

A Precise Determination of α_s from the Heavy Jet Mass Distribution

Miguel Benitez



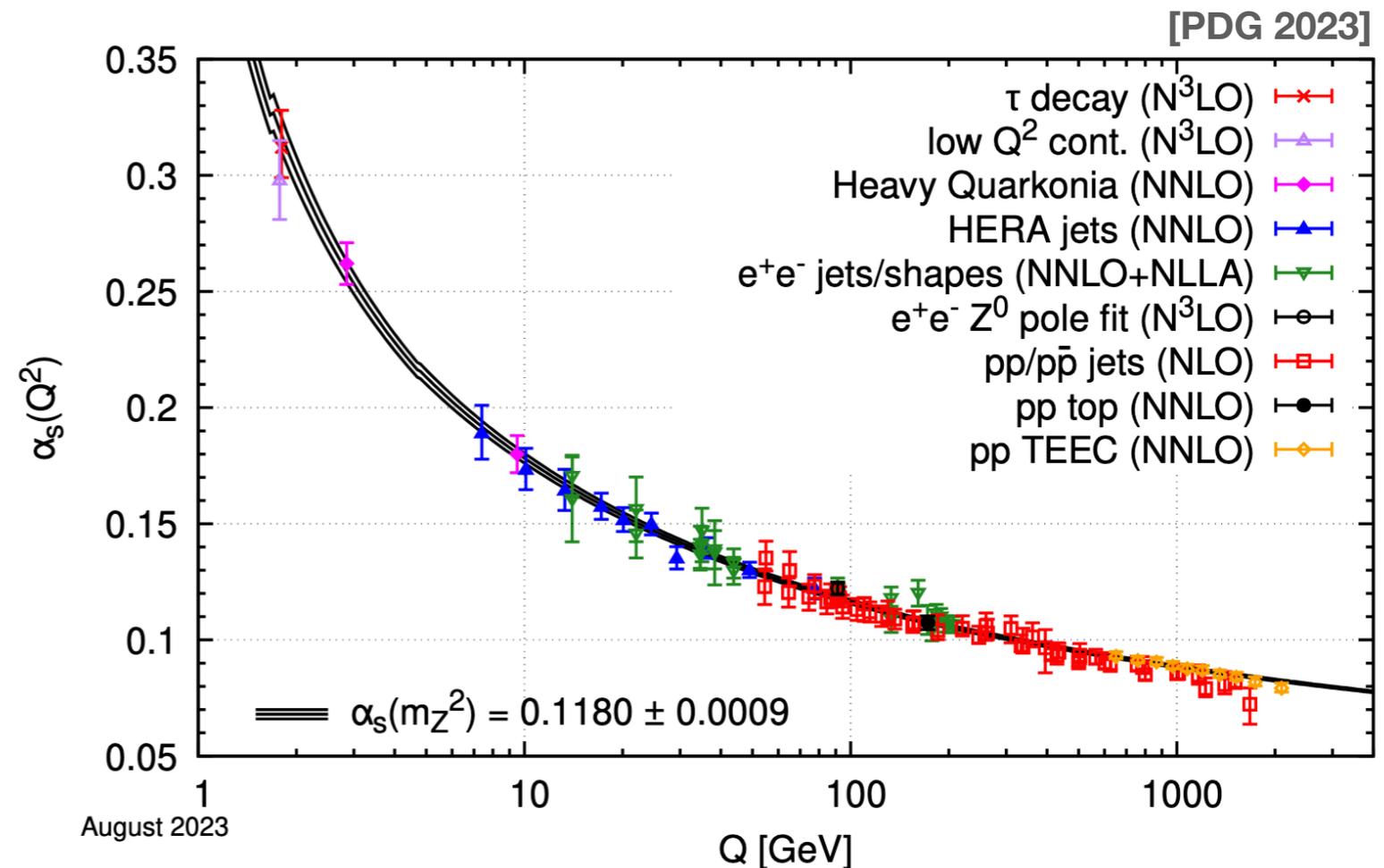
IUFFyM
Instituto Universitario
de Física Fundamental y Matemáticas

VNiVERSiDAD D SALAMANCA

**MB, Arindam Bhattacharya, André H. Hoang, Vicent Mateu,
Matthew D. Schwartz, Iain W. Stewart and Xiaoyuan Zhang – 2502.12253**

The strong coupling constant – An Overview

- Strong coupling constant α_s a free parameter of the Standard Model
- α_s runs with energy \rightarrow allows to relate extractions at different Q
- Use collider experiments to measure it
- Current precision at the percent level
(compare to QED coupling – known to 10^{-8})
- PDG world average represents combination of results that meet criteria
- Current world average

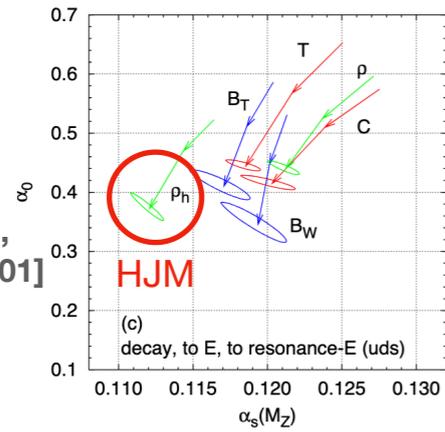


$$\alpha_s(m_Z^2) = 0.1180 \pm 0.0009 \quad (\text{PDG 2023 average})$$

α_s from e^+e^- event shapes

[See also prior work of Catani, Trentadue, Turnock, Webber 1993]

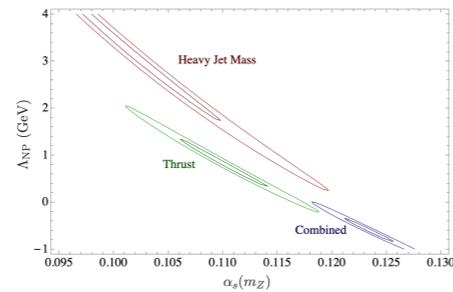
[Salam, Wicke 2001]



Secondly fits for the heavy-jet mass (a very non-inclusive variable) lead to values for α_s which are about 10% smaller than for inclusive variables like the thrust or the mean jet mass. This needs to be understood. It could be due to a difference in the behaviour of the perturbation series at higher orders.

[See also Dissertori et al 2007]

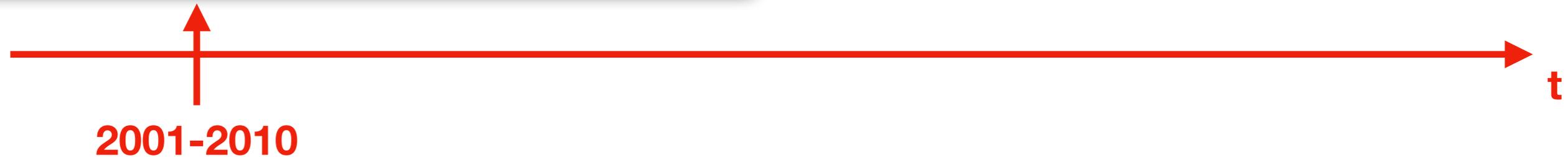
NNLL resummation with NNLO matching



Event Shape	$\alpha_s(m_Z)$	Λ_{NP} (GeV)	$\chi^2/\text{d.o.f.}$
Thrust	0.1101	0.821	66.9/47
Heavy Jet Mass	0.1017	3.17	60.4/43
Combined	0.1236	-0.621	453/92

[Chien, Schwartz 2010]

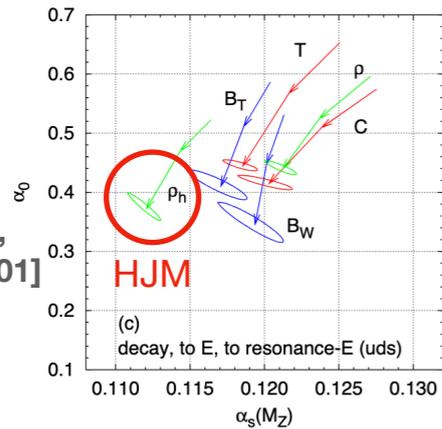
These studies used, in parts, only 25% of data bins:
 $0.08 < \rho < 0.18$



α_s from e^+e^- event shapes

[See also prior work of Catani, Trentadue, Turnock, Webber 1993]

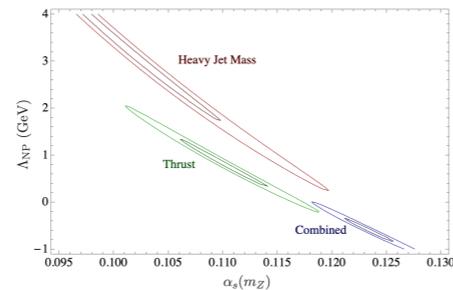
[Salam, Wicke 2001]



Secondly fits for the heavy-jet mass (a very non-inclusive variable) lead to values for α_s which are about 10% smaller than for inclusive variables like the thrust or the mean jet mass. This needs to be understood. It could be due to a difference in the behaviour of the perturbation series at higher orders.

[See also Dissertori et al 2007]

NNLL resummation with NNLO matching



Event Shape	$\alpha_s(m_Z)$	Λ_{NP} (GeV)	$\chi^2/\text{d.o.f.}$
Thrust	0.1101	0.821	66.9/47
Heavy Jet Mass	0.1017	3.17	60.4/43
Combined	0.1236	-0.621	453/92

[Chien, Schwartz 2010]

These studies used, in parts, only 25% of data bins:
 $0.08 < \rho < 0.18$



Thrust at N3LL with Power Corrections and a Precision Global Fit for $\alpha_s(m_Z)$

[Abbate, Fickinger, Hoang, Mateu, Stewart 2010]

Power corrections in the dispersive model for a determination of the strong coupling constant from the thrust distribution

[Gehrmann, Luisoni, Monni 2012]

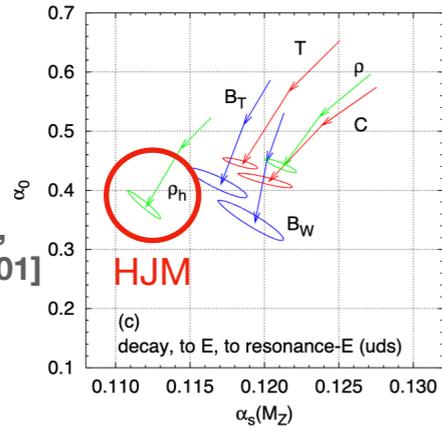
Precise Determination of α_s from the C-parameter Distribution

[Hoang, Kolodrubetz, Mateu, Stewart 2015]

α_s from e^+e^- event shapes

[See also prior work of Catani, Trentadue, Turnock, Webber 1993]

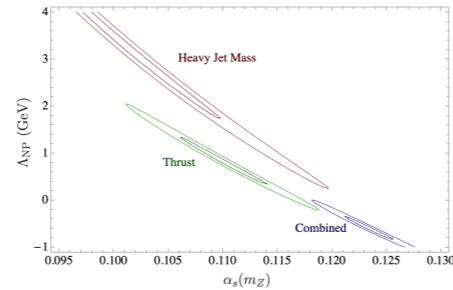
[Salam, Wicke 2001]



Secondly fits for the heavy-jet mass (a very non-inclusive variable) lead to values for α_s which are about 10% smaller than for inclusive variables like the thrust or the mean jet mass. This needs to be understood. It could be due to a difference in the behaviour of the perturbation series at higher orders.

[See also Dissertori et al 2007]

NNLL resummation with NNLO matching

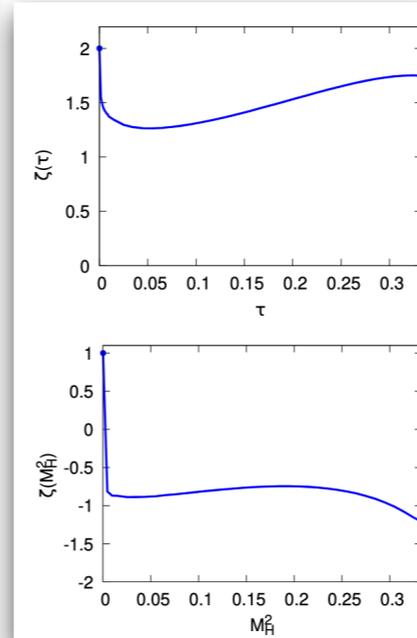


Event Shape	$\alpha_s(m_Z)$	Λ_{NP} (GeV)	$\chi^2/\text{d.o.f.}$
Thrust	0.1101	0.821	66.9/47
Heavy Jet Mass	0.1017	3.17	60.4/43
Combined	0.1236	-0.621	453/92

[Chien, Schwartz 2010]

These studies used, in parts, only 25% of data bins:
 $0.08 < \rho < 0.18$

[Caola et al. 2021, 2022]
[Nason, Zanderighi 2023, 2025]



Thrust at N3LL with Power Corrections and a Precision Global Fit for $\alpha_s(m_Z)$

[Abbate, Fickinger, Hoang, Mateu, Stewart 2010]

Power corrections in the dispersive model for a determination of the strong coupling constant from the thrust distribution

[Gehrmann, Luisoni, Monni 2012]

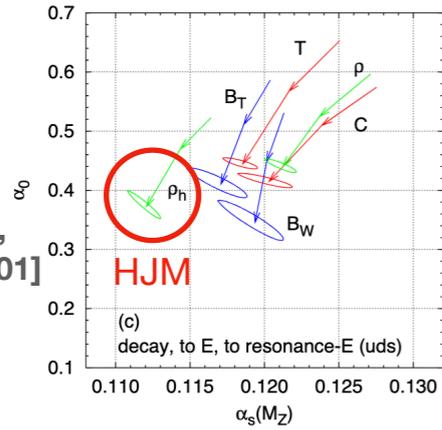
Precise Determination of α_s from the C-parameter Distribution

[Hoang, Kolodrubetz, Mateu, Stewart 2015]

α_s from e^+e^- event shapes

[See also prior work of Catani, Trentadue, Turnock, Webber 1993]

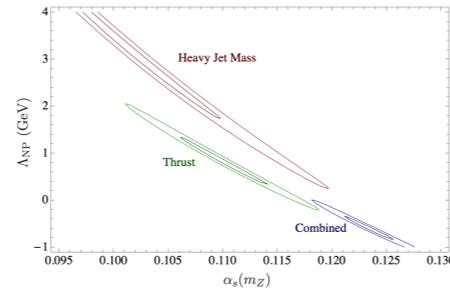
[Salam, Wicke 2001]



Secondly fits for the heavy-jet mass (a very non-inclusive variable) lead to values for α_s which are about 10% smaller than for inclusive variables like the thrust or the mean jet mass. This needs to be understood. It could be due to a difference in the behaviour of the perturbation series at higher orders.

[See also Dissertori et al 2007]

NNLL resummation with NNLO matching

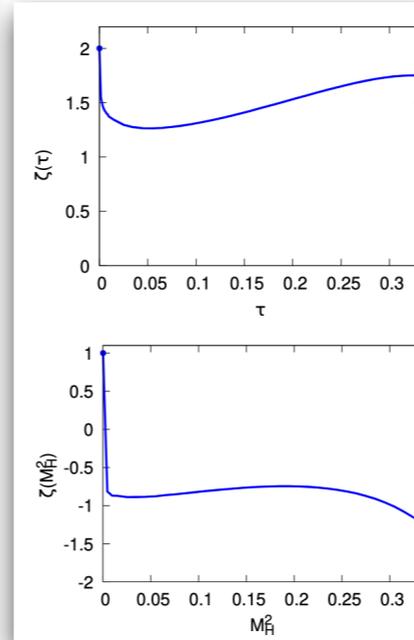


Event Shape	$\alpha_s(m_Z)$	Λ_{NP} (GeV)	$\chi^2/d.o.f.$
Thrust	0.1101	0.821	66.9/47
Heavy Jet Mass	0.1017	3.17	60.4/43
Combined	0.1236	-0.621	453/92

[Chien, Schwartz 2010]

These studies used, in parts, only 25% of data bins: $0.08 < \rho < 0.18$

[Caola et al. 2021, 2022]
[Nason, Zanderighi 2023, 2025]



Thrust at N3LL with Power Corrections and a Precision Global Fit for $\alpha_s(m_Z)$

[Abbate, Fickinger, Hoang, Mateu, Stewart 2010]

Power corrections in the dispersive model for a determination of the strong coupling constant from the thrust distribution

[Gehrmann, Luisoni, Monni 2012]

Precise Determination of α_s from the C-parameter Distribution

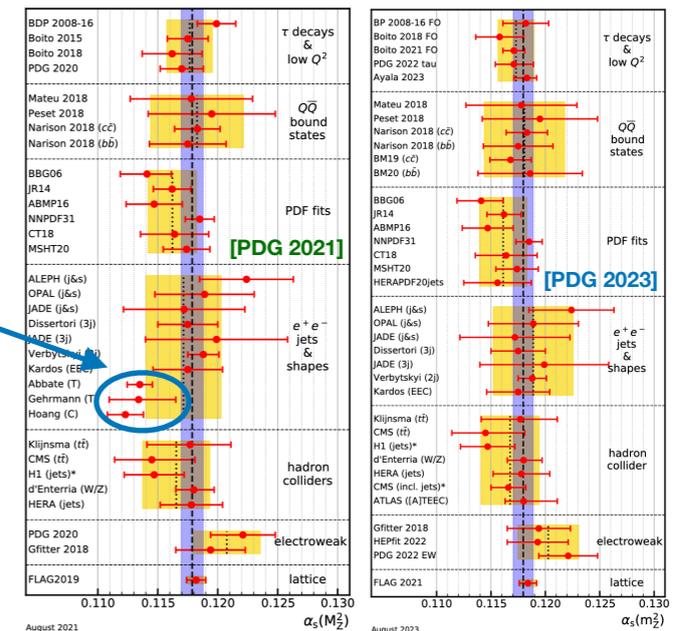
[Hoang, Kolodrubetz, Mateu, Stewart 2015]

The last edition of the PDG removed three extractions from event shapes

[Abbate, Fickinger, Hoang, Mateu, Stewart 2010]

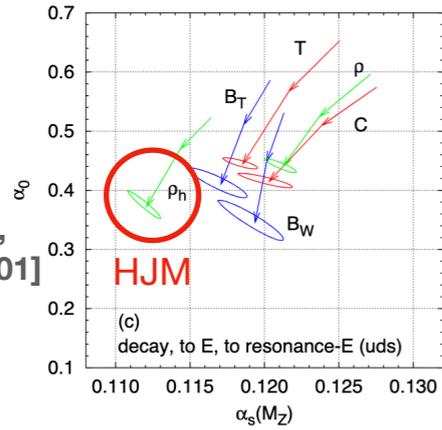
[Gehrmann, Luisoni, Monni 2012]

[Hoang, Kolodrubetz, Mateu, Stewart 2015]



α_s from e^+e^- event shapes

[See also prior work of Catani, Trentadue, Turnock, Webber 1993]

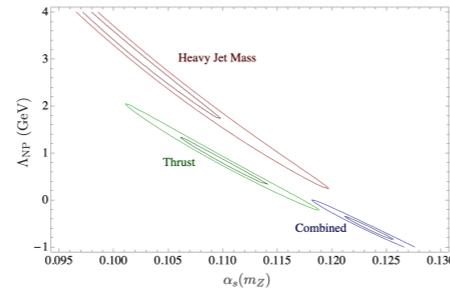


[Salam, Wicke 2001]

Secondly fits for the heavy-jet mass (a very non-inclusive variable) lead to values for α_s which are about 10% smaller than for inclusive variables like the thrust or the mean jet mass. This needs to be understood. It could be due to a difference in the behaviour of the perturbation series at higher orders.

[See also Dissertori et al 2007]

NNLL resummation with NNLO matching

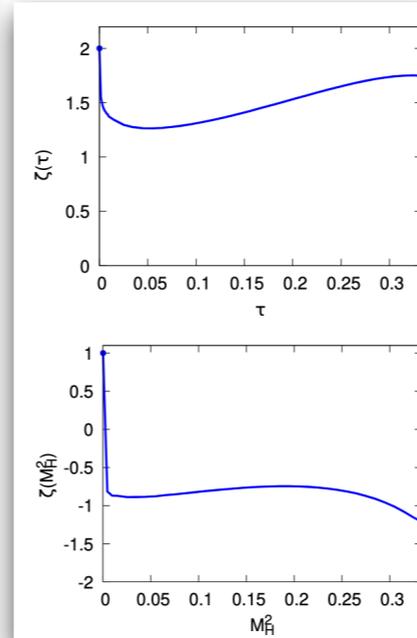


Event Shape	$\alpha_s(m_Z)$	Λ_{NP} (GeV)	$\chi^2/d.o.f.$
Thrust	0.1101	0.821	66.9/47
Heavy Jet Mass	0.1017	3.17	60.4/43
Combined	0.1236	-0.621	453/92

[Chien, Schwartz 2010]

These studies used, in parts, only 25% of data bins: $0.08 < \rho < 0.18$

[Caola et al. 2021, 2022]
[Nason, Zanderighi 2023, 2025]



On Determining $\alpha_s(m_Z)$ from Dijets in e^+e^- Thrust

Miguel A. Benitez^a, André H. Hoang^b, Vicent Mateu^a, Iain W. Stewart^{b,c} and Gherardo Vita^d

A Precise Determination of α_s from the Heavy Jet Mass Distribution

Miguel A. Benitez¹, Arindam Bhattacharya², André H. Hoang³, Vicent Mateu¹, Matthew D. Schwartz², Iain W. Stewart^{3,4} and Xiaoyuan Zhang²

This talk

2001-2010

2010-2015

2023

2023

2024/2025

Thrust at N3LL with Power Corrections and a Precision Global Fit for $\alpha_s(m_Z)$

[Abbate, Fickinger, Hoang, Mateu, Stewart 2010]

Power corrections in the dispersive model for a determination of the strong coupling constant from the thrust distribution

[Gehrmann, Luisoni, Monni 2012]

Precise Determination of α_s from the C-parameter Distribution

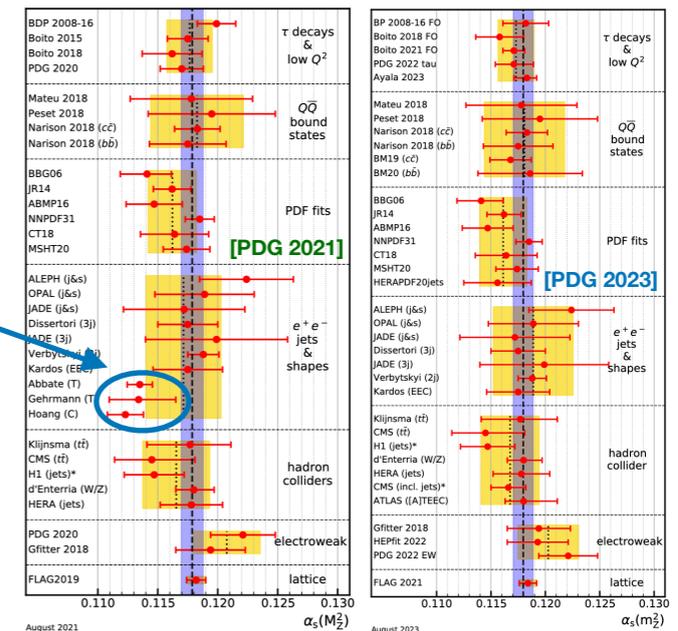
[Hoang, Kolodrubetz, Mateu, Stewart 2015]

The last edition of the PDG removed three extractions from event shapes

[Abbate, Fickinger, Hoang, Mateu, Stewart 2010]

[Gehrmann, Luisoni, Monni 2012]

[Hoang, Kolodrubetz, Mateu, Stewart 2015]



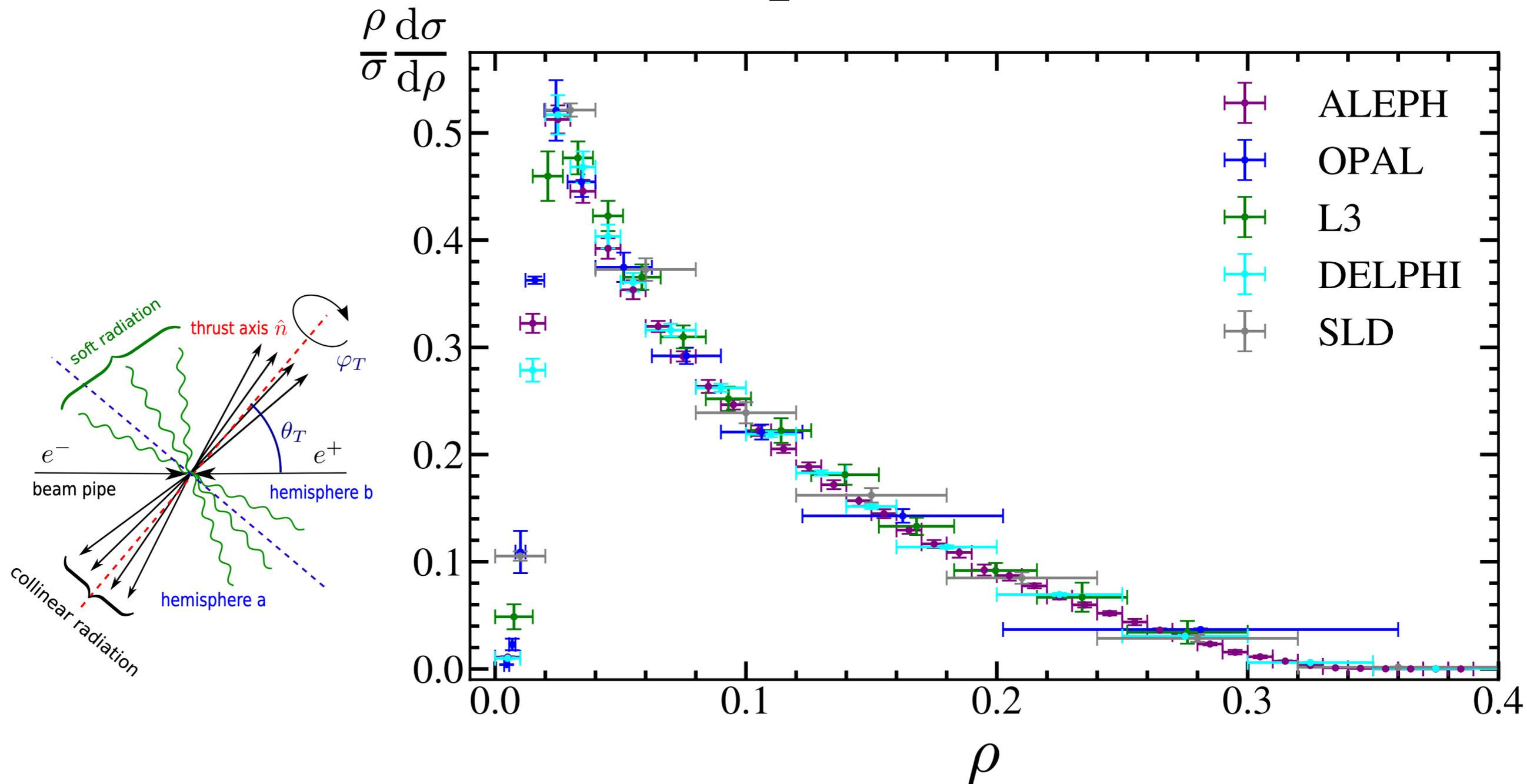
The HJM distribution

- HJM defined as $\rho = \max(s_1, s_2)/Q^2$

[Clavelli 1979] [Chandramohan, Clavelli 1981] [Clavelli, Wyler 1981]
- Experimental Distribution at $Q = m_Z$

$$s_1 = \left(\sum_{i \in L} p_i^\mu \right)^2, \quad s_2 = \left(\sum_{i \in R} p_i^\mu \right)^2$$

Hemisphere jet invariant masses



The HJM distribution

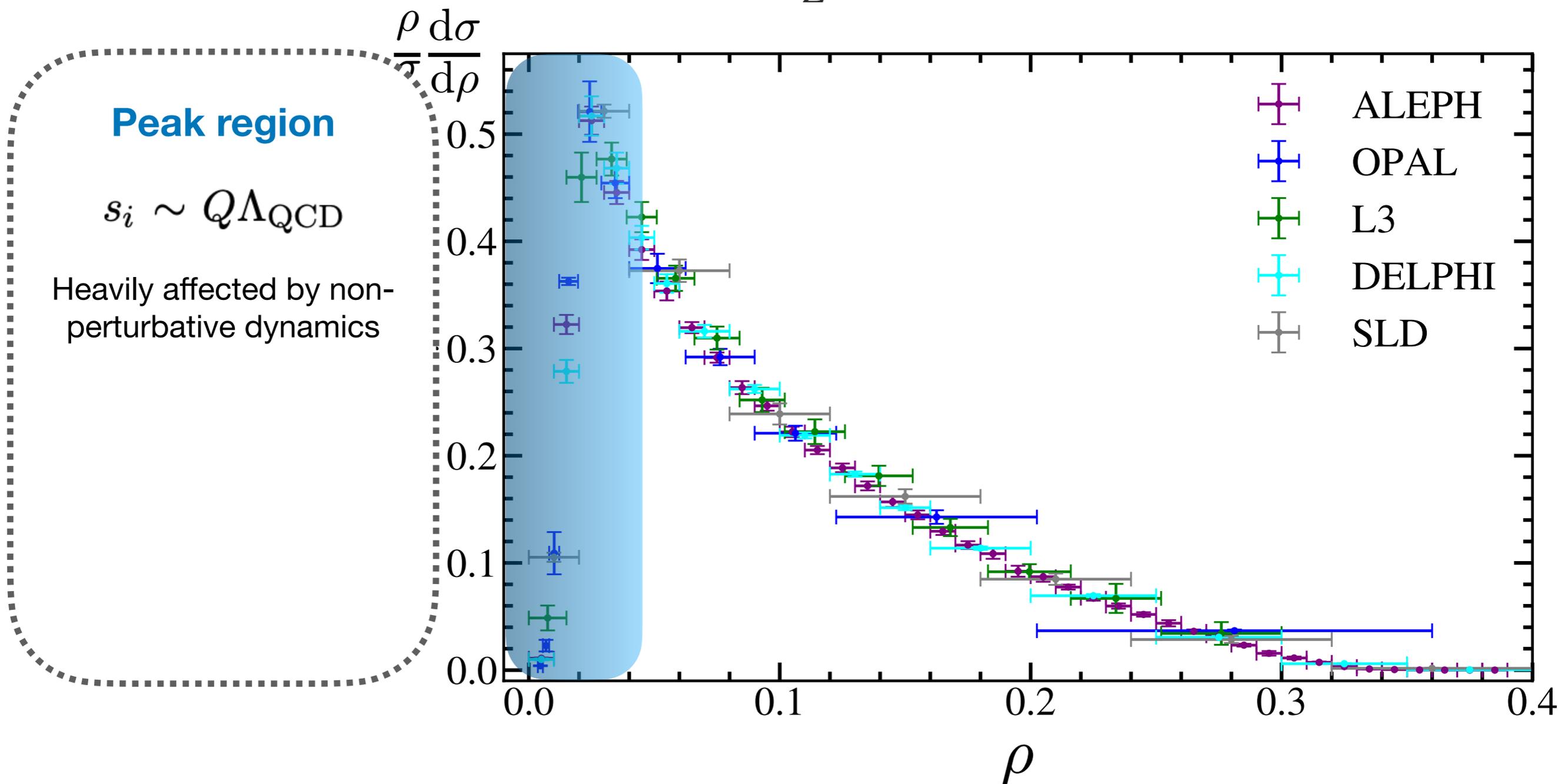
- HJM defined as $\rho = \max(s_1, s_2)/Q^2$

[Clavelli 1979] [Chandramohan, Clavelli 1981] [Clavelli, Wyler 1981]

$$s_1 = \left(\sum_{i \in L} p_i^\mu \right)^2, \quad s_2 = \left(\sum_{i \in R} p_i^\mu \right)^2$$

Hemisphere jet invariant masses

- Experimental Distribution at $Q = m_Z$



The HJM distribution

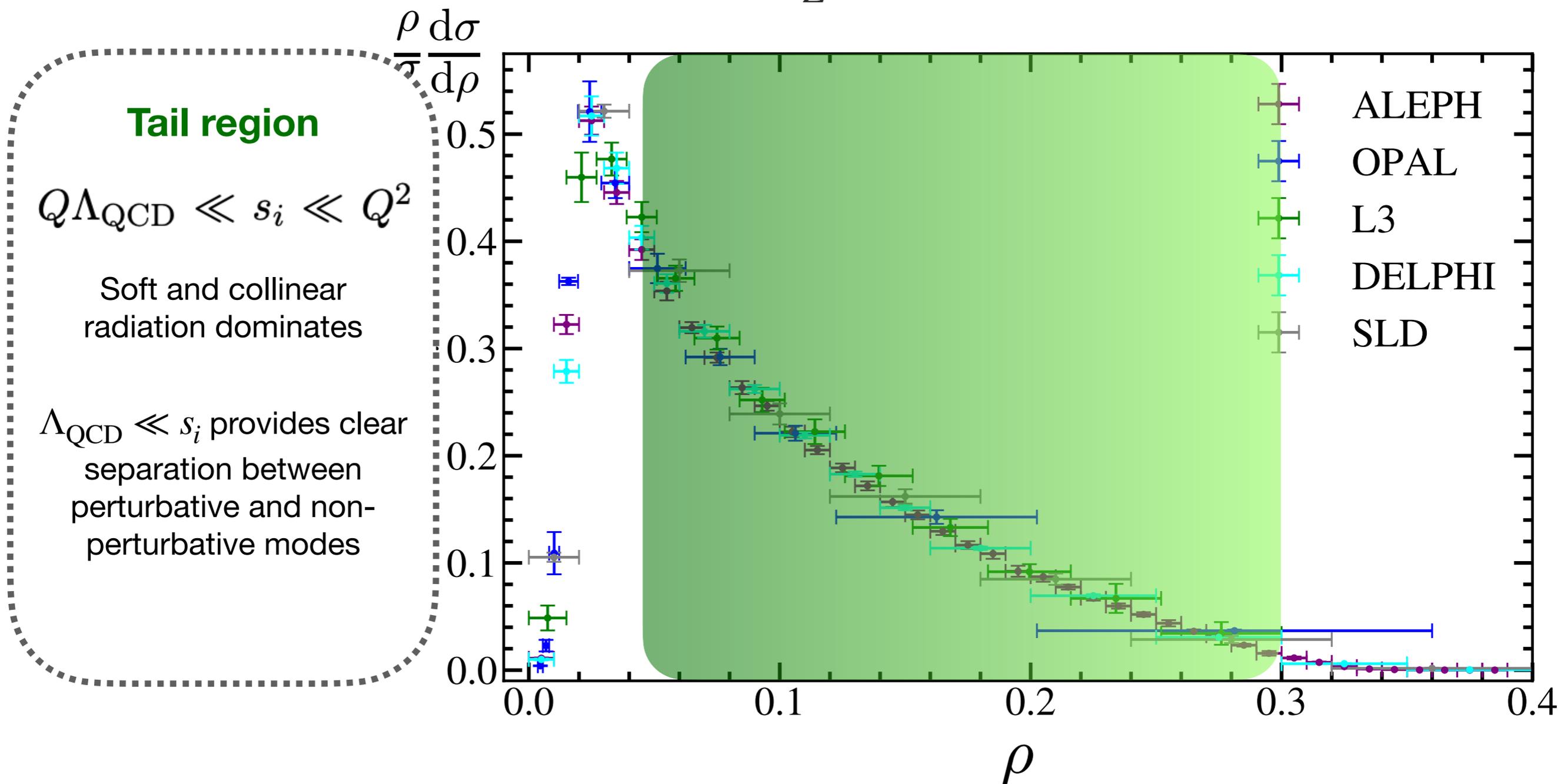
- HJM defined as $\rho = \max(s_1, s_2)/Q^2$

[Clavelli 1979] [Chandramohan, Clavelli 1981] [Clavelli, Wyler 1981]

$$s_1 = \left(\sum_{i \in L} p_i^\mu \right)^2, \quad s_2 = \left(\sum_{i \in R} p_i^\mu \right)^2$$

Hemisphere jet invariant masses

- Experimental Distribution at $Q = m_Z$



The HJM distribution

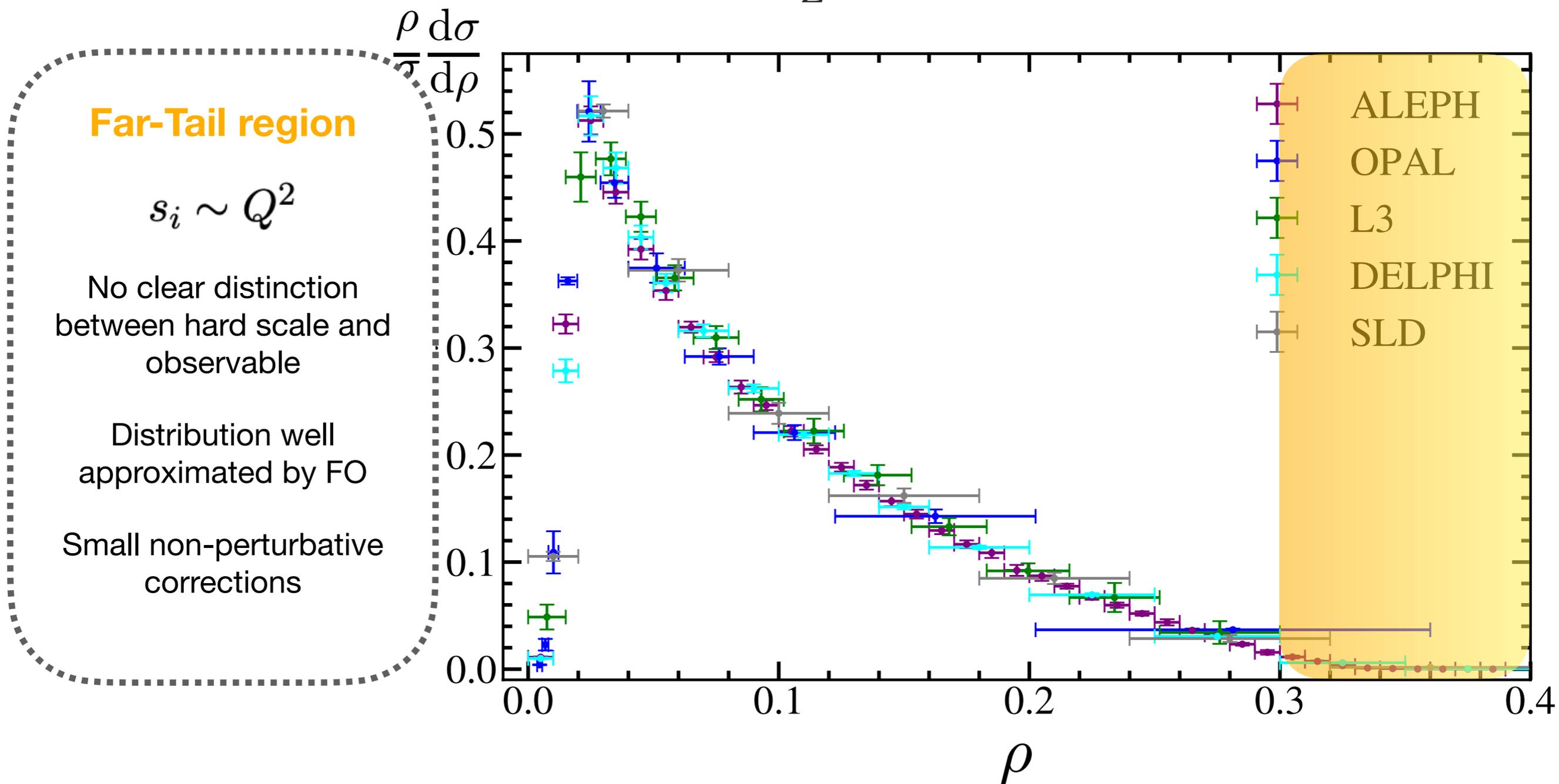
- HJM defined as $\rho = \max(s_1, s_2)/Q^2$

[Clavelli 1979] [Chandramohan, Clavelli 1981] [Clavelli, Wyler 1981]

$$s_1 = \left(\sum_{i \in L} p_i^\mu \right)^2, \quad s_2 = \left(\sum_{i \in R} p_i^\mu \right)^2$$

Hemisphere jet invariant masses

- Experimental Distribution at $Q = m_Z$



The HJM distribution

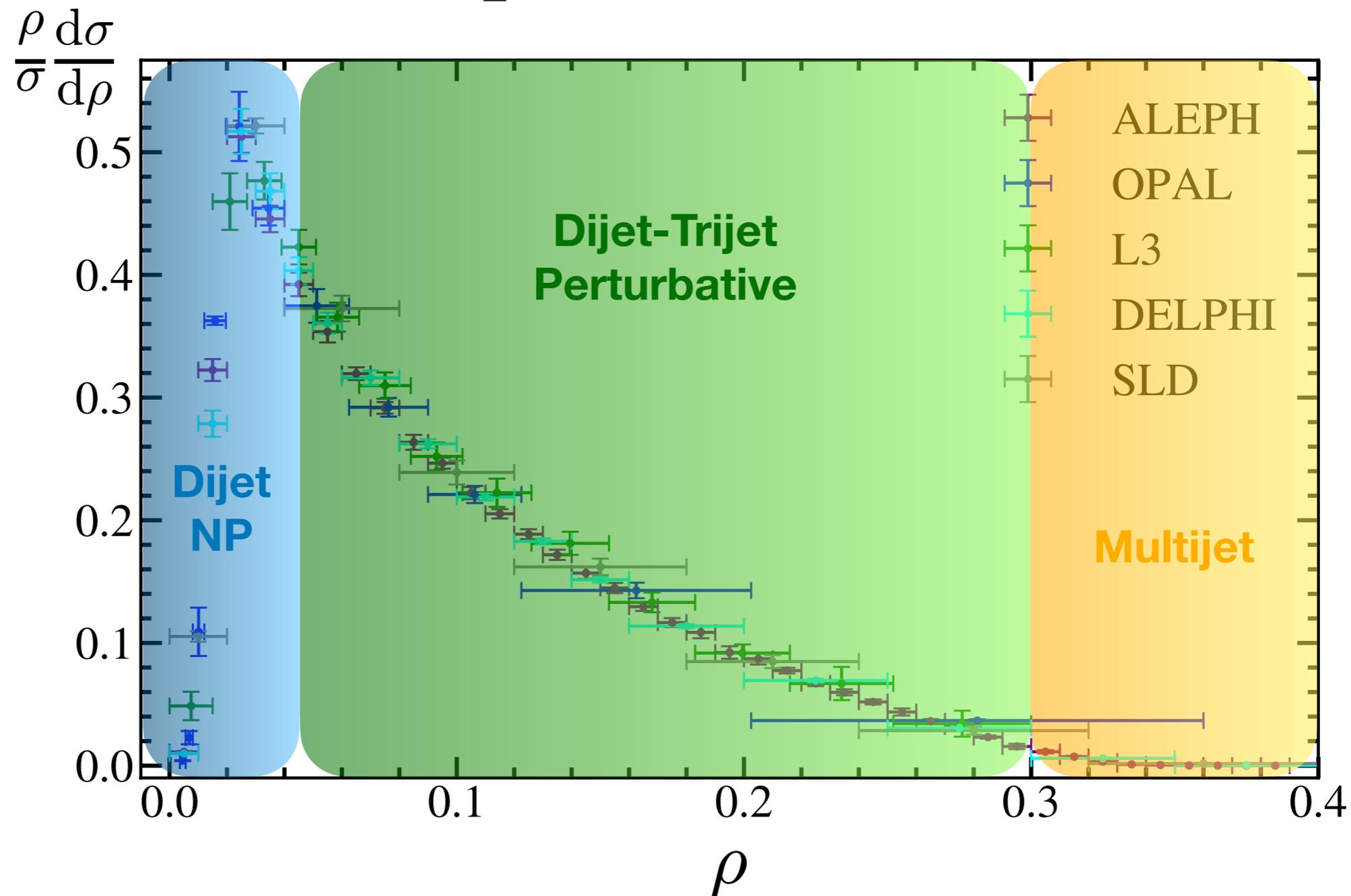
- HJM defined as $\rho = \max(s_1, s_2)/Q^2$

[Clavelli 1979] [Chandramohan, Clavelli 1981] [Clavelli, Wyler 1981]

$$s_1 = \left(\sum_{i \in L} p_i^\mu \right)^2, \quad s_2 = \left(\sum_{i \in R} p_i^\mu \right)^2$$

Hemisphere jet invariant masses

- Experimental Distribution at $Q = m_Z$



The HJM distribution

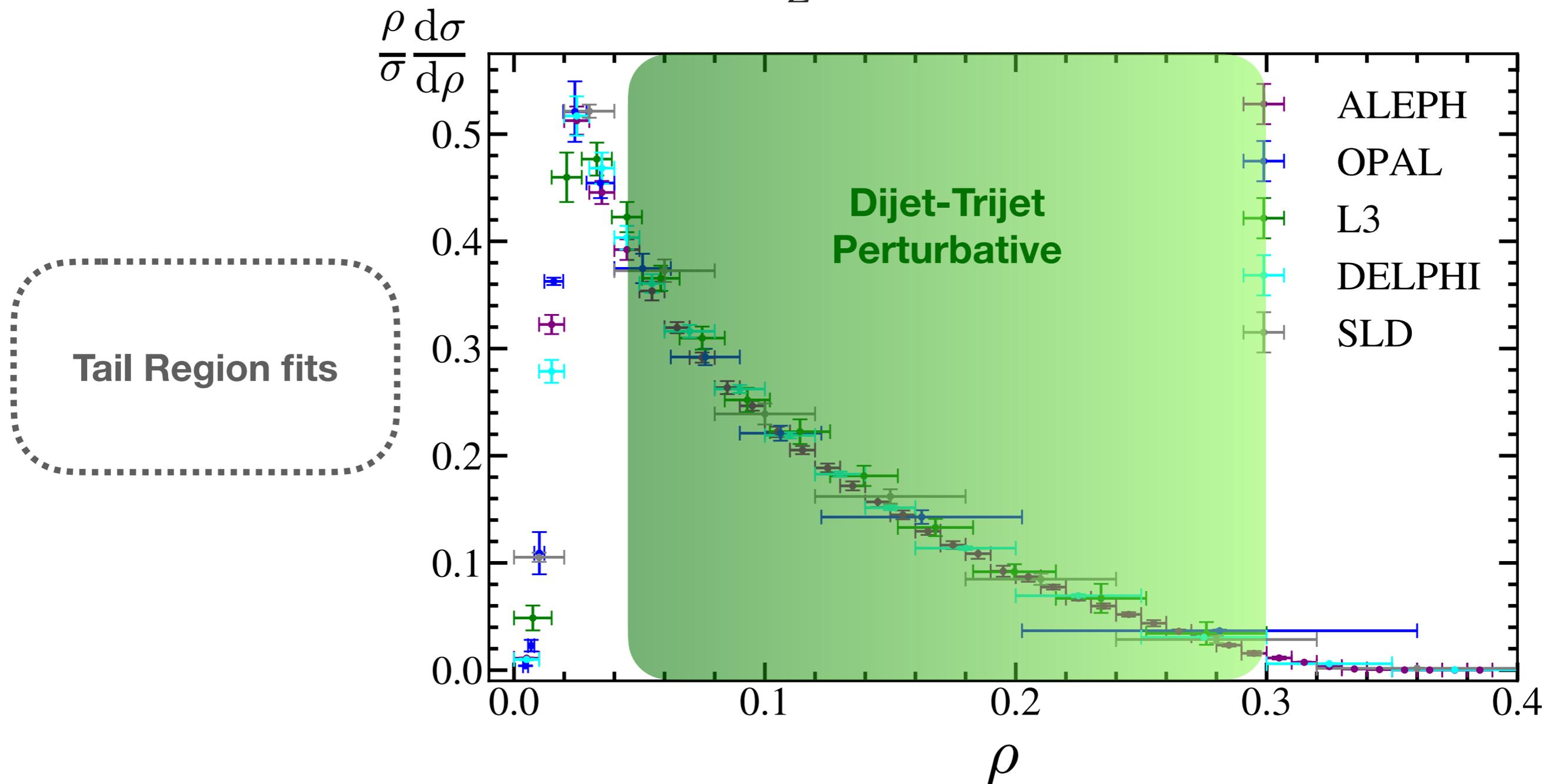
- HJM defined as $\rho = \max(s_1, s_2)/Q^2$

[Clavelli 1979] [Chandramohan, Clavelli 1981] [Clavelli, Wyler 1981]

$$s_1 = \left(\sum_{i \in L} p_i^\mu \right)^2, \quad s_2 = \left(\sum_{i \in R} p_i^\mu \right)^2$$

Hemisphere jet invariant masses

- Experimental Distribution at $Q = m_Z$



The HJM distribution — dijet resummation

[Hoang, Mateu, Schwartz, Stewart to appear]

- HJM distribution expressed as integral over **doubly differential dihemisphere mass distribution**

$$\frac{d\sigma}{d\rho} = 2 Q^2 \int_0^{Q^2 \rho} ds \frac{d^2\sigma}{ds_1 ds_2}(s_1 = Q^2 \rho, s_2 = s) = 2 Q^2 \Xi(Q^2 \rho, Q^2 \rho)$$

mixed distribution-cumulative
cross-section

Non-perturbative double differential distribution can be factorized as

$$\frac{d\sigma}{ds_1 ds_2} = \int dk_1 dk_2 \frac{d\hat{\sigma}}{ds_1 ds_2}(s_1 - Q k_1, s_2 - Q k_2) F(k_1, k_2)$$

Translates to

$$\begin{aligned} \frac{d\sigma}{d\rho} &= 2 Q^2 \int dk_1 dk_2 \hat{\Xi}(Q^2 \rho - Q k_1, Q^2 \rho - Q k_2) F(k_1, k_2) \\ &= 2 Q^2 \int dk_1 dk_2 \frac{d\hat{\sigma}}{ds_1 ds_2}(Q^2 \rho - Q k_1, Q^2 \rho - Q k_2) F^\Xi(k_1, k_2) \end{aligned}$$

Two-dimensional non-perturbative shape function

In tail region, OPE gives rise to power corrections

$$\Omega_{i,j} = \Omega_{j,i} = \int dk_1 dk_2 k_1^i k_2^j F(k_1, k_2)$$

As for thrust, first moment $\Omega_{1,0}$ non-perturbative fit parameter

Partonic distribution can be split into singular and non-singular terms

$$\frac{d^2\hat{\sigma}}{ds_1 ds_2} = \frac{d\hat{\sigma}_s}{ds_1 ds_2} + \frac{d^2\hat{\sigma}_{ns}}{ds_1 ds_2}$$

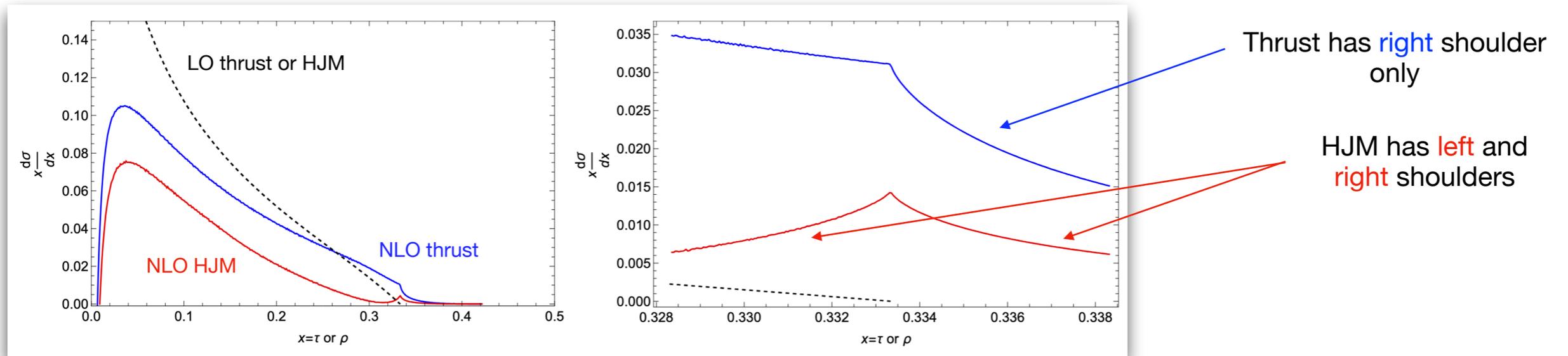
Singular

Nonsingular

$$\frac{d\hat{\sigma}_{ns}}{d\rho}(Q, \mu_{ns}) = \frac{d\hat{\sigma}_{full}^{FO}}{d\rho}(Q, \mu_{ns}) - \frac{d\hat{\sigma}_s^{FO}}{d\rho}(Q, \mu_{ns})$$

$$\begin{aligned} \frac{1}{\sigma_0} \frac{d^2\hat{\sigma}_s}{ds_1 ds_2} &= H(Q, \mu) \int dl_1 dl_2 J(s_1 - Q l_1 - Q \bar{\Delta}(R, \mu), \mu) \\ &\quad \times J(s_2 - Q l_2 - Q \bar{\Delta}(R, \mu), \mu) e^{\delta(R, \mu) \left(\frac{\partial}{\partial \ell_1} + \frac{\partial}{\partial \ell_2} \right)} \hat{S}(\ell_1, \ell_2, \mu) \\ &= H(Q, \mu_H) U_H(Q, \mu_H, \mu_J) \int dl_1 dl_2 J(s_1 - Q l_1 - Q \bar{\Delta}(R, \mu_S), \mu_J) \\ &\quad J(s_2 - Q l_2 - Q \bar{\Delta}(R, \mu_S), \mu_J) \int dl'_1 dl'_2 U_S(\ell_1 - \ell'_1, \mu_J, \mu_S) \\ &\quad \times U_S(\ell_2 - \ell'_2, \mu_J, \mu_S) e^{\delta(R, \mu_S) \left(\frac{\partial}{\partial \ell'_1} + \frac{\partial}{\partial \ell'_2} \right)} \hat{S}(\ell'_1, \ell'_2, \mu_S) \end{aligned}$$

The HJM distribution – dijet + shoulder resummation



[Bhattacharya, Schwartz, Zhang 2022]

- Around symmetric trijet limit $\rho \rightarrow 1/3$, distribution factorizes as

$$d\sigma_{\text{sh}}^{\text{pert}} = H_{\text{sh}} \times J_1 \times J_2 \times J_3 \otimes S_{1,2,3}$$

[Bhattacharya, Michel, Schwartz, Stewart, Zhang 2023]

- Matching between the dijet, fixed-order and shoulder regions done by writing full cross section as

$$d\sigma = \left[d\sigma_{\text{dij}} - d\sigma_{\text{dij}}^{\text{sing}} \right] + d\sigma_{\text{FO}} + \left[d\sigma_{\text{sh}} - d\sigma_{\text{sh}}^{\text{sing}} \right]$$

- Model power corrections around the symmetric trijet limit with non-perturbative shift parameter Θ_1

$$\frac{d\sigma_{\text{sh}}}{d\rho}(\rho) = \frac{d\sigma_{\text{sh}}^{\text{pert}}}{d\rho} \left(\rho - \frac{\Theta_1}{Q} \right)$$

Fit procedure

- Use χ^2 function including theoretical and experimental uncertainties

Experiment

35 GeV < Q < 207 GeV
(700 experimental datapoints)

Minimal Overlap Model treats correlations of systematic uncertainties on experimental measurements

$$\sigma_{ij}^{\text{exp}} = \delta_{ij}(\Delta_i^{\text{stat}})^2 + \delta_{D_i D_j} \min(\Delta_i^{\text{sys}}, \Delta_j^{\text{sys}})^2$$

Theory

Theory uncertainties assessed through renormalization scale variation
→ not Gaussian + highly correlated

Employ flat random scan: M = 5000 sets of k ≤ 17 parameters generated, each produces theory prediction for data-point x_i

Determine $\bar{x}_i = (x_i^{\text{max}} + x_i^{\text{min}})/2$ and $\Delta_i^{\text{theo}} = (x_i^{\text{max}} - x_i^{\text{min}})/2$

Correlation coefficient r_{ij} among bins $r_{ij}^{\text{theo}} = \frac{\langle (x_i - \bar{x}_i)(x_j - \bar{x}_j) \rangle}{\sqrt{\langle (x_i - \bar{x}_i)^2 \rangle} \sqrt{\langle (x_j - \bar{x}_j)^2 \rangle}}$

Theory covariance matrix results from scaling correlation coefficient by 1 - σ uncertainties

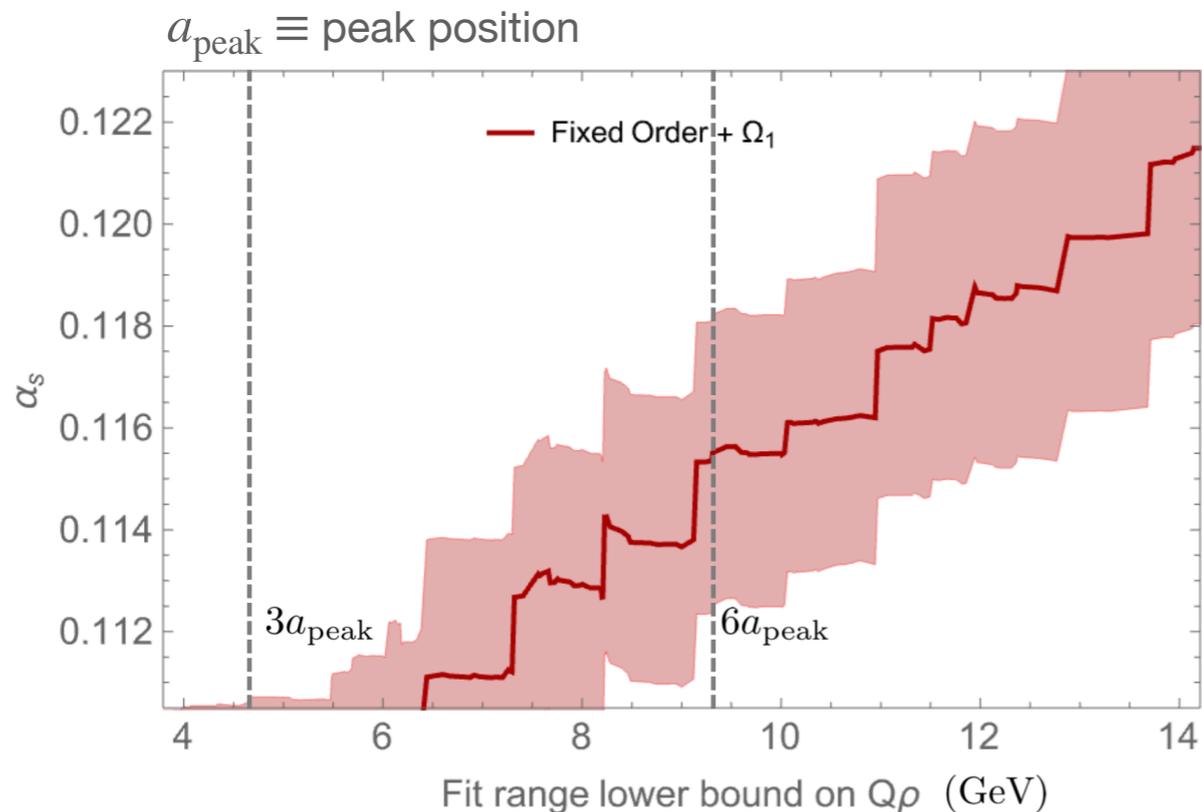
$$\sigma_{ij}^{\text{theo}} = \Delta_i^{\text{theo}} \Delta_j^{\text{theo}} r_{ij}^{\text{theo}}$$

- Total covariance matrix = sum of theoretical and experimental: $\sigma_{ij}^{\text{tot}} = \sigma_{ij}^{\text{theo}} + \sigma_{ij}^{\text{exp}}$

- χ^2 reads:
$$\chi^2 = \sum_{i,j=1}^{N_{\text{bins}}} (\bar{x}_i - x_i^{\text{exp}}) (\bar{x}_j - x_j^{\text{exp}}) (\sigma_{\text{tot}}^{-1})_{ij}$$

Fit results – Fixed Order

- Results for α_s using fit range $a/Q \leq \rho \leq 0.3$ for different a

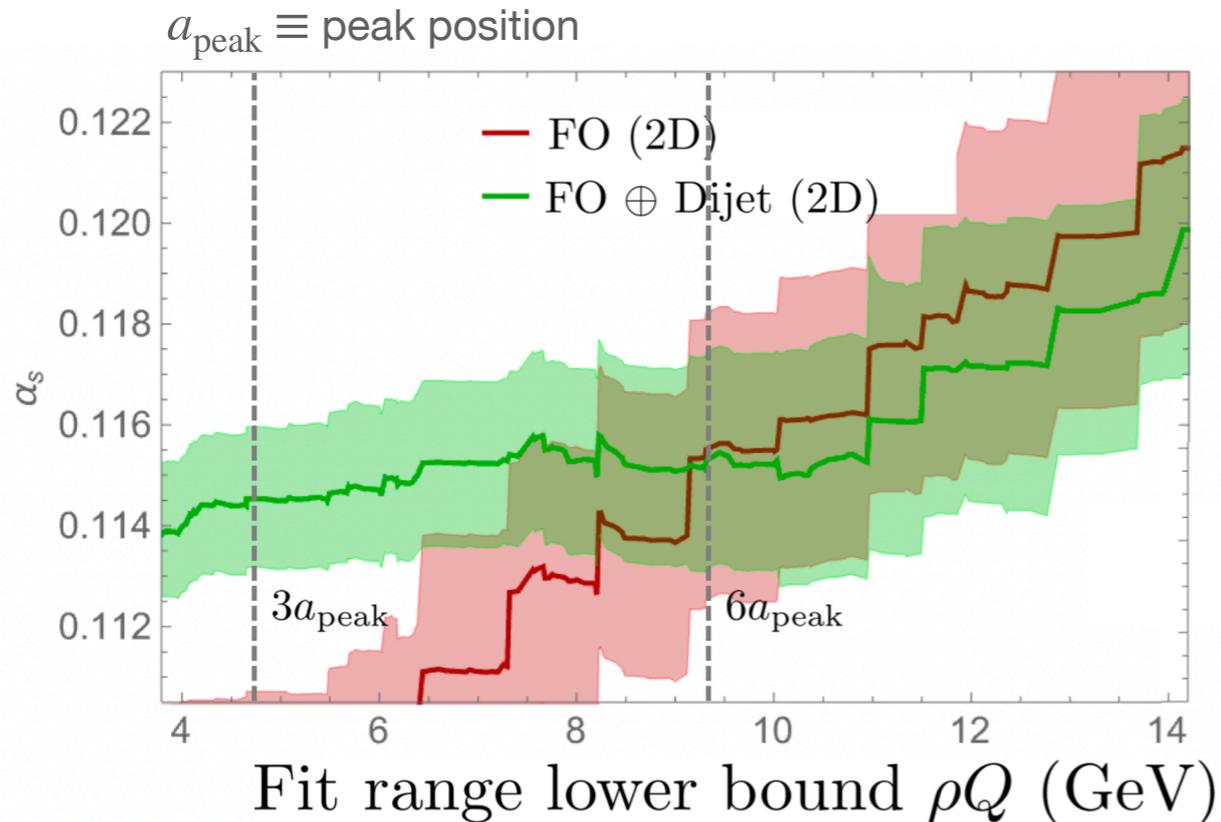


- Results for α_s very sensitive to fit range
- Large **fit range uncertainty** even with restriction $a \in [5a_{\text{peak}}, 8a_{\text{peak}}]$
- Impossible to extract sensible value of α_s without arbitrary choice of fit range

Model	$\alpha_s(m_Z)$	th+exp	Ω_1^ρ	Θ_1	fit range	χ^2/dof	Ω_1^ρ [GeV]	Θ_1 [GeV]
Fixed Order 2D	0.1166 ± 0.0034	± 0.0014	± 0.0027	–	± 0.0015	1.108	0.06 ± 0.13	–

Fit results – Dijet resummation

- Results for α_s using fit range $a/Q \leq \rho \leq 0.3$ for different a

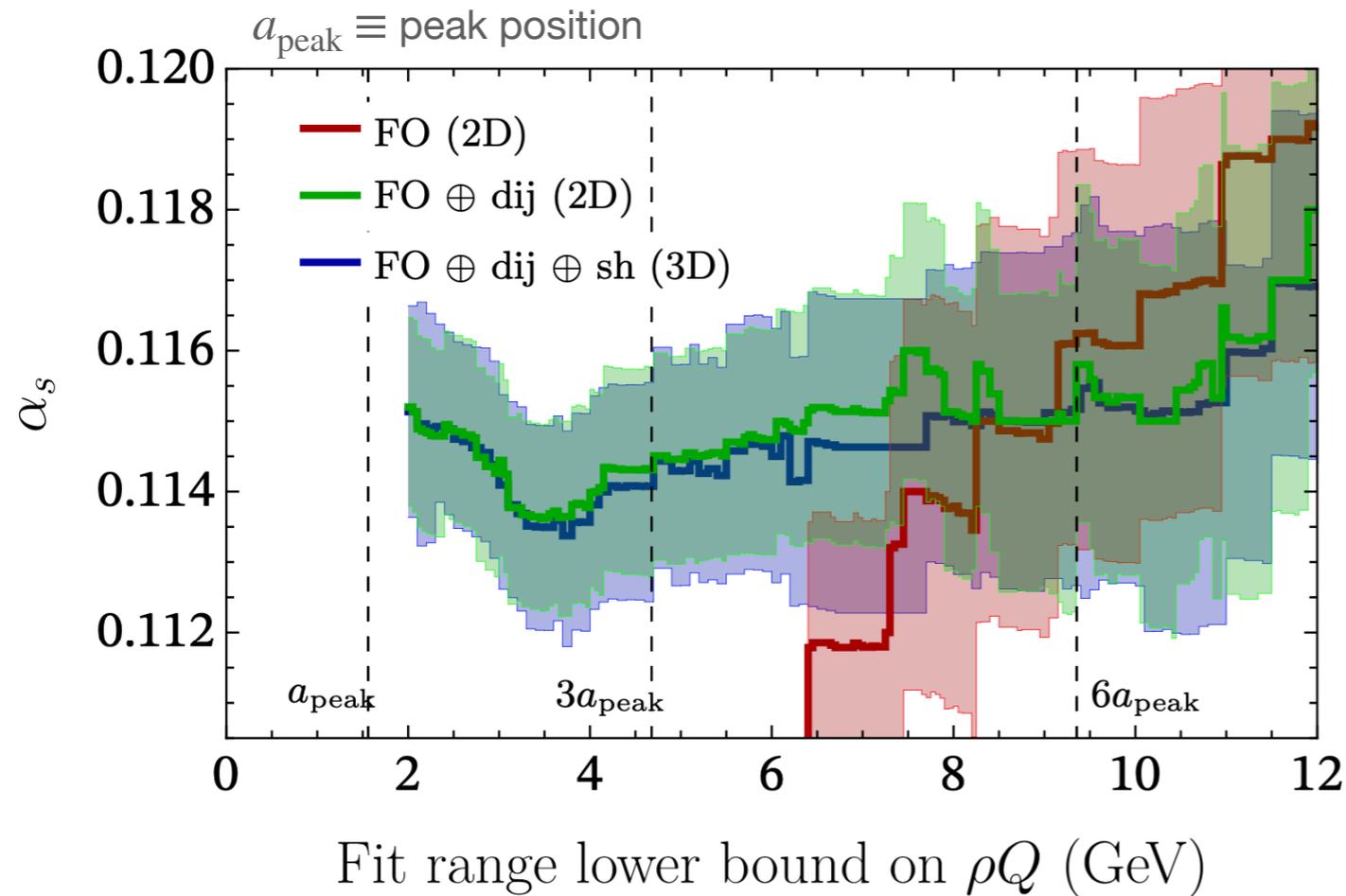


- Fit value remarkably insensitive to fit range
- Small **fit range uncertainty** for $a \in [3a_{\text{peak}}, 6a_{\text{peak}}]$
- Data prefers positive power correction (rightward shift of distribution)

Model	$\alpha_s(m_Z)$	th+exp	Ω_1^ρ	Θ_1	fit range	χ^2/dof	Ω_1^ρ [GeV]	Θ_1 [GeV]
Fixed Order 2D	0.1166 ± 0.0034	± 0.0014	± 0.0027	–	± 0.0015	1.108	0.06 ± 0.13	–
FO + dijet 2D	0.1148 ± 0.0018	± 0.0010	± 0.0014	–	± 0.0004	1.055	0.53 ± 0.09	–

Fit results – Dijet + Shoulder resummation

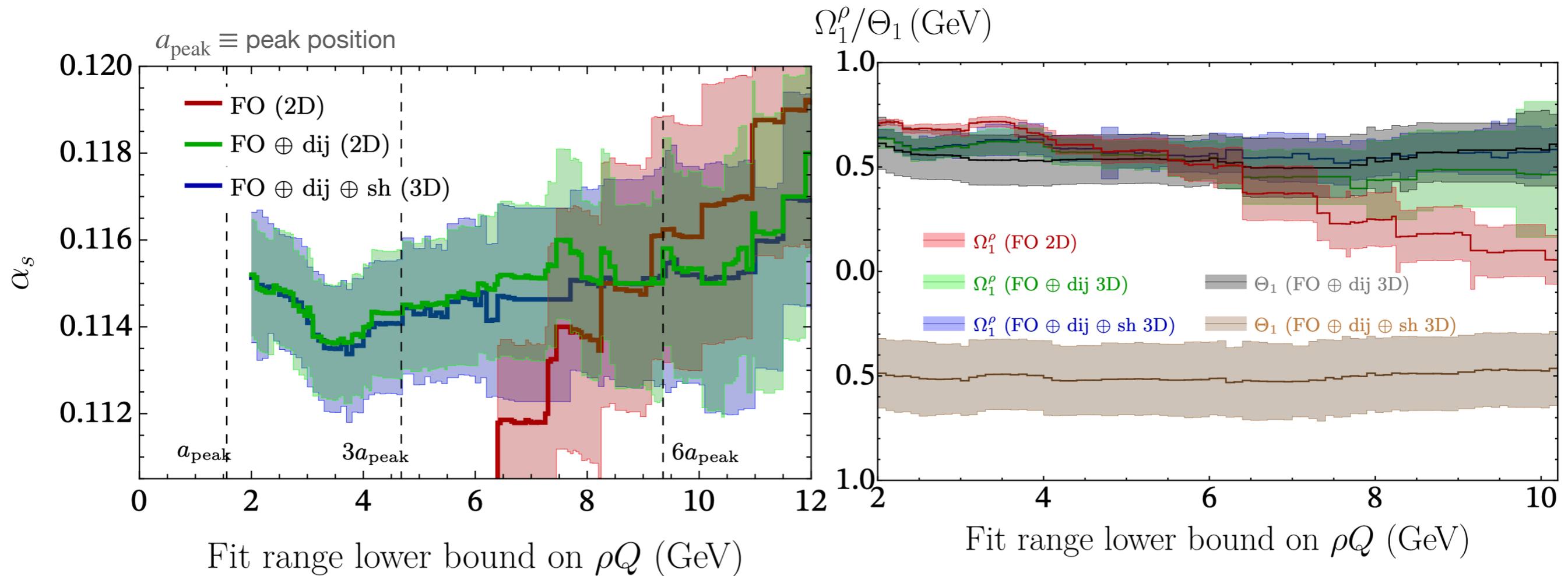
- Results for α_s using fit range $a/Q \leq \rho \leq 0.3$ for different a



- Fit value remarkably insensitive to fit range
- Small fit range uncertainty for $a \in [3a_{\text{peak}}, 6a_{\text{peak}}]$
- **But what about the power corrections?**

Fit results – Dijet + Shoulder resummation

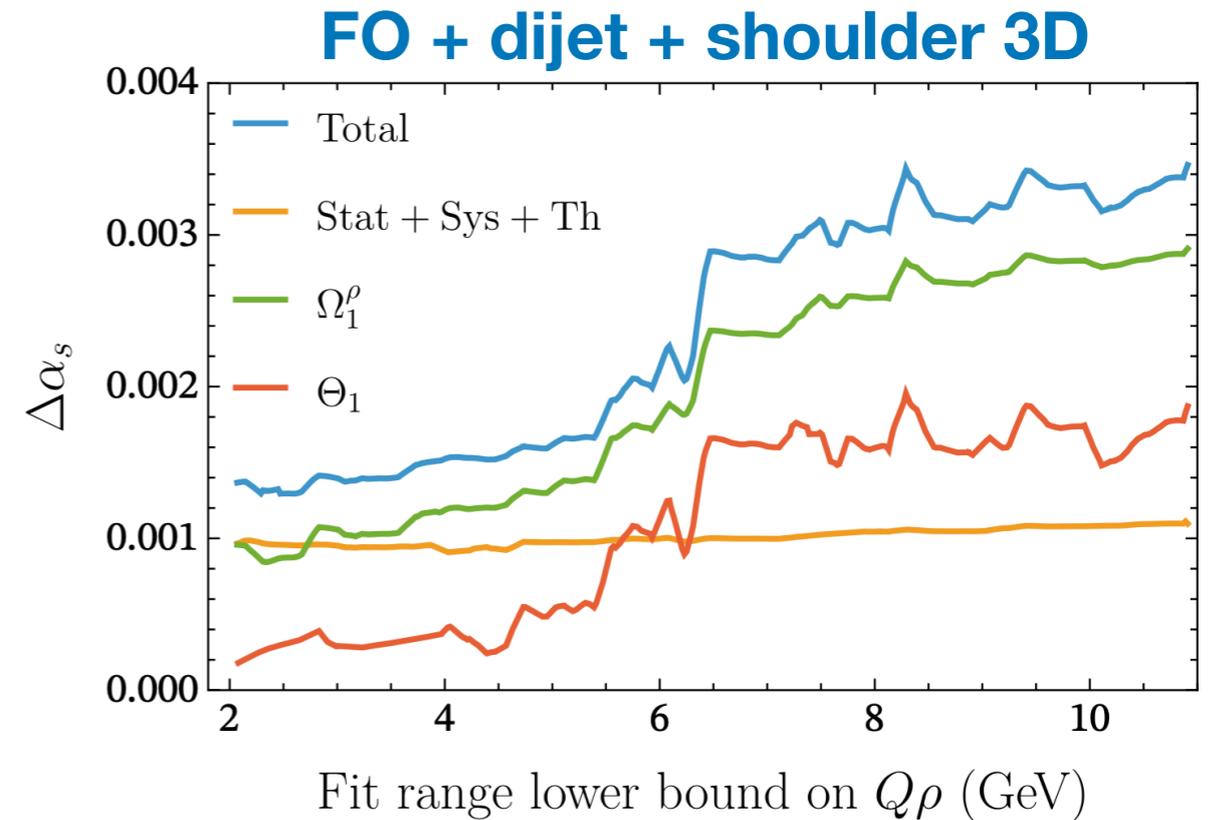
- Results for α_s , Ω_1^ρ and Θ_1 using fit range $a/Q \leq \rho \leq 0.3$ for different a



Data favors negative power corrections only with shoulder!

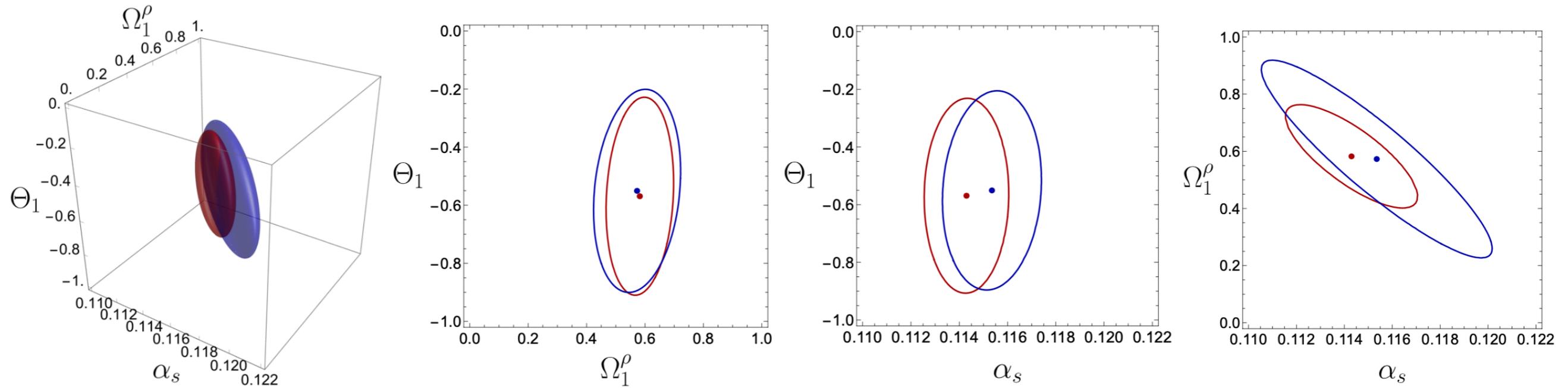
Fit results – Breakdown

- More small ρ data \rightarrow fit uncertainty similar in size to uncertainty induced by Ω_1^ρ , uncertainty arising from Θ_1 comparatively small
- Linear rise of uncertainty tied to Ω_1^ρ when increasing lower bound of the fit range
- Large contribution to overall uncertainty induced by $\Omega_1^\rho \rightarrow$ strong $\alpha_s - \Omega_1^\rho$ correlation



Model	$\alpha_s(m_Z)$	th+exp	Ω_1^ρ	Θ_1	fit range	χ^2/dof	Ω_1^ρ [GeV]	Θ_1 [GeV]
Fixed Order 2D	0.1166 ± 0.0034	± 0.0014	± 0.0027	–	± 0.0015	1.108	0.06 ± 0.13	–
FO + dijet 2D	0.1148 ± 0.0018	± 0.0010	± 0.0014	–	± 0.0004	1.055	0.53 ± 0.09	–
FO + dijet 3D	0.1156 ± 0.0024	± 0.0010	± 0.0021	± 0.0004	± 0.0007	1.052	0.52 ± 0.08	0.53 ± 0.13
FO + dijet + shoulder 3D	0.1145 ± 0.0020	± 0.0009	± 0.0018	± 0.0001	± 0.0003	1.043	0.57 ± 0.09	-0.50 ± 0.17

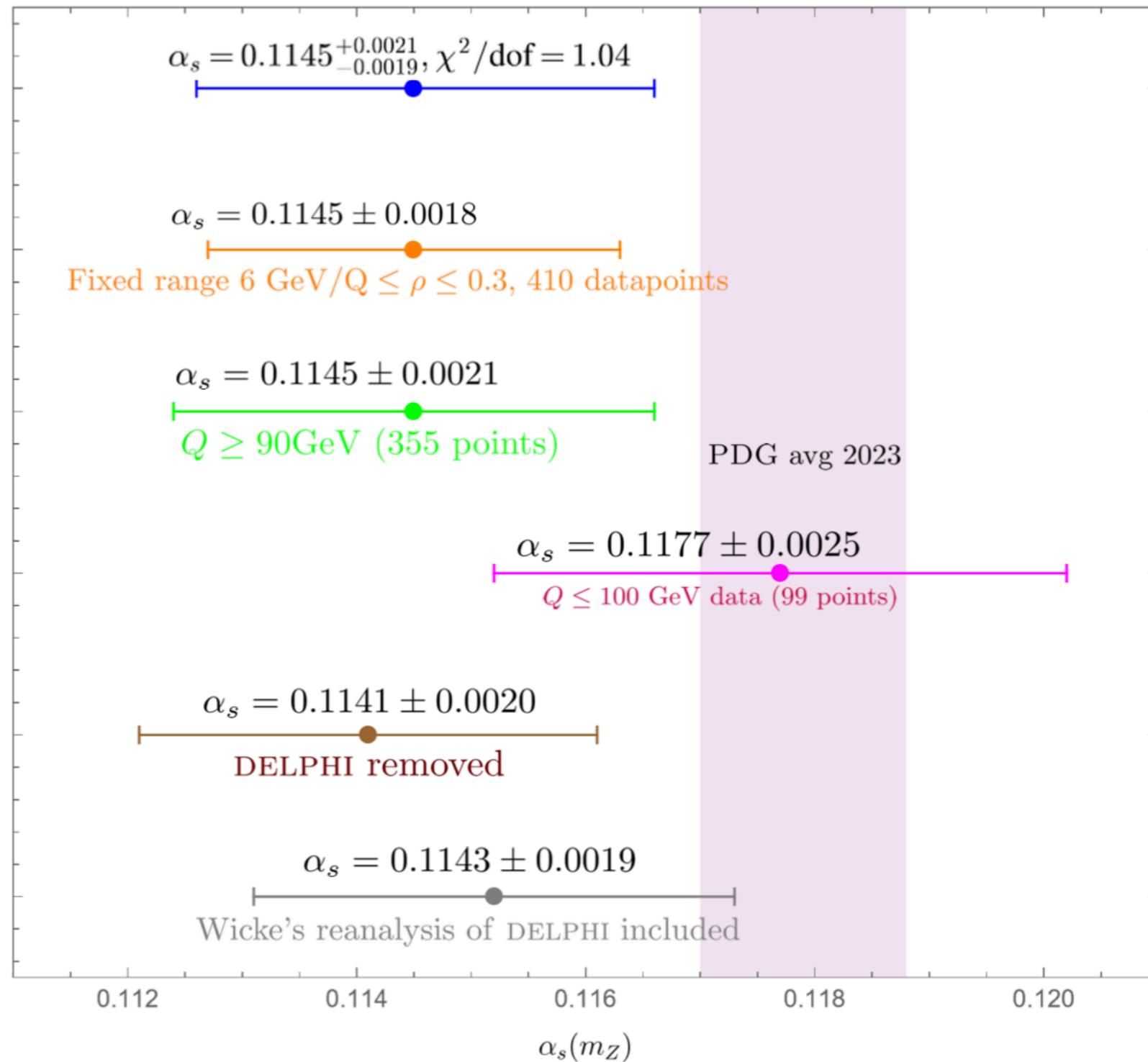
Correlations among $(\alpha_s, \Omega_1^\rho, \Theta_1)$



$$Q\rho > 3a_{\text{peak}} \quad Q\rho > 6a_{\text{peak}}$$

- Strongest correlation between $\alpha_s - \Omega_1^\rho$
- Raising lower bound of fit range increase correlation of $\alpha_s - \Omega_1^\rho$
- Strong $\alpha_s - \Omega_1^\rho$ correlation also reflected in overall uncertainty of α_s — largest contribution arises due to Ω_1^ρ

Cross Checks



Summary

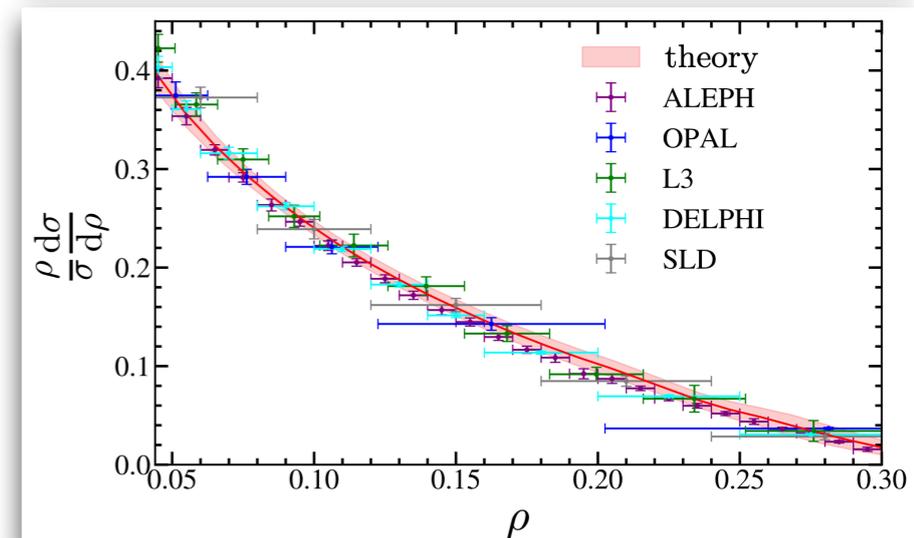
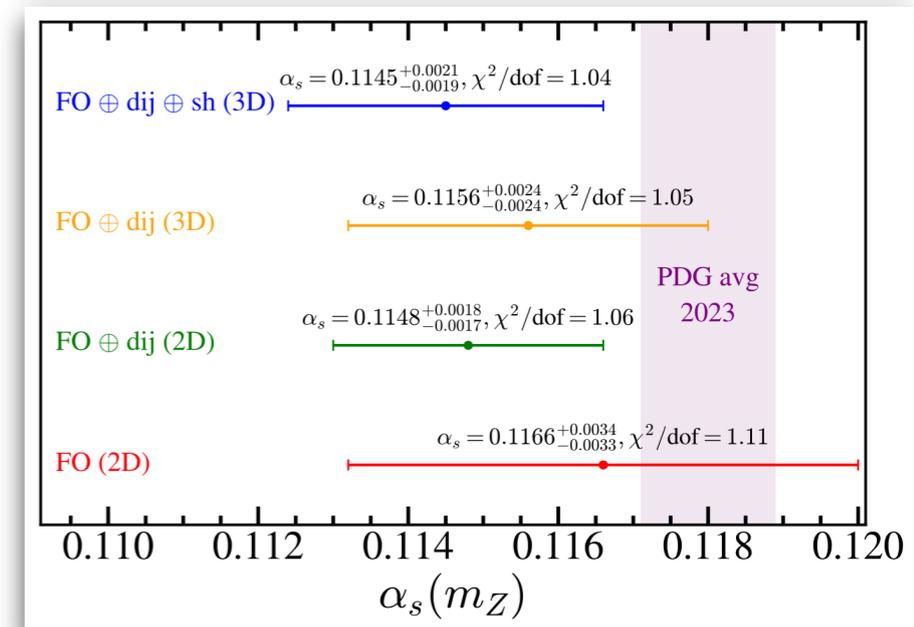
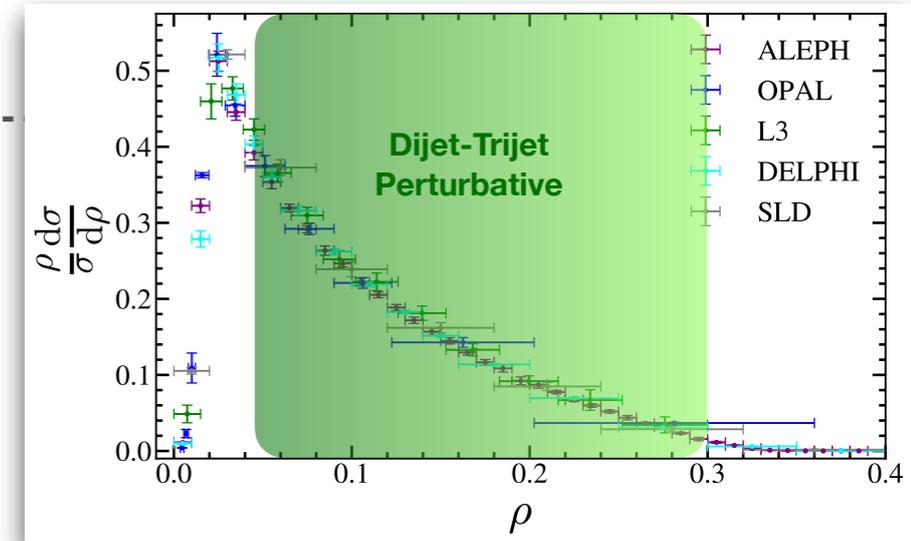
- Provided comprehensive analysis of available data on HJM
- Innovations include
 - Improved treatments of dijet/OPE and trijet/shoulder region
 - Inclusion of theory correlations during fitting
 - Careful attention to the range of data used for fitting
- Found fits are minimally sensitive to fit range when including resummation, in contrast to fixed-order perturbation theory (essentially linear dependence on lower bound)
- Found evidence for negative power correction in tail of distribution only if Sudakov shoulder resummation is included
- Our extracted value is

$$\alpha_s(m_Z) = 0.1145^{+0.0021}_{-0.0019}$$

$$\Omega_1^\rho = 0.57 \pm 0.09 \text{ GeV}, \quad \Theta_1 = -0.50 \pm 0.17 \text{ GeV}$$

$$\chi^2/\text{dof} = 1.04$$

perfectly compatible with Thrust and C-parameter results



Back-Up

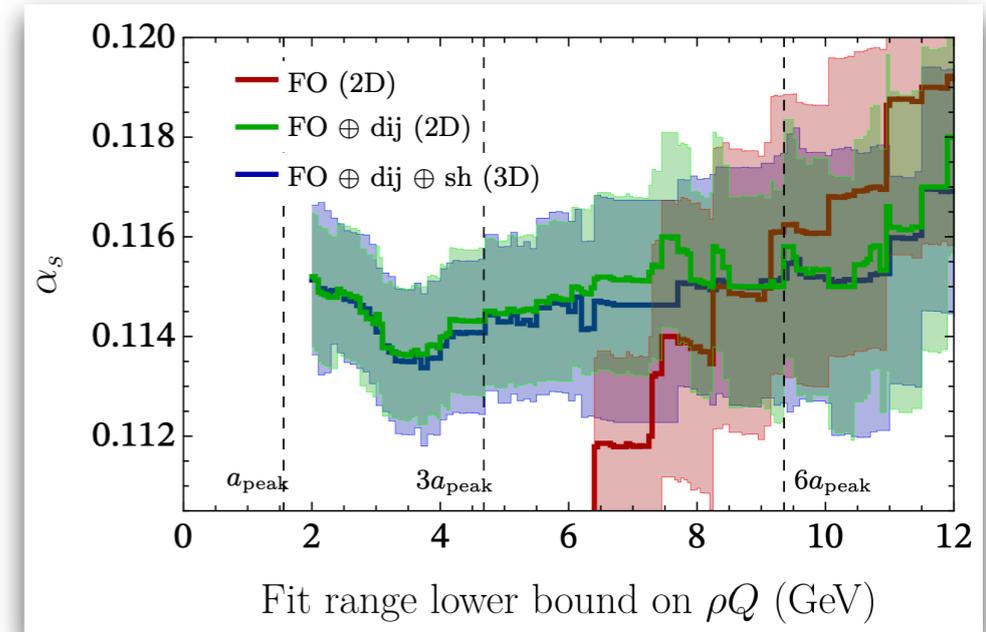
More details on fits

- To extract value of α_s from our fits we perform weighted average over different choices for lower bound $a \leq \rho Q$ with weight

$$w_a = \sigma_a^{-2} (\sum_a \sigma_a^{-2})^{-1}$$

($\sigma_a \equiv$ uncertainty for given fit range)

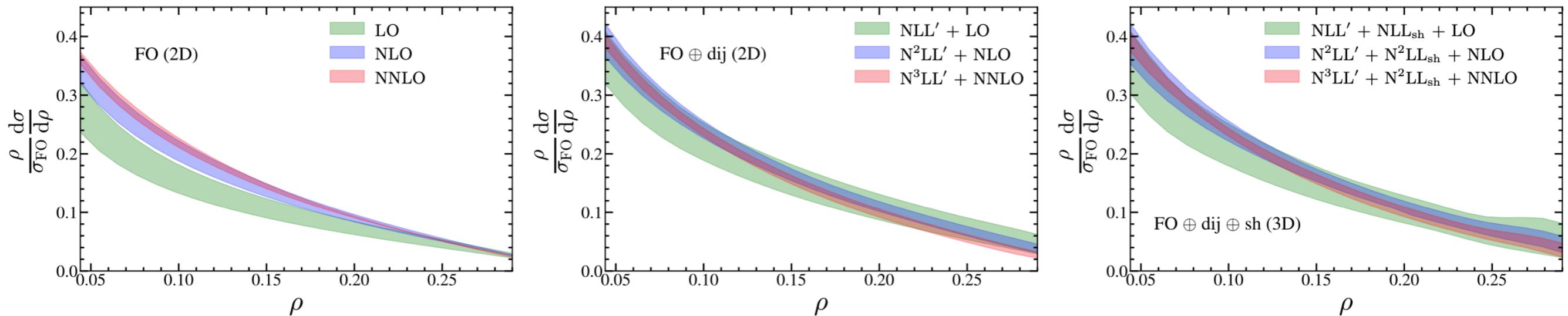
- For resummed results consider $a \in [3a_{\text{peak}}, 6a_{\text{peak}}]$
- For fixed order results consider $a \in [5a_{\text{peak}}, 8a_{\text{peak}}]$
- th+exp**: includes experimental systematical and statistical uncertainty as well as perturbative uncertainty from variation of all theory parameters
- fit-range**: given by standard deviation among best-fit values for various lower bounds considered



$a_{\text{peak}} \equiv$ peak position

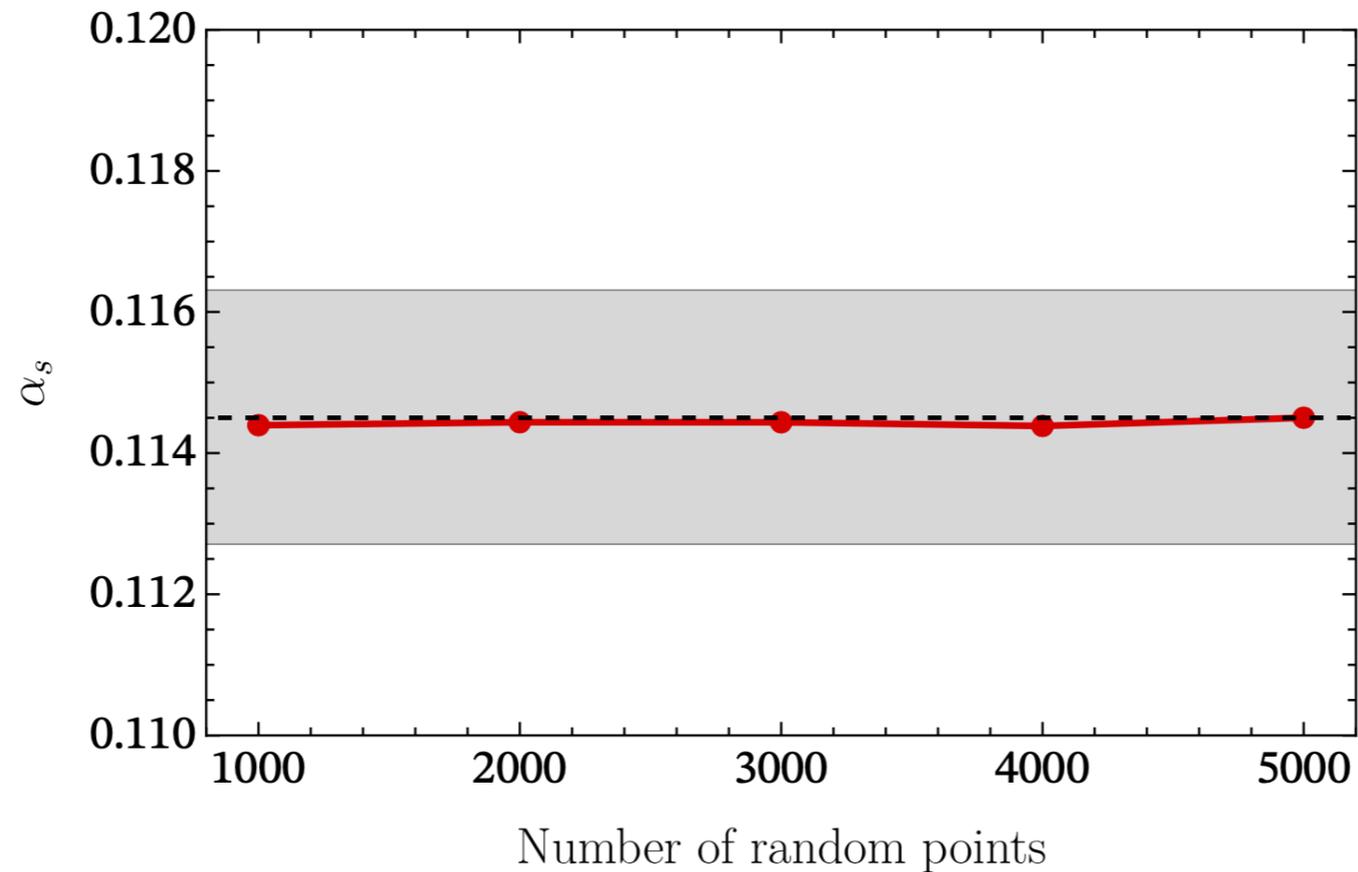
Model	$\alpha_s(m_Z)$	th+exp	Ω_1^p	Θ_1	fit range	χ^2/dof	Ω_1^p [GeV]	Θ_1 [GeV]
Fixed Order 2D	0.1166 ± 0.0034	± 0.0014	± 0.0027	–	± 0.0015	1.108	0.06 ± 0.13	–
FO + dijet 2D	0.1148 ± 0.0018	± 0.0010	± 0.0014	–	± 0.0004	1.055	0.53 ± 0.09	–
FO + dijet 3D	0.1156 ± 0.0024	± 0.0010	± 0.0021	± 0.0004	± 0.0007	1.052	0.52 ± 0.08	0.53 ± 0.13
FO + dijet + shoulder 3D	0.1145 ± 0.0020	± 0.0009	± 0.0018	± 0.0001	± 0.0003	1.043	0.57 ± 0.09	-0.50 ± 0.17

Convergence



- Including dijet resummation yields substantially better convergent behavior
- Near $\rho \approx 0.25$ visibly less overlap between the different orders even when including dijet resummation
- Improved convergence around $\rho \approx 0.25$ when including shoulder resummation

Random Scan



- Increasing random point density yields fluctuations of central values of α_s that stay well within fit uncertainty