Universität Siegen

HEP Experimental High Energy Physics
CPPS Center for Particle Physics Siegen

Quantum entanglement and other observables with $t\bar{t}$

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Beyond Flavour Physics
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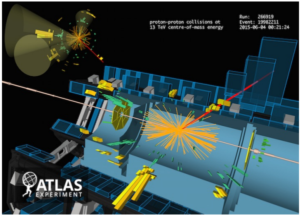
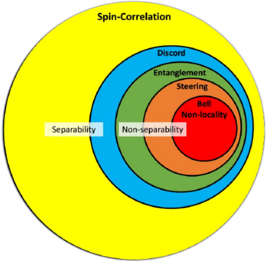


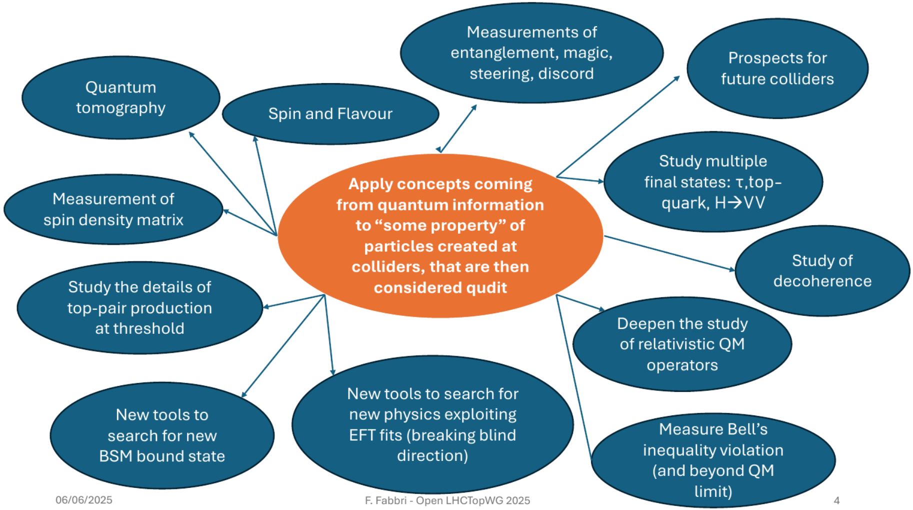
Introduction

- Top quark is a useful tool for SM and BSM → high statistics with top quark pair production
    - Spin correlation measurements can test SM prediction and a possible tool to see BSM effects
    - Can also test some of the core properties of quantum mechanics → unique tests of actual quantum behaviour of quarks at LHC

- Available/Planed measurements of:
    - spin correlations
    - quantum discord
    - entanglement
    - Bell non-locality (maybe impossible...)



More general picture: What is going on?



Apply concepts coming from quantum information to "some property" of particles created at colliders, that are then considered qudit

- Quantum tomography
- Spin and Flavour
- Measurements of entanglement, magic, steering, discord
- Prospects for future colliders
- Study multiple final states: τ,top-quark, H→VV
- Study of decoherence
- Deepen the study of relativistic QM operators
- Measure Bell's inequality violation (and beyond QM limit)
- New tools to search for new physics exploiting EFT fits (breaking blind direction)
- New tools to search for new BSM bound state
- Study the details of top-pair production at threshold
- Measurement of spin density matrix

06/06/2025 F. Fabbri - Open LHCTopWG 2025 4

Spin correlations in $t\bar{t}$

Top and antitop produced in pairs → their spins S_i and S_j correlated = two qubit system

$$\rho = \frac{1}{4}\left(1 \otimes 1 + \sum_{i=1}^{3} B_i\, \sigma_i \otimes 1 + \sum_{j=1}^{3} \bar{B}_j\, 1 \otimes \sigma_j + \sum_{i=1}^{3}\sum_{j=1}^{3} C_{ij}\, \sigma_i \otimes \sigma_j\right)$$

- 15 parameters in total that describe the quantum state of the top pair

$$\langle S_i \rangle = B_i, \quad \langle \bar{S}_j \rangle = \bar{B}_j, \quad \langle S_i \bar{S}_j \rangle = C_{ij}$$

Quantum Tomography **→ measurement of 15 parameters:**

- ➔ **6 polarisations**
- ➔ **9 correlations**

$$B = \begin{pmatrix} B_i \\ \dots \\ \dots \end{pmatrix} \qquad C = \begin{pmatrix} C_{ij} & \dots & \dots \\ \dots & \dots & \dots \\ \dots & \dots & \dots \end{pmatrix}$$

Measuring spin correlations

- Spin information is preserved through the top quark decay

$$\underbrace{\frac{1}{m_t}}_{\text{production}10^{-27}\text{ s}} < \underbrace{\frac{1}{\Gamma_t}}_{\text{lifetime}10^{-25}\text{ s}} < \underbrace{\frac{1}{\Lambda_{\text{QCD}}}}_{\text{hadronization}10^{-24}\text{ s}} < \underbrace{\frac{m_t}{\Lambda^2}}_{\text{spin-flip}10^{-21}\text{ s}}$$

- The spin of the particles measured at colliders can not be measured directly
  - It is extracted thanks to the direction of the final state particles
  - Relation created by the weak decay
  - The *strength* of the relation is dictated by the spin analysing power
- The spin analysing power depends on the flavour of the particle
- The elements of the spin density matrix are extracted averaging on angular distributions and accounting for the spin analysing power

Measuring spin correlations

- The bases to measure the angular distributions are
  - Helicity bases → defined in the parent particle frame

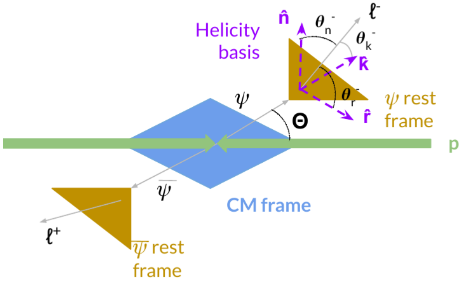
Helicity basis

$$\hat{k} = \text{top direction}, \quad \hat{r} = \frac{\hat{p} - \hat{k}\cos\theta}{\sin\theta}, \quad \hat{n} = \frac{\hat{p}\times\hat{k}}{\sin\theta}$$

  - Beam basis → defined in the laboratory frame

Beamline basis

$$(\hat{x}, \hat{y}, \hat{z}) \text{ with } \hat{x} = (1,0,0), \quad \hat{y} = (0,1,0), \quad \hat{z} = (0,0,1)$$



Measuring spin correlations

- Relation between angular distribution of the top quark pair decay products and top spin axis as

$$\frac{1}{\Gamma_T}\frac{d\Gamma}{d\cos\chi_i} = \frac{1+\alpha_i\cos\chi_i}{2}$$

$$\alpha_i = \begin{cases} +1.0 & \ell^+ \text{ or } \bar{d}\text{-quark} \\ -0.31 & \bar{\nu} \text{ or } u\text{-quark} \\ -0.41 & b\text{-quark} \end{cases}$$

at LO

Spin correlation and entanglement

There are four maximally entangled states

$$|\Phi^{\pm}\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\uparrow\rangle \pm |\downarrow\downarrow\rangle \right),$$

$$|\Psi^{\pm}\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle \pm |\downarrow\uparrow\rangle \right).$$

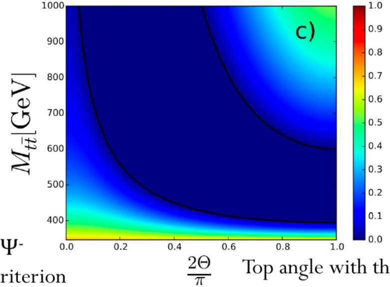
at high $M_{t\bar{t}}$ triplet vector state

(Φ^{+}-Φ^{-}, Ψ^{+}, Φ^{+}+Φ^{-})

Peres-Horodecki criterion

$$\Delta_E = C_{nn} - C_{rr} - C_{kk} = 3\tilde{D} > 1$$

$$\tilde{D} > \frac{1}{3}$$



Top angle with the beam axis

at low $M_{t\bar{t}}$ singlet pseudoscalar state Ψ^{-}

Peres-Horodecki criterion

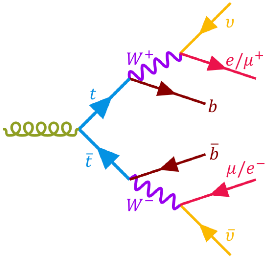
$$\Delta_E = Tr(C) = -3D > 1$$

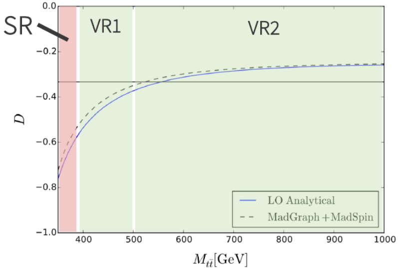
$$D < -\frac{1}{3}$$

Plot from Afik, De Nova
EPJP136(2021)9,907
hep-ph:2003.02280

Quantum entanglement in dileptonic $t\bar{t}$ [Nature 633 (2024) 542]

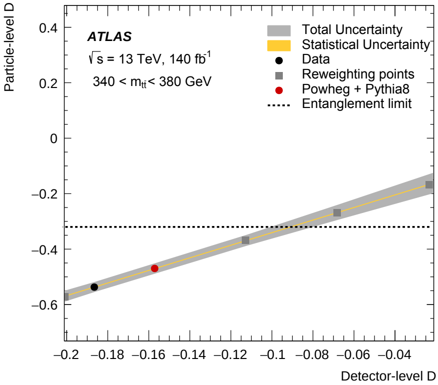
- $e\mu$ final state: very clean (90% purity)
- Boost the leptons in their parent top quark's rest frame
- Sufficient condition for entanglement from spin correlation matrix, using diagonal elements: $\Delta = C_{33} + |C_{11} + C_{22}| > 1$
- Measure entanglement proxy $D = -\Delta/3 = -Tr[C]/3$ extracted from angle between charged leptons
  $\frac{1}{\sigma}\frac{d\sigma}{d\cos\phi} = \frac{1}{2}(1 - D\cos\phi)$
- Define $m_{t\bar{t}}$ entanglement signal region and validation regions

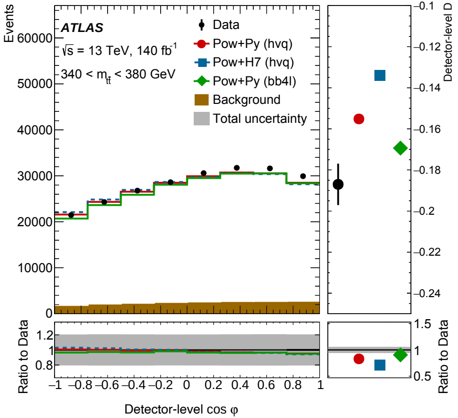




Analysis procedure

- *Calibration curve* method: use the nominal MC to map the detector-level D value (average of distribution) to the fiducial particle-level D.
  - Build the curve by sampling different D values.
- Systematics are propagated with their own curves, quadratic envelope.





A closer look to the uncertainties

“Backgrounds”: mostly $Z\rightarrow\tau\tau$, which leads to a flat cos(φ) distribution

(spin information from taus is lost)

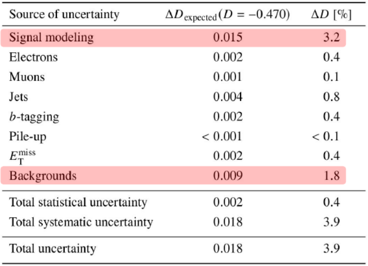
Calibrating to fiducial particle-level reduces the parton shower uncertainty (Pythia vs Herwig)

→ full details in the paper.

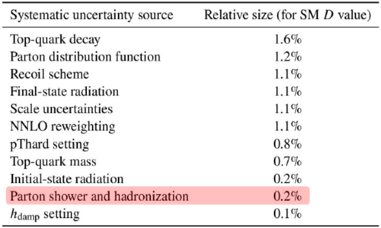
Signal modelling:

by far the largest contribution

arXiv:2311.07288



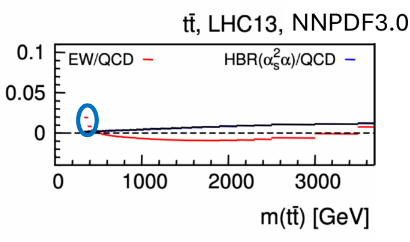
Source of uncertainty	$\Delta D_{\text{expected}}(D=-0.470)$	ΔD [%]
Signal modeling	0.015	3.2
Electrons	0.002	0.4
Muons	0.001	0.1
Jets	0.004	0.8
b-tagging	0.002	0.4
Pile-up	< 0.001	< 0.1
$E_{\text{T}}^{\text{miss}}$	0.002	0.4
Backgrounds	0.009	1.8
Total statistical uncertainty	0.002	0.4
Total systematic uncertainty	0.018	3.9
Total uncertainty	0.018	3.9



Systematic uncertainty source	Relative size (for SM D value)
Top-quark decay	1.6%
Parton distribution function	1.2%
Recoil scheme	1.1%
Final-state radiation	1.1%
Scale uncertainties	1.1%
NNLO reweighting	1.1%
pThard setting	0.8%
Top-quark mass	0.7%
Initial-state radiation	0.2%
Parton shower and hadronization	0.2%
h_{damp} setting	0.1%

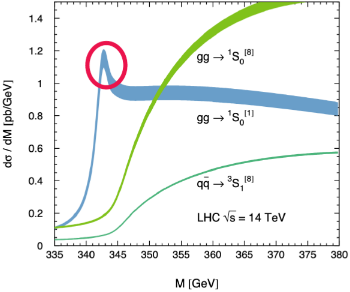
Sources of $t\bar{t}$ mismodelling

NLO EW



Cross-section enhancement
(10.1007/JHEP10(2017)186)

Bound state



Spin-, colour-singlet enhancement

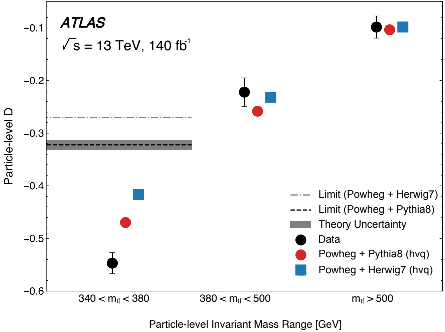
- CMS measurements considered them
  - EW corrections with Hathor
  - Added pseudo-scalar colour singlet predicted by non-relativistic QCD → affects $m_{t\bar{t}}$ and spin correlations at threshold

Since then improved model available! (Fuks' talk)

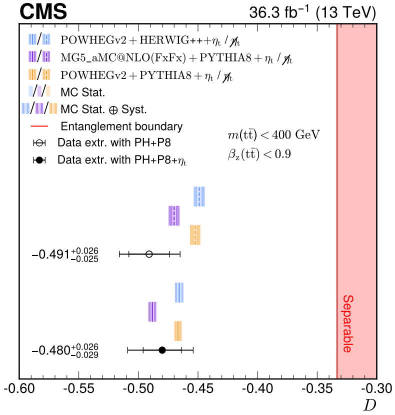
Observation of top quark entanglement

[Nature 633 (2024) 542, Rep. Prog. Phys. 87 (2024) 117801]

- $D < -1/3$ established at 5σ level

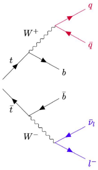


non-relativistic QCD effects close to threshold not included in MC generators → would only affect predictions, not calibration

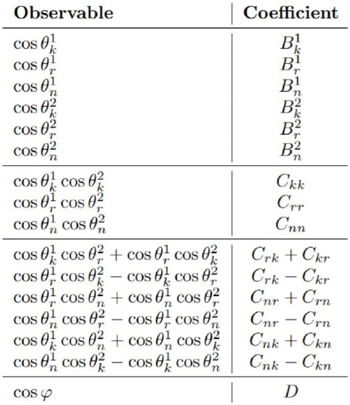


$\sim 1.5\sigma$ tension with the expectation if *toponium* is not included

Taking it a step further [Phys. Rev. D 110 (2024) 112016]



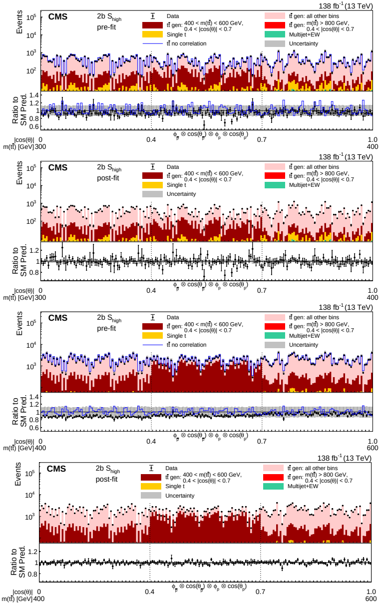
- Measuring the correlation matrix in single-lepton $t\bar{t}$ events
- All coefficients of polarization vectors and correlation matrix from fit to the angles of the down-type quark and the charged lepton in the W boson decays.
- Challenging identification of down-type quark in W decay
- Using NN to reconstruct the $t\bar{t}$ system
- Δ from the full matrix, or from two proxies: D and $\tilde{D} = 3(C_{33} - C_{11} - C_{22})$ for high masses



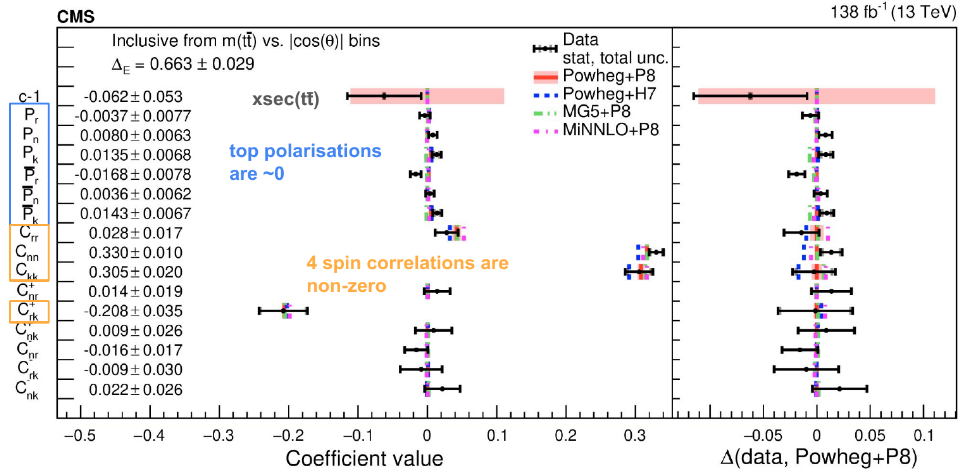
Observable	**Coefficient**
$\cos\theta_k^1$	B_k^1
$\cos\theta_r^1$	B_r^1
$\cos\theta_n^1$	B_n^1
$\cos\theta_k^2$	B_k^2
$\cos\theta_r^2$	B_r^2
$\cos\theta_n^2$	B_n^2
$\cos\theta_k^1 \cos\theta_k^2$	C_{kk}
$\cos\theta_r^1 \cos\theta_r^2$	C_{rr}
$\cos\theta_n^1 \cos\theta_n^2$	C_{nn}
$\cos\theta_k^1 \cos\theta_r^2 + \cos\theta_r^1 \cos\theta_k^2$	$C_{rk} + C_{kr}$
$\cos\theta_r^1 \cos\theta_k^2 - \cos\theta_k^1 \cos\theta_r^2$	$C_{rk} - C_{kr}$
$\cos\theta_r^1 \cos\theta_n^2 + \cos\theta_n^1 \cos\theta_r^2$	$C_{nr} + C_{rn}$
$\cos\theta_n^1 \cos\theta_r^2 - \cos\theta_r^1 \cos\theta_n^2$	$C_{nr} - C_{rn}$
$\cos\theta_k^1 \cos\theta_n^2 + \cos\theta_n^1 \cos\theta_k^2$	$C_{nk} + C_{kn}$
$\cos\theta_n^1 \cos\theta_k^2 - \cos\theta_k^1 \cos\theta_n^2$	$C_{nk} - C_{kn}$
$\cos\varphi$	D

Measuring the full set of coefficients

- Pre- and post-fit distributions comparing the data (points) to the POWHEG + PYTHIA simulation (stacked histograms) for the full matrix measurement in bins of $mt\bar{t}$ vs. $|\cos\theta|$



Spin density matrix

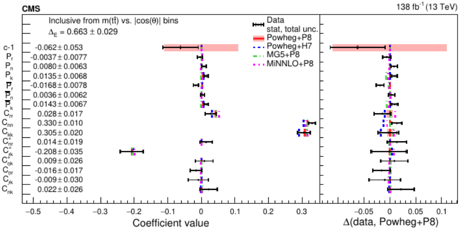


xsec($t\bar{t}$)

top polarisations are ~0

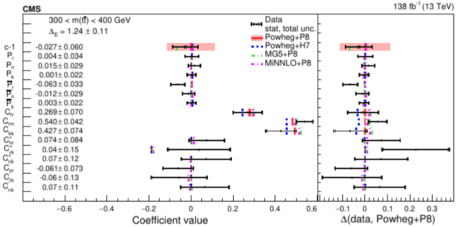
4 spin correlations are non-zero

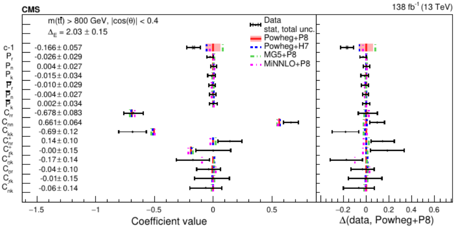
Spin density matrix



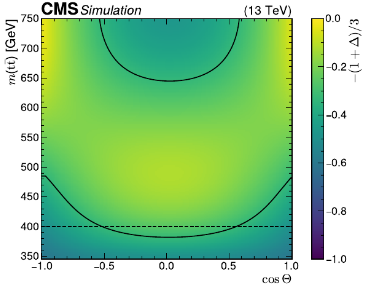
- Only C^+_{rk} is invariant under P and C transformation →only non-zero off-diagonal element
- Diagonal elements indicate the transition from a dominant spin-singlet state at low to a triplet-state at high $m(\mathrm{t\bar{t}})$
- All coefficients in good agreement with SM values
- Access to full density matrix :

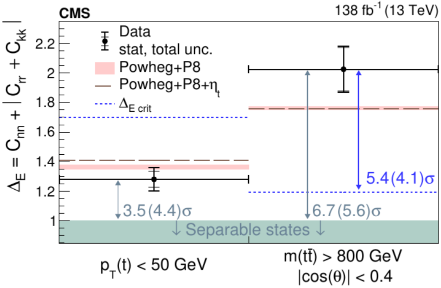
$$\rho = \frac{1}{4}\left(\mathbb{1}_4 + \sum_i P_i\sigma_i\otimes\mathbb{1}_2 + \sum_j \bar{P}_j\mathbb{1}_2\otimes\sigma_j + \sum_{ij} C_{ij}\sigma_i\otimes\sigma_j\right)$$





Quantum entanglement at high $m_{t\bar{t}}$ (from full SDM)

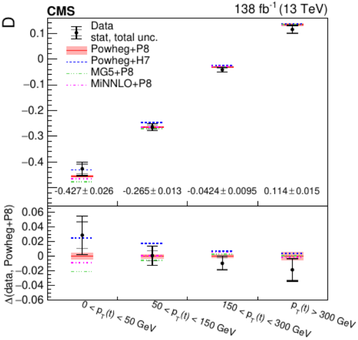


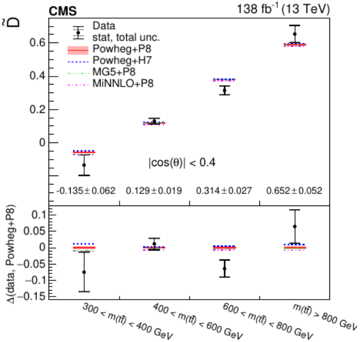


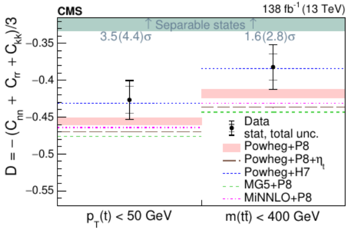
- At the threshold and at high $m(t\bar{t})$ with low $\cos(\theta_t)$ $t\bar{t}$ is expected to be produced in entangled states
- Criterion for entanglement (based on Peres-Horodecki criterion): $\Delta E = C_{nn} + |C_{rr} + C_{kk}| > 1$

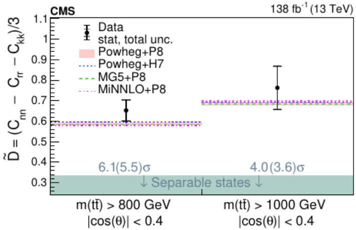
Assuming that the $t\bar{t}$ system is described by QM, this is the first observation of an entangled quantum state at high $m(t\bar{t})$

Quantum entanglement from D (threshold) and $\tilde{D}$ (high mass)









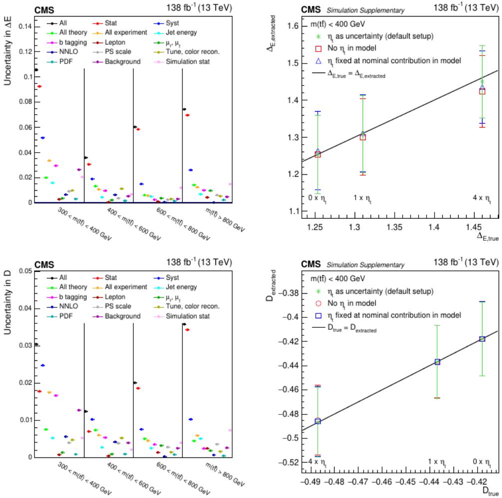
A closer look to the uncertainties

Uncertainties and tests

- measurements are mostly statistically limited
  systematic uncertainties in D are larger; more assumption about modeling and detector effects made
- toponium signal injection tests show that the correct values can be obtained.
  however, the signal is within uncertainties

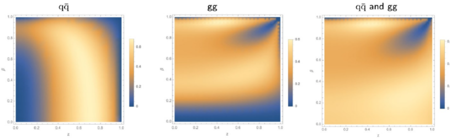
toponium simulated as a pseudo-scalar particle with mass 343 GeV, $\Gamma = 2m_t$, and production cross section of 6.4 pb

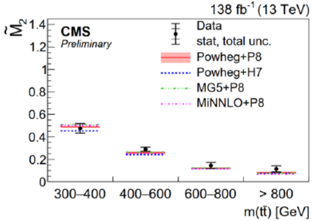
- tests with altered injected coefficients successfully performed in many regions of phase space

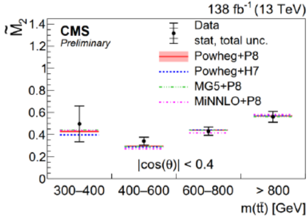


Magic

- Property related to the advantage of implementing a quantum state on a quantum computer.
- Measured by CMS starting from the differential measurement of the spin density matrix in the $t\bar{t}$ l+jets final state (CMS-PAS-TOP-25-001)
- Take home messages:
    - Magic agrees with the SM
    - Magic is non-linear:
    - Non-zero magic of the mixed states does not imply non-zero magic for the individual quantum states
    - Values of magic, discord etc... also depend on the coordinate systems. This is not a problem, but their interpretation is not as straight forward







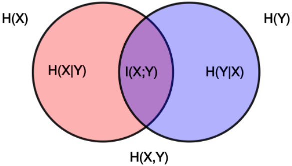
Discord

- Discord captures the non-classicality of correlations by measuring differences in the total mutual information

$$I(X;Y) = H(X) + H(Y) - H(X,Y)$$

$$J(X;Y) = H(X) - H(X|Y)$$

$$I(X;Y) \overset{?}{=} J(X;Y)$$ [classically they are the same]



Discord → $\mathcal{D}_A = S(\rho_B) - S(\rho) + \min_{\hat{\mathbf{n}}} p_{\hat{\mathbf{n}}} S(\rho_{\hat{\mathbf{n}}}) + p_{-\hat{\mathbf{n}}} S(\rho_{-\hat{\mathbf{n}}})$

  - In general: **$0 \leq D_A \leq 1$** and **D_A != D_B**

$$S(\rho) = -\mathrm{Tr}\rho \log_2 \rho \qquad \rho_{\hat{\mathbf{n}}} = \frac{1 + \mathbf{B}^+_{\hat{\mathbf{n}}} \cdot \sigma}{2}, \; \mathbf{B}^+_{\hat{\mathbf{n}}} = \frac{\mathbf{B}^+ + \mathbf{C} \cdot \hat{\mathbf{n}}}{1 + \hat{\mathbf{n}} \cdot \mathbf{B}^-}, \; p_{\hat{\mathbf{n}}} = \frac{1 + \hat{\mathbf{n}} \cdot \mathbf{B}^-}{2}$$

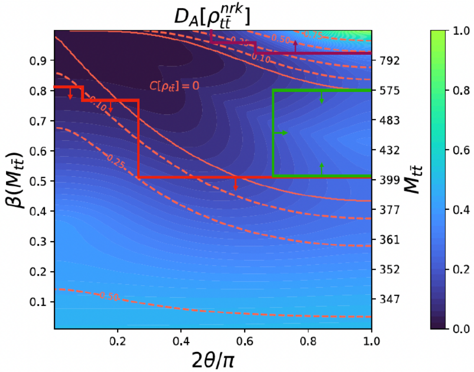
- The problem with Discord is that it is non convex:
  - Mixture of states with zero discord lead to non-zero discord
  - Hard to interpret a measurement of non zero discord

Discord

- Solutions proposed:
  - Only perform discord measurements in regions with non-zero entanglement
  - Perform measurements in regions where all sub-state have non-zero discord
- Shown results for discord measurement in $t\bar{t}$ final states in a separable region of the phase space but with reduced classical correlations

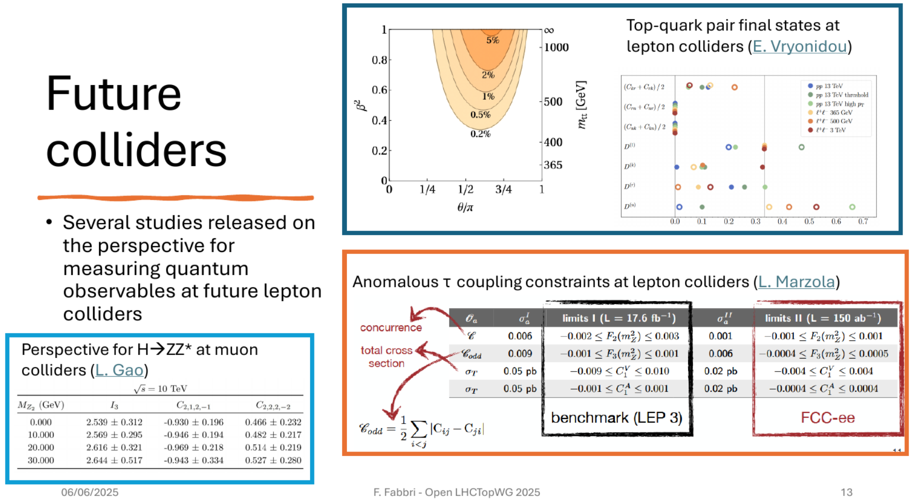
Three signal regions:

- Threshold
- Separable
- Boosted



Courtesy: N. McGinnis

Beyond LHC and $t\bar{t}$



Future colliders

- Several studies released on the perspective for measuring quantum observables at future lepton colliders

Top-quark pair final states at lepton colliders (E. Vryonidou)

Perspective for H→ZZ* at muon colliders (L. Gao)

$\sqrt{s} = 10$ TeV

M_{Z_2} (GeV)	I_3	$C_{2,1,2,-1}$	$C_{2,2,2,-2}$
0.000	2.539 ± 0.312	-0.930 ± 0.196	0.466 ± 0.232
10.000	2.569 ± 0.295	-0.946 ± 0.194	0.482 ± 0.217
20.000	2.616 ± 0.321	-0.969 ± 0.218	0.514 ± 0.219
30.000	2.644 ± 0.517	-0.943 ± 0.334	0.527 ± 0.280

Anomalous τ coupling constraints at lepton colliders (L. Marzola)

concurrence

total cross section

$\mathscr{O}_a$	σ_a^I	limits I (L = 17.6 fb^{-1})	σ_a^{II}	limits II (L = 150 ab^{-1})
$\mathscr{C}$	0.006	$-0.002 \leq F_2(m_Z^2) \leq 0.003$	0.001	$-0.001 \leq F_2(m_Z^2) \leq 0.001$
$\mathscr{C}_{odd}$	0.009	$-0.001 \leq F_3(m_Z^2) \leq 0.001$	0.006	$-0.0004 \leq F_3(m_Z^2) \leq 0.0005$
σ_T	0.05 pb	$-0.009 \leq C_1^V \leq 0.010$	0.02 pb	$-0.004 \leq C_1^V \leq 0.004$
σ_T	0.05 pb	$-0.001 \leq C_1^A \leq 0.001$	0.02 pb	$-0.0004 \leq C_1^A \leq 0.0004$

benchmark (LEP 3)

FCC-ee

$$\mathscr{C}_{odd} = \frac{1}{2}\sum_{i<j}|C_{ij} - C_{ji}|$$

06/06/2025

F. Fabbri - Open LHCTopWG 2025

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Courtesy: F. Fabbri

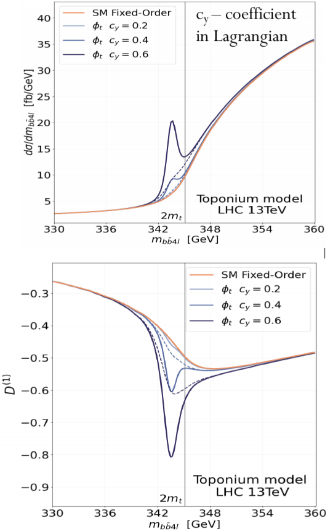
Summary and outlook

- Top quark polarization and spin correlation measurement is interesting in its own right as a test of the SM, but it also provides new opportunities for testing quantum mechanics (QM) at high energies using the decay products of unstable particles as probes
- Angular distributions of the top and antitop quarks used to measure their polarization and spin correlation matrix
- In some regions of phase space top and antitop get entangled, which can be demonstrated using Peres-Horodecki criterion based on their spin correlation matrix
- Large interest from the exp. and theo community to use $t\bar{t}$ for further tests (see Workshop on Quantum observables for Collider Physiscs)
- Discussions on interplay of these measurements and BSM, mostly in the context of EFT
- Effort to exploit other topologies (e.g. Higgs decays) ongoing

BACK UP

Signal modelling

- NLO POWHEG+Pythia8
- Include EW corrections with HATHOR (*Comput. Phys. Commun. 182 (2011) 10)*
- NNLO (*Phys. Rev. Lett. 127 (2021) 062001)*
  - Dilepton: p_T reweighting to match the top quark p_T spectrum from a fixed order ME calculation at NNLO
  - Lepton+jets: NN-based reweighting to match NNLO distributions at reco level
- Add "toponium" (pseudo-scalar color singlet predicted by non-relativistic QCD)
  - M(toponium)-344 GeV, σ=~6.5pb
    - Sumino, Fujii, Hagiwara, Murayama & Ng (PRD`93)
    - Jezabek, Kuhn & Teubner (Z.Phys.C`92)
    - B. Fuks et al. (PRD 104 (2021) 034023)
  - affects the invariant mass distribution and the **spin correlations** at the threshold

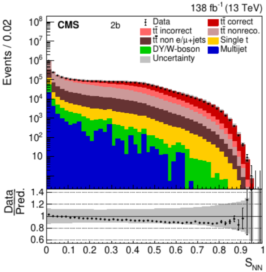


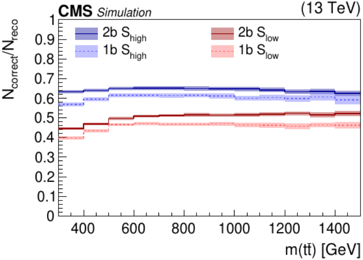
c_y – coefficient in Lagrangian

F. Maltoni et al. JHEP03(2024)099

$t\bar{t}$ reconstruction (l+jets)

Use dense neural network for identification of the top decay products (7 layers, 220 nodes)
– inputs: [ℓ, $p_{\mathrm{T}}^{\mathrm{miss}}$, b_ℓ, b_h, j_{down}, j_{up}, additional jets]; momentum and b-tagging information for jets
– present all permutations for the jets from $\mathrm{t\bar{t}}$ to NN and train for high score if the 4 jets are at the correct positions
half of the time there is a c-jet in the W decay; in average, down-type jets are softer (65% correctly identified)

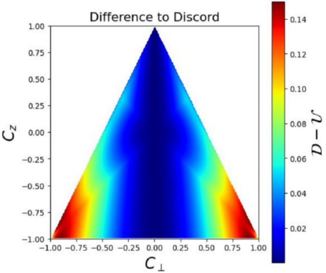




S_{low}: 0.1–0.36, S_{high}: > 0.36

New (in the context of HEP) observables

- Discord captures the non-classicality of correlations by measuring differences in the total mutual information
    - Experimentally very challenging due to a minimization over projective measurements
- Suggestion to instead measure local quantum uncertainty (LQU)
    - a local measurement on one part can disturb the global state
    - If the minimum of LQU is non-zero the state should be discordant
    - In case of qubit the formula for LQU is very simple and can be derived by the spin density matrix
    - LQU seems to provide a lower bound of discord
        - Results based on the CMS spin density matrix measurement already provided



Courtesy: B. Ravina

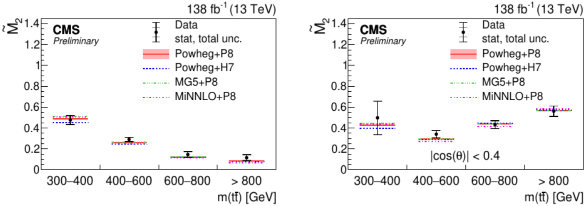
Magic

- In quantum information science pure eigen-states of unitary operators (mostly tensor products of Pauli matrices) are called stabilizer states. For these magic is zero.
- Non-stabilizer states have enhanced properties for quantum computing [*D. Gottesman*]

A generalized definition of magic for mixed states [*C. White*]:

$$\tilde{M}_2 = -\log_2\left(\frac{1+\sum_{i\in n,k,r}[(P_i^4+\bar{P}_i^4)]+\sum_{i,j\in n,k,r}C_{ij}^4}{1+\sum_{i\in n,k,r}[(P_i^2+\bar{P}_i^2)]+\sum_{i,j\in n,k,r}C_{ij}^2}\right)$$

This can be calculated from the measured spin correlations:



Courtesy: O. Hendrichs