# Anatomy of Yukawa Matrices and SMEFT couplings with AutoEFT

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- The SM gauge-kinetic sector is flavour-blind

flavour symmetry: $U(3)_Q \times U(3)_U \times U(3)_D \times U(3)_L \times U(3)_E$

for 3 copies (”generations”) of 5 types of fermion multiplets $(Q, U, D, L, E)$

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- flavour-symmetry breaking encoded in SM Yukawa sector:

$$\mathcal{L}_{\rm yuk} = -Y_U^{ab}\,\bar{Q}^a \tilde{H} U^b - Y_D^{ab}\,\bar{Q}^a H D^b - Y_E^{ab}\,\bar{L}^a H E^b + \text{h.c.}$$

$\rightarrow$ 54 real parameters in 3 complex $3 \times 3$ Yukawa matrices

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$$U(1)_B \times U(1)_e \times U(1)_\mu \times U(1)_\tau$$

$\Rightarrow$ $54 - (5 \cdot 9 - 4) = 13$ observable flavour parameters:

6 quark masses, 3 lepton masses, 3 CKM angles, 1 CP phase

## Standard Model:

- $54 \to 13$ flavour parameters from Yukawas
- "natural" value for top-Yukawa:

$$y_t \sim \mathcal{O}(1)\,, \qquad \text{while } y_{f\neq t} \ll 1$$

- small CKM angles (from $(U_L, D_L)$ rotations)

$$\theta_{12} \ll \theta_{23} \ll \theta_{13} \sim \theta_{12}\theta_{23}$$

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## Beyond the SM

- Any generic extension of the SM adds (many!) new flavour-specific couplings
- No a-priori information about ”natural” size of dimensionless couplings
- So far, no direct hint for NP mass scale
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## Example: SMEFT

Assuming unbroken baryon- and lepton-#:

- 2499 new parameters @ dim-6

  most of them related to flavour-specific 2- and 4-fermion operators

[Buchmüller/Wyler ’86, Grzadkowski et al. 2010]

## Assumptions about the NP flavour sector

perfect flavour symmetry

$U(3)^5$ @high scale

- $+$ flavour-blind observables constrain NP scale
- $-$ new flavour effects from RG, only
- $-$ working assumption not RG-invariant

perfect flavour symmetry

$U(3)^5$ @high scale

generic $\mathcal{O}(1)$ couplings

no symmetry constraints

+ no specific flavour assumptions
− flavour constraints imply huge NP scales
− dim-4 Yukawas are fine-tuned (natural?)

**perfect flavour symmetry**

$U(3)^5$ @high scale

**Minimal Flavour Violation**

expand in $Y_U, Y_D, Y_E$ from SM

- $+$ RG covariant
- $+$ SM-like suppression of rare flavour decays
- $\pm$ correlates high-energy and flavour observ.
  still too restrictive?
- $-$ reversed logic? (IR $\mapsto$ UV)

[D'Ambrosio et al. 2002, Buras 2003]

**generic $\mathcal{O}(1)$ couplings**

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expand in $Y_U, Y_D, Y_E$ from SM

**Froggatt-Nielsen Scenario**

a single $U(1)$ broken@UV

- \+ RG covariant
- \+ self-consistent flavour power counting
- − (too?) large NP flavour effects

[Froggatt/Nielsen ’79]

**generic $\mathcal{O}(1)$ couplings**

no symmetry constraints

**perfect flavour symmetry**

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expand in $Y_U, Y_D, Y_E$ from SM

$\updownarrow$ ?

**Froggatt-Nielsen Scenario**

a single $U(1)$ broken@UV

**generic $\mathcal{O}(1)$ couplings**

no symmetry constraints

## Froggatt-Nielsen Approach:

[Froggatt/Nielsen 1979]

- fermion generations have different charges under $U(1)_{\rm FN}$ (breaks $U(3)^5$)
- spontaneously broken by VEV of a charged scalar field ("spurion")

$\rightarrow$ power-counting parameter

$$\lambda = \frac{\langle \phi \rangle}{\Lambda_{\rm UV}} \equiv \theta_{\rm Cabibbo} \simeq 0.2$$

- hierarchical Yukawa entries from # of necessary spurion insertions

$$y_{ij} \sim \lambda^{|\Delta q_{ij}|}$$

dictated by charge *differences*

$\rightarrow$ FN charges also fix the scaling of SMEFT operators

see e.g. [Bordone/Cata/TF 2019]

## The approximate flavour symmetry of the SM

- quark flavour symmetry of the SM gauge sector: $G_0 = U(3)^3/U(1)_B$
- lepton flavour symmetry of the SM gauge sector: $H_0 = U(3)^2/U(1)^3_{L_i}$
- But: top-Yukawa coupling is O(1) $\rightarrow$ *explicit* symmetry breaking

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$\Rightarrow$ Starting point: *Approximate* flavour symmetry of quarks in the SM

$$G_1 = SU(2)_Q \times U(2)_U \times U(3)_D \times U(1)_{(Q_3,U_3)}$$

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$$G_1 = SU(2)_Q \times U(2)_U \times U(3)_D \times U(1)_{(Q_3,U_3)}$$

- allowed dim-4 operators with $\mathcal{O}(1)$ couplings:

Higgs self-coupling: $(HH^\dagger)^2$

Top Yukawa: $\bar{Q}_3\,\widetilde{H}\,U_3 + \text{h.c.}$

Lesson: All operators consistent with $G_1 \times H_0$ may have $\mathcal{O}(1)$ coefficients

## AutoEFT package:

- generate set of operators allowed by user-defined symmetries
  here: SM + flavour symmetries

[Harlander/Schaaf 2023]

dim-6 operators with $G_1 \times H_0$ symmetry:

- 22 operators without quark fields
- 27 two-quark operators
- 33 four-quark operators

→ sufficient for (flavour-diagonal) Higgs and top observables

→ non-diagonal flavour transitions: include operators that violate $G_1 \times H_0$

| dim-6 | generic | $H_0$ | $G_1 \times H_0$ | $U(3)^5$ |
|---|---|---|---|---|
| no-quark | 258 | 22 | **22** | 22 |
| 2-quark | 1251 | 261 | **27** | 11 |
| 4-quark | 990 | 990 | **33** | 14 |
| $\Sigma$ | 2499 | 1273 | **82** | 47 |

- To be specific, we consider the standard Wolfenstein scaling for the CKM angles,

$$\theta_{12} \sim \lambda\,, \quad \theta_{23} \sim \lambda^2\,, \quad \theta_{13} \sim \lambda^3$$

together with

$$y_b \sim \lambda^2\,, \quad y_c \sim \lambda^3\,, \quad y_s \sim \lambda^4\,, \quad y_{u,d} \sim \lambda^6\,,$$

- adjusting the according FN charges would lead to

$$Y_U \sim \begin{pmatrix} \lambda^6 & \lambda^4 & \lambda^3 \\ \lambda^5 & \lambda^3 & \lambda^2 \\ \lambda^3 & \lambda & \boxed{1} \end{pmatrix}\,, \qquad Y_D \sim \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^5 \\ \lambda^5 & \lambda^4 & \lambda^4 \\ \lambda^3 & \lambda^2 & \lambda^2 \end{pmatrix} \qquad \uparrow V_{\rm CKM} \downarrow$$

$$\leftarrow \; V_U \; \rightarrow \qquad\qquad \leftarrow \; V_D \; \rightarrow$$

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$$\leftarrow \; V_U \; \rightarrow \qquad\qquad \leftarrow \; V_D \; \rightarrow$$

## Step 1: Realizing $(Y_U)_{32} \sim \lambda$

$$Y_U \sim \begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \boxed{\lambda} & \boxed{1} \end{pmatrix}$$

### Our approach:

- introduce spurions as *smallest representations* required to break appropriate subgroups of $G_1$
- implement spurions in AutoEFT

- break $SU(2)_U$:

  charged doublet: $S_1 \sim y_c/\theta_{23} \sim \lambda$

- residual flavour symmetry

$$G_1 \to G_2 = SU(2)_Q \times U(3)_D \times U(1)^2$$

| dim-6 | $H_0$ | $\mathcal{O}(\lambda)$ | $\mathcal{O}(1)$ |
|---|---|---|---|
| no-quark | 22 | | 22 |
| 2-quark | 261 | **14** | 27 |
| 4-quark | 990 | **18** | 33 |

New operators for $t_R \to c_R$ transitions at $\mathcal{O}(\lambda)$ !

[for comparison: MFV would not induce any new operators at $\mathcal{O}(\lambda)$ ]

## Step 2: Realizing $(Y_U)_{23}$, $(Y_D)_{32}$, $(Y_D)_{33} \sim \lambda^2$

$$Y_U \sim \begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \boxed{\lambda^2} \\ \cdot & \boxed{\lambda} & \boxed{1} \end{pmatrix} \quad Y_D \sim \begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & & \end{pmatrix}$$

- break $SU(2)_Q$:

  charged doublet: $S_2 \sim y_t \theta_{23} \sim \lambda^2$

## Step 2: Realizing $(Y_U)_{23},\ (Y_D)_{32},\ (Y_D)_{33} \sim \lambda^2$

$$Y_U \sim \begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & & \boxed{\lambda^2} \\ \cdot & \boxed{\lambda} & \boxed{1} \end{pmatrix} \qquad Y_D \sim \begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & & \\ \cdot & \boxed{\lambda^2} & \boxed{\lambda^2} \end{pmatrix}$$

- break $SU(2)_Q$:

  charged doublet: $S_2 \sim y_t \theta_{23} \sim \lambda^2$

- break $SU(3)_D$:

  two charged triplets: $S_{3,4} \sim y_b \sim \lambda^2$

$\rightarrow$ large $\mathcal{O}(1)$ transitions $b_R \rightarrow s_R$

## Step 2: Realizing $(Y_U)_{23}$, $(Y_D)_{32}$, $(Y_D)_{33} \sim \lambda^2$

$$Y_U \sim \begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \lambda^3 & \boxed{\lambda^2} \\ \cdot & \boxed{\lambda} & \boxed{1} \end{pmatrix} \qquad Y_D \sim \begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \lambda^4 & \lambda^4 \\ \cdot & \boxed{\lambda^2} & \boxed{\lambda^2} \end{pmatrix}$$

- break $SU(2)_Q$:

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  $\rightarrow$ large $\mathcal{O}(1)$ transitions $b_R \rightarrow s_R$

- residual flavour symmetry

$$G_2 \rightarrow G_3 = U(1)^3$$

also realizes $(Y_U)_{22}$ and $(Y_D)_{22}$ and $(Y_D)_{23}$

| dim-6 | $H_0$ | $\mathcal{O}(\lambda^2)$ | $\mathcal{O}(\lambda)$ | $\mathcal{O}(1)$ |
|---|---|---|---|---|
| no-quark | 22 | | | 22 |
| 2-quark | 261 | **41** | 14 | 27 |
| 4-quark | 990 | **42** | 18 | 33 |

## Including effects of order $\lambda^6$

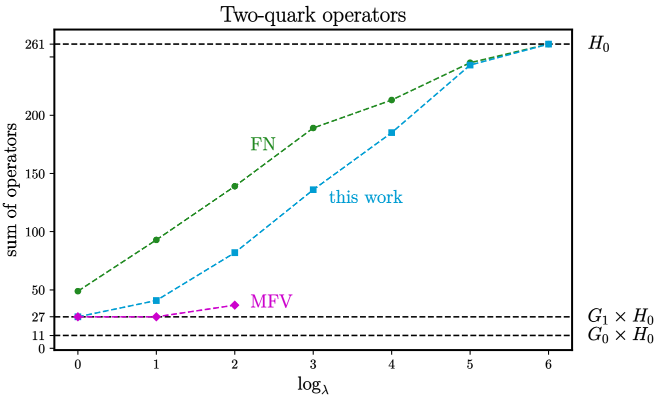
- Further break $G_3 \to \{\}$ by charged singlets

| dim-6 | $H_0$ | $\mathcal{O}(\lambda^6)$ | $\mathcal{O}(\lambda^5)$ | $\mathcal{O}(\lambda^4)$ | $\mathcal{O}(\lambda^3)$ | $\mathcal{O}(\lambda^2)$ | $\mathcal{O}(\lambda)$ | $\mathcal{O}(1)$ |
|---|---|---|---|---|---|---|---|---|
| no-quark | 22 | | | | | | | 22 |
| 2-quark | 261 | 18 | 58 | 49 | 54 | 41 | 14 | 27 |
| 4-quark | 990 | 116 | 158 | 109 | 66 | 42 | 18 | 33 |
| sum | 1273 | $\leftarrow$ 825 $\leftarrow$ | 691 $\leftarrow$ | 475 $\leftarrow$ | 317 $\leftarrow$ | 197 $\leftarrow$ | 114 $\leftarrow$ | 82 |

- 2-quark operators ”saturate” at $\mathcal{O}(\lambda^6)$
- still $\approx 40\%$ of the 4-quark operators are suppressed by more than $\lambda^6$

# Comparing with FN (and MFV)

– preliminary –




| Scenario | Symmetry | Spurions |
|---|---|---|
| FN | $U(1)_{\rm FN}$ | 1 singlet |
| this work | $G_1$ | 2 triplets |
| | | 2 doublets |
| | | 3 singlets |
| MFV | $G_0$ | 2 bi-triplets |

- stronger $\lambda$ suppression compared to FN
- weaker $\lambda$ suppression compared to MFV

…implementation of MFV in AutoEFT non-trivial, due to non-homogeneous power-counting …w.i.p.

- we have provided an example for a flavour scenario inbetween MFV and FN
  - RG-covariant
  - self-consistent power-counting in $\lambda$
  - based on simple spurion representations with homogeneous $\lambda$ scaling
  - implemented in AutoEFT

- can be used as a benchmark in global SMEFT-based fits
  - order of magnitude for Wilson coefficients is pre-defined
  - meaningful constraints on possible NP scale from collider and flavour data

- further benchmarks to be constructed:
  - $\rightarrow$ more MFV-like: also allow for bi-fundamental spurion representations
  - $\rightarrow$ more FN-like: break some sub-symmetries by assigning different $U(1)$ charges

– THE END –

# Yukawas as spurions of MFV

[D'Ambrosio et al. 2002]

- promote quark Yukawa matrices to spurion fields, transforming as bi-fundamentals under the $G_0$ flavour symmetry,

$$Y_U \to V_Q\, Y_U\, V_U^\dagger\,, \qquad Y_D \to V_Q\, Y_D\, V_D^\dagger$$

with $V_Q \in U(3)_Q$, $V_U \in U(3)_U$, $V_D \in U(3)_D$

- use $Y_U$ and $Y_D$ to make NP operators formally invariant under $G_0$, e.g.

$$c_{ij}\, \bar{Q}^i\, (\ldots)\, U^j$$

$$\to \qquad c_{ij} = \left(\#Y_U + \#Y_U Y_U^\dagger Y_U + \#Y_D Y_D^\dagger Y_U + \ldots\right)_{ij}$$

with prefactors $\# \sim \mathcal{O}(1)$.

- in the SM basis, set

$$Y_U \to \langle Y_U \rangle = \mathrm{diag}(y_u, y_c, y_t)\,, \qquad Y_D \to \langle Y_D \rangle = V_{\mathrm{CKM}}\, \mathrm{diag}(y_d, y_s, y_b)$$

# Example for AutoEFT output

## Spurion Flavour Symmetry @ $d = 6$

**Standard Model with $G_1 \times H_0$ flavour symmetry $O(1)$**

Generated by `AutoEFT` *1.1.0-a1*

| family | types | terms | operators |
|---|---|---|---|
| ${F_{\mathrm{L}}}^3 + \text{h.c.}$ | 4 | 4 | 4 |
| ${F_{\mathrm{L}}}^2\phi^2 + \text{h.c.}$ | 8 | 8 | 8 |
| $F_{\mathrm{L}}{\psi_{\mathrm{L}}}^2\phi + \text{h.c.}$ | 6 | 6 | 6 |
| ${\psi_{\mathrm{L}}}^2{\psi_{\mathrm{R}}}^2$ | 28 | 49 | 49 |
| $\psi_{\mathrm{L}}\phi^2\psi_{\mathrm{R}}D$ | 7 | 10 | 10 |
| $\phi^4 D^2$ | 1 | 2 | 2 |
| ${\psi_{\mathrm{L}}}^2\phi^3 + \text{h.c.}$ | 2 | 2 | 2 |
| $\phi^6$ | 1 | 1 | 1 |
| 12 | 57 | 82 | 82 |

$$
\begin{aligned}
\hat{H}_6 = {} & {G_{\mathrm{L}}}^3 + {W_{\mathrm{L}}}^3 + B_{\mathrm{L}}W_{\mathrm{L}}HH^\dagger + {B_{\mathrm{L}}}^2HH^\dagger + {G_{\mathrm{L}}}^2HH^\dagger + {W_{\mathrm{L}}}^2HH^\dagger + B_{\mathrm{L}}T_{\mathrm{L}}{t_{\mathrm{R}}}^\dagger H + G_{\mathrm{L}}T_{\mathrm{L}}{t_{\mathrm{R}}}^\dagger H \\
& + W_{\mathrm{L}}T_{\mathrm{L}}{t_{\mathrm{R}}}^\dagger H + 2\,L_{\mathrm{L}}Q_{\mathrm{L}}{L_{\mathrm{L}}}^\dagger{Q_{\mathrm{L}}}^\dagger + 2\,L_{\mathrm{L}}T_{\mathrm{L}}{L_{\mathrm{L}}}^\dagger{T_{\mathrm{L}}}^\dagger + L_{\mathrm{L}}{d_{\mathrm{R}}}^\dagger{L_{\mathrm{L}}}^\dagger d_{\mathrm{R}} + L_{\mathrm{L}}{e_{\mathrm{R}}}^\dagger{L_{\mathrm{L}}}^\dagger e_{\mathrm{R}} + L_{\mathrm{L}}{t_{\mathrm{R}}}^\dagger{L_{\mathrm{L}}}^\dagger t_{\mathrm{R}} \\
& + L_{\mathrm{L}}{u_{\mathrm{R}}}^\dagger{L_{\mathrm{L}}}^\dagger u_{\mathrm{R}} + 2\,{L_{\mathrm{L}}}^2{L_{\mathrm{L}}}^{\dagger 2} + 4\,Q_{\mathrm{L}}T_{\mathrm{L}}{Q_{\mathrm{L}}}^\dagger{T_{\mathrm{L}}}^\dagger + 2\,Q_{\mathrm{L}}{d_{\mathrm{R}}}^\dagger{Q_{\mathrm{L}}}^\dagger d_{\mathrm{R}} + Q_{\mathrm{L}}{e_{\mathrm{R}}}^\dagger{Q_{\mathrm{L}}}^\dagger e_{\mathrm{R}} + 2\,Q_{\mathrm{L}}{t_{\mathrm{R}}}^\dagger{Q_{\mathrm{L}}}^\dagger t_{\mathrm{R}} \\
& + 2\,Q_{\mathrm{L}}{u_{\mathrm{R}}}^\dagger{Q_{\mathrm{L}}}^\dagger u_{\mathrm{R}} + 4\,{Q_{\mathrm{L}}}^2{Q_{\mathrm{L}}}^{\dagger 2} + 2\,T_{\mathrm{L}}{d_{\mathrm{R}}}^\dagger{T_{\mathrm{L}}}^\dagger d_{\mathrm{R}} + T_{\mathrm{L}}{e_{\mathrm{R}}}^\dagger{T_{\mathrm{L}}}^\dagger e_{\mathrm{R}} + 2\,T_{\mathrm{L}}{t_{\mathrm{R}}}^\dagger{T_{\mathrm{L}}}^\dagger t_{\mathrm{R}} + 2\,T_{\mathrm{L}}{u_{\mathrm{R}}}^\dagger{T_{\mathrm{L}}}^\dagger u_{\mathrm{R}} \\
& + 2\,{T_{\mathrm{L}}}^2{T_{\mathrm{L}}}^{\dagger 2} + {d_{\mathrm{R}}}^\dagger{e_{\mathrm{R}}}^\dagger d_{\mathrm{R}}e_{\mathrm{R}} + 2\,{d_{\mathrm{R}}}^\dagger{t_{\mathrm{R}}}^\dagger d_{\mathrm{R}}t_{\mathrm{R}} + 2\,{d_{\mathrm{R}}}^\dagger{u_{\mathrm{R}}}^\dagger d_{\mathrm{R}}u_{\mathrm{R}} + 2\,{d_{\mathrm{R}}}^{\dagger 2}{d_{\mathrm{R}}}^2 + {e_{\mathrm{R}}}^\dagger{t_{\mathrm{R}}}^\dagger e_{\mathrm{R}}t_{\mathrm{R}} + {e_{\mathrm{R}}}^\dagger{u_{\mathrm{R}}}^\dagger e_{\mathrm{R}}u_{\mathrm{R}} \\
& + {e_{\mathrm{R}}}^{\dagger 2}{e_{\mathrm{R}}}^2 + 2\,{t_{\mathrm{R}}}^\dagger{u_{\mathrm{R}}}^\dagger t_{\mathrm{R}}u_{\mathrm{R}} + {t_{\mathrm{R}}}^{\dagger 2}{t_{\mathrm{R}}}^2 + 2\,{u_{\mathrm{R}}}^{\dagger 2}{u_{\mathrm{R}}}^2 + 2\,L_{\mathrm{L}}HH^\dagger{L_{\mathrm{L}}}^\dagger D + 2\,Q_{\mathrm{L}}HH^\dagger{Q_{\mathrm{L}}}^\dagger D \\
& + 2\,T_{\mathrm{L}}HH^\dagger{T_{\mathrm{L}}}^\dagger D + {d_{\mathrm{R}}}^\dagger HH^\dagger d_{\mathrm{R}}D + {e_{\mathrm{R}}}^\dagger HH^\dagger e_{\mathrm{R}}D + {t_{\mathrm{R}}}^\dagger HH^\dagger t_{\mathrm{R}}D + {u_{\mathrm{R}}}^\dagger HH^\dagger u_{\mathrm{R}}D + 2\,H^2H^{\dagger 2}D^2 \\
& + T_{\mathrm{L}}{t_{\mathrm{R}}}^\dagger H^2H^\dagger + H^3H^{\dagger 3} + \text{h.c.}
\end{aligned}
$$

+ more details …