Quantum Tomography:
Spin states & Flavor Oscillations

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Beyond Flavor Physics

University of Siegen, June 23, 2025


K. Cheng, TH, M. Low, A. Wu: arXiv: 2311.09166; 2407.01672; 2410.08303; 2507.xxxxx

Disclaimer:

In HEP experiments, we do not "test quantum mechanics" in the EPR (Einstein-Podolsky-Rosen) sense.

Our goals:

In the framework of QFT, in the HE regime at colliders,

- We lay out the QM predictions / information: Entanglement, Bell variables, Discord, Magic …
- Hope to establish the quantum tomography.
- Understand quantum nature at this unprecedented regime & seek for BSM effects.

Quantum State & Quantum Tomography

For a state vector $|\phi_i\rangle$

Density matrix

a state

$$\rho = \sum_i n_i \, |\phi_i\rangle \, \langle\phi_i|$$

an observable

$$\langle \mathcal{O} \rangle = \mathrm{Tr}(\mathcal{O}\rho)$$

For a **pure state**: $n_i = 1$; for a **mixed state**: $\boldsymbol{\Sigma}_i \, n_i = 1$.

For a **single qubit** (*i.e.*, a doublet of spin, iso-spin etc.):

$$\rho = \frac{1}{2}\Big(\mathbb{I}_2 + \sum_i B_i \sigma_i\Big)$$

For a **bipartite system** (*i.e.*, ½⊗½)

$$\rho = \frac{1}{4}\Big(\mathbb{I}_4 + \sum_i \left(B_i^{\mathcal{A}} \left(\sigma_i \otimes \mathbb{I}_2\right) + B_i^{\mathcal{B}} \left(\mathbb{I}_2 \otimes \sigma_i\right)\right) + \sum_{i,j} C_{ij} \left(\sigma_i \otimes \sigma_j\right)\Big)$$

$B_i^{A,B}$ the polarizations, C_{ij} the spin-correlation matrix.

The 15 coefficients for the bipartite → **Quantum Tomography, which encodes the full QI.**

Quantum Information:
Entanglement & Bell Inequality

Entanglement is a genuine QM feature,
no classical correspondence!

If a state can be written as:
then it is "separable"
and thus not entangled.

$$|\psi_{AB}\rangle = |\psi_A\rangle \otimes |\psi_B\rangle$$

$$\rho_{AB} = \sum_\omega p_\omega \rho_\omega^A \otimes \rho_\omega^B$$

i.e. separable, not entangled:

$$|\psi\rangle = |00\rangle = |0\rangle \otimes |0\rangle$$

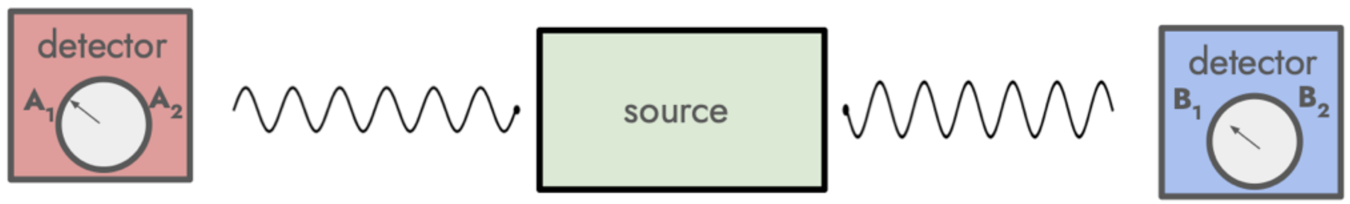
Non-separable, entangled:

$$|\psi\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$

Bell inequality (1964) :

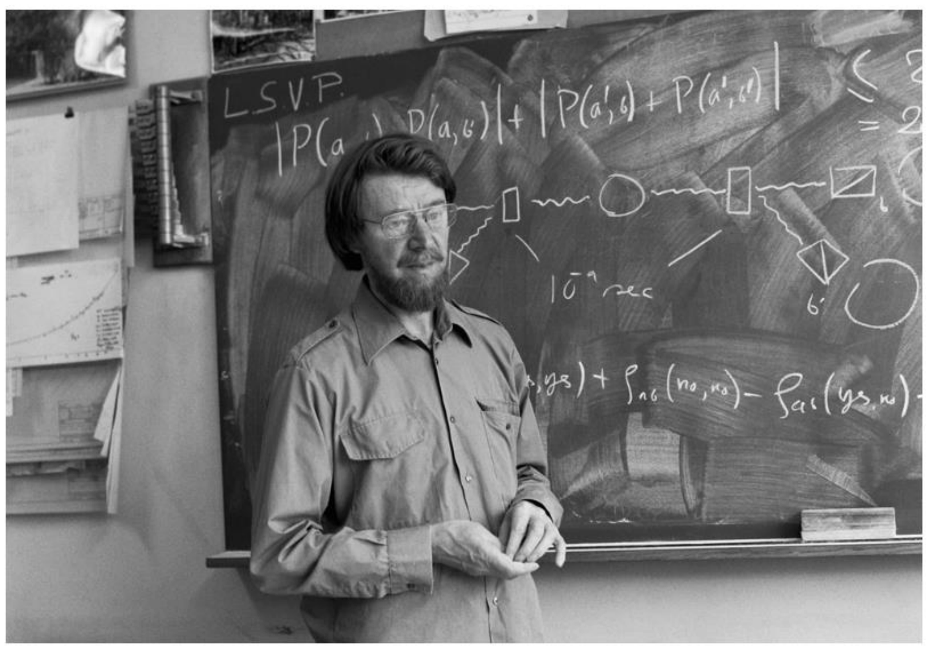
$$\left|\langle \hat{A}_1\hat{B}_1\rangle + \langle \hat{A}_1\hat{B}_2\rangle + \langle \hat{A}_2\hat{B}_1\rangle - \langle \hat{A}_2\hat{B}_2\rangle\right| \leq 2$$

[Clauser et al, *PRL* 23, 880 (1969)]



Alice

Bob



Quantum Information:
Bell Inequality violation & (many) observables

Bell inequality violation in QM: $\left|\vec{a}_1 \cdot C \cdot (\vec{b}_1 - \vec{b}_2) + \vec{a}_2 \cdot C \cdot (\vec{b}_1 + \vec{b}_2)\right| > 2$

With the quantum tomography C_{ij} :

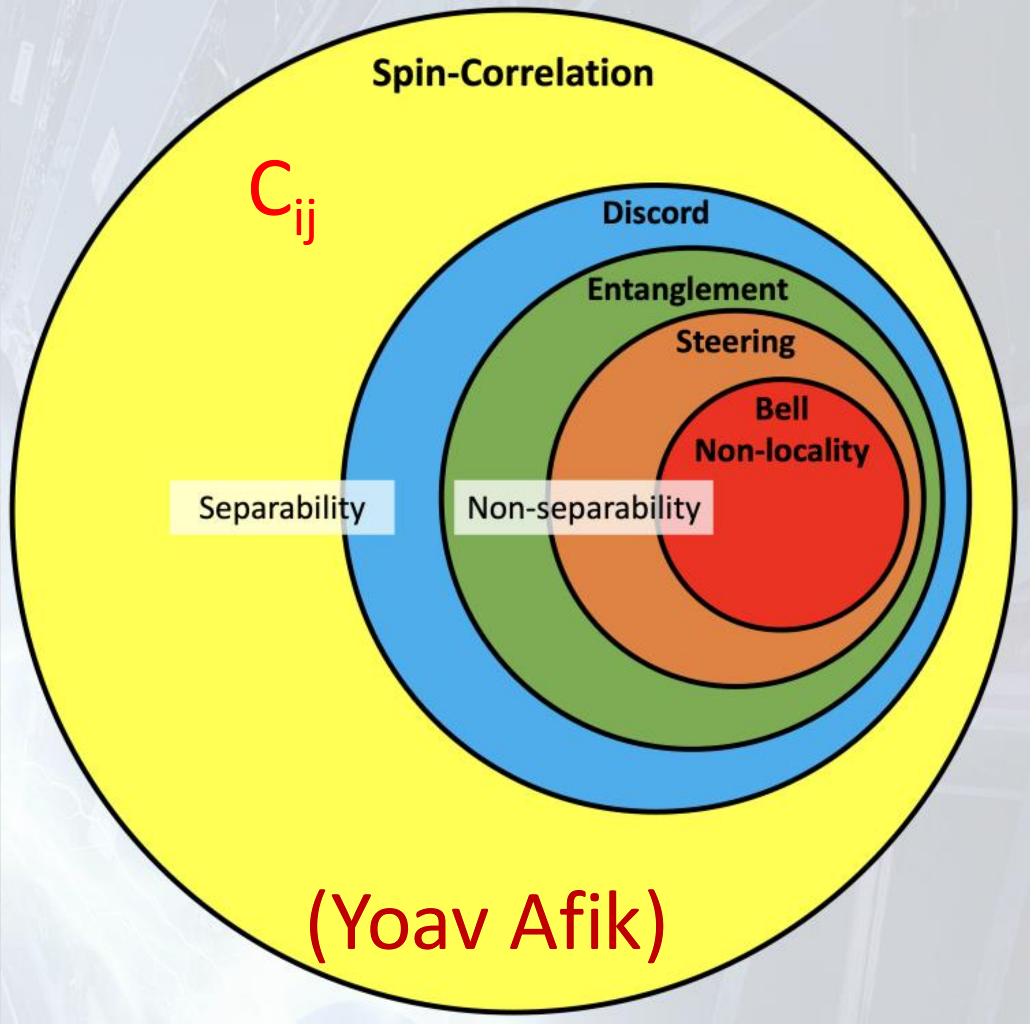
→ much more quantum information:

"Concurrence": (entanglement CHSH):

$$0 < C(\rho) < 1$$

"Discord": the difference between the total mutual information and the classical mutual information (Shannon entropy vs Von Neumann entropy)

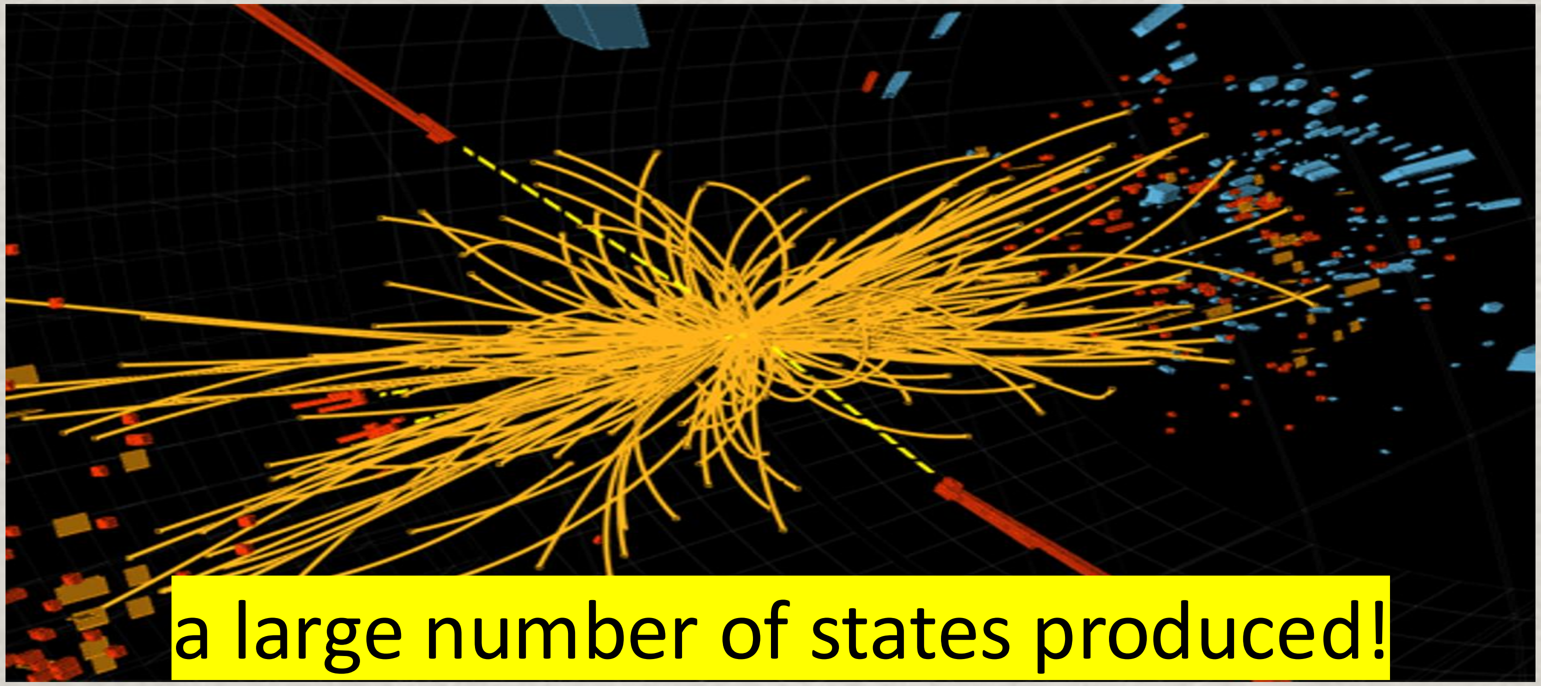
$$0 < D(\rho_A) < 1$$



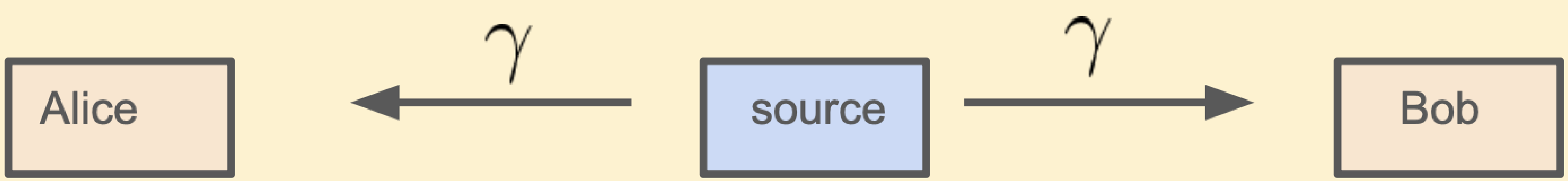
"Magic": something else beyond classical limit for quantum computers (non-stabiliziness; Renyi entropy) : $M = -\log_2[\zeta(\rho)]$

… …

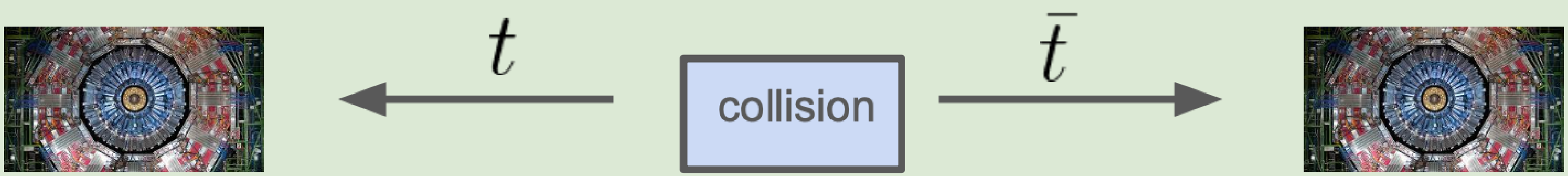
Quantum Tomography @ Colliders



- Low energy photon experiment



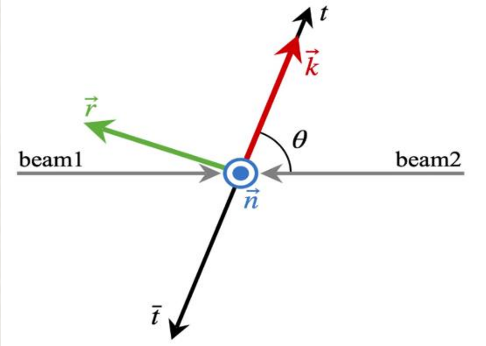
- At LHC, treat the spin of each particle as a qubit



Quantum Tomography for spin states

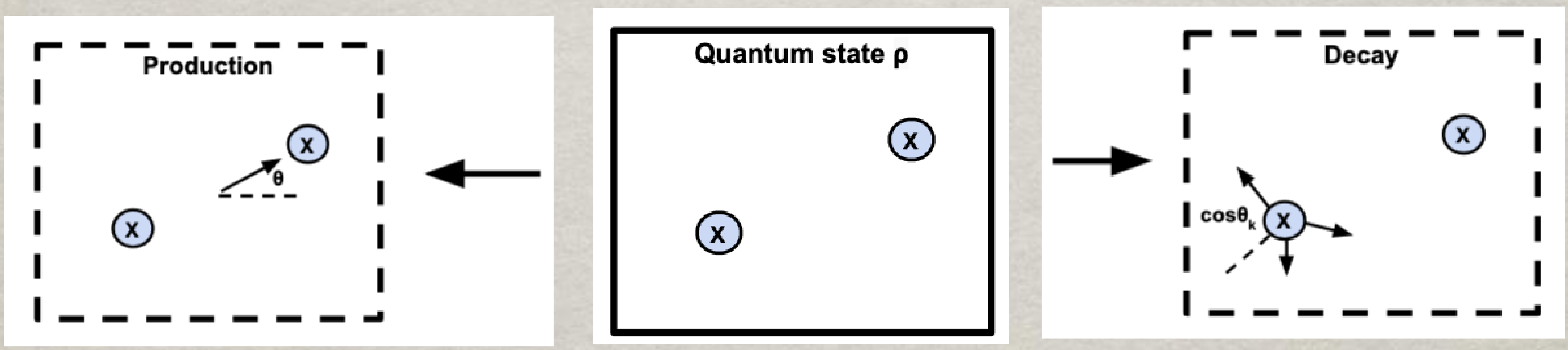
- All “classical observables”: energy, angles …
- No direct spin measurement on event-by-event

→ inferred by angular distributions,
thus statistically: “**fictitious states**”!



Y. Afik, J. de Nova, arXiv:2003.02280; K. Cheng, TH, M. Low, arXiv:2407.01672.

Two paths to proceed to quantum tomography



$$\rho = \frac{1}{4}\Big(\mathbb{I}_4 + \sum_i \left(B_i^{\mathcal{A}}\,(\sigma_i \otimes \mathbb{I}_2) + B_i^{\mathcal{B}}\,(\mathbb{I}_2 \otimes \sigma_i)\right) + \sum_{i,j} C_{ij}\,(\sigma_i \otimes \sigma_j)\Big)$$

Both the state before decay & the final state decay products inherit the SAME quantum information!

(1) Top decay & spin correlation

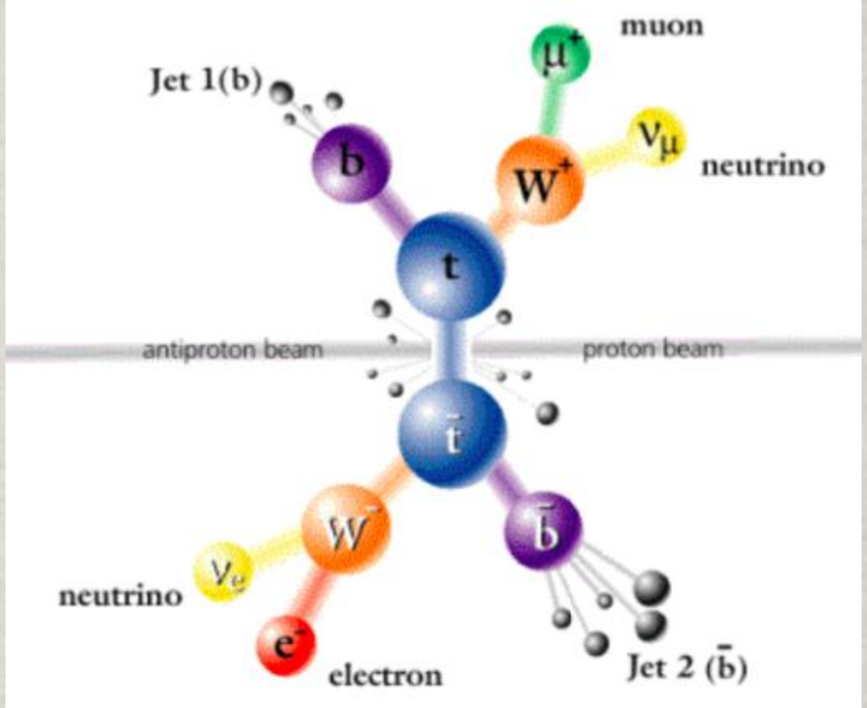
decaying to $A_{1,2,3}$, $B_{1,2,3}$

$$\sigma(XY \to t\bar{t} \to (A_1A_2A_3)(B_1B_2B_3)) =$$

$$\int d\Omega^A d\Omega^B \left(\frac{d\Gamma_{ab}}{d\Omega^A}\right) R_{ab,\bar{a}\bar{b}} \left(\frac{d\bar{\Gamma}_{\bar{a}\bar{b}}}{d\Omega^B}\right)$$

$$\frac{d\Gamma_{ab}}{d\Omega} \propto \delta_{ab} + \kappa\, \sigma^i_{ab}\, \Omega^i$$

Spin analyzing power



$$\Rightarrow \quad \frac{1}{\sigma}\frac{d\sigma}{d\Omega^A d\Omega^B} = \frac{1}{(4\pi)^2}\left(1 + \kappa^A P_i^A \Omega_i^A + \kappa^B P_i^B \Omega_i^B + \kappa^A \kappa^B \Omega_i^A C_{ij} \Omega_j^B\right)$$

Direction of A, B

$$\Rightarrow \quad \frac{1}{\sigma}\frac{d\sigma}{d(\cos\theta_i^A \cos\theta_j^B)} = -\frac{1 + \kappa^A\kappa^B C_{ij}\cos\theta_i^A\cos\theta_j^B}{2} \log\left|\cos\theta_i^A\cos\theta_j^B\right|$$

Polar angle of A with respect to the i-th axis

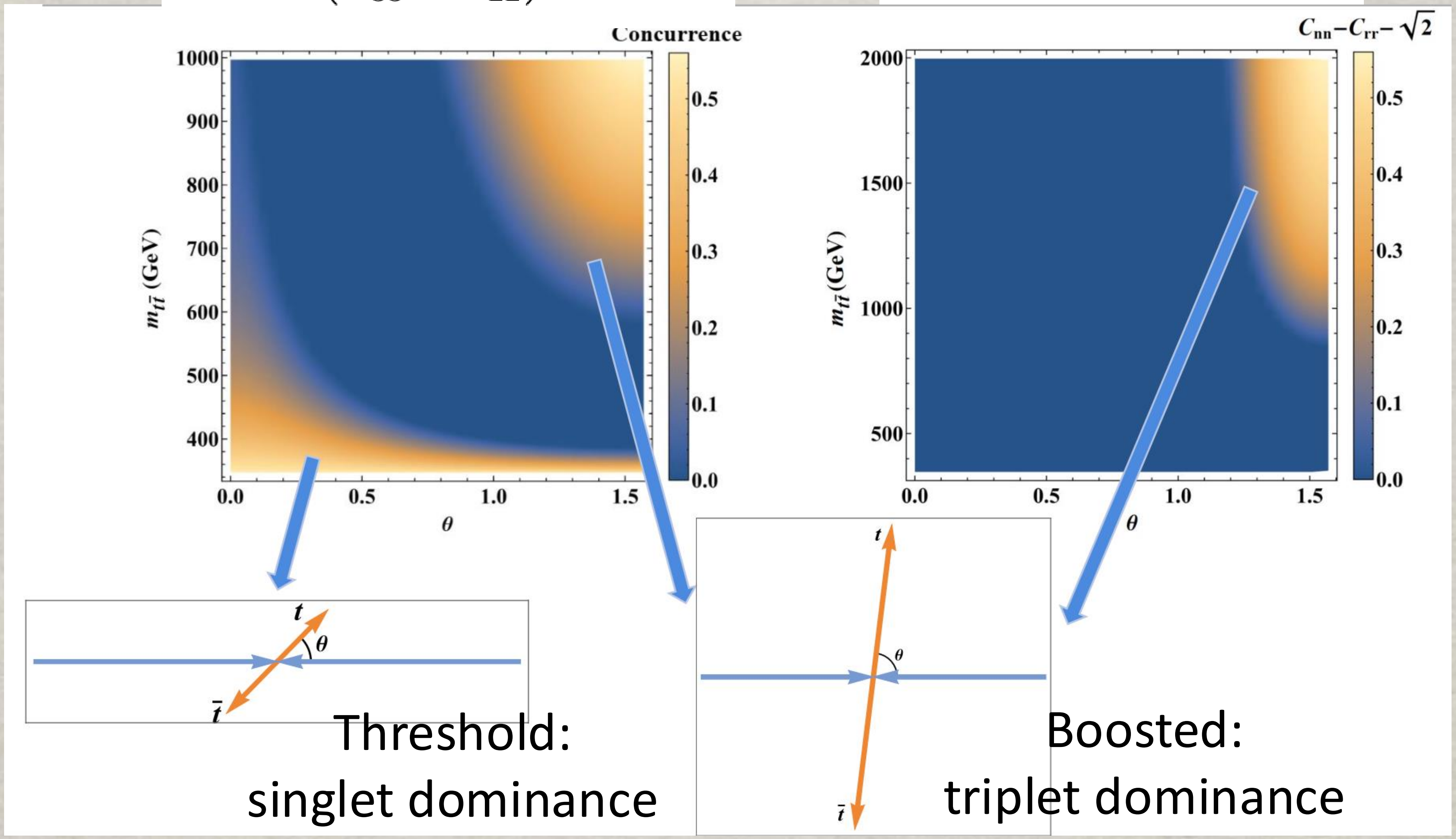
$$\Rightarrow \quad C_{ij} = \frac{4}{\kappa^A\kappa^B}\frac{N(\cos\theta_i^A\cos\theta_j^B > 0) - N(\cos\theta_i^A\cos\theta_j^B < 0)}{N(\cos\theta_i^A\cos\theta_j^B > 0) + N(\cos\theta_i^A\cos\theta_j^B < 0)}$$

Theory & Observation:

(Many theory papers; ATLAS & CMS publications.)

$$\mathcal{C} = \frac{1}{2}(C_{11} + C_{33} - C_{22} - 1),$$
$$\mathcal{B} = \sqrt{2}(C_{33} - C_{22}).$$

$\mathcal{C} > 0, \qquad \mathcal{B} > 2,$
for entanglement



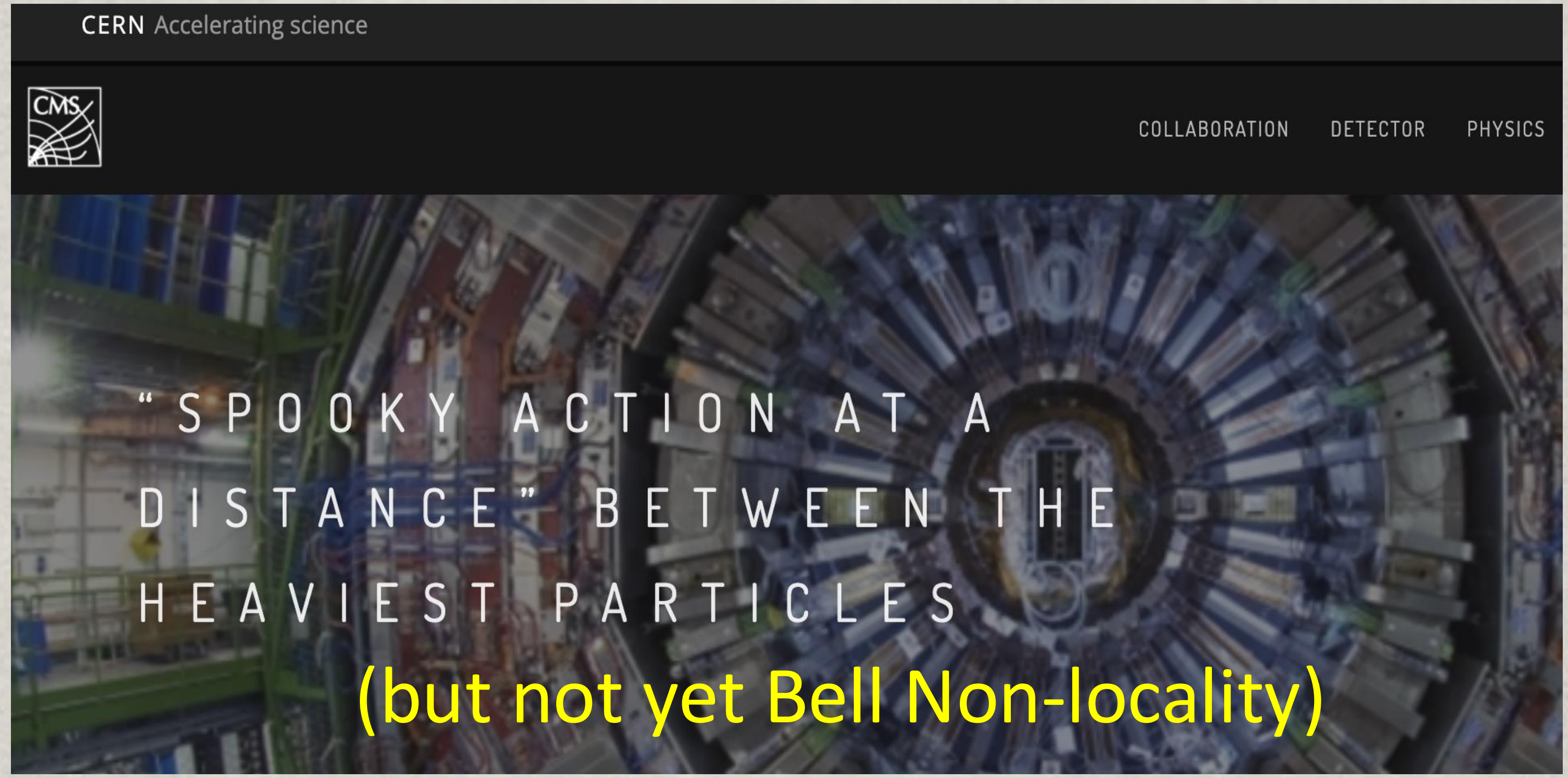
- Threshold:

high rate, low sensitivity

- Highly boosted:

Low rate, high sensitivity

CERN press release on Sept. 19, 2024



(but not yet Bell Non-locality)

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Article | Open access | Published: 18 September 2024

Observation of quantum entanglement with top quarks at the ATLAS detector

The ATLAS Collaboration

Nature **633**, 542–547 (2024) | Cite this article

75k Accesses | **11** Citations | **485** Altmetric | Metrics

LHC experiments at CERN observe quantum entanglement at the highest energy yet

The results open up a new perspective on the complex world of quantum physics

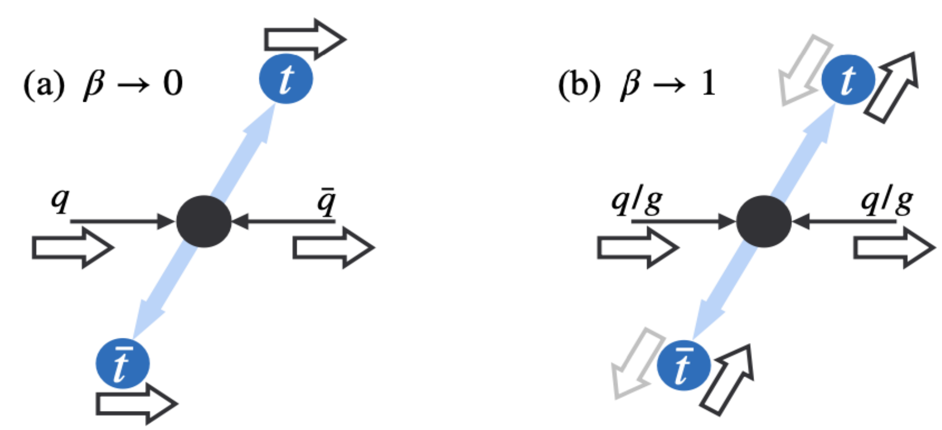
18 SEPTEMBER, 2024

Artist's impression of a quantum-entangled pair of top quarks. (Image: CERN)

Also CMS: arXiv: 2503.22382

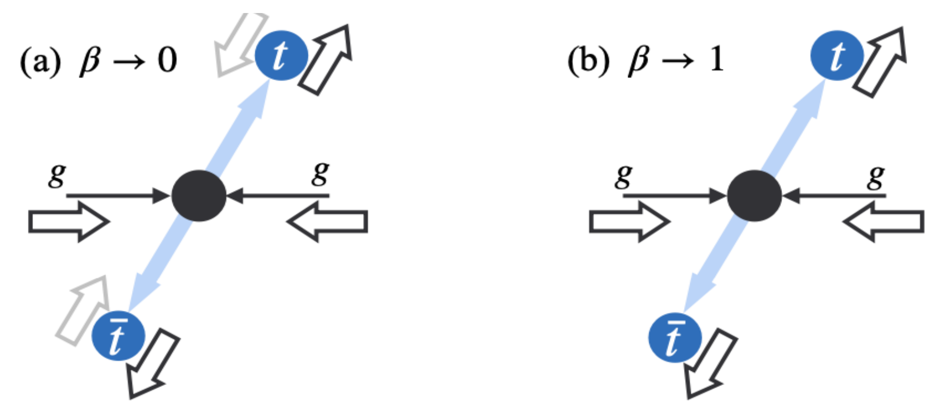
(2). Kinematic Approach for 2 → 2 Production

QCD production	$\overline{\sum}\lvert\mathcal{M}\rvert^2$	spin correlation matrix C_{ij}	ξ
$q\bar{q} \to t\bar{t}$	$\kappa_q\left(2-\beta^2 s_\theta^2\right)$	$\begin{pmatrix} \frac{(2-\beta^2)s_\theta^2}{2-\beta^2 s_\theta^2} & 0 & -\frac{2c_\theta s_\theta\sqrt{1-\beta^2}}{2-\beta^2 s_\theta^2} \\ 0 & \frac{-\beta^2 s_\theta^2}{2-\beta^2 s_\theta^2} & 0 \\ -\frac{2c_\theta s_\theta\sqrt{1-\beta^2}}{2-\beta^2 s_\theta^2} & 0 & \frac{2c_\theta^2+\beta^2 s_\theta^2}{2-\beta^2 s_\theta^2} \end{pmatrix}$	$\tan\xi = \frac{1}{\gamma}\tan\theta$
$g_L g_R \to t\bar{t}$	$\kappa_g \beta^2 s_\theta^2(2-\beta^2 s_\theta^2)$		$\tan\xi = \frac{1}{\gamma}\tan\theta$



All C_{ij} encoded in the production kinematics:
θ & **β**$=(1-4m_t^2/m_{tt}^2)^{1/2}$

$g_L g_L/g_R g_R \to t\bar{t}$	$\kappa_g(1-\beta^4)$	$\begin{pmatrix} \frac{\beta^2-1}{\beta^2+1} & 0 & 0 \\ 0 & \frac{\beta^2-1}{\beta^2+1} & 0 \\ 0 & 0 & -1 \end{pmatrix}$	$\xi = 0$



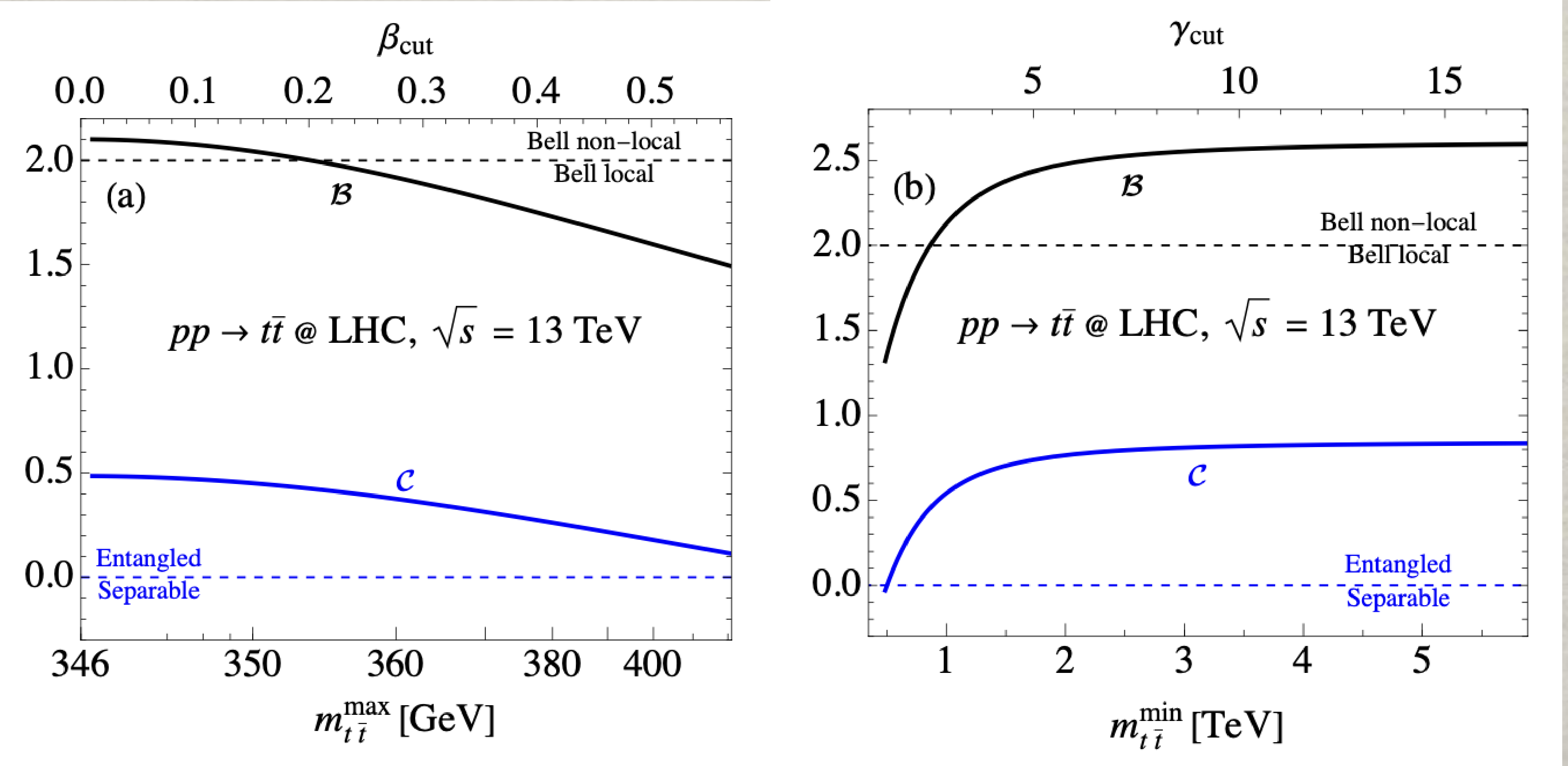
K. Cheng, TH, M. Low, 2410.08303.

Quantum Entanglement from production: without decay measurement

$$\rho(\Theta,\beta) = \frac{L_{q\bar{q}}|\mathcal{M}_{q\bar{q}\to t\bar{t}}|^2 \rho_{q\bar{q}\to t\bar{t}} + L_{gg}|\mathcal{M}_{gg\to t\bar{t}}|^2 \rho_{gg\to t\bar{t}}}{L_{q\bar{q}}|\mathcal{M}_{q\bar{q}\to t\bar{t}}|^2 + L_{gg}|\mathcal{M}_{gg\to t\bar{t}}|^2},$$

$$F_{\mathcal{C}}(\Theta,\beta) = \frac{C_{11}(\Theta,\beta) + C_{22}(\Theta,\beta) - C_{33}(\Theta,\beta) - 1}{2} = \frac{\beta^2 s_\Theta^2}{2 - \beta^2 c_\Theta}$$

$$F_{\mathcal{B}}(\Theta,\beta) = \sqrt{2}|C_{22}(\Theta,\beta) - C_{33}(\Theta,\beta)| = \frac{2\sqrt{2}}{2 - \beta^2 s_\Theta^2}$$



Quantum Entanglement from production: With stable particles

Entanglement in Drell-Yan Production

$pp \to Z \to \mu^+\mu^-$

$$C_{ij} = \begin{pmatrix} \frac{s_\Theta^2((2-\beta^2)g_V^2-\beta^2 g_A^2)}{\beta^2 g_A^2(1+c_\Theta^2)-4\beta g_A g_V c_\Theta + g_V^2(2-\beta^2 s_\Theta^2)} & 0 & -\frac{2g_V s_\Theta\sqrt{1-\beta^2}(g_V c_\Theta - g_A\beta)}{\beta^2 g_A^2(1+c_\Theta^2)-4\beta g_A g_V c_\Theta + g_V^2(2-\beta^2 s_\Theta^2)} \\ 0 & \frac{(g_V^2-g_A^2)\beta^2 s_\Theta^2}{\beta^2 g_A^2(1+c_\Theta^2)-4\beta g_A g_V c_\Theta + g_V^2(2-\beta^2 s_\Theta^2)} & 0 \\ -\frac{2g_V s_\Theta\sqrt{1-\beta^2}(g_V c_\Theta - g_A\beta)}{\beta^2 g_A^2(1+c_\Theta^2)-4\beta g_A g_V c_\Theta + g_V^2(2-\beta^2 s_\Theta^2)} & 0 & \frac{g_V^2(2c_\Theta^2+\beta^2 s_\Theta^2)+g_A^2\beta^2(1+c_\Theta^2)-4g_V g_A\beta}{\beta^2 g_A^2(1+c_\Theta^2)-4\beta g_A g_V c_\Theta + g_V^2(2-\beta^2 s_\Theta^2)} \end{pmatrix}$$

Taking **β** =1 →

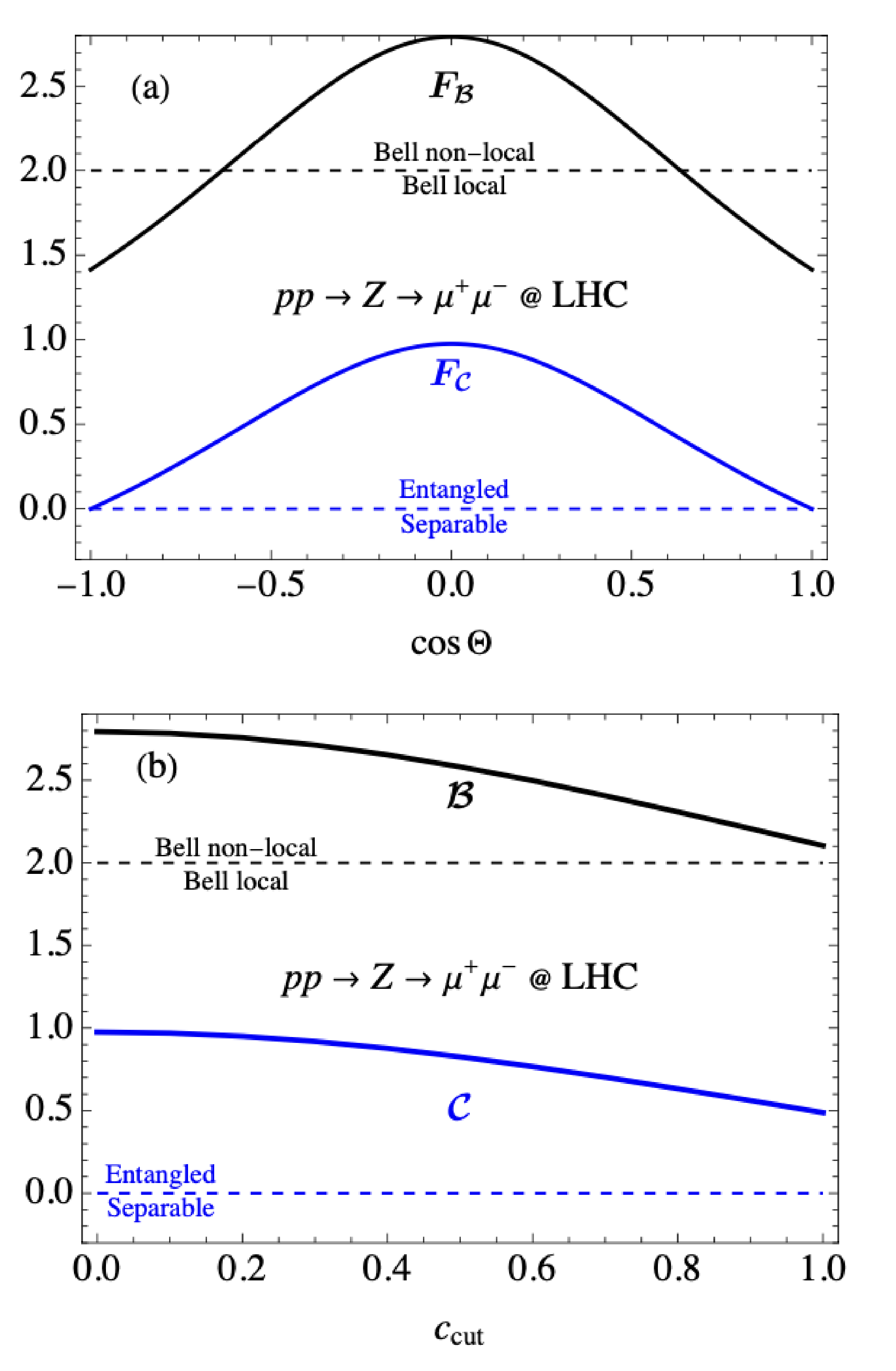
$$C_{ij} = \begin{pmatrix} \frac{s_\Theta^2(g_V^2-g_A^2)}{(1+c_\Theta^2)(g_A^2+g_V^2)} & 0 & 0 \\ 0 & -\frac{s_\Theta^2(g_V^2-g_A^2)}{(1+c_\Theta^2)(g_A^2+g_V^2)} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$F_{\mathcal{C}}(\Theta, \beta=1) = \frac{1}{2}(|C_{rr}-C_{nn}|+C_{kk}-1) = \frac{s_\Theta^2(g_A^2-g_V^2)}{(1+c_\Theta^2)(g_A^2+g_V^2)}$$

$$F_{\mathcal{B}}(\Theta, \beta=1) = \sqrt{2}(C_{kk}+C_{nn}) = \frac{2\sqrt{2}\,(g_A^2+g_V^2 c_\Theta^2)}{(1+c_\Theta^2)\,(g_A^2+g_V^2)}.$$

K. Cheng, TH, M. Low, arXiv:2410.08303

Entanglement in Drell-Yan Production
$pp \to Z \to \mu^+\mu^-$



Such a simple process, simple kinematics!

K. Cheng, TH, M. Low, arXiv:2410.08303

Quantum Tomography in flavor-oscillations

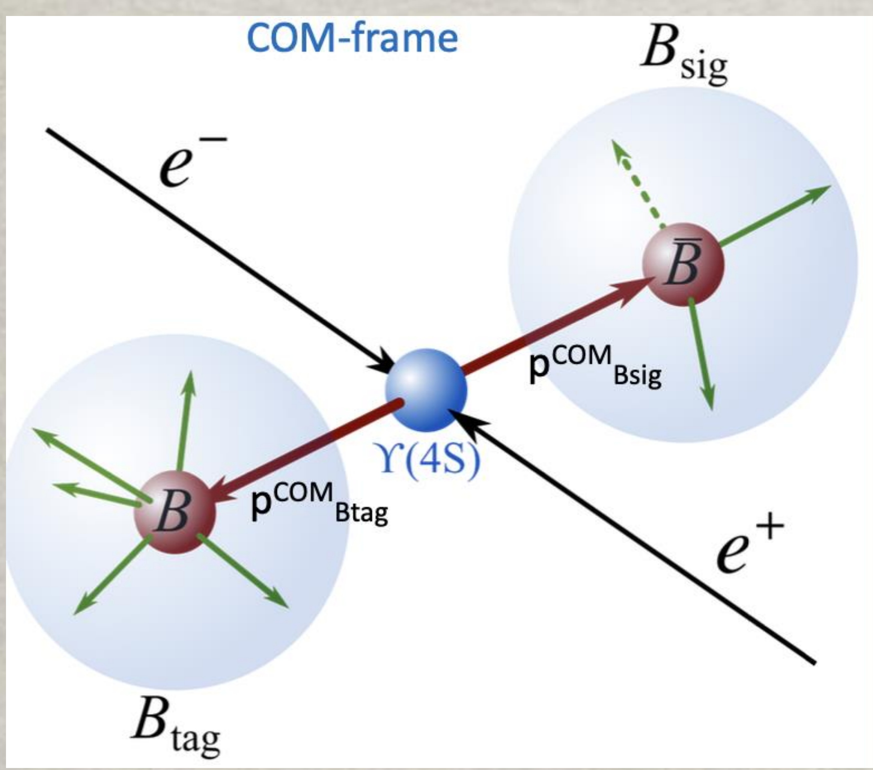
For a neutral meson – anti-meson system: $|M^0\rangle - |\bar{M}^0\rangle$
such as $|B^0\rangle - |\bar{B}^0\rangle,\ |D^0\rangle - |\bar{D}^0\rangle,\ |K^0\rangle - |\bar{K}^0\rangle$

There is a quantum-mechanical mixing/oscillation

$$|M_1\rangle = p|M\rangle + q|\bar{M}\rangle, \qquad \text{with } (m_1, \Gamma_1),$$
$$|M_2\rangle = p|M\rangle - q|\bar{M}\rangle, \qquad \text{with } (m_2, \Gamma_2),$$

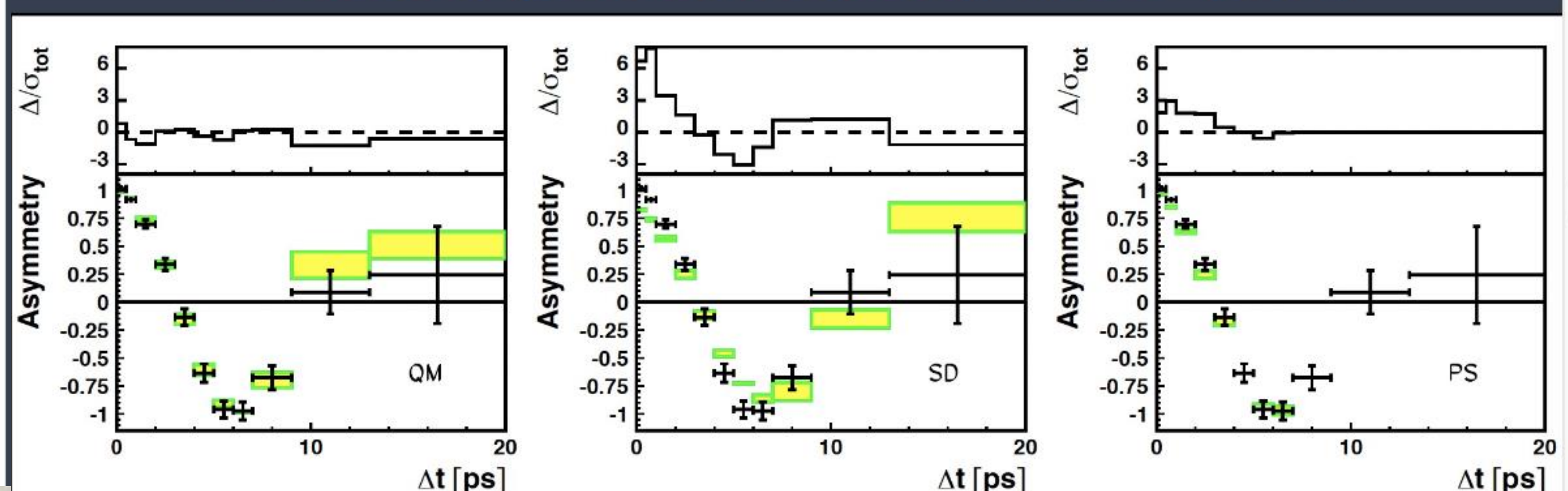
$$\mathcal{H} = \begin{pmatrix} m - i\frac{\Gamma}{2} & H_{12} \\ H_{21} & m - i\frac{\Gamma}{2} \end{pmatrix}$$

Belle measurements on entanglement & Bell non-locality test



$$|\Psi(t)\rangle = \frac{e^{-t/\tau_{B0}}}{\sqrt{2}} \left[|B^0(\vec{p})\overline{B}^0(-\vec{p})\rangle - |\overline{B}^0(\vec{p})B^0(-\vec{p})\rangle \right]$$

A. Go *et al.* (Belle Collaboration), PRL **99** (2007)



QM fits well $\chi^2/n_{dof} = 5/11$

SD disfavoured: 13σ $\chi^2/n_{dof} = 174/11$

PS disfavoured: 5.1σ $\chi^2/n_{dof} = 31/11$

SD=Spontaneous Disentanglement
PS=Pompili-Selleri hidden variable model

Quantum Tomography in flavor-oscillations

K. Cheng, TH, M. Low, A. Wu, arXiv:2507.xxxxx.

For a neutral meson – anti-meson system: $|M^0\rangle - |\bar{M}^0\rangle$

such as $|B^0\rangle - |\bar{B}^0\rangle,\ |D^0\rangle - |\bar{D}^0\rangle,\ |K^0\rangle - |\bar{K}^0\rangle$

we can treat them as 2-qubit systems:

$$|M_1\rangle = p|M\rangle + q|\bar{M}\rangle, \qquad \text{with } (m_1,\, \Gamma_1),$$
$$|M_2\rangle = p|M\rangle - q|\bar{M}\rangle, \qquad \text{with } (m_2,\, \Gamma_2),$$

$$\mathcal{H} = \begin{pmatrix} m - i\frac{\Gamma}{2} & H_{12} \\ H_{21} & m - i\frac{\Gamma}{2} \end{pmatrix}$$

$$U(t) = \begin{pmatrix} \frac{1}{2}\left(e^{-\Gamma_1 t/2 - i m_1 t} + e^{-\Gamma_2 t/2 - i m_2 t}\right) & \frac{q}{2p}\left(e^{-\Gamma_1 t/2 - i m_1 t} - e^{-\Gamma_2 t/2 - i m_2 t}\right) \\ \frac{p}{2q}\left(e^{-\Gamma_1 t/2 - i m_1 t} - e^{-\Gamma_2 t/2 - i m_2 t}\right) & \frac{1}{2}\left(e^{-\Gamma_1 t/2 - i m_1 t} + e^{-\Gamma_2 t/2 - i m_2 t}\right) \end{pmatrix}$$

$$\frac{N(t)}{N_0} = \mathrm{tr}\left(U(t)\rho_{_M} U(t)^\dagger\right),$$

$$\rho_{_M}(t) = \frac{U(t)\rho_{_M} U(t)^\dagger}{\mathrm{tr}\left(U(t)\rho_{_M} U(t)^\dagger\right)}$$

A Single State M

The density matrix: $\rho_M = \frac{\mathbb{1}_2 + R_i \sigma_i}{2}$

The Bloch vectors R_i specify the quantum state (quantum tomography).

Flavor eigen-states $|M^0\rangle, |\bar{M}^0\rangle$ by Bloch vectors $R^{init} = (0,0,\pm 1)$

so that: $\sigma_z |M\rangle = +|M\rangle$ $\sigma_z |\bar{M}\rangle = -|\bar{M}\rangle$

Mass eigen-states $|M_1\rangle, |M_2\rangle$ by Bloch vectors $R^{init} = (\pm 1,0,0)$

State evolution: $\frac{d\vec{R}(t)}{dt} = -\vec{X} \times \vec{R}(t)$ $\vec{X} = (\Delta m, 0, 0)$

Oscillatory Precession solution:

$$\begin{aligned} R_x(t) &= R_x, \\ R_y(t) &= R_y \mathrm{c}_t + R_z \mathrm{s}_t, \\ R_z(t) &= R_z \mathrm{c}_t - R_y \mathrm{s}_t, \end{aligned}$$

$\mathrm{c}_t = \cos(\Delta m t)$, $\mathrm{s}_t = \sin(\Delta m t)$

Thus: $R_y^{init}(0)$ or $R_z^{init}(0) \rightarrow R_{y,z}(t)$, but not $R_x(t)$.

In practice, consider decays

(a) Decaying to non-CP states (e.g., $B^0 \to X^- \ell^+ \nu$)

$M \to f$ with $CP|f\rangle = |\bar{f}\rangle \neq |f\rangle$:

project to $|M\rangle, |\bar{M}\rangle$ with $P_{M/\bar{M}} = \frac{1 \pm R_z}{2}$

$$\Gamma_{M(t)\to f/\bar{f}} = \frac{1 \pm R_z(t)}{2}\Gamma_{M\to f}$$

$$\Gamma_{M(t)\to f} - \Gamma_{M(t)\to \bar{f}} = R_z(t)\Gamma_{M\to f}$$

$$\frac{d(N_f - N_{\bar{f}})}{dt} = N(t)R_z(t)\Gamma_{M\to f} = N_0 e^{-\Gamma t}(R_z \mathrm{c}_t - R_y \mathrm{s}_t)\Gamma_{M\to f}$$

Thus: $R_y(0)$ or $R_z(0)$ → $R_{y,z}(t)$, but not $R_x(t)$.

(b) Decaying to CP eigen-states (e.g., $B^0 \to J/\psi K_S$)

$M \to f_\eta$ with $CP|f_\eta\rangle = \eta|f_\eta\rangle$, $\eta = \pm 1$:

project to $|M_1\rangle, |M_2\rangle$ with $P_{M_1/M_2} = \frac{1 \pm R_x}{2}$

$$\Gamma_{M(t)\to f_+} = \frac{1 + R_x(t)}{2}\Gamma_{M_1\to f_+} = (1 + R_x(t))\Gamma_{M\to f_+}$$

$$\frac{d(N_f + N_{\bar{f}})}{dt} = N(t)\Gamma_{M\to f} = N_0 e^{-\Gamma t}(\mathrm{ch}_t - R_x \mathrm{sh}_t)\Gamma_{M\to f}$$

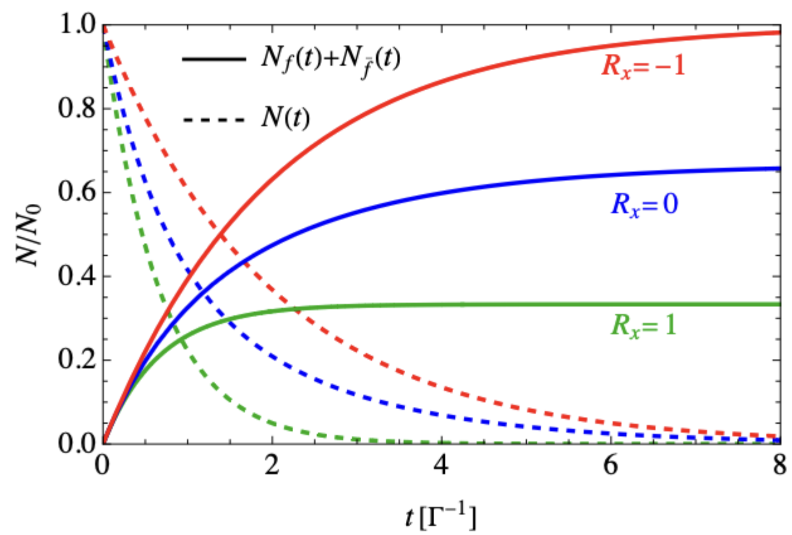


FIG. 1. In percentage, the total number of mesons (dashed lines) and decay products (solid lines) as a function of time, with different Bloch vectors at $t = 0$. Calculated in a toy example with $\Gamma_f = \Gamma_{f_+}$

Thus $R_x(t)$ determined → full quantum tomography $R_i(t)$!

A M^0 – anti-M^0 Pair System:

The density matrix for a 2-qubit system:

$$\rho_{MM} = \frac{\mathbb{1}_4 + R_i^{\mathcal{A}}\sigma_i \otimes \mathbb{1}_2 + R_i^{\mathcal{B}}\mathbb{1}_2 \otimes \sigma_i + C_{ij}\sigma_i \otimes \sigma_j}{4}$$

The Bloch vectors $R_i^{A,B}$, correlation matrix C_{ij} specify the full quantum tomography.

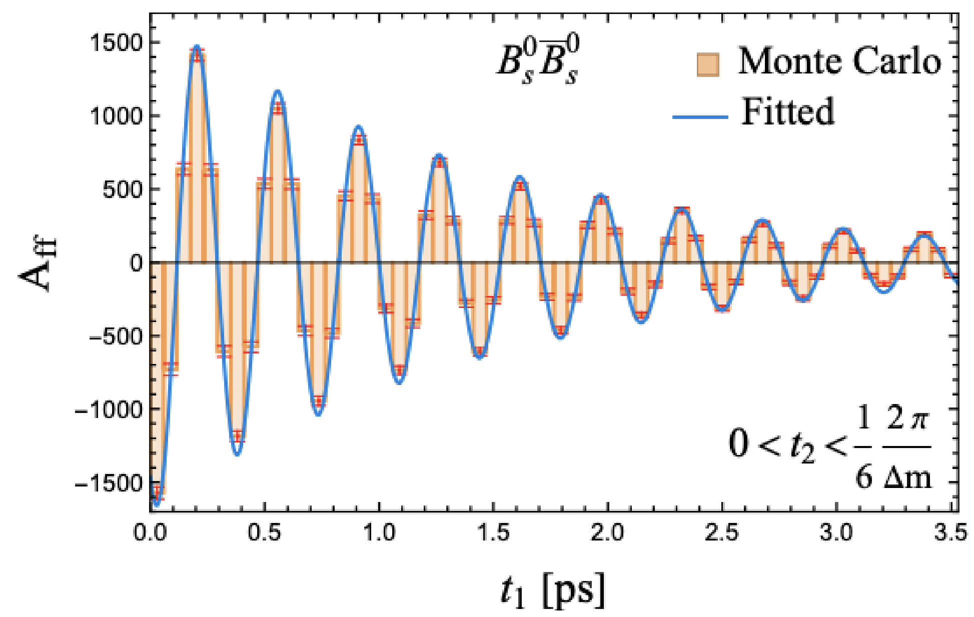
Define the correlated decay events:

$$N_{\text{tot}} = N_{ff} + N_{\bar{f}f} + N_{f\bar{f}} + N_{\bar{f}\bar{f}}$$
$$A_{ff} = N_{ff} - N_{\bar{f}f} - N_{f\bar{f}} + N_{\bar{f}\bar{f}} = N_{\text{like}} - N_{\text{unlike}}$$
$$A_f^{\mathcal{A}} = N_{ff} - N_{\bar{f}f} + N_{f\bar{f}} - N_{\bar{f}\bar{f}}$$
$$A_f^{\mathcal{B}} = N_{ff} + N_{\bar{f}f} - N_{f\bar{f}} - N_{\bar{f}\bar{f}}$$

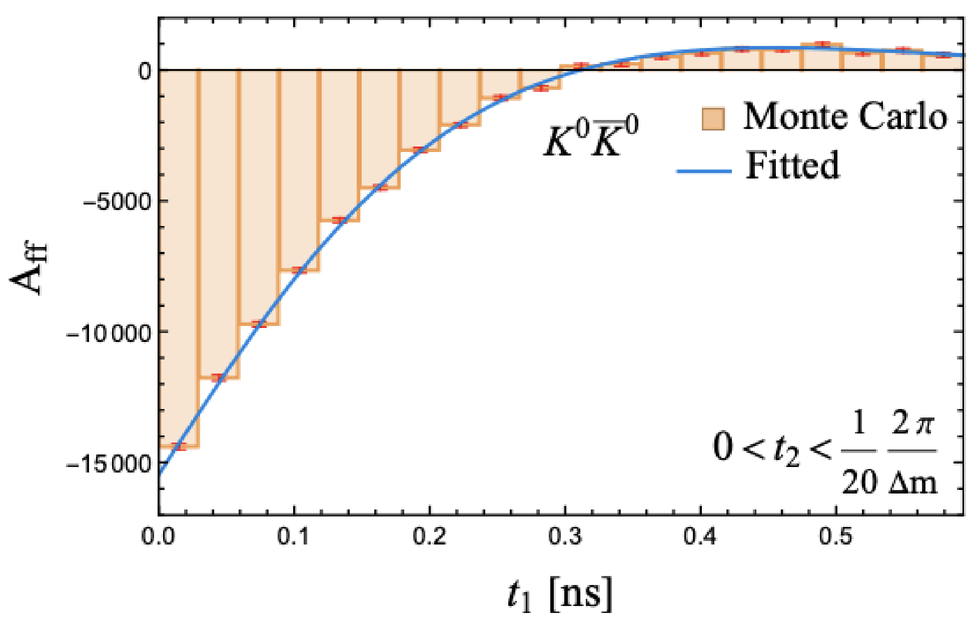
$$\frac{\mathrm{d}N_{\text{tot}}}{\mathrm{d}t_1\mathrm{d}t_2} = N_0\Gamma^2_{B_0\to f}\, e^{-\Gamma(t_1+t_2)}\left(\mathrm{ch}_{t_1}\mathrm{ch}_{t_2} - \mathrm{ch}_{t_1}\mathrm{sh}_{t_2}R_x^{\mathcal{A}} - \mathrm{sh}_{t_1}\mathrm{ch}_{t_2}R_x^{\mathcal{B}} + C_{xx}\mathrm{sh}_{t_1}\mathrm{sh}_{t_2}\right) + \mathcal{O}(\epsilon)$$
$$\frac{\mathrm{d}A_{ff}}{\mathrm{d}t_1\mathrm{d}t_2} = N_0\Gamma^2_{B_0\to f}\, e^{-\Gamma(t_1+t_2)}\left(\mathrm{s}_{t_1}\mathrm{s}_{t_2}C_{yy} - \mathrm{c}_{t_1}\mathrm{s}_{t_2}C_{zy} - \mathrm{s}_{t_1}\mathrm{c}_{t_2}C_{yz} + \mathrm{c}_{t_1}\mathrm{c}_{t_2}C_{zz}\right) + \mathcal{O}(\epsilon)$$
$$\frac{\mathrm{d}A_f^{\mathcal{A}}}{\mathrm{d}t_1\mathrm{d}t_2} = N_0\Gamma^2_{B_0\to f}\, e^{-\Gamma(t_1+t_2)}\left(\mathrm{ch}_{t_2}(\mathrm{c}_{t_1}R_z^{\mathcal{A}} - \mathrm{s}_{t_1}R_y^{\mathcal{A}}) - \mathrm{sh}_{t_2}(\mathrm{c}_{t_1}C_{zx} - \mathrm{s}_{t_1}C_{yx})\right) + \mathcal{O}(\epsilon)$$
$$\frac{\mathrm{d}A_f^{\mathcal{B}}}{\mathrm{d}t_1\mathrm{d}t_2} = N_0\Gamma^2_{B_0\to f}\, e^{-\Gamma(t_1+t_2)}\left(\mathrm{ch}_{t_1}(\mathrm{c}_{t_2}R_z^{\mathcal{B}} - \mathrm{s}_{t_2}R_y^{\mathcal{B}}) - \mathrm{sh}_{t_1}(\mathrm{c}_{t_2}C_{xz} - \mathrm{s}_{t_2}C_{xy})\right) + \mathcal{O}(\epsilon)$$

($\boldsymbol{\varepsilon}$ is the CP-violation parameter -- very small)

Reconstruction of quantum tomography



(a) First sub-figure



(b) Second sub-figure

	$B_s^0 \bar{B}_s^0$ fitted	$K^0 \bar{K}^0$ fitted	Obs.
$R_x^{\mathcal{A}}$	-0.01 ± 0.06	-0.002 ± 0.006	$N_{\rm tot}$
$R_x^{\mathcal{B}}$	-0.01 ± 0.06	-0.003 ± 0.006	
$R_y^{\mathcal{A}}$	0.000 ± 0.003	0.005 ± 0.006	$A_f^{\mathcal{A}}$
$R_z^{\mathcal{A}}$	0.000 ± 0.003	0.003 ± 0.005	
$R_y^{\mathcal{B}}$	0.000 ± 0.003	0.005 ± 0.006	$A_f^{\mathcal{B}}$
$R_z^{\mathcal{B}}$	0.001 ± 0.003	0.002 ± 0.004	
C_{xx}	-1.2 ± 1.0	-1.005 ± 0.012	$N_{\rm tot}$
C_{yx}	0.00 ± 0.06	0.005 ± 0.008	$A_f^{\mathcal{A}}$
C_{zx}	0.00 ± 0.05	0.006 ± 0.006	
C_{xy}	0.00 ± 0.05	0.006 ± 0.007	$A_f^{\mathcal{B}}$
C_{xz}	0.00 ± 0.05	0.004 ± 0.006	
C_{yy}	-1.001 ± 0.004	-1.003 ± 0.008	A_{ff}
C_{yz}	0.001 ± 0.003	0.000 ± 0.007	
C_{zy}	0.000 ± 0.003	0.000 ± 0.006	
C_{zz}	-1.000 ± 0.003	-1.001 ± 0.003	
Concurrence	1.1 ± 0.5	1.005 ± 0.007	

TABLE I. The central value and statistic uncertainty of Bloch vectors and correlation matrix of $B_s^0 \bar{B}_s^0$ and $K^0 \bar{K}^0$ when produced, both fitted from 10^6 semi-leptonic decay events, together with the observables that give the best sensitivity. The statistical uncertainties scale as $1/\sqrt{N}$.

Discussions:

- The complete flavor density matrix of meson pair can be reconstructed!
  - We can ask about concurrence, Bell, discord and magic, etc.
  - Larger $\Delta m/\Gamma$, better sensitvity on y, z components.
  - Larger $\Delta\Gamma/\Gamma$, better senstivity on x components

	B_s^0	B_d^0	D^0	K^0
$\Delta m/\text{ps}^{-1}$	17.76	0.506	9.2×10^{-3}	5.29×10^{-3}
Γ/ps^{-1}	0.662	0.658	2.44	5.59×10^{-3}
$\Delta\Gamma/\text{ps}^{-1}$	0.082	2.6×10^{-3}	0.030	0.0111

Belle II ← complementarity → LHCb

- Large data sample B_s, B_d : ~ $5\text{x}10^{10}$
- well-defined state $\Upsilon(4s),\Upsilon(5s)$

- Larger data sample
- Many bb pairs in detectable region
- Not well-defined states fragmentation?

CP violation effects:

- Rather small on quantum entanglement
- Additional information, combine with channels
- More precision

Conclusions

- Collider experiments produce a large number of quantum states at the unprecedented energy regime.
- Various quantum numbers to explore: spin, flavor …
- Hope to establish the quantum tomography & QI: Entanglement, Bell variables, Discord, Magic …
- Understand quantum & seek for BSM effects.

Quantum Information meets High-Energy Physics: Input to the update of the European Strategy for Particle Physics

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QIS @ colliders → new observables & rich physics!

Backups

Collapse the superposition

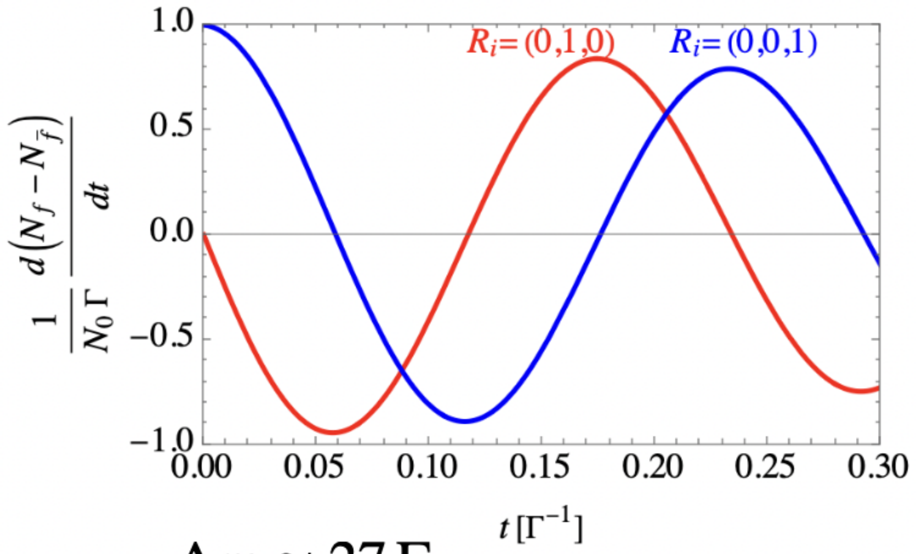
- Two kinds of decay final states, CP eigenstate or not
  - $M \to f$ with $CP\,|f\rangle = |\bar{f}\rangle \neq |f\rangle$:   project to $|M\rangle, |\bar{M}\rangle$ with $P_{M/\bar{M}} = \dfrac{1 \pm R_z}{2}$
- Decay rate asymmetry to $f, \bar{f}$:

$$\Gamma_{M(t)\to f/\bar{f}} = \frac{1 \pm R_z(t)}{2}\Gamma_{M\to f} \qquad (N_f - N_{\bar{f}}) \sim \langle \sigma_z \rangle = R_z$$

- Observable:

$$\frac{1}{N_0}\frac{d(N_f - N_{\bar{f}})}{dt} = e^{-\Gamma t}\left(R_z \cos(\Delta m t) - R_y \sin(\Delta m t)\right)\Gamma_{M\to f}$$

Meson flavor state when it is produced, $t = 0$



$\Delta m \approx 27\,\Gamma$,
slightly damped oscillation

- Both R_y and R_z are obtained as they oscillated into each other.

Observables in semileptonic decay channel

- Reconstruct ρ_{MM} at $t = 0$.

$$\rho_{MM} = \frac{\mathbb{1}_4 + R_i^{\mathcal{A}} \sigma_i \otimes \mathbb{1}_2 + R_i^{\mathcal{B}} \mathbb{1}_2 \otimes \sigma_i + C_{ij} \sigma_i \otimes \sigma_j}{4}$$

- One meson:
  - $N_f - N_{\bar{f}} \Longrightarrow R_y, R_z$
  - $N_f + N_{\bar{f}} \Longrightarrow R_x$

$$\frac{1}{N_0} \frac{d(N_f - N_{\bar{f}})}{dt} = e^{-\Gamma t} \left(R_z \cos(\Delta m t) - R_y \sin(\Delta m t) \right) \Gamma_{M \to f}$$

$$\frac{1}{N_0} \frac{d(N_f + N_{\bar{f}})}{dt} = e^{-\Gamma t} (\cosh(\Delta \Gamma t/2) - R_x \sinh(\Delta \Gamma t/2)) \Gamma_{M \to f}$$

- Meson pair:
  - Four observables from the correlation between the above two

$\mathscr{H}_A \otimes \mathscr{H}_B$ $\quad$ $N_{f\bar{f}}$: meson A decay to f **and** meson B decay to $\bar{f}$

$I_2 \otimes I_2 \longrightarrow N_{\text{tot}} = N_{ff} + N_{\bar{f}f} + N_{f\bar{f}} + N_{\bar{f}\bar{f}}$

$\sigma_z \otimes \sigma_z \longrightarrow A_{ff} = N_{ff} - N_{\bar{f}f} - N_{f\bar{f}} + N_{\bar{f}\bar{f}} = N_{\text{like}} - N_{\text{unlike}}$

$\sigma_z \otimes I_2 \longrightarrow A_f^{\mathcal{A}} = N_{ff} + N_{f\bar{f}} - N_{\bar{f}f} - N_{\bar{f}\bar{f}}$

$I_2 \otimes \sigma_z \longrightarrow A_f^{\mathcal{B}} = N_{ff} - N_{f\bar{f}} + N_{\bar{f}f} - N_{\bar{f}\bar{f}}$

Collapse the superposition

— x direction in the Bloch vector space

- Conside a meson that only decay to flavor eigenstate $|f\rangle$ (such as semileptonic) or CP-even eigenstate $|f_+\rangle$

$$\Gamma_{M(t)\to f_+} = \frac{1+R_x(t)}{2}\Gamma_{M_1\to f_+} = (1+R_x(t))\Gamma_{M\to f_+}$$

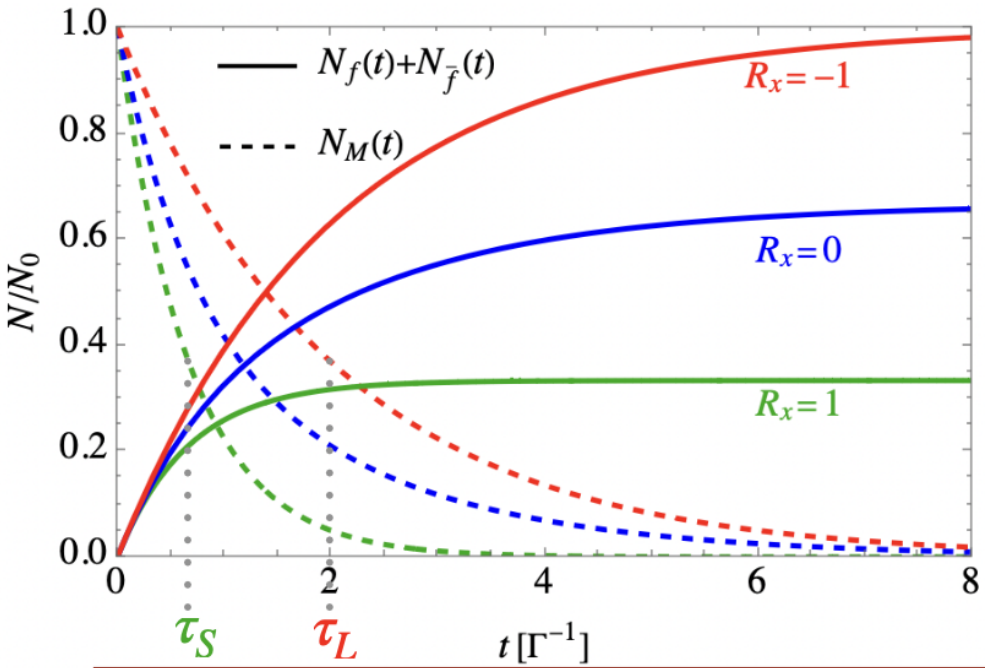
- Both $M(t) \to f_+$ and $M(t) \to f/\bar{f}$ depend on R_x

affected by branching fraction

  - $R_x = 1$, M_1 can decay to f_+
  - $R_x = -1$, M_2 can't decay to f_+, more f

$$\frac{1}{N_0}\frac{d(N_f+N_{\bar{f}})}{dt} = e^{-\Gamma t}(\cosh(\Delta\Gamma t/2) - R_x \sinh(\Delta\Gamma t/2))\Gamma_{M\to f}$$

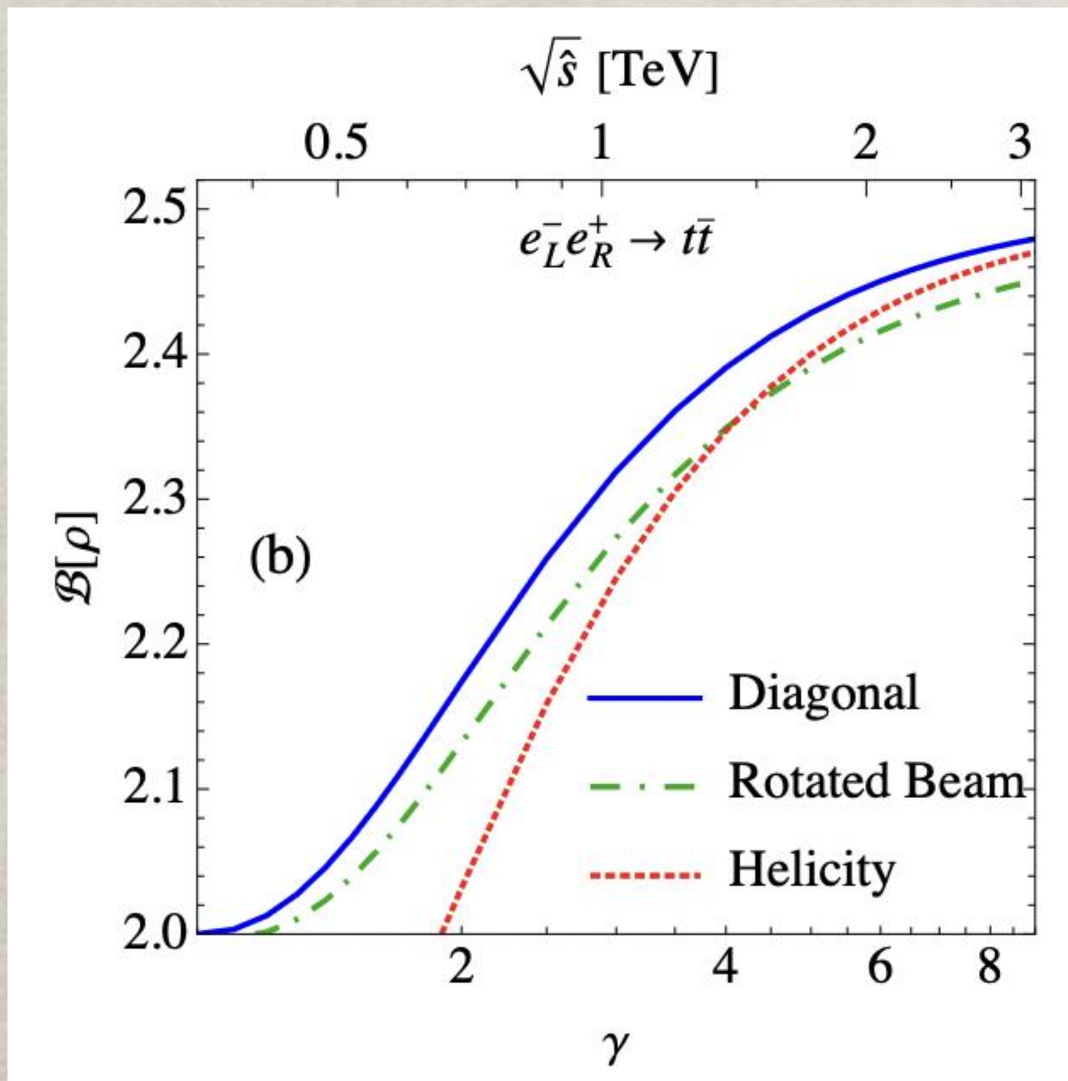
- Semi-leptonic channel is enough. e.g. $B_s \to \ell^+\nu_\ell X + h.c.$

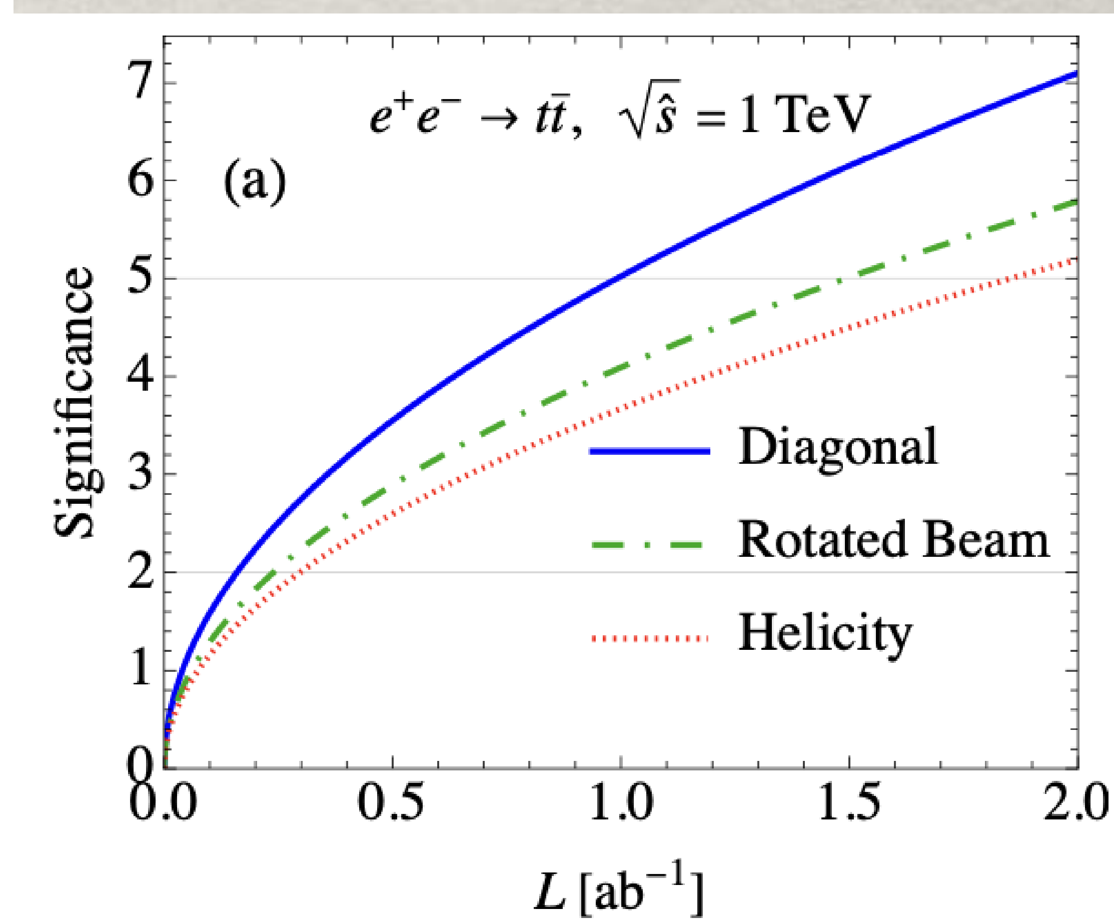


difference between each lines $\propto$ $\Delta\Gamma$
i.e., decay to CP eigenstates.

Kinematic approach equally applicable to $e^+e^- \rightarrow f\bar{f}$

$$C_{ij} \sim \begin{pmatrix} s_\theta^2\left(f_V^2(2-\beta^2) - f_A^2\beta^2\right) & 0 & -2(f_V^2 c_\theta \pm f_V f_A \beta) s_\theta \sqrt{1-\beta^2} \\ 0 & (f_A^2 - f_V^2)\beta^2 s_\theta^2 & 0 \\ -2(f_V^2 c_\theta \pm f_V f_A \beta) s_\theta \sqrt{1-\beta^2} & 0 & f_V^2(2c_\theta^2 + \beta^2 s_\theta^2) + f_A^2\beta^2(1+c_\theta^2) \pm 4 f_V f_A \beta \end{pmatrix}$$





Observability: Error Estimation

Kinematic Approach:

$$\frac{\Delta\mathcal{C}}{\mathcal{C}} \leq \frac{1}{4\langle F_{\mathcal{C}}(\Theta,\beta)\rangle}\frac{1}{\sqrt{N}}$$

$$\frac{\Delta\mathcal{B}}{\mathcal{B}} \leq \frac{1}{\langle F_{\mathcal{B}}(\Theta,\beta)\rangle}\frac{1}{\sqrt{N}}.$$

Decay Approach:

$$\frac{\Delta\mathcal{C}}{\mathcal{C}} = \frac{3\sqrt{3}}{(|C_{11}+C_{22}|-C_{33}-1)}\frac{1}{\sqrt{N}}$$

$$\frac{\Delta\mathcal{B}}{\mathcal{B}} = \frac{3\sqrt{2}}{|C_{22}-C_{33}|}\frac{1}{\sqrt{N}}.$$

	cuts	$\mathcal{C}$	$\Delta\mathcal{C}^{\text{stat}}$	$\mathcal{B}$	$\Delta\mathcal{B}^{\text{stat}}$
Production	$m_{t\bar{t}} < 350\,\text{GeV}$	0.45	1.0×10^{-4}	2.04	1.4×10^{-4}
Decay			1.4×10^{-2}		6.3×10^{-2}
Production	$m_{t\bar{t}} > 1.5\,\text{TeV}$	0.70	2.6×10^{-3}	2.37	3.8×10^{-3}
Decay	$\lvert\cos\Theta\rvert < 0.5$		5.6×10^{-2}		0.26

TABLE I. Statistical uncertainties on the $\mathcal{C}$ and $\mathcal{B}$ measurements for $pp\to t\bar{t}$ with represetative selection cuts. The production rate is given by $N_{t\bar{t}} = \mathcal{L}\sigma_{pp\to t\bar{t}}$ with 300 fb^{-1} luminosity. The di-leptonic decay branching fractions are included in the decay approach without other kinematic cuts.

Kinematic approach is optimal!

Ultimately, systematic dominance!

perhaps ~ 1% level