BEYOND FLAVOUR PHYSICS

SHADOW TOMOGRAPHY FOR COLLIDER EXPERIMENTS

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HCN, G. Tetlalmatzi-Xolocotzi, C. Diez Pardos, O. Gühne, M. Kleinman, *in preparation*

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Article

Observation of quantum entanglement with top quarks at the ATLAS detector

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The ATLAS Collaboration*✉

Entanglement is a key feature of quantum mechanics[1–3], with applications in fields such as metrology, cryptography, quantum information and quantum computation[4–8]. It has been observed in a wide variety of systems and length scales,

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Quantum Entropy and Special Relativity

Asher Peres, Petra F. Scudo, and Daniel R. Terno
Department of Physics, Technion–Israel Institute of Technology, 32000 Haifa, Israel
(Received 7 March 2002; published 22 May 2002)

We consider a single free spin-$\frac{1}{2}$ particle. The reduced density matrix for its spin is not covariant under Lorentz transformations. The spin entropy is not a relativistic scalar and has no invariant meaning.

DOI: 10.1103/PhysRevLett.88.230402 PACS numbers: 03.65.Ta, 03.30.+p

... entanglement is eventually dependent on the reference frame

see also Peres and Terno, RMP 2004

Article

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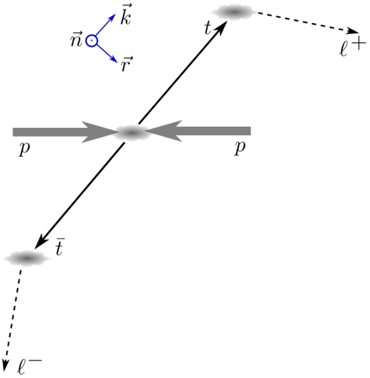


ments as matter–antimatter pairs. A pair of top–antitop quarks ($t\bar{t}$) is a two-qubit system in which the spin quantum state is described by the spin density matrix ρ:

$$\rho = \frac{1}{4}\left[I_4 + \sum_i \left(B_i^+ \sigma^i \otimes I_2 + B_i^- I_2 \otimes \sigma^i\right) + \sum_{i,j} C_{ij} \sigma^i \otimes \sigma^j \right].$$

[…]

a way that the normalized differential cross-section (σ) of the process may be written as[27]

$$\frac{1}{\sigma}\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega_+ \mathrm{d}\Omega_-} = \frac{1 + \mathbf{B}^+ \cdot \hat{\mathbf{q}}_+ - \mathbf{B}^- \cdot \hat{\mathbf{q}}_- - \hat{\mathbf{q}}_+ \cdot C \cdot \hat{\mathbf{q}}_-}{(4\pi)^2},$$

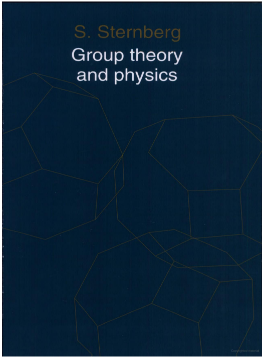
where $\hat{\mathbf{q}}_+$ is the antilepton direction in the rest frame of its parent top quark and $\hat{\mathbf{q}}_-$ is the lepton direction in the rest frame of its parent antitop quark; and Ω_+ is the solid angle associated with the antilepton and Ω_- is the solid angle associated with the lepton. The vectors $\mathbf{B}^\pm$ deter-

… some attention

Transformation of the spin density operator?

Not **entanglement**, rather **the reduced density operator**

$\rho \to ???$



No non-trivial finite dimensional unitary rep. of the Lorentz group

Transformation of spin and momentum

Under the Lorentz transformation

$$|p\rangle \otimes |s\rangle \to |\Lambda p\rangle \otimes D_\sigma[W(\Lambda, p)]\,|s\rangle$$

with the Wigner's rotation

$$W(\Lambda, p) = L^{-1}(\Lambda p)\Lambda L(p)$$

where $L(p)$ boosts a rest particle to four-momentum p

F. Scheck, *Quantum Field Theory* (Springer)

For a pair of spin

For each pair of momenta (p_1, p_2) one has a density operator

$$\rho^{p_1,p_2}$$

which transform under Λ to

$$(D_\sigma[W(\Lambda, p_1)] \otimes D_\sigma[W(\Lambda, p_2)])\rho^{p_1,p_2}(D^\dagger_\sigma[W(\Lambda, p_1)] \otimes D^\dagger_\sigma[W(\Lambda, p_2)])$$

In QI, local unitary, entanglement is conserved

Reduced density operator in a new frame

The reduced spin density operator

$$\int \mathrm{d}p_1 \mathrm{d}p_2 \rho^{p_1,p_2}$$

transforms under Λ as

$$\int \mathrm{d}p_1 \mathrm{d}p_2 (D_\sigma[W(\Lambda, p_1)] \otimes D_\sigma[W(\Lambda, p_2)]) \rho^{p_1,p_2} (D_\sigma^\dagger[W(\Lambda, p_1)] \otimes D_\sigma^\dagger[W(\Lambda, p_2)])$$

Not a group transformation, entanglement is also not conserved

see also Peres and Terno, RMP 2004

... this trouble only begins when one **traces out** available momenta

Spin-momentum ensemble

Instead of tracing out the momenta, consider (unnormalised) ensemble

$$\{\rho^{p_1,p_2}\}$$

of spin density operators as **a whole**

- momenta available anyway
- entanglement is conserved under Lorentz transform

All momenta-dependent observables X^{p_1,p_2} can be estimated as

$$\langle X \rangle = \int \mathrm{d}p_1 \mathrm{d}p_2 \, \mathrm{tr}[\rho^{p_1,p_2} X^{p_1,p_2}]$$

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Entanglement of spin-momentum ensemble

Entanglement witness W are defined to be

$$\mathrm{tr}[\rho W] \geq 0$$

for all unentangled states ρ

One can choose different witnesses at different momentum W^{p_1,p_2}, if

$$\int \mathrm{d}p_1 \mathrm{d}p_2 \, \mathrm{tr}[\rho^{p_1,p_2} W^{p_1,p_2}] < 0$$

there is some entanglement in the **ensemble**

Effective density operator

Fixed particular W_0 and choose

$$W^{p_1,p_2} = (U_1^{p_1} \otimes U_2^{p_2})^\dagger W_0 (U_1^{p_1} \otimes U_2^{p_2})$$

then

$$\langle W \rangle = \mathrm{tr}(\rho_{\mathrm{eff}} W_0)$$

where

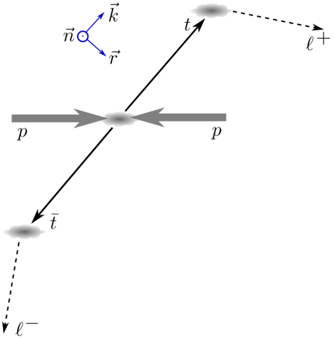
$$\rho_{\mathrm{eff}} = \int \mathrm{d}p_1 \mathrm{d}p_2 (U_1^{p_1} \otimes U_2^{p_2}) \rho^{p_1,p_2} (U_1^{p_1} \otimes U_2^{p_2})^\dagger$$

These include

- ☞ the standard reduced density operator (QI community)
- ☞ center of mass reduced density operator (ATLAS, Nature 2024)
- ☞ 'fictitious' density operators (Afik et al, Quantum 2022)

each can tell about **entanglement of the ensemble**, but **not all**

Characterising the whole ensemble?



Too many density operators?

$$\{\rho^{p_1,p_2}\}$$

Data collection contains **single** events

$$(p_1^{(k)}, p_2^{(k)}, \hat{\mathrm{n}}_1^{(k)}, \hat{\mathrm{n}}_2^{(k)})$$

Tomography not possible!

Shadow tommography of spin-momentum ensemble

ARTICLES
https://doi.org/10.1038/s41567-020-0932-7

nature physics

Check for updates

Predicting many properties of a quantum system from very few measurements

Hsin-Yuan Huang[1,2], Richard Kueng[1,2,3] and John Preskill[1,2,4]

Shadow tomography converts a single shot measurement of spin to a **single shot estimate** of density operator called **classical shadow**

$$\hat{\rho}^{p_1,p_2}_{\hat{n}_1,\hat{\mathrm{n}}_2} = \hat{\rho}^{p_1}_{\hat{\mathrm{n}}_1} \otimes \hat{\rho}^{p_2}_{\hat{\mathrm{n}}_2}$$

Noisy, not positive, but converge rightly when estimating observables

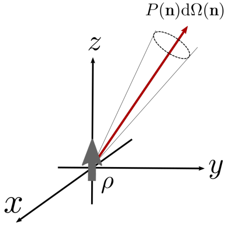
A closer look at the measurement of spin

Decay of a polarised particle at rest

$$P(\hat{\mathrm{n}}) = \mathrm{tr}(\rho F_{\hat{\mathrm{n}}})$$

Covariant under rotation R

$$F_{R\hat{\mathrm{n}}} = D_\sigma[R] F_{\hat{\mathrm{n}}} D_\sigma^\dagger[R]$$



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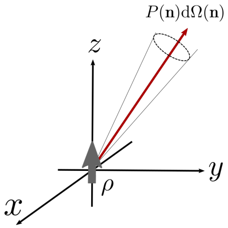
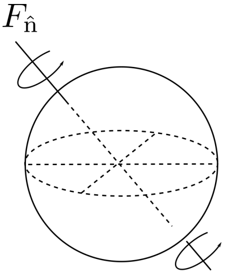
This implies

$$F_{\hat{\mathrm{n}}} = \sum_{m=-\sigma}^{+\sigma} \alpha_m \left|\hat{\mathrm{n}}, m\right\rangle \left\langle\hat{\mathrm{n}}, m\right|$$





HCN et al, `arXiv:2003.12553`

Classical shadows and their symmetry

Abtract definition of classical shadows

$$\int \mathrm{d}\Omega(\hat{\mathrm{n}})\, \mathrm{tr}(\rho F_{\hat{\mathrm{n}}})\hat{\rho}_{\hat{\mathrm{n}}} = \rho$$

for all ρ. Notice

- flexible in choices
- general not positive
- averaging converges to ρ

The classical shadow can inherit the same symmetry as measurement

$$\hat{\rho}_{\hat{\mathrm{n}}} = \sum_{m=-\sigma}^{+\sigma} \beta_m \left|\hat{\mathrm{n}}, m\right\rangle \left\langle\hat{\mathrm{n}}, m\right|$$

where β_m can be explicitly computed from α_m

HCN et al, PRL 2022

Use of classical shadows

- Tomography of all effective density operators

$$\rho_{\mathrm{eff}} \approx \frac{1}{M} \sum_k (U_1^{p_1^{(k)}} \otimes U_2^{p_2^{(k)}}) \hat{\rho}_{\hat{\mathrm{n}}_1^{(k)}, \hat{\mathrm{n}}_2^{(k)}}^{p_1^{(k)}, p_2^{(k)}} (U_1^{p_1^{(k)}} \otimes U_2^{p_2^{(k)}})^\dagger$$

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- Estimate of momentum-dependent spin observable X^{p_1,p_2}

$$\langle X\rangle \approx \frac{1}{M}\sum_k \mathrm{tr}(\hat{\rho}^{p_1^{(k)},p_2^{(k)}}_{\hat{\mathrm{n}}_1^{(k)},\hat{\mathrm{n}}_2^{(k)}} X^{p_1^{(k)},p_2^{(k)}})$$

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- Test entanglement in the whole ensemble

Use of classical shadows

- Tomography of all effective density operators

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- Test entanglement in the whole ensemble
- If events come from different channels $I \in \{1, 2\}$, one can classify classical shadows $\hat{\rho}^{p_1,p_2}_{\hat{\text{n}}_1,\hat{\text{n}}_2}$ to the most probable channel

Summary & outlook

- ☞ Tracing out available momenta brings conceptual difficulty
- ☞ Shadow tomography allows for processing full information
- ☞ Constraints/properties of ρ^{p_1,p_2} can be analysed as a whole

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The abstract relationship of QI and HEP

- it is not only about entanglement, it is about **density operators**
- it is not only about observables, but **methods of processing**

HCN, G. Tetlalmatzi-Xolocotzi, C. Diez Pardos, O. Gühne, M. Kleinman, *in preparation*

Thank you for your attention!