge field theory of dynamical fields A^μ and ψ
  $\rightsquigarrow$ minimal coupling photon field with charged fermions (matter)

$$\mathcal{L}_{\text{QED}} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{2\xi}(\partial_\mu A^\mu)^2 + \bar{\psi}(i\partial\!\!\!/ + qeA\!\!\!/ - m)\psi$$

- QED is a renormalizable QFT
  $\rightsquigarrow$ all UV divergences absorbed into field/parameter renormalizations

the road ahead

- lift gauge symmetry to construction principle for weak & strong forces
  $\rightsquigarrow$ $SU(2)$ and $SU(3)$ symmetries
- however, while QED is parity conserving, the weak force is not
  $\rightsquigarrow$ charged weak interaction maximally parity violating
  [Wu experiment 1956; Nobel price Lee, Yang 1957]
  $\rightsquigarrow$ need to consider chiral fermions

Towards EW SM: chiral fermions

- consider massless Dirac fermion ψ satisfying $i\partial\!\!\!/\psi = 0$

  $\rightsquigarrow$ based on $\{\gamma_5, \gamma^\mu\} = 0$ it then also holds $i\partial\!\!\!/\gamma_5\psi = 0$
- define superpositions $\psi_{R/L}$ of definite chirality ($\gamma_5\psi_{R/L} = \pm\psi_{R/L}$)

$$\boxed{\psi_L = \frac{1}{2}(1-\gamma_5)\,\psi = P_L\psi} \quad \text{and} \quad \boxed{\psi_R = \frac{1}{2}(1+\gamma_5)\,\psi = P_R\psi}$$

- Lagrangian for chiral fermions (using $P_L P_R = P_R P_L = 0$, $P^2_{R/L} = P_{R/L}$)

$$\boxed{\mathcal{L} = i\bar{\psi}\partial\!\!\!/\psi = i\bar{\psi}_L\partial\!\!\!/\psi_L + i\bar{\psi}_R\partial\!\!\!/\psi_R}$$

  $\rightsquigarrow$ exhibits invariance under *global* phase trafo $\psi_{L/R} \to \exp\left(i\alpha_{L/R}\right)\psi_{L/R}$

$$\partial_\mu J^\mu_{L/R} = 0 \quad \text{with} \quad J^\mu_{L/R} = \bar{\psi}_{L/R}\gamma^\mu\psi_{L/R}$$

- however, mass term breaks chiral symmetry

$$\boxed{m\bar{\psi}\psi = m\bar{\psi}_L\psi_R + m\bar{\psi}_R\psi_L}$$

  $\rightsquigarrow$ breaks $SU(2)_L$ gauge invariance, need different source for mass term

Towards EW SM: non-abelian gauge symmetry I

- consider doublet of fermions and a unitary transformation U

$$\Psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \to \Psi' = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \equiv U\Psi$$

with

$$UU^\dagger = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \begin{pmatrix} \alpha^* & \gamma^* \\ \beta^* & \delta^* \end{pmatrix} = \mathbb{1}_{2\times 2}$$

$\rightsquigarrow$ 4 constraints for 4 complex elements: leaves 4 real parameters α_i

$\rightsquigarrow$ employ traceless, hermitian Pauli matrices τ_i with: $[\tau_i, \tau_j] = 2\epsilon_{ijk}\tau_k$

$$\begin{aligned} U &= \exp\left(i\alpha_0 \mathbb{1}_{2\times 2} + i\alpha_1\tau_1 + i\alpha_2\tau_2 + i\alpha_3\tau_3\right) \\ &= \exp\left(i\alpha_0\right)\exp\left(i\alpha_1\tau_1 + i\alpha_2\tau_2 + i\alpha_3\tau_3\right) \end{aligned}$$

- define *unit-determinant* matrices $V = \exp\left(-i\alpha_0\right)U$

$$\det(V) = \exp\left(i\mathsf{Tr}(\vec{\alpha}\cdot\vec{\tau})\right) = \exp(0) = 1$$

$$\boxed{V = \exp\left(i\vec{\alpha}\cdot\vec{\tau}\right) \approx \mathbb{1}_{2\times 2} + i\vec{\alpha}\cdot\vec{\tau} + \mathcal{O}(\vec{\alpha}^2) \in SU(2)}$$

$\rightsquigarrow$ note, $V_1V_2 \neq V_2V_1$, $SU(2)$ is non-abelian

Towards EW SM: non-abelian gauge symmetry II

- consider small *local* $SU(2)$ transformation, i.e. $\alpha_i = \alpha_i(x) = \frac{g}{2}\omega_i(x)$

$$\Psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \to \Psi' = \left(\mathbb{1}_{2\times 2} + i\frac{g}{2}\vec{\omega}\cdot\vec{\tau}\right)\Psi$$

- as for QED minimal coupling to vector fields W_1^μ, W_2^μ, W_3^μ

$$\boxed{D^\mu = \partial^\mu + i\frac{g}{2}\vec{W}^\mu\cdot\vec{\tau}}$$

- for *covariant derivative* it has to hold

$$D^\mu\Psi \to D'^\mu\Psi' = \left(\mathbb{1}_{2\times 2} + i\frac{g}{2}\vec{\omega}\cdot\vec{\tau}\right)D^\mu\Psi$$

$\rightsquigarrow$ dictates gauge transformation of fields W_i^μ

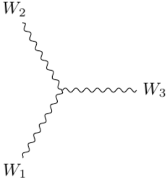
$$\boxed{\vec{W}'^\mu = \vec{W}^\mu - \partial^\mu\vec{\omega} - g\vec{\omega}\times\vec{W}^\mu}$$

- gauge invariant kinetic terms for the vector fields

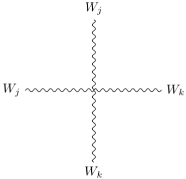
$$\boxed{\begin{aligned} \mathcal{L}_{SU(2)}^{\text{gauge}} &= -\tfrac{1}{4}G_i^{\mu\nu}G_{i,\mu\nu} \\ G_i^{\mu\nu} &= \partial^\mu W_i^\nu - \partial^\nu W_i^\mu - g\epsilon_{ijk}W_j^\mu W_k^\nu \end{aligned}}$$

- $\mathcal{L}^{\text{gauge}}_{SU(2)}$ contributes to W^μ_i propagator and gives rise to self interactions

$$V_{W_iW_jW_k} = -\frac{1}{2}g\epsilon_{ijk}\left(\partial^\mu W_i^\nu - \partial^\nu W_i^\mu\right)W_{j,\mu}W_{k,\nu}$$



$$V_{W_jW_kW_lW_m} = -\frac{1}{4}g^2\epsilon_{ijk}\epsilon_{ilm}W_j^\mu W_k^\nu W_{l,\mu}W_{m,\nu}$$



Towards EW SM: weak interactions

- represent two charge states of lh lepton/quark species as $SU(2)$ doublet
- describe weak interaction by $SU(2)$ gauge theory: weak isospin $I_w = \frac{1}{2}$

$$\Psi_L^l \equiv P_L \Psi^l = \begin{pmatrix} \psi_{\nu_e} \\ \psi_e \end{pmatrix}_L , \quad \Psi_L^q = \begin{pmatrix} \psi_u \\ \psi_d \end{pmatrix}_L , \quad \Psi_L \to \exp\left(ig\frac{1}{2}\vec{\omega}\cdot\vec{\tau}\right)\Psi_L$$

- right-handed fermions have $I_w = 0$, invariant under isospin trafo

$$\psi_{e,R} \to \exp(0)\psi_{e,R} = \psi_{e,R}$$

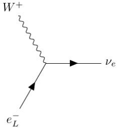
- interaction via minimal coupling (fermion mass term forbidden)

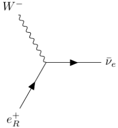
$$\mathcal{L}_{\text{int}} \subset \frac{g}{2}\bar{\Psi}_L^l \gamma^\mu \vec{W}_\mu \cdot \vec{\tau}\Psi_L^l = \frac{g}{4}\bar{\Psi}^l \gamma^\mu (1-\gamma_5)\vec{W}_\mu \cdot \vec{\tau}\Psi^l$$

with

$$\vec{W}_\mu \cdot \vec{\tau} = \begin{pmatrix} W_3^\mu & W_1^\mu - iW_2^\mu \\ W_1^\mu + iW_2^\mu & -W_3^\mu \end{pmatrix}$$

defining $W^{\mu,\pm} = \frac{1}{\sqrt{2}}(W_1^\mu \mp iW_2^\mu)$





The Glashow, Weinberg, Salam theory (1961-1968)

- unifying electro-magnetic and weak interactions
- electroweak symmetry given by $SU(2)_L \otimes U(1)_Y$, weak hypercharge Y

$$\boxed{Y_w = 2(q_f - I_{w,3})}$$

particle			q_f	$I_{w,3}$	Y_w	
$\nu_{e,L}$	$\nu_{\mu,L}$	$\nu_{\tau,L}$	0	$\frac{1}{2}$	-1	Ψ_L^l
e_L	μ_L	τ_L	-1	$-\frac{1}{2}$	-1	
~~$\nu_{e,R}$~~	~~$\nu_{\mu,R}$~~	~~$\nu_{\tau,R}$~~	0	0	0	discard right-handed neutrinos in SM
e_R	μ_R	τ_R	-1	0	-2	
u_L	c_L	t_L	$+\frac{2}{3}$	$\frac{1}{2}$	$\frac{1}{3}$	Ψ_L^q
d_L	s_L	b_L	$-\frac{1}{3}$	$-\frac{1}{2}$	$\frac{1}{3}$	
u_R	c_R	t_R	$+\frac{2}{3}$	0	$\frac{4}{3}$	
d_R	s_R	b_R	$-\frac{1}{3}$	0	$-\frac{2}{3}$	

- weak isospin singlets of three fermion generations

$$\psi_R \in \{e_R, \mu_R, \tau_R, u_R, c_R, t_R, d_R, s_R, b_R\}$$

The Electroweak Standard Model Lagrangian

- kinetic terms for the gauge fields

$$\mathcal{L}_{\text{EW}}^{\text{gauge}} = \mathcal{L}_{SU(2)_L}^{\text{gauge}} + \mathcal{L}_{U(1)_Y}^{\text{gauge}} = -\frac{1}{4} G_i^{\mu\nu} G_{i,\mu\nu} - \frac{1}{4} B^{\mu\nu} B_{\mu\nu}$$

- kinetic terms for leptons and quarks & minimal coupling to gauge fields

$$\mathcal{L}_{\text{EW}}^{\text{matter}} = \sum_{\Psi_L} \bar{\Psi}_L i \not{D} \Psi_L + \sum_{\psi_R} \bar{\psi}_R i \not{D} \psi_R$$

with the covariant derivatives for left-/right-chiral fermions

$$\begin{aligned} D_\mu \psi_R &= \left(\partial_\mu + i\frac{g_1}{2} Y_w B_\mu\right) \psi_R \\ D_\mu \Psi_L &= \left(\mathbb{1}_{2\times 2}\left(\partial_\mu + i\frac{g_1}{2} Y_w B_\mu\right) + i\frac{g_2}{2} \vec{W}_\mu \cdot \vec{\tau}\right) \Psi_L \end{aligned}$$

- GWS Lagrangian exhibits exact $SU(2)_L \otimes U(1)_Y$ symmetry

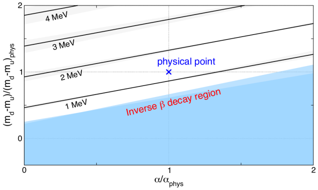
  however, fermion & boson mass terms would break gauge invariance

Interlude: the fate of symmetries I

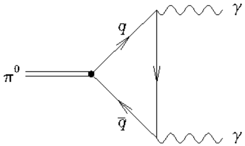
the various ways of symmetry breaking

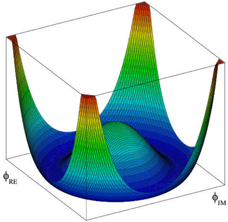
- *explicit* breaking by (small) terms that are not invariant
  - $\rightsquigarrow$ for example proton/neutron isospin broken by finite quark masses
- symmetry of classical theory might have an *anomaly* at quantum level
  - $\rightsquigarrow$ global axial $U(1)$ symmetry broken, $\pi^0 \to \gamma\gamma$ decay
- symmetry might be *hidden*: Lagrangian invariant, but ground state not
  - $\rightsquigarrow$ symmetry not manifest in the spectrum of physical states
  - $\rightsquigarrow$ e.g. via vacuum expectation value of scalar field(s): Higgs-mechanism

$m_n - m_p = 1.51(16)(23)$ MeV



[Borsanyi et al. Science 347 (2015) 1452]





Interlude: the fate of symmetries II

hidden symmetry

- let Q be symmetry charge from Noether theorem, global vacuum state trafo

$$|0\rangle \to e^{i\alpha Q}|0\rangle\,, \text{invariance}\,,\ \text{i.e. } e^{i\alpha Q}|0\rangle = |0\rangle\ \forall\, \alpha,\ \ \text{implies}\ \ Q|0\rangle = 0$$

  $\rightsquigarrow$ vacuum is unique, symmetry manifest

- assume new state $|\alpha\rangle$ is reached, i.e. $Q|0\rangle \neq 0$, however, $\dot{Q} = i[H,Q] = 0$

$$H|\alpha\rangle = He^{i\alpha Q}|0\rangle = e^{i\alpha Q}H|0\rangle = e^{i\alpha Q}E_0|0\rangle = E_0|\alpha\rangle\ \forall\ \alpha$$

  $\rightsquigarrow$ continuous symmetry results in family of degenerate ground states

- what's the meaning of states obtained from continuous symmetry trafo?
  - excitations about ground state get quantized, interpreted as particles
  - minimal excitation given by particle's mass

    $\rightsquigarrow$ zero energy excitations correspond to massless particles

Goldstone's theorem (1961): for a continuous symmetry of the Lagrangian, that is not a symmetry of the vacuum, there must exist one or more massless bosons, called Goldstone bosons

The Higgs mechanism

- add minimal scalar sector to the GWS theory to arrange for mass terms
  $\rightsquigarrow$ complex $SU(2)$ doublet of weak hypercharge $Y_w = +1$

$$\Phi = \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix}$$

- kinetic terms, potential and couplings to the $SU(2)_L \otimes U(1)_Y$ gauge fields

$$\boxed{\mathcal{L}_{\mathsf{HG}} = (D^\mu \Phi)^\dagger (D_\mu \Phi) - V(\Phi)}$$

$$\begin{aligned} D^\mu \Phi &= \left(\mathbb{1}_{2\times 2} \left(\partial_\mu + i\frac{g_1}{2} Y_w B_\mu \right) + i\frac{g_2}{2} \vec{W}_\mu \cdot \vec{\tau} \right) \Phi \\ V(\Phi) &= -\mu^2 \Phi^\dagger \Phi + \lambda \left(\Phi^\dagger \Phi \right)^2 \quad \text{with} \quad \mu^2, \lambda \in \mathbb{R} > 0 \end{aligned}$$

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- Yukawa interactions with fermion fields (here for 1st generation)

$$\boxed{\mathcal{L}_{\mathsf{HF}} = -f_u \bar{\Psi}^u_L \widetilde{\Phi} u_R - f_d \bar{\Psi}^d_L \Phi d_R - f_e \bar{\Psi}^e_L \Phi e_R + h.c.}$$

with $\widetilde{\Phi} = i\tau_2 \Phi^*$ (charge conjugate to Φ), $f_{u,d,e}$ arbitrary coupling constants

The Higgs mechanism: fermion masses

- consider electron contribution for definiteness

$$f_e \bar{\Psi}^e_L \Phi e_R + h.c. = f_e \bar{\nu}_{e,L} \varphi_1 e_R - f_e \bar{e}_L \varphi_2 e_R + h.c.$$

$\rightsquigarrow$ provides electron mass for $\langle\Phi\rangle_0 = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$ with $f_e v/\sqrt{2} \equiv m_e$

$\rightsquigarrow$ φ_2 neutral ($Q = 0$), φ_1 carries positive charge ($Q = +1$)

- $\langle\Phi\rangle_0$ vacuum expectation value of Higgs field Φ

$\rightsquigarrow$ non-trivial minimum of potential picked (SSB)

$$\frac{dV}{d\Phi^\dagger} = \Phi(-\mu^2 + 2\lambda\Phi^\dagger\Phi) \stackrel{!}{=} 0 \quad \rightsquigarrow \quad \langle\Phi^\dagger\Phi\rangle_0 = \frac{1}{2}\sqrt{\frac{\mu^2}{\lambda}} \equiv \frac{v^2}{2}$$

- $SU(2)_L \otimes U(1)_Y$ symmetry hidden by non-trivial vacuum state $\langle\Phi\rangle_0$

$\rightsquigarrow$ τ_1, τ_2 and $\tau_3 - \frac{1}{2}Y_w$ generators broken, i.e. $e^{i\alpha\tau_1}\langle\Phi\rangle_0 \neq \langle\Phi\rangle_0$ etc

$\rightsquigarrow$ only linear combination $Q = \tau_3 + \frac{1}{2}Y_w$ preserves vacuum state (*cf.* p7)

$$SU(2)_L \otimes U(1)_Y \quad \stackrel{\text{SSB}}{\longrightarrow} \quad U(1)_Q + 3 \text{ Goldstone bosons}$$

The Higgs mechanism: gauge boson masses

- vector field masses from minimal coupling terms $(\partial_\mu \langle\Phi\rangle_0 = 0)$

$$\begin{aligned}\mathcal{L}_{\mathsf{HG}}(\langle\Phi\rangle_0) &= \left(\frac{vg_2}{2}\right)^2 W^+_\mu W^{-,\mu}\\ &\quad + \frac{v^2}{8}\,(W_{3,\mu},\, B_\mu) \begin{pmatrix} g_2^2 & -g_1 g_2 \\ -g_1 g_2 & g_1^2 \end{pmatrix} \begin{pmatrix} W_3^\mu \\ B^\mu \end{pmatrix}\end{aligned}$$

- 2nd term diagonalizes for basis states rotated by Weinberg angle θ_W

$$\begin{aligned} Z_\mu &= \cos\theta_W W^3_\mu - \sin\theta_W B_\mu \\ A_\mu &= \sin\theta_W W^3_\mu + \cos\theta_W B_\mu \end{aligned} \quad \text{with} \quad \tan\theta_W = \frac{g_1}{g_2}$$

- can read-off masses for the $U(1)_Q$ photon (A_μ), the Z^0 and $W^\pm$

$$M_\gamma = 0\,,\quad M_W = \frac{v}{2} g_2\,,\quad M_Z = \frac{v}{2}\sqrt{g_1^2 + g_2^2}$$

- note, the $W^\pm$-to-Z^0 mass ratio fixed by

$$\frac{M_W}{M_Z} = \cos\theta_W$$

The Higgs mechanism: gauge boson fermion interactions

- the interactions of the gauge bosons with fermionic matter become

$$\mathcal{L}_{\text{EW}}^{\text{matter}} = \sum_f \bar{\psi}_f i\partial\!\!\!/ \psi_f + \frac{g_2}{2\sqrt{2}}(J_\mu^+ W^{-,\mu} + h.c.) + \frac{g_2}{2\cos\theta_W} J_\mu^Z Z^\mu + e J_\mu^{\text{em}} A^\mu$$

with the currents given by

$$J_\mu^+ = \sum_{l\in\{e,\mu,\tau\}} \bar{\psi}_{\nu_l}\gamma_\mu(1-\gamma_5)\psi_l + (\bar{\psi}_u, \bar{\psi}_c, \bar{\psi}_t)\gamma_\mu(1-\gamma_5)V_{\text{CKM}}\begin{pmatrix}\psi_d\\ \psi_s\\ \psi_b\end{pmatrix}$$

with $$V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub}\\ V_{cd} & V_{cs} & V_{cb}\\ V_{td} & V_{ts} & V_{tb}\end{pmatrix}$$ Cabibbo–Kobayashi–Maskawa matrix

$$J_\mu^Z = \sum_f \bar{\psi}_f\gamma_\mu(v_f - a_f\gamma_5)\psi_f$$ with $v_f = I_{w,3} - 2q_f\sin^2\theta_W\,,\quad a_f = I_{w,3}$

$$J_\mu^{\text{em}} = \sum_f q_f\bar{\psi}_f\gamma_\mu\psi_f$$

The Higgs mechanism: ... and finally the Higgs boson

- consider excitations of the Higgs field above the ground state
- in unitary gauge, i.e. $\Phi \to \exp(i\alpha_i\tau_i)\Phi$, the Higgs doublet reads

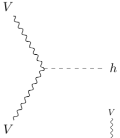
$$\Phi(x) = \left(\begin{array}{c} 0 \\ \frac{v+h(x)}{\sqrt{2}} \end{array} \right)$$

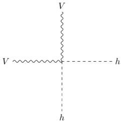
$\rightsquigarrow$ note, Ψ_L and $\vec{W}^\mu$ transform accordingly

- mass for the Higgs boson from potential $V(\Phi)$

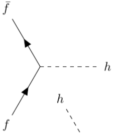
$$\boxed{M_h = \sqrt{2}\mu = v\sqrt{2\lambda}}$$

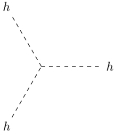
- $\mathcal{L}_{\mathsf{HG}}$ and $\mathcal{L}_{\mathsf{FG}}$ induce kinetic and interaction terms for Higgs boson h





$$\begin{array}{lcl}
\mathcal{L}_{WWh} & = & \frac{g_2}{4} W^{-,\mu} W^+_\mu (2vh + h^2) \\
\mathcal{L}_{ZZh} & = & \frac{\sqrt{g_1^2+g_2^2}}{8} Z^\mu Z_\mu (2vh + h^2) \\
\mathcal{L}_{hhh} & = & \lambda(vh^3 + h^4) \\
\mathcal{L}_{f\bar{f}h} & = & -\frac{m_f}{v} h\bar{\psi}_f\psi_f
\end{array}$$





Summary lecture II

- EW Standard Model gauge group $SU(2)_L \otimes U(1)_Y$
  - $\rightsquigarrow$ non-trivial ground state for $SU(2)$ scalar field doublet (SSB)
  - $\rightsquigarrow$ gauge symmetry hidden in the spectrum of the theory
  - $\rightsquigarrow$ finite masses for matter fermions and $W^{\pm}$, Z^0 bosons
- strong interaction modelled by $SU(3)_c$ color symmetry
  - $\rightsquigarrow$ parity conserving, massless gauge bosons – the gluons