Theory improvement

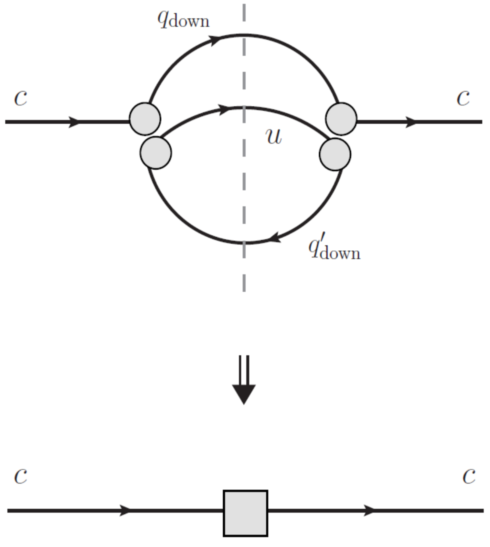
BEAUTY vs CHARM

Discussion

HEAVY QUARK EXPANSION (HQE) – systematic **expansion in Λ_{QCD}/m_Q and α_s**

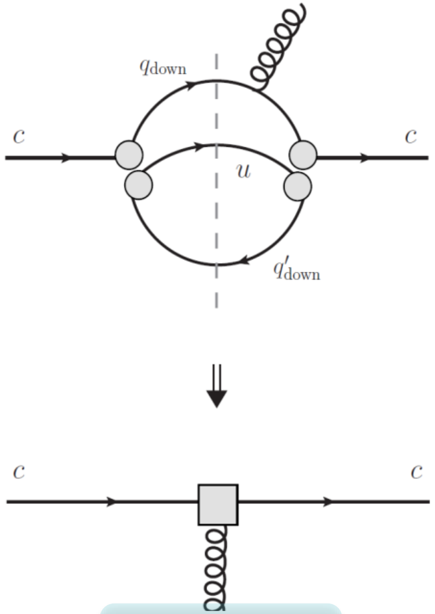
$$\mathcal{T} = \left(\mathcal{C}_3 \mathcal{O}_3 + \frac{\mathcal{C}_5}{m_Q^2} \mathcal{O}_5 + \frac{\mathcal{C}_6}{m_Q^3} \mathcal{O}_6 + \ldots \right) + 16\pi^2 \left(\frac{\tilde{\mathcal{C}}_6}{m_Q^3} \tilde{\mathcal{O}}_6 + \frac{\tilde{\mathcal{C}}_7}{m_Q^4} \tilde{\mathcal{O}}_7 + \ldots \right)$$

TWO-QUARK LEADING NON-SPECTATOR CONTRIBUTION



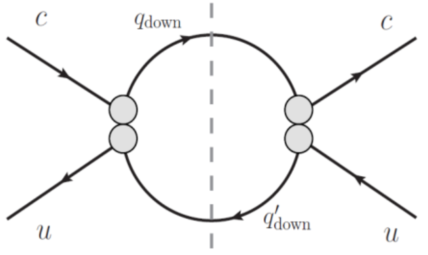
$\mathcal{O}_3$

TWO-QUARK NON-LEADING NON-SPECTATOR CONTRIBUTION



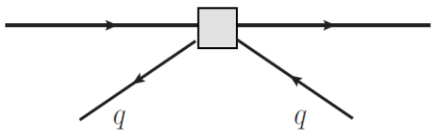
$\mathcal{O}_5\ \mathcal{O}_6$ etc

FOUR-QUARK SPECTATOR CONTRIBUTIONS – **ENHANCED FOR c**

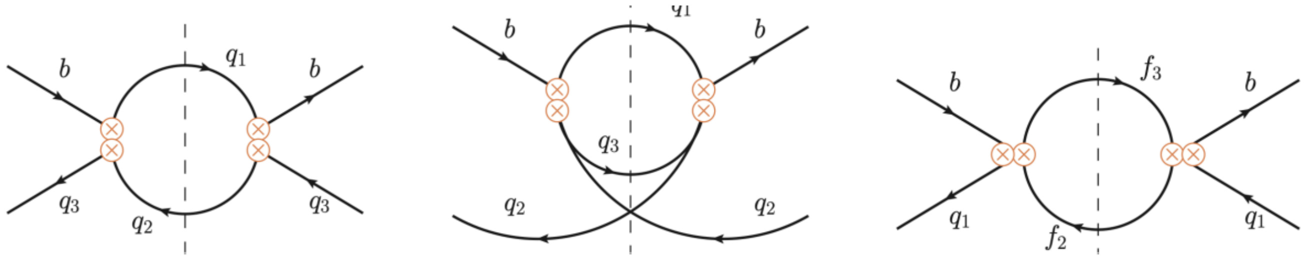


light-quarks come into game

$\Downarrow$



$\tilde{\mathcal{O}}_6\ \tilde{\mathcal{O}}_7$ etc



$$16\pi^2 \frac{\Lambda^3_{\rm QCD}}{m_b^3} \sim 0.2 \quad \text{but} \quad 16\pi^2 \frac{\Lambda^3_{\rm QCD}}{m_c^3} \sim 6$$

Th. Mannel

Gratrex, Melic, Nisandzic,2204.11935

	decay / H_c	CE NL $c \to s\bar{d}u$	SCS-s NL $c \to s\bar{s}u$	SCS-d NL $c \to d\bar{d}u$	DCS NL $c \to d\bar{s}u$	CE SL $c \to s\bar{l}\nu_l$	CS SL $c \to d\bar{l}\nu_l$
mesons	$D^0(\bar{u}c)$	$\tilde\Gamma_{\rm WE}$	$\tilde\Gamma_{\rm WE}$	$\tilde\Gamma_{\rm WE}$	$\tilde\Gamma_{\rm WE}$	-	-
	$D^+(\bar{d}c)$	$\tilde\Gamma_{\rm PI}$	-	$\tilde\Gamma_{\rm PI} + \tilde\Gamma_{\rm WA}$	$\tilde\Gamma_{\rm WA}$	-	$\tilde\Gamma^{\rm SL}_{\rm WA}$
	$D_s^+(\bar{s}c)$	$\tilde\Gamma_{\rm WA}$	$\tilde\Gamma_{\rm PI} + \tilde\Gamma_{\rm WA}$	-	$\tilde\Gamma_{\rm PI}$	$\tilde\Gamma^{\rm SL}_{\rm WA}$	-
baryons	$\Lambda_c^+(udc)$	$\tilde\Gamma_{\rm exc} + \tilde\Gamma_{\rm int-}$	$\tilde\Gamma_{\rm int-}$	$\tilde\Gamma_{\rm exc} + \tilde\Gamma_{\rm int-} + \tilde\Gamma_{\rm int+}$	$\tilde\Gamma_{\rm int-} + \tilde\Gamma_{\rm int+}$	-	$\tilde\Gamma^{\rm SL}_{\rm int+}$
	$\Xi_c^+(usc)$	$\tilde\Gamma_{\rm int-} + \tilde\Gamma_{\rm int+}$	$\tilde\Gamma_{\rm exc} + \tilde\Gamma_{\rm int-} + \tilde\Gamma_{\rm int+}$	$\tilde\Gamma_{\rm int-}$	$\tilde\Gamma_{\rm exc} + \tilde\Gamma_{\rm int-}$	$\tilde\Gamma^{\rm SL}_{\rm int+}$	-
	$\Xi_c^0(dsc)$	$\tilde\Gamma_{\rm exc} + \tilde\Gamma_{\rm int+}$	$\tilde\Gamma_{\rm exc} + \tilde\Gamma_{\rm int+}$	$\tilde\Gamma_{\rm exc} + \tilde\Gamma_{\rm int+}$	$\tilde\Gamma_{\rm exc} + \Gamma_{\rm int+}$	$\tilde\Gamma^{\rm SL}_{\rm int+}$	$\tilde\Gamma^{\rm SL}_{\rm int+}$
	$\Omega_c^0(ssc)$	$\tilde\Gamma_{\rm int+}$	$\tilde\Gamma_{\rm exc} + \tilde\Gamma_{\rm int+}$	-	$\tilde\Gamma_{\rm exc}$	$\tilde\Gamma^{\rm SL}_{\rm int+}$	-

HQE: beauty vs. charm

- The HQE is a double expansion in $\alpha_s(m_Q)$ and Λ_{QCD}/m_Q
- In the beauty system ($m_b \sim 4.5$GeV):

$$\alpha_s(m_b) \sim 0.22 \qquad \frac{\Lambda_{QCD}}{m_b} \sim 0.10$$

  - Applicability of the HQE is well justified

- In the charm system ($m_c \sim 1$GeV):

$$\alpha_s(m_c) \sim 0.33 \qquad \frac{\Lambda_{QCD}}{m_c} \sim 0.30$$

  - Can we still *reliably* apply the HQE?

Must compare HQE predictions with measurements

The observables

- Compute total widths

$$\Gamma(B) = \Gamma_3 + \Gamma_5 \frac{\langle \mathcal{O}_5 \rangle}{m_b^2} + \Gamma_6 \frac{\langle \mathcal{O}_6 \rangle}{m_b^3} + \ldots + 16\pi^2 \left[\tilde{\Gamma}_6 \frac{\langle \tilde{\mathcal{O}}_6 \rangle}{m_b^3} + \tilde{\Gamma}_7 \frac{\langle \tilde{\mathcal{O}}_7 \rangle}{m_b^4} + \ldots \right]$$

  - Strong dependence on value of b-quark mass in Γ_3

- And lifetime ratios

$$\tau(B^+_{(s)})/\tau(B_d) = 1 + \left[\delta\Gamma(B_d)^{\mathrm{HQE}} - \delta\Gamma(B^+_{(s)})^{\mathrm{HQE}}\right] \tau(B^+_{(s)})^{\mathrm{exp}}$$

THIS IS NOT HELPING MUCH FOR c-HADRONS SINCE
4-q NON_SPECTATOR CONTRIBUTIONS DOMINATE !

TWO- QUARK
Dim-5 & Dim–6 operators:

$$\mu_\pi^2(H) = \frac{-1}{2m_H}\langle H|\bar{c}_v(iD)^2 c_v|H\rangle\,,$$

$$\mu_G^2(H) = \frac{1}{2m_H}\langle H|\bar{c}_v\frac{1}{2}\sigma\cdot(g_sG)c_v|H\rangle\,,$$

$$\boxed{\rho_D^3(H)} = \frac{1}{2m_H}\langle H|\bar{c}_v(iD_\mu)(iv\cdot D)(iD^\mu)c_v|H\rangle$$

FOUR_QUARK
Dim-6 operators:

$$Q_1^{(qq')} = (\bar{c}^i\gamma_\mu(1-\gamma_5)q^j)(\bar{q}'^j\gamma^\mu(1-\gamma_5)u^i)\,,$$

$$Q_2^{(qq')} = (\bar{c}^i\gamma_\mu(1-\gamma_5)q^i)(\bar{q}'^j\gamma^\mu(1-\gamma_5)u^j)\,,$$

$$\boxed{Q_{\rm SL}^{(q\ell)}} = (\bar{c}\gamma_\mu(1-\gamma_5)q)(\bar{\ell}\gamma^\mu(1-\gamma_5)\nu_\ell)\,,$$

\+ color-octet operators

\+ µ-running and mixing

FOUR_QUARK
Dim-7 operators:

$$P_1^q = m_q(\bar{c}_i(1-\gamma_5)q_i)(\bar{q}_j(1-\gamma_5)c_j)\,,$$

$$P_2^q = \frac{1}{m_Q}(\bar{c}_i\overleftarrow{D}_\rho\gamma_\mu(1-\gamma_5)D^\rho q_i)(\bar{q}_j\gamma^\mu(1-\gamma_5)c_j)\,,$$

$$P_3^q = \frac{1}{m_Q}(\bar{c}_i\overleftarrow{D}_\rho(1-\gamma_5)D^\rho q_i)(\bar{q}_j(1+\gamma_5)c_j)\,,$$

$$S_1^q = m_q(\bar{c}_i(1-\gamma_5)t^a_{ij}q_j)(\bar{q}_k(1-\gamma_5)t^a_{kl}c_l)\,,$$

$$S_2^q = \frac{1}{m_Q}(\bar{c}_i\overleftarrow{D}_\rho\gamma_\mu(1-\gamma_5)t^a_{ij}D^\rho q_j)(\bar{q}_k\gamma^\mu(1-\gamma_5)t^a_{kl}c_l)\,,$$

$$S_3^q = \frac{1}{m_Q}(\bar{c}_i\overleftarrow{D}_\rho(1-\gamma_5)t^a_{ij}D^\rho q_j)(\bar{q}_k(1+\gamma_5)t^a_{kl}c_l)\,.$$

\+ color-octet operators
\+ non-local operators - *reabsorbed into* dim6 matrix elements (proved for mesons)

D-MESONS

King, Lenz, Piscopo, Rauh,Rusov, Vlahos, 2109.13219 (c-mesons)
Gratrex, Melic, Nisandzic,2204.11935 (c-mesons & c-baryons)

LO

Mass scheme	$\Gamma_3^{(0)}$	$\Gamma_3^{(1)}$	$\Gamma_\pi^{(0)}$	$\Gamma_G^{(0)}$	$\Gamma_{\rm Darwin}^{(0)}$
Pole	$1.49^{+0.17}_{-0.14}$	$1.62^{+0.26}_{-0.22}$	$(-0.17^{+0.02}_{-0.02})\frac{\mu_\pi^2}{0.5\,\mathrm{GeV}^2}$	$(0.01^{+0.07}_{-0.07})\frac{\mu_G^2}{0.25\,\mathrm{GeV}^2}$	$(0.44^{+0.09}_{-0.09})\frac{\rho_D^3}{0.1\,\mathrm{GeV}^3}$
$\overline{\mathrm{MS}}$	$0.69^{+0.08}_{-0.07}$	$1.28^{+0.37}_{-0.29}$	$(-0.10^{+0.01}_{-0.01})\frac{\mu_\pi^2}{0.5\,\mathrm{GeV}^2}$	$(0.00^{+0.05}_{-0.05})\frac{\mu_G^2}{0.25\,\mathrm{GeV}^2}$	$(0.43^{+0.08}_{-0.08})\frac{\rho_D^3}{0.1\,\mathrm{GeV}^3}$
Kinetic	$1.10^{+0.13}_{-0.11}$	$1.65^{+0.41}_{-0.32}$	$(-0.14^{+0.01}_{-0.02})\frac{\mu_\pi^2}{0.5\,\mathrm{GeV}^2}$	$(0.01^{+0.06}_{-0.06})\frac{\mu_G^2}{0.25\,\mathrm{GeV}^2}$	$(0.44^{+0.09}_{-0.08})\frac{\rho_D^3}{0.1\,\mathrm{GeV}^3}$
MSR	$0.93^{+0.11}_{-0.09}$	$1.54^{+0.41}_{-0.32}$	$(-0.13^{+0.01}_{-0.01})\frac{\mu_\pi^2}{0.5\,\mathrm{GeV}^2}$	$(0.01^{+0.06}_{-0.06})\frac{\mu_G^2}{0.25\,\mathrm{GeV}^2}$	$(0.44^{+0.09}_{-0.08})\frac{\rho_D^3}{0.1\,\mathrm{GeV}^3}$

	D^0
WE	$-(0.01+0.22\,x)B_1^q+(0.01+0.21\,x)B_2^q-(2.27+0.93\,x)\epsilon_1^q+(2.30+0.68\,x)\epsilon_2^q$
	D^+
PI	$-(1.25+1.02\,x)B_1^q-0.17\,x\,B_2^q+(7.46+5.24\,x)\epsilon_1^q-0.28\,x\,\epsilon_2^q$
WA	$-(0.13+0.05\,x)B_1^q+(0.13+0.05\,x)B_2^q-(0.02+0.06\,x)\epsilon_1^q+(0.02+0.06\,x)\epsilon_2^q$
SL	$-(0.08+0.01\,x)B_1^q+(0.08+0.01\,x)B_2^q-0.03\,x\,\epsilon_1^q+0.02\,x\,\epsilon_2^q$
	D_s^+
PI	$-(0.09+0.08\,x)B_1^s-0.01\,xB_2^s+(0.55+0.39\,x)\epsilon_1^s-0.02\,x\epsilon_2^s$
WA	$(-3.66-1.38\,x)B_1^s+(3.66+1.38\,x)B_2^s-(0.47+1.74\,x)\epsilon_1^s+(0.47+1.60\,x)\epsilon_2^s$
SL	$-(2.08+0.26\,x)B_1^s+(2.09+0.27\,x)B_2^s-0.72\,x\epsilon_1^s+0.67\,x\epsilon_2^s$

Table 12: Contributions of valence dimension-six spectator operators to the decay widths of charmed mesons, in units ps^{-1}, in the $\overline{\mathrm{MS}}$ scheme. The different contributions are separated by the topologies defined in figure 2. The HQET bag parameters have been left unevaluated, but are assumed to be renormalized at the scale $\mu_0 \sim 1.5\,\mathrm{GeV}$. Their coefficients correspond to the scale $\mu = 1.5\,\mathrm{GeV}$. The factor $x = 1$ denotes the $\mathcal{O}(\alpha_s)$ contributions. Semileptonic contributions involve both e and μ channels.

Observable	Pole	$\overline{\mathrm{MS}}$	Kinetic	MSR
$\Gamma(D^0)$	$1.71^{+0.41+0.39}_{-0.47-0.36}$	$1.43^{+0.36+0.48}_{-0.40-0.40}$	$1.77^{+0.40+0.53}_{-0.45-0.45}$	$1.68^{+0.38+0.53}_{-0.43-0.44}$
$\Gamma(D^+)$	$-0.07^{+0.76+0.31}_{-0.68-0.20}$	$-0.27^{+0.66+0.03}_{-0.88-0.04}$	$-0.07^{+0.73+0.20}_{-0.66-0.14}$	$-0.13^{+0.71+0.13}_{-0.64-0.11}$
$\tilde{\Gamma}(D_s^+)$	$1.71^{+0.49+0.44}_{-0.60-0.40}$	$1.43^{+0.42+0.49}_{-0.52-0.41}$	$1.77^{+0.47+0.55}_{-0.58-0.47}$	$1.67^{+0.46+0.55}_{-0.56-0.46}$

Lenz, Piscopo, Rusov, 2208.02643 (b-mesons)
Gratrex, Lenz, Melic, Nisandzic,Piscopo, Rusov 2301.07698 (b-baryons)

$$\Gamma(B_d^0) = \Gamma_0\Big[(\underbrace{5.97}_{\rm LO} - \underbrace{0.44}_{\Delta\rm NLO}) - 0.14\,\frac{\mu_\pi^2(B)}{\rm GeV^2} - 0.24\,\frac{\mu_G^2(B)}{\rm GeV^2} - \boxed{1.35}\,\frac{\rho_D^3(B)}{\rm GeV^3}$$

$$- (\underbrace{0.012}_{\rm LO} + \underbrace{0.022}_{\Delta\rm NLO})\,\tilde B_1^q + (\underbrace{0.012}_{\rm LO} + \underbrace{0.020}_{\Delta\rm NLO})\,\tilde B_2^q - (\underbrace{0.74}_{\rm LO} + \underbrace{0.03}_{\Delta\rm NLO})\,\tilde B_3^q$$

$$+ (\underbrace{0.78}_{\rm LO} - \underbrace{0.01}_{\Delta\rm NLO})\,\tilde B_4^q - 0.14\,\tilde\delta_1^{qq'} + 0.02\,\tilde\delta_2^{qq'} - 2.29\,\tilde\delta_3^{qq'} + 0.00\,\tilde\delta_4^{qq'}$$

$$- 0.01\,\tilde\delta_1^{sq} + 0.01\,\tilde\delta_2^{sq} - 0.69\,\tilde\delta_3^{sq} + 0.78\,\tilde\delta_4^{sq}\Big],$$

SMALL coeff. In front of dim-6 , dim-7
$B_i \sim 1$, $\varepsilon_i \sim 0$ and $\delta_i \sim 10$ ^(-3)

King, Lenz, Piscopo, Rauh,Rusov, Vlahos, 2109.13219 (c-mesons)
Gratrex, Melic, Nisandzic,2204.11935 (c-mesons & c-baryons)

$$\Gamma(D^0) = \Gamma_0\Big[\underbrace{6.15}_{c_3^{\rm LO}} + \underbrace{2.95}_{\Delta c_3^{\rm NLO}} - 1.66\,\frac{\mu_\pi^2(D)}{\rm GeV^2} + 0.13\,\frac{\mu_G^2(D)}{\rm GeV^2} + \boxed{23.6}\,\frac{\rho_D^3(D)}{\rm GeV^3}$$

$$\boxed{- 1.60\,\tilde B_1^q + 1.53\,\tilde B_2^q} - 21.0\,\tilde\epsilon_1^q + 19.2\,\tilde\epsilon_2^q + \boxed{\underbrace{0.00}_{\rm dim-7, VIA}}$$

$$- 10.7\,\tilde\delta_1^{qq} + 1.53\,\tilde\delta_2^{qq} + 54.6\,\tilde\delta_3^{qq} + 0.13\,\tilde\delta_4^{qq} - 29.2\,\tilde\delta_1^{sq} + 28.8\,\tilde\delta_2^{sq} + 0.56\,\tilde\delta_3^{sq} + 2.36\,\tilde\delta_4^{sq}\Big]$$

RELATIVELY
LARGE coeff in front of two-quark matrix elem.

LARGE coeff. In front of dim-6, dim-7
$B_i \sim 1$, $\varepsilon_i \sim 0$ and $\delta_i \sim 10$ ^(-3)

$$\Gamma(D^+) = \Gamma_0 \Big[\underbrace{6.15}_{c_3^{\rm LO}} + \underbrace{2.95}_{\Delta c_3^{\rm NLO}} - 1.66 \, \frac{\mu_\pi^2(D)}{\rm GeV^2} + 0.13 \, \frac{\mu_G^2(D)}{\rm GeV^2} + 23.6 \, \frac{\rho_D^3(D)}{\rm GeV^3}$$

$$\boxed{- 16.9 \, \tilde{B}_1^q + 0.56 \, \tilde{B}_2^q} + 84.0 \, \tilde{\epsilon}_1^q - 1.34 \, \tilde{\epsilon}_2^q + \boxed{\underbrace{6.76}_{\rm dim-7}}$$

$$- 0.06 \, \tilde{\delta}_1^{qq} + 0.06 \, \tilde{\delta}_2^{qq} - 16.8 \, \tilde{\delta}_3^{qq} + 16.9 \, \tilde{\delta}_4^{qq} - 29.3 \, \tilde{\delta}_1^{sq} + 28.8 \, \tilde{\delta}_2^{sq} + 0.56 \, \tilde{\delta}_3^{sq} + 2.36 \, \tilde{\delta}_4^{sq} \Big]$$

COMPARISON OF PARAMETERS

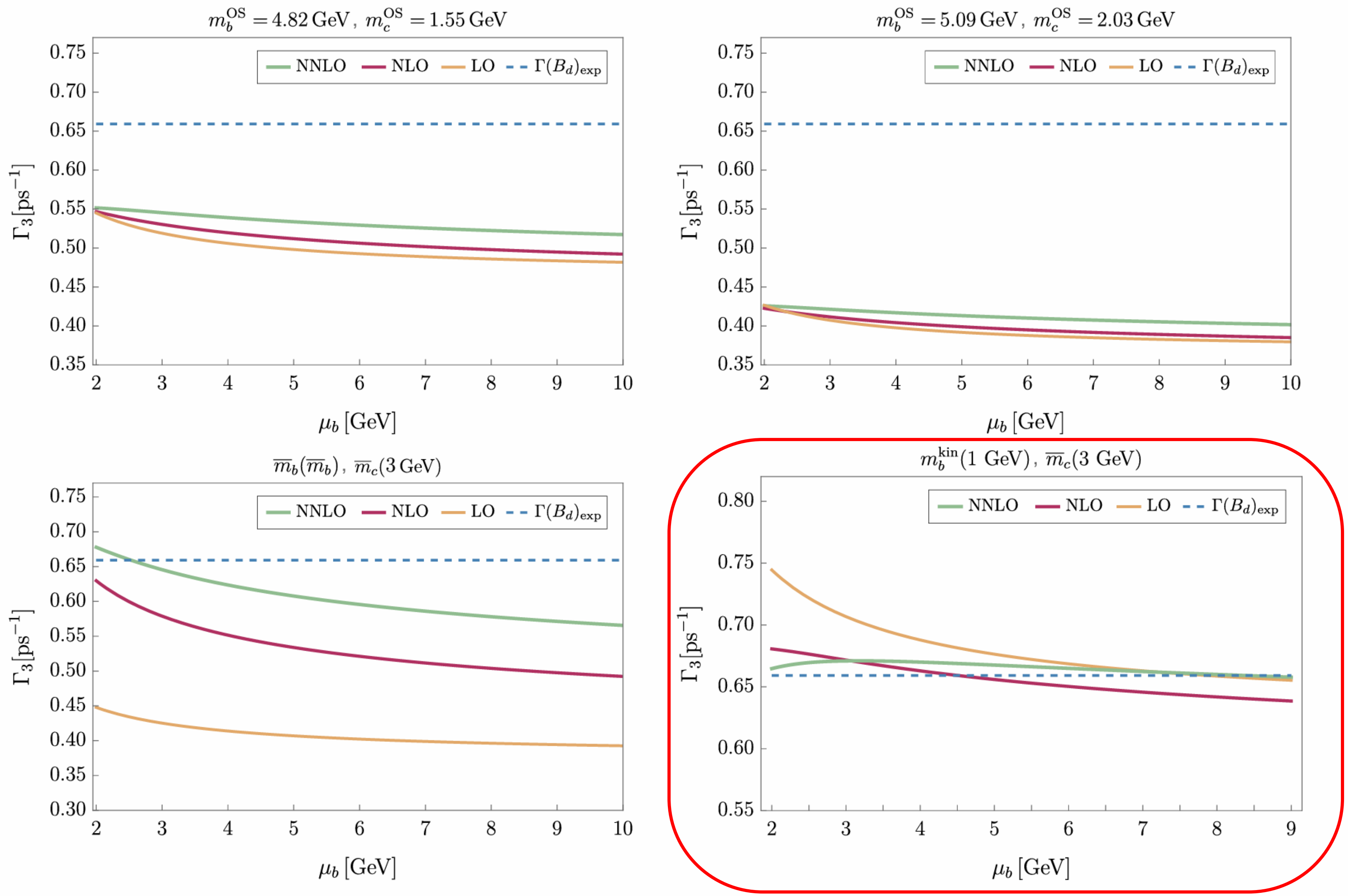
Parameter	$B^{+,0}$	$D^{+,0}$
f_{B_q} [GeV]	0.1900 ± 0.0013	0.2120 ± 0.0007
$\bar{\Lambda}_q$ [GeV]	0.5 ± 0.1	0.465 ± 0.198
$\mu_\pi^2(B_q)$ [GeV^2]	0.477 ± 0.056 0.43 ± 0.24	
$\mu_G^2(B_q)$ [GeV^2]	0.294 ± 0.054 0.38 ± 0.07	0.34 ± 0.10
$\rho_D^3(B_q)$ [GeV^3]	$\boxed{0.185 \pm 0.031}$ $\boxed{0.03 \pm 0.02}$	$\boxed{0.075 \pm 0.034}$

HQET, $\mu_0 = 1.5\,\text{GeV}$	$\tilde{B}_1$	$\tilde{B}_2$	$\tilde{\epsilon}_1$	$\tilde{\epsilon}_2$
$B^{+,0}$	$1.0026^{+0.0198}_{-0.0106}$	$0.9982^{+0.0052}_{-0.0066}$	$-0.0165^{+0.0209}_{-0.0346}$	$-0.0004^{+0.0200}_{-0.0326}$
$D^{+,0}$	$1.0026^{+0.0198}_{-0.0106}$	$0.9982^{+0.0052}_{-0.0066}$	$-0.0165^{+0.0209}_{-0.0346}$	$-0.0004^{+0.0200}_{-0.0326}$

$$2m_H\rho_D^3 = \langle H|\bar{h}_v(iD_\mu)(iv\cdot D)(iD^\mu)h_v|H\rangle + \mathcal{O}(1/m_c)$$
$$= g_s^2\langle H|\Big(-\frac{1}{9}\mathcal{O}_1^q + \frac{2}{9}\mathcal{O}_2^q + \frac{1}{12}\mathcal{T}_1^q - \frac{1}{6}\mathcal{T}_2^q\Big)|H\rangle + \mathcal{O}(1/m_c),$$

HQET, $\mu_0 = 1.5\,\text{GeV}$	$\tilde{\delta}_1$	$\tilde{\delta}_2$	$\tilde{\delta}_3$	$\tilde{\delta}_4$
$\langle B_q|\tilde{O}^q|B_q\rangle$	$0.0026^{+0.0142}_{-0.0092}$	$-0.0018^{+0.0047}_{-0.0072}$	$-0.0004^{+0.0015}_{-0.0024}$	$0.0003^{+0.0012}_{-0.0008}$
$\langle B_s|\tilde{O}^q|B_s\rangle$	$0.0025^{+0.0144}_{-0.0093}$	$-0.0018^{+0.0047}_{-0.0072}$	$-0.0004^{+0.0015}_{-0.0024}$	$0.0003^{+0.0012}_{-0.0008}$
$\langle B_q|\tilde{O}^s|B_q\rangle$	$0.0023^{+0.0140}_{-0.0091}$	$-0.0017^{+0.0046}_{-0.0070}$	$-0.0004^{+0.0015}_{-0.0023}$	$0.0003^{+0.0012}_{-0.0008}$
$\langle D_q|\tilde{O}^q|D_q\rangle$	$0.0026^{+0.0142}_{-0.0092}$	$-0.0018^{+0.0047}_{-0.0072}$	$-0.0004^{+0.0015}_{-0.0024}$	$0.0003^{+0.0012}_{-0.0008}$
$\langle D_s|\tilde{O}^q|D_s\rangle$	$0.0025^{+0.0144}_{-0.0093}$	$-0.0018^{+0.0047}_{-0.0072}$	$-0.0004^{+0.0015}_{-0.0024}$	$0.0003^{+0.0012}_{-0.0008}$
$\langle D_q|\tilde{O}^s|D_q\rangle$	$0.0023^{+0.0140}_{-0.0091}$	$-0.0017^{+0.0046}_{-0.0070}$	$-0.0004^{+0.0015}_{-0.0023}$	$0.0003^{+0.0012}_{-0.0008}$

Mass-scheme dependence



For b- hadrons:
kinetic mass scheme for m_b
and MS scheme for m_c

For c- hadrons ?
kinetic mass scheme for m_c,
and MS scheme for m_s ??

[Egner, Fael, Lenz, MLP, Rusov, Schönwald, Steinhauser '24]

Possible improvements??

- resummations ?
- ratios/combinations of results?
- better determination of parameters and combination with experiments?
- another scheme for m_c? (doubly-heavy baryon lifetimes?)

Dulibic, Gratrex, Melic, Nisandzic, 2305.02243

Beneke, 2108.04861

charm	$\overline{m}_c$	1-loop	2-loop	3-loop	4-loop	Sum
m_c	1.280	0.211	0.202	0.282	0.510	$2.486^{+0.126}_{-0.109}$
$m_{c,\mathrm{RS}}$	1.280	-0.017	0.037	0.026	0.005	$1.331^{+0.006}_{-0.005}$
$m_{c,\mathrm{MSR}}$	1.280	0.046	0.022	0.010	-0.002	$1.356^{+0.004}_{-0.004}$
$m_{c,\mathrm{PS}}$	1.280	0.046	0.052	0.034	-0.019	$1.393^{+0.006}_{-0.006}$
$m_{c,\mathrm{PS}*}$	1.280	0.046	0.052	0.034	0.002	$1.414^{+0.009}_{-0.008}$
$m_{c,\mathrm{kin}}$	1.280	-0.073	-0.062	-0.017	—	$1.128^{+0.008}_{-0.009}$

RS, MRS = renormalon-subtracted masses

$$\delta m_{\mathrm{RS}}(\mu_f) = \mu_f N \sum_{n=1}^{\infty} \tilde{c}_n^{(\mathrm{as})}\, \alpha_s^n(\mu_f)$$

PS, PS* = potential-subtracted masses

$$m_{\mathrm{PS}}(\mu_f) = m + \frac{1}{2} \int_{|\boldsymbol{q}|<\mu_f} \frac{d^3\boldsymbol{q}}{(2\pi)^3}\, \tilde{V}(\boldsymbol{q})$$

kin = kinetic mass

$$m_{\mathrm{kin}}(\mu_f) = m \underbrace{-[\bar{\Lambda}(\mu_f)]_{\mathrm{pert}} - \left[\frac{\mu_\pi^2(\mu_f)}{2m_{\mathrm{kin}}(\mu_f)}\right]_{\mathrm{pert}}}_{-\delta m_{\mathrm{kin}}(\mu_f)} + \ldots$$