Spectator Effects from nonperturbatively renormalized Lattice-HQET

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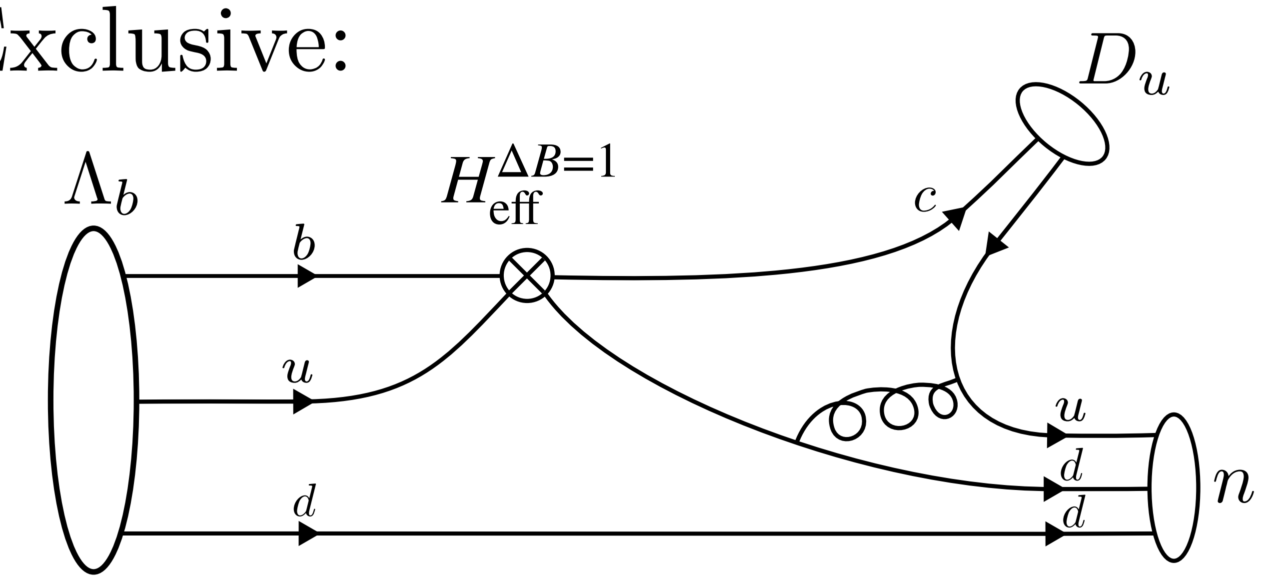
(with Will Detmold and Stefan Meinel)

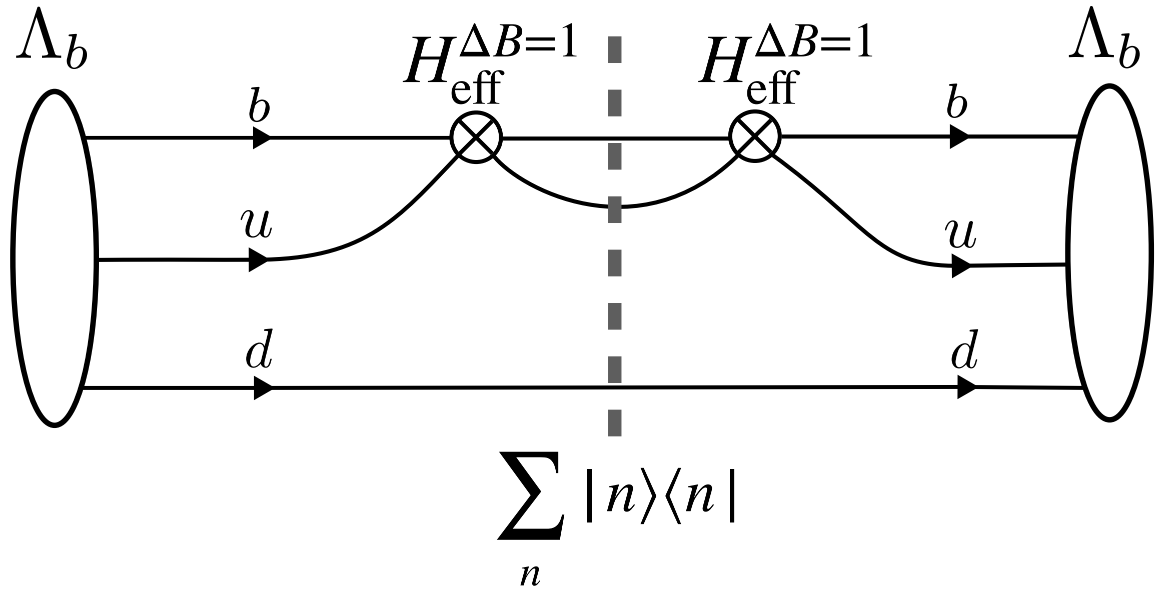
[J.Lin, W. Detmold, S. Meinel: 2212.09275, 2404.16191, and In Preparation]

Spectator Effects in the OPE

Exclusive:



Inclusive (via optical theorem):



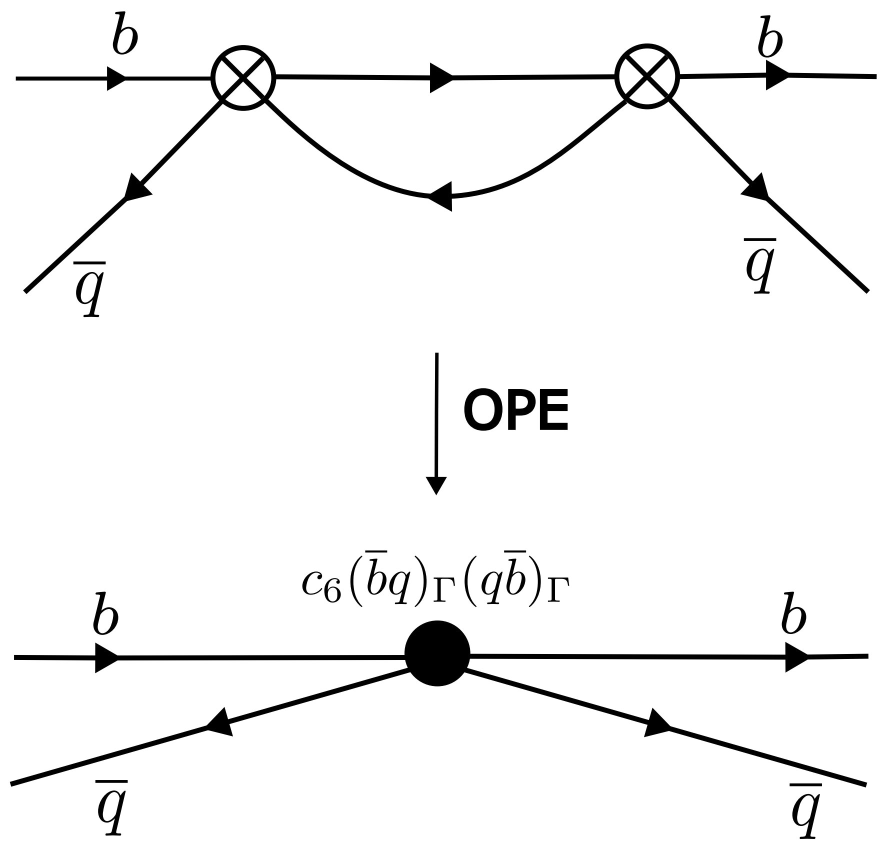
Operator-Product-Expansion expands two currents as a sum over local currents:

$$\Gamma(H \to X) = \frac{1}{2m_B}\langle H|\hat{T}|H\rangle = \frac{1}{2m_B}\langle H|\mathrm{Im}\ i\int d^4x T\left(H_{\text{eff}}^{\Delta B=1}(x)H_{\text{eff}}^{\Delta B=1}(0)\right)|H\rangle$$

$$= \underbrace{C_1\langle H|\bar{b}b|H\rangle + C_2\langle H|\bar{b}\partial_\perp^2 b|H\rangle}_{} + \ldots + \boxed{C_{\text{S.E.}}\langle H|(\bar{b}\Gamma_L q)(\bar{q}\Gamma_R b)|H\rangle} + \ldots$$

(can be extracted from heavy hadron spectra)

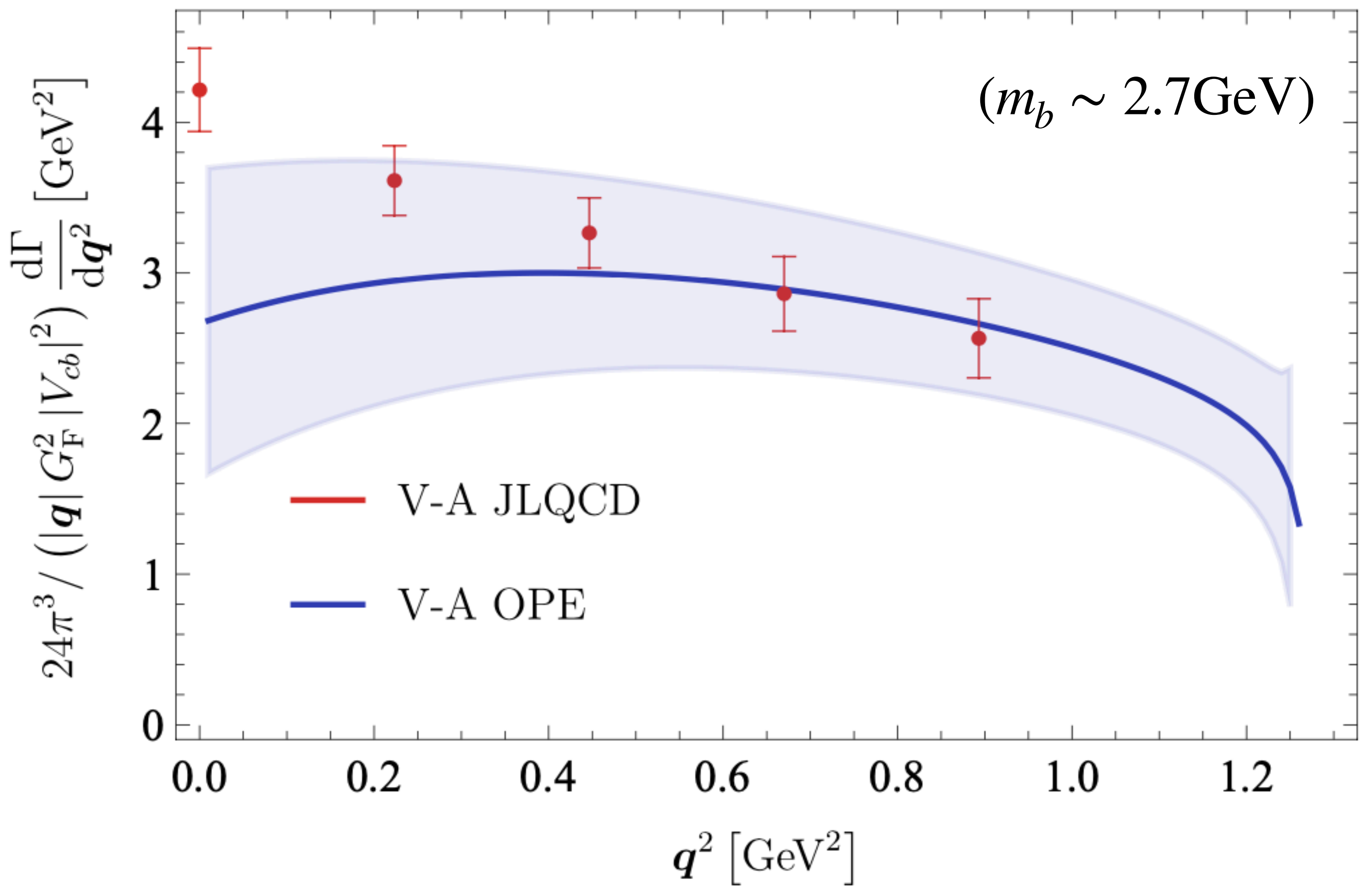
(phase-space enhanced)



Theory predictions for Inclusive Lifetimes

Four-point correlation functions
(analytic continuation/spectral reconstruction)

[Hansen et al 1704.08993, Hashimoto 1703.01881, Gambino et al 2005.13730, and others]



[Gambino et al 2203.11762]

Operator Product Expansion

$$\hat{T} = \frac{G_F^2|V_{cb}|^2 m_b^5}{192\pi^3}\left[c^{(3)}\bar{b}b + \frac{1}{m_b^2}c^{(5)}g_s\bar{b}\sigma^{\mu\nu}G_{\mu\nu}b + \frac{1}{m_b^3}\sum_k c_k^{(6)}O_k^{(6)} + O(1/m_b^4)\right]$$

Gradient Flow Schemes

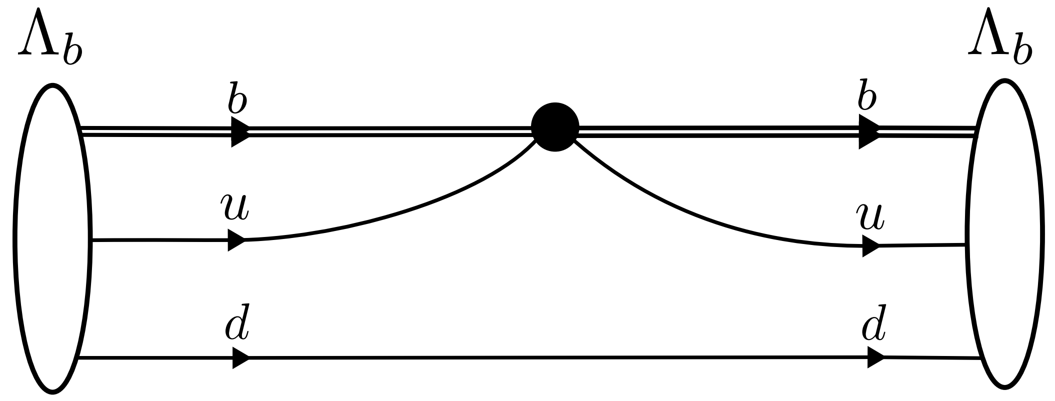
[Black et al, 2310.18059]

Fits to physical parameters as functions of m_b

[Gambino et al, 1704.06105]

Lattice - HQET

Now!



[Pierro, Sachrajda 9805028]

In this talk : isospin non-singlet dim-6 operators are the focus:

$$O_1 = \left(\bar{Q}\gamma_\mu P_L q\right)\left(\bar{q}\gamma_\mu P_L Q\right) \qquad O_3 = \left(\bar{Q}T^A\gamma_\mu P_L q\right)\left(\bar{q}T^A\gamma^\mu P_L Q\right)$$

$$O_2 = \left(\bar{Q}P_L q\right)\left(\bar{q}P_R Q\right) \qquad O_4 = \left(\bar{Q}T^A P_L q\right)\left(\bar{q}T^A P_R Q\right)$$

Ensembles

[RBC-UKQCD 2+1f DWF ensembles]

Ensemble	Lattice Volume	a^{-1} (GeV)	m_π (MeV)	N_{cfgs}	N_p	N_s	
24I	$24^3 \times 64(\times 16)$	1.785(5)	339.7(1.3)	247	444	2133	
32I	$32^3 \times 64(\times 16)$	2.383(9)	302.4(1.2)	142	373	303	
32Ifine	$32^3 \times 64(\times 12)$	3.148(17)	371(5)	113	1062	2230	
48	$48^3 \times 192o(\times 12)$	3.5(1)	280	13	30	0	(step-scaling purposes)

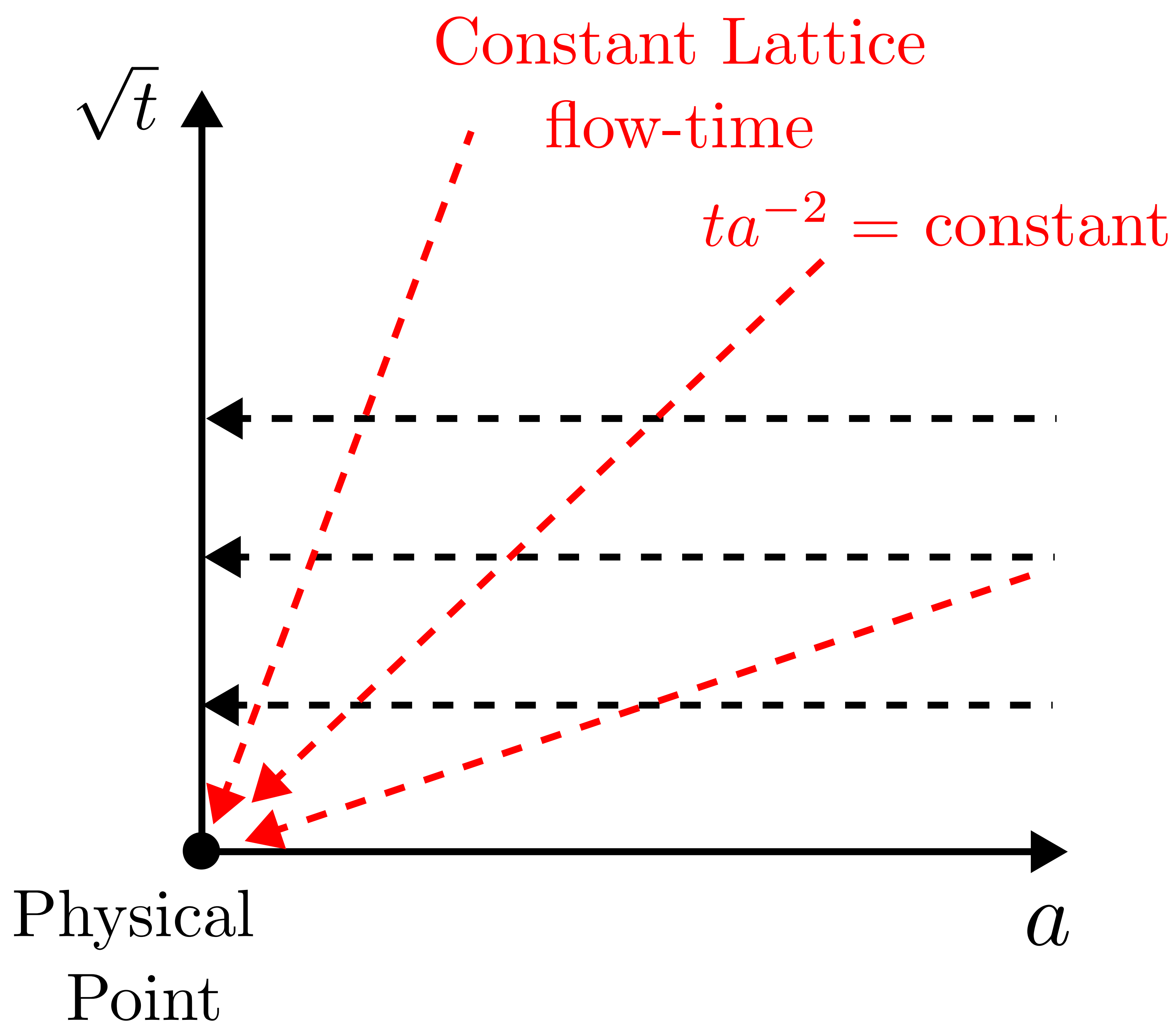
Continuum symmetries are discretised

(chiral symmetry) $D\gamma_5 + \gamma_5 D = 0 \longrightarrow D\gamma_5 + \gamma_5 D = aD\gamma_5 D$ (Ginsparg-Wilson relation)

(Using Ginsparg-Wilson action prevents mixing with operators in other chiral representations)

The role of Gradient Flow

- Gradient flow to a physical flow time t replaces any mixings in a^{-1} with $t^{-\frac{1}{2}}$

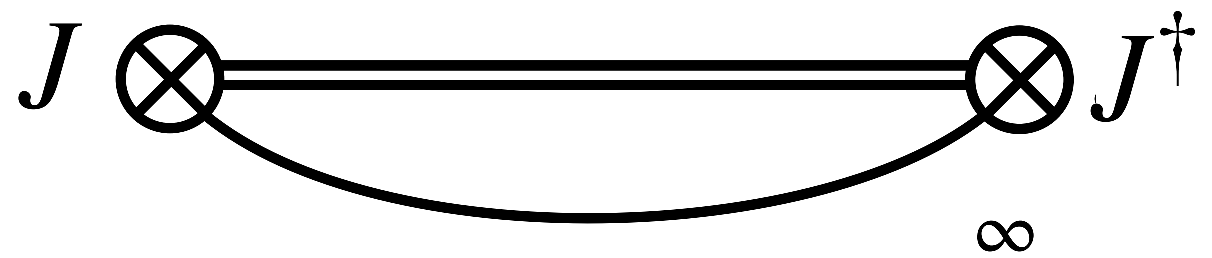


- At fixed physical flow time, OPE power divergences are mollified.

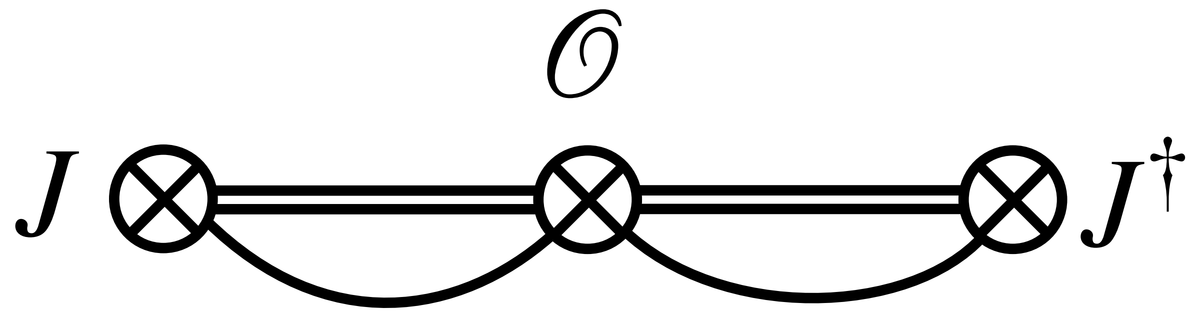
- For Lattice-HQET where there is a static quark self-energy divergence $O(\alpha_S)\bar{h}h/a$, this divergence is removed by Gradient Flow and signal to noise drastically improves. *Gauge fields are Gradient-Flowed, quark fields are unflowed in our study.*

- At fixed lattice flow time, the flow scale vanishes in the continuum limit.

Fitting Procedure (schematic)

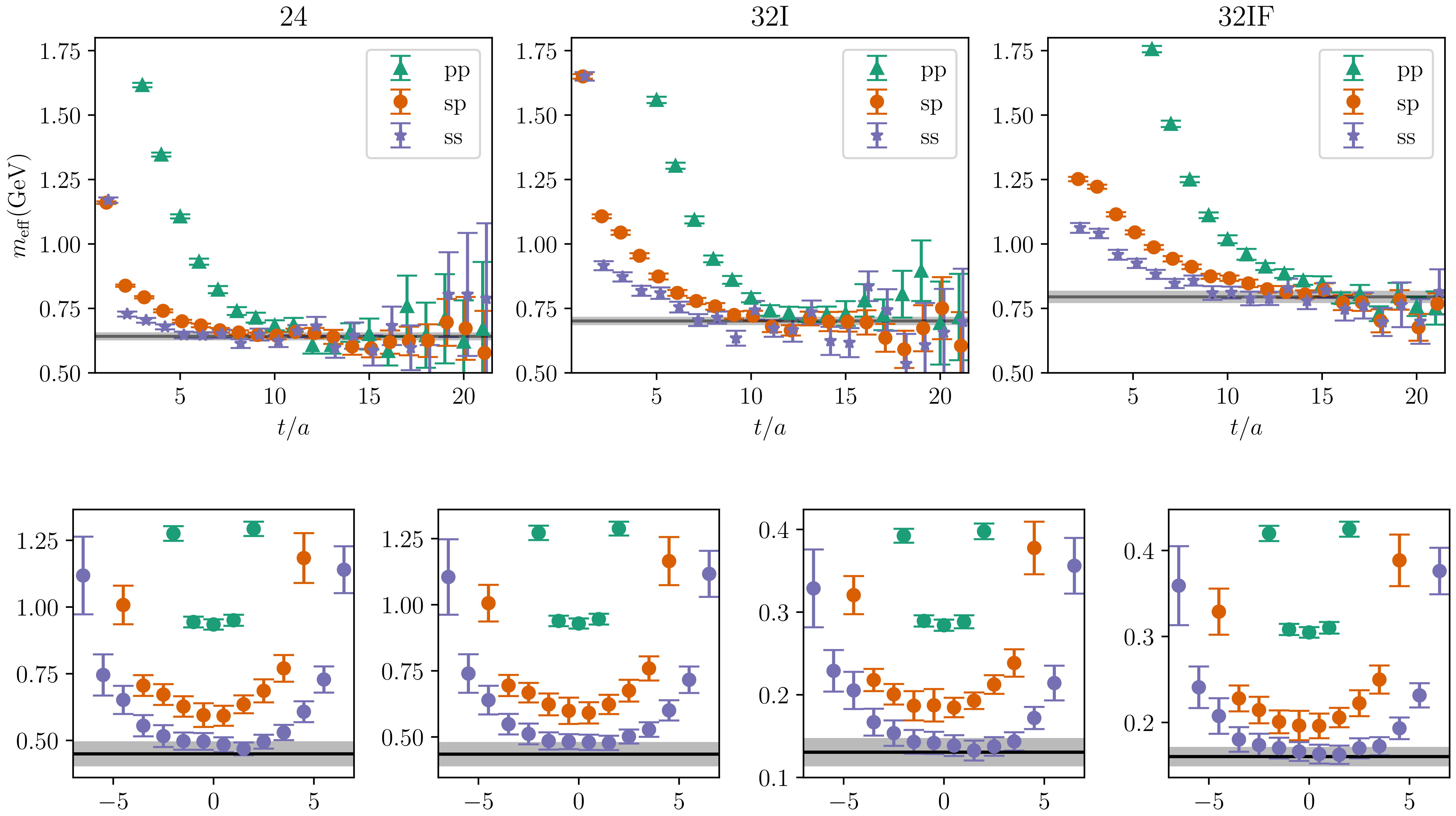


$$= \sum_{n=1}^{\infty} Z_n^* Z_n e^{-E_n t}$$



$$= \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} Z_n^* Z_m e^{-E_n t_1} \langle n | \mathcal{O} | m \rangle e^{-E_m t_2}$$

Fits $\longrightarrow$



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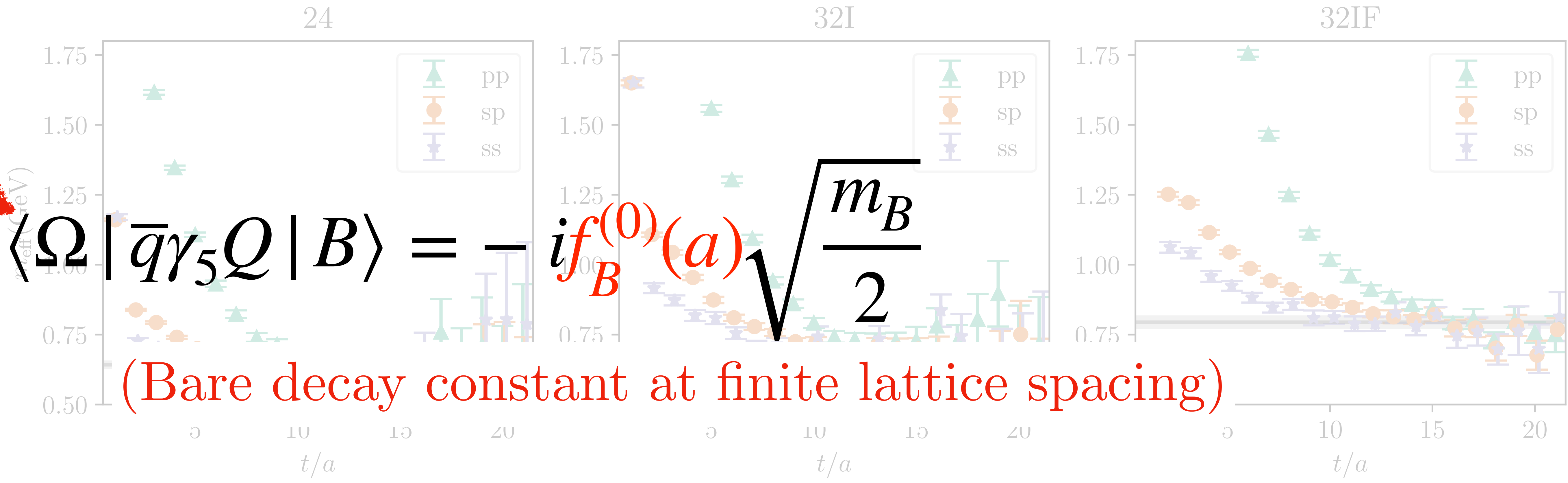
Fitting Procedure (schematic)

$$= \sum_{n=1}^{\infty} Z_n^* Z_n e^{-E_n t}$$

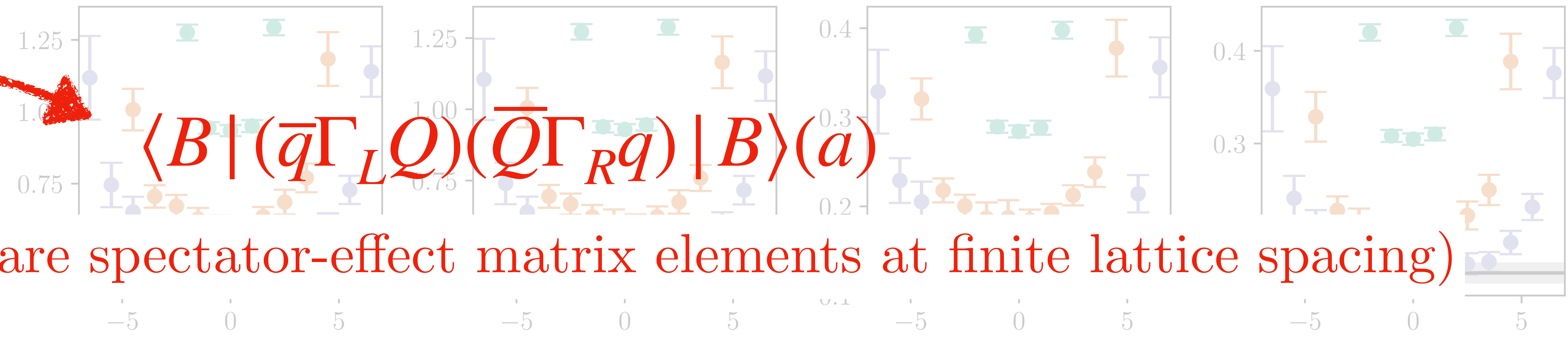


$$\langle \Omega | \bar{q}\gamma_5 Q | B \rangle = -\, i f_B^{(0)}(a) \sqrt{\frac{m_B}{2}}$$

(Bare decay constant at finite lattice spacing)

Fits

$$= \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} Z_n^* Z_m e^{-E_n t_1} \langle n | \mathcal{O} | m \rangle e^{-E_m t_2}$$



$$\langle B | (\bar{q}\Gamma_L Q)(\bar{Q}\Gamma_R q) | B \rangle (a)$$

(Bare spectator-effect matrix elements at finite lattice spacing)

$\boxed{\text{How to renormalise?}}$

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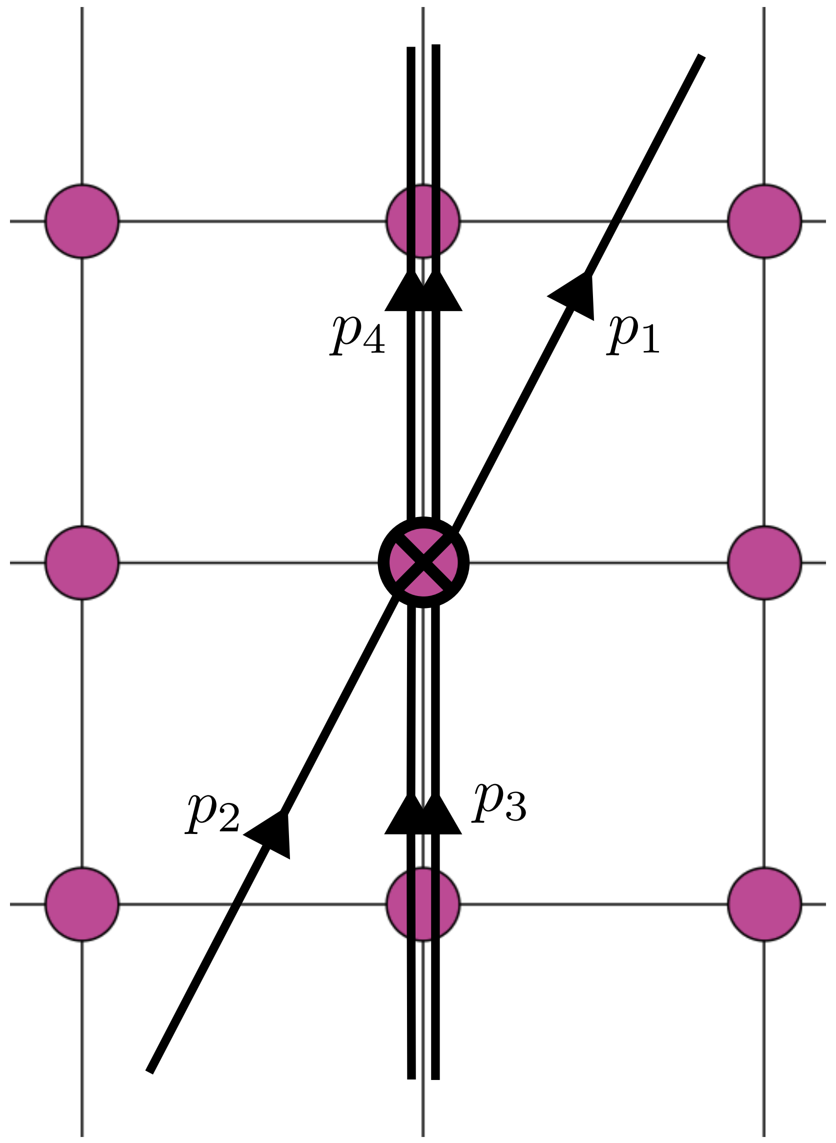
Renormalizing Lattice-HQET

Lattice Perturbation Theory

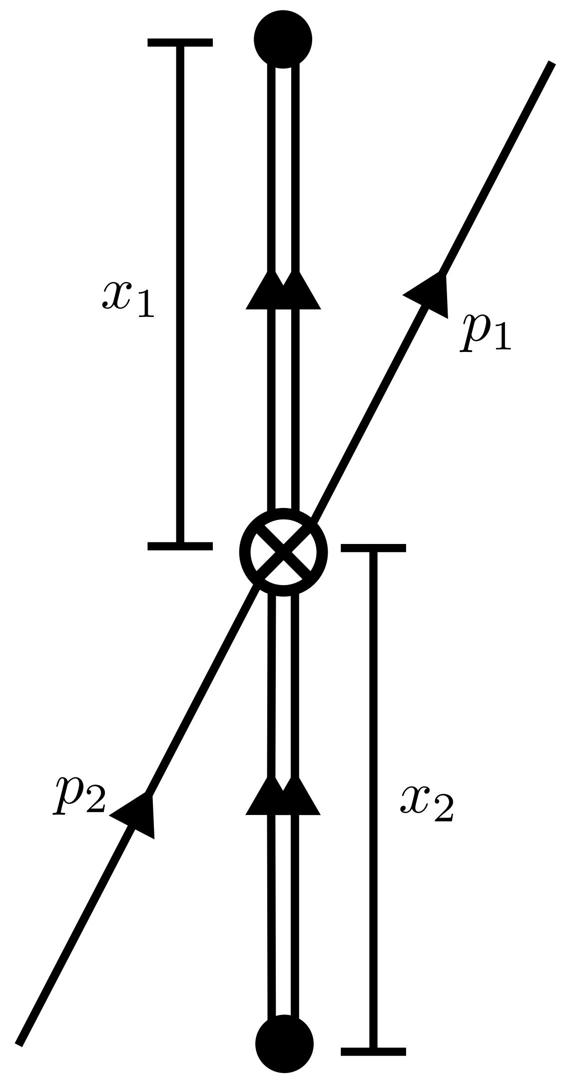


- [Ishikawa et al, 1101.1072]

  Potentially poor convergence properties

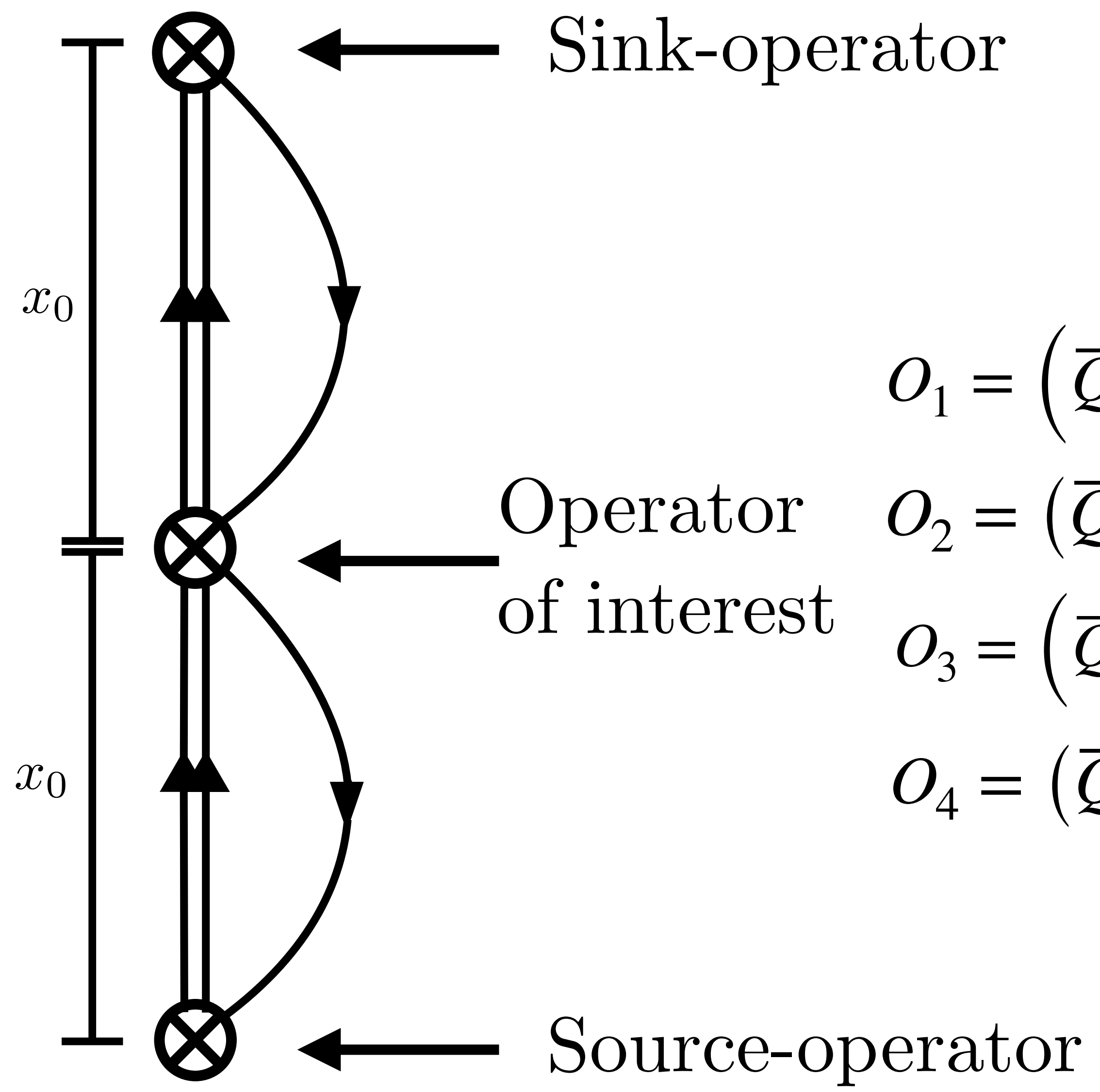
Nonperturbative Renormalization

RI-xMOM



- Long history for light 4q operators (RIMOM)
- Gauge-fixed

X-Space

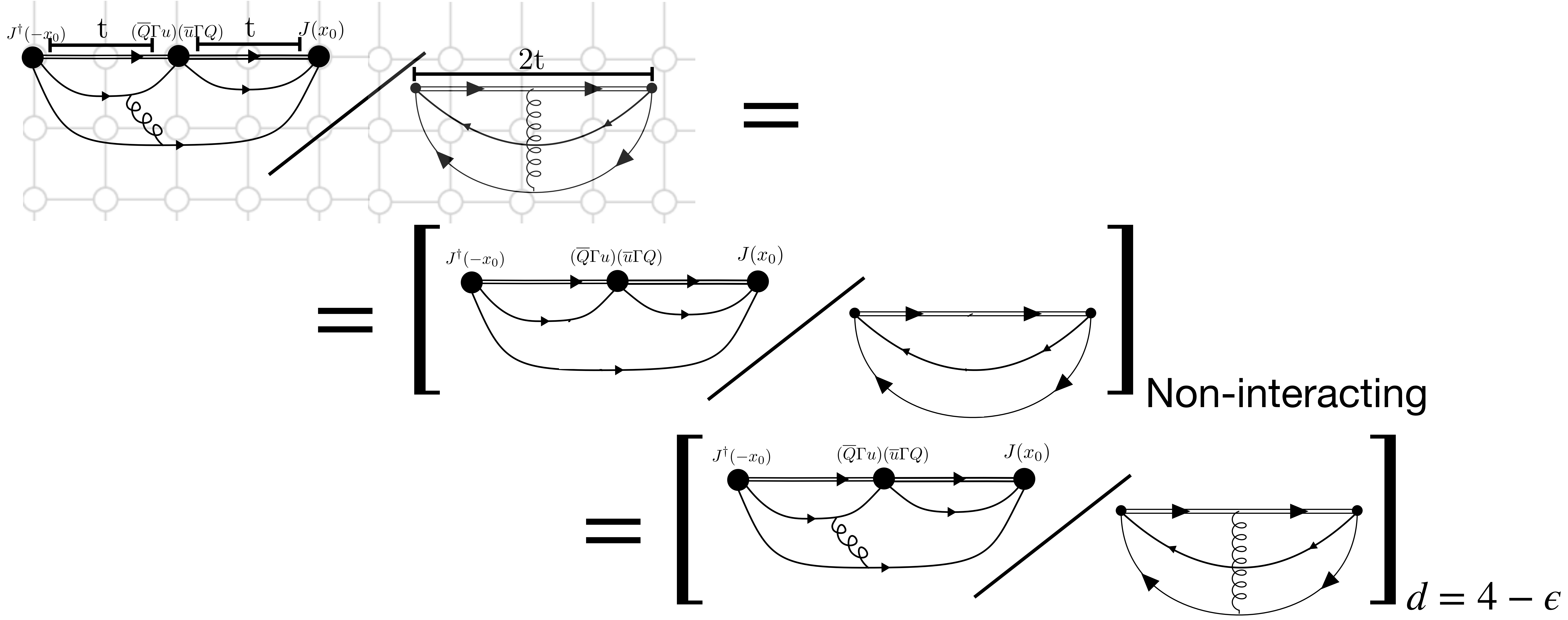


$$O_1 = \left(\bar{Q}\gamma_\mu P_L q\right)\left(\bar{q}\gamma_\mu P_L Q\right)$$
$$O_2 = \left(\bar{Q}P_L q\right)\left(\bar{q}P_R Q\right)$$
$$O_3 = \left(\bar{Q}T^A\gamma_\mu P_L q\right)\left(\bar{q}T^A\gamma^\mu P_L Q\right)$$
$$O_4 = \left(\bar{Q}T^A P_L q\right)\left(\bar{q}T^A P_R Q\right)$$

- We recently proposed X-space schemes and calculated NLO matchings to $\overline{\text{MS}}$

[J.Lin, W. Detmold, S. Meinel: 2404.16191]

The Scheme



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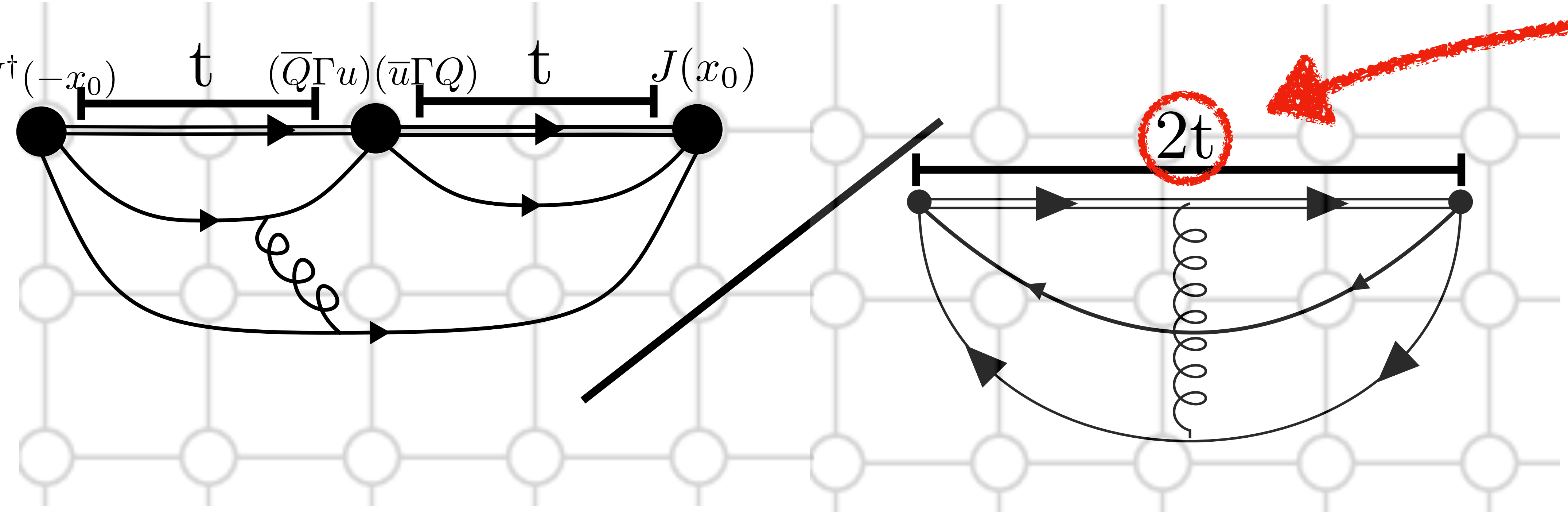

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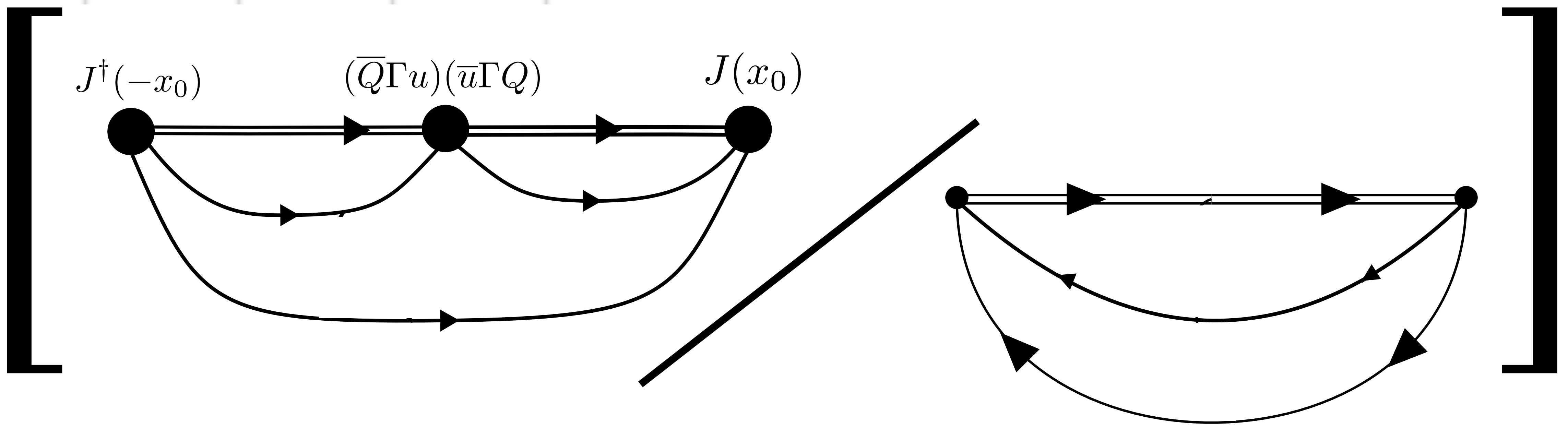
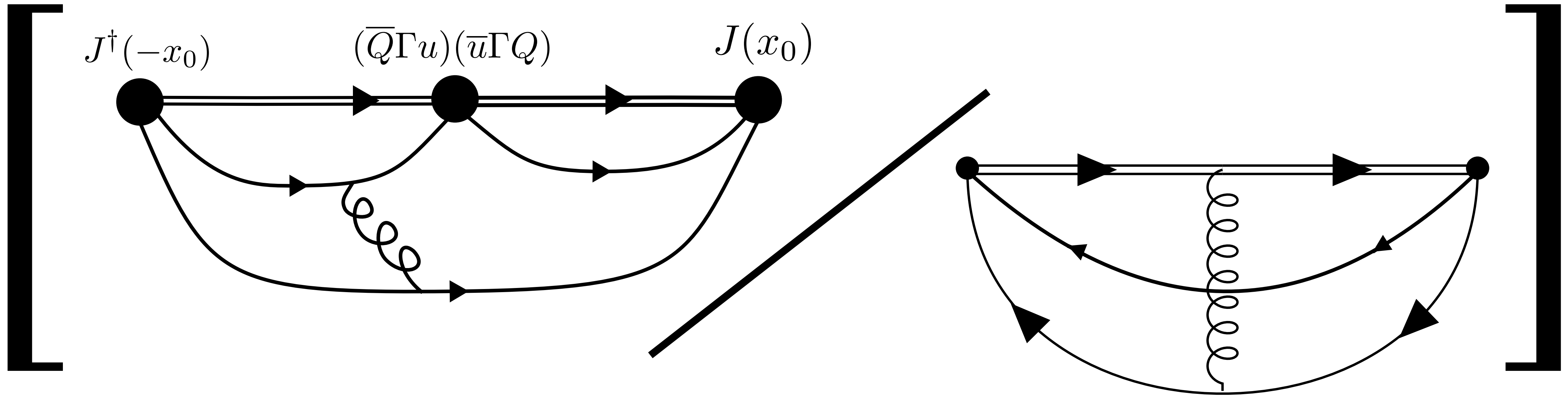
Time-separation between currents sets the scale of the renormalized operators, $\mu \approx t^{-1}$

=



Non-interacting

=



$d = 4 - \epsilon$

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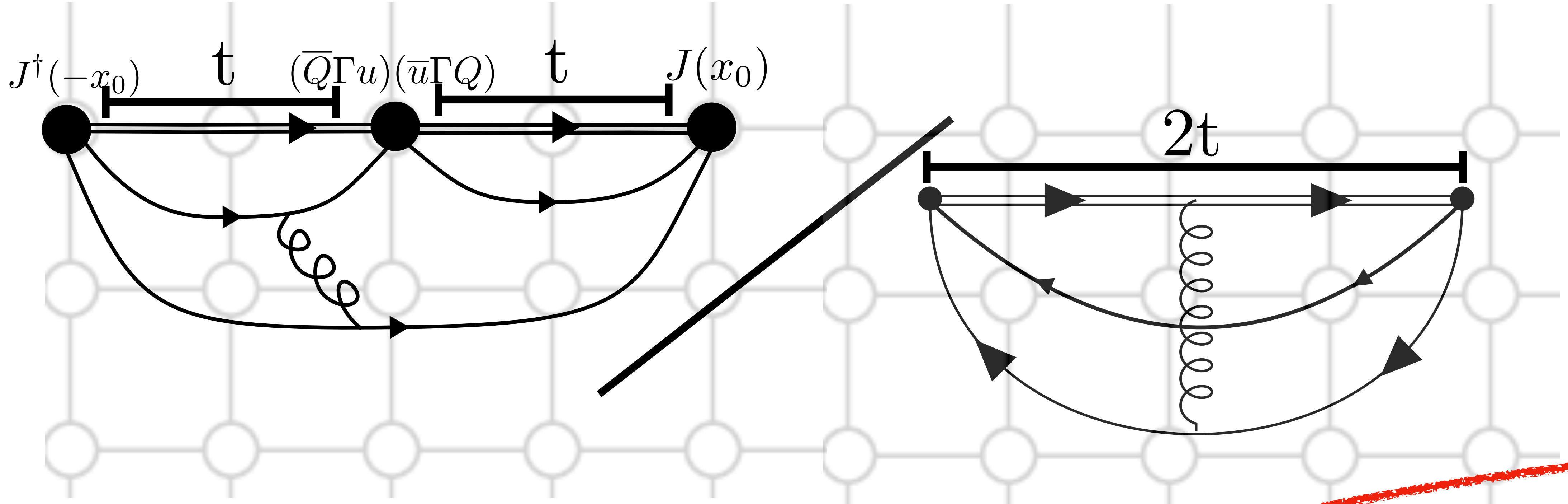
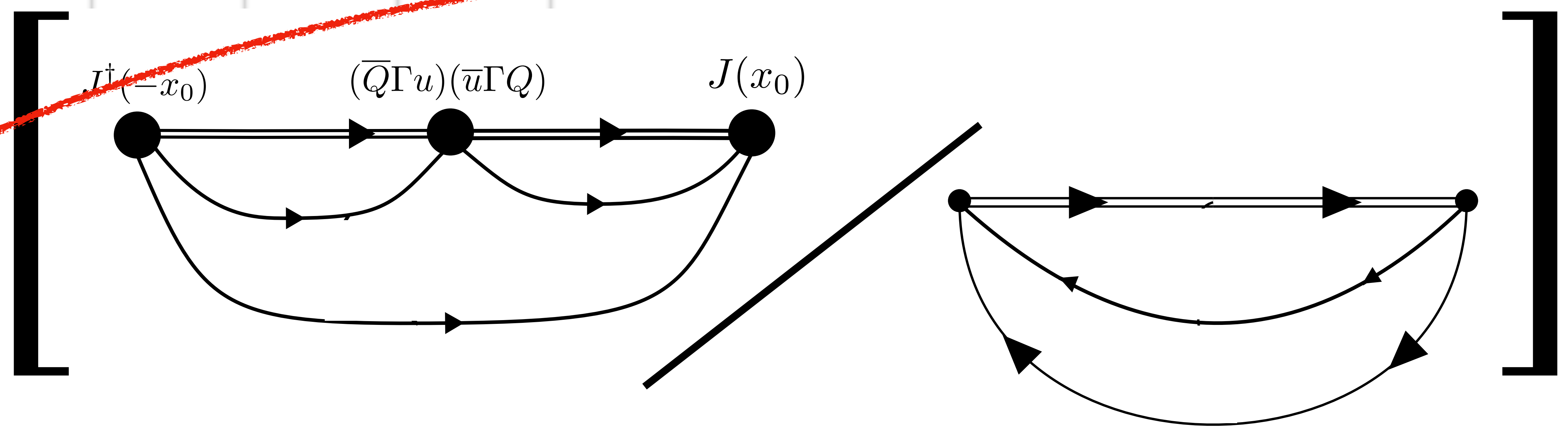


$=$

Requires some perturbative calculations

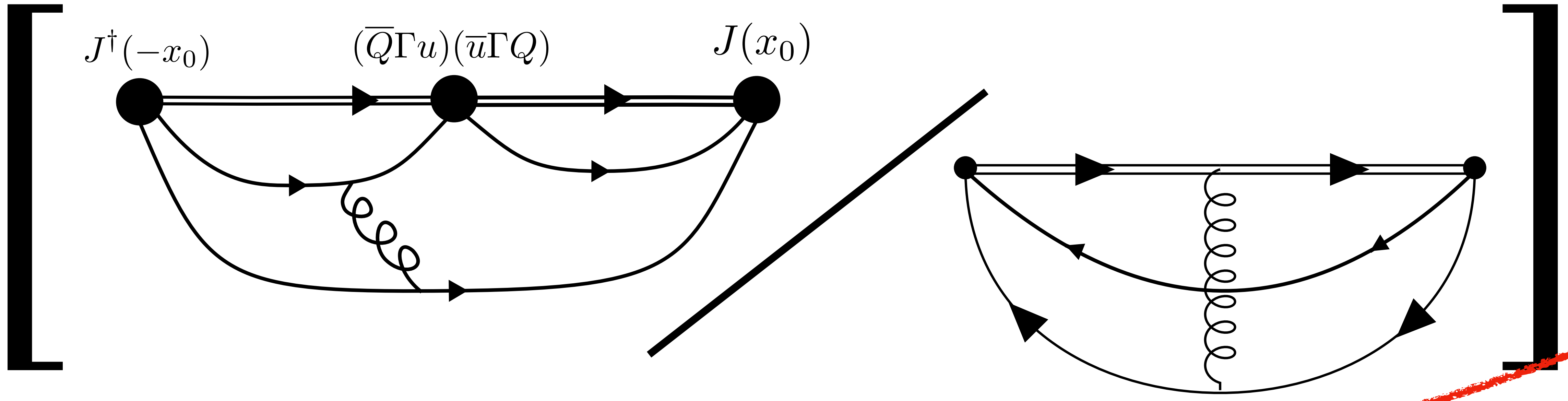
$=$



Non-interacting

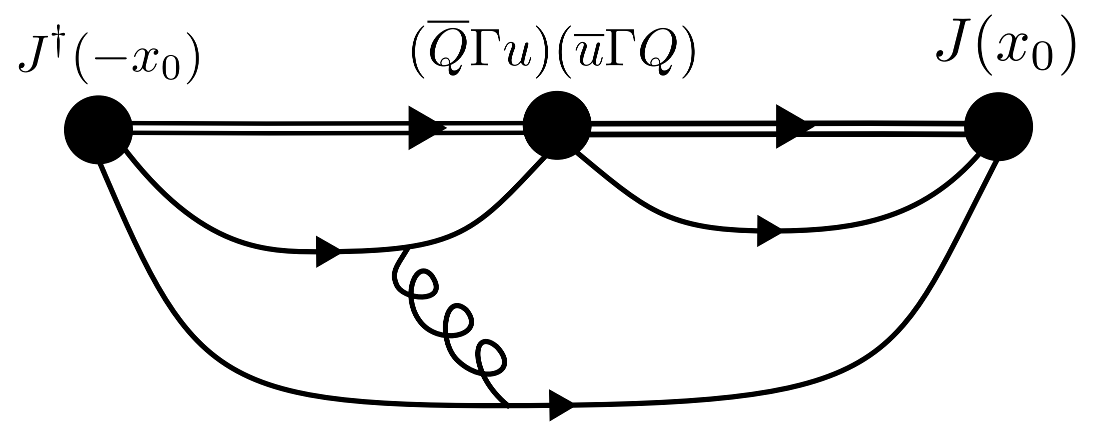
$=$

$=$



$d = 4 - \epsilon$

Perturbative Matching



$$\downarrow$$

$$\int \frac{d^d p_L}{(2\pi)^d}\frac{d^d p_R}{(2\pi)^d}\frac{d^d k}{(2\pi)^d}\frac{e^{ip_L x_L - ip_R x_R}\, p_R^\alpha (p_R-k)^\beta (p_L-k)^\rho p_L^\delta}{(-p_L^2)(-(p_L-k)^2)(-(p_R-k)^2)(-p_R^2)(-k^2)}$$

$$\downarrow$$

Integration by parts

$$\downarrow$$

Two-loop master integrals

- $\Delta Q = 0$, isospin nonsinglet operators relevant for Spectator Effects

$$O_1 := (\overline{Q}\gamma_\mu P_L q)(\overline{q}\gamma_\mu P_L Q),$$
$$O_2 := (\overline{Q} P_L q)(\overline{q} P_R Q),$$
$$O_3 := (\overline{Q}\gamma_\mu P_L T^A q)(\overline{q}\gamma_\mu P_L T^A Q),$$
$$O_4 := (\overline{Q} P_L T^A q)(\overline{q} P_R T^A Q),$$

$$C^{(\overline{\mathrm{MS}};X)}_{ij,n\in\{1,2,3,4\}} := \sum_k Z^{(\overline{\mathrm{MS}})}_{ik} (Z^{(X)})^{-1}_{kj,n\in\{1,2,3,4\}} = \mathbb{1} + \qquad \left(\beta := e^{2\gamma_E}\frac{\mu^2 t^2}{16}\right)$$

$$\alpha_S(\mu)\begin{pmatrix} \frac{\log(\beta)}{\pi} + \frac{4\pi}{9} + \frac{8}{3\pi} & 0 & -\frac{3\log(\beta)}{4\pi} + \pi - \frac{3}{8\pi} & -\frac{5}{4\pi} \\ 0 & \frac{\log(\beta)}{\pi} + \frac{4\pi}{9} + \frac{8}{3\pi} & -\frac{1}{16\pi} & -\frac{3\log(\beta)}{4\pi} + \pi - \frac{7}{8\pi} \\ -\frac{\log(\beta)}{6\pi} + \frac{2\pi}{27} - \frac{5}{18\pi} & -\frac{1}{9\pi} & \frac{\log(\beta)}{8\pi} + \frac{7\pi}{18} + \frac{5}{8\pi} & -\frac{1}{12\pi} \\ -\frac{1}{36\pi} & -\frac{\log(\beta)}{6\pi} + \frac{2\pi}{27} - \frac{5}{18\pi} & -\frac{3}{16\pi} & \frac{\log(\beta)}{8\pi} + \frac{7\pi}{18} + \frac{23}{24\pi} \end{pmatrix}$$

- $\Delta Q = 2$ operators relevant for $B - \overline{B}$ mixing

$$O_1 := (\overline{Q}_+ P_L q)(\overline{Q}_- P_L q),$$
$$O_2 := (\overline{Q}_+ P_L T^A q)(\overline{Q}_- P_L T^A q),$$
$$O_3 := (\overline{Q}_+ P_L q)(\overline{Q}_- P_R q),$$
$$O_4 := (\overline{Q}_+ P_L T^A q)(\overline{Q}_- P_R T^A q).$$

$$C^{(\overline{\mathrm{MS}},X)}_{\{O_1,O_2\}} = \mathbb{1} + \frac{\alpha_S}{\pi}\begin{pmatrix} \frac{7\log(\beta)}{9} + \frac{4\pi^2}{9} + \frac{23}{9} & \frac{2\log(\beta)}{3} - \frac{\pi^2}{3} + \frac{4}{3} \\ \frac{4\log(\beta)}{27} - \frac{2\pi^2}{27} + \frac{8}{27} & \frac{5\log(\beta)}{9} + \frac{5\pi^2}{9} + \frac{19}{9} \end{pmatrix}$$

$$C^{(\overline{\mathrm{MS}},X)}_{\{O_3,O_4\}} = \mathbb{1} + \frac{\alpha_S}{\pi}\begin{pmatrix} \log(\beta) + \frac{4\pi^2}{9} + \frac{25}{9} & \frac{3\log(\beta)}{4} - \frac{\pi^2}{3} + \frac{7}{6} \\ \frac{\log(\beta)}{6} - \frac{2\pi^2}{27} + \frac{7}{27} & \frac{3\log(\beta)}{4} + \frac{5\pi^2}{9} + \frac{43}{18} \end{pmatrix}$$

[J.Lin, W. Detmold, S. Meinel: 2404.16191]

$$\int \frac{d^d p_L}{(2\pi)^d}\frac{d^d p_R}{(2\pi)^d}\frac{d^d k}{(2\pi)^d}\frac{e^{ip_L x_L - ip_R x_R}}{(-p_L^2)(-(p_L - k \ldots}$$

Integration by parts

Two-loop master integrals

- $\Delta B = 0$, isospin nonsinglet operators relevant for Spectator Effects

$O_1 := (\overline{Q}\gamma_\mu P_L q)(\overline{q}\gamma_\mu P_L Q)$,

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$$\begin{pmatrix} \frac{\log(\beta)}{\pi} + \frac{4\pi}{9} + \frac{8}{3\pi} & 0 & -\frac{3\log(\beta)}{4\pi} + \pi - \frac{3}{8\pi} & -\frac{5}{4\pi} \\ \ldots & \ldots & -\frac{1}{16\pi} & -\frac{3\log(\beta)}{4\pi} + \pi - \frac{7}{8\pi} \\ \ldots & \ldots & \ldots + \frac{7\pi}{18} + \frac{5}{8\pi} & -\frac{1}{12\pi} \\ \ldots & \ldots & -\frac{3}{16\pi} & \frac{\log(\beta)}{8\pi} + \frac{7\pi}{18} + \frac{23}{24\pi} \end{pmatrix}$$

$$C_{\{O_1,O_2\}} \ldots \frac{\ldots}{\pi}\begin{pmatrix} \ldots & \frac{2\log(\beta)}{3} - \frac{\pi^2}{3} + \frac{4}{3} \\ \frac{4\log(\beta)}{27} - \frac{2\pi^2}{27} + \frac{8}{27} & \frac{5\log(\beta)}{9} + \frac{5\pi^2}{9} + \frac{19}{9} \end{pmatrix}$$

$O_2 := (\overline{Q}_+ P_L T^A q)(\overline{Q}_- P_L T^A q)$,

$O_3 := (\overline{Q}_+ P_L q)(\overline{Q}_- P_R q)$,

$O_4 := (\overline{Q}_+ P_L T^A q)(\overline{Q}_- P_R T^A q)$.

$$C^{(\overline{\mathrm{MS}},X)}_{\{O_3,O_4\}} = \mathbb{1} + \frac{\alpha_S}{\pi}\begin{pmatrix} \log(\beta) + \frac{4\pi^2}{9} + \frac{25}{9} & \frac{3\log(\beta)}{4} - \frac{\pi^2}{3} + \frac{7}{6} \\ \frac{\log(\beta)}{6} - \frac{2\pi^2}{27} + \frac{7}{27} & \frac{3\log(\beta)}{4} + \frac{5\pi^2}{9} + \frac{43}{18} \end{pmatrix}$$

Upshot

- X-space schemes offer the advantage that they are gauge-invariant *and* don't require separate renormalization condition for static-quark energy.

[J.Lin, W. Detmold, S. Meinel: 2404.16191]

Step-scaling

$$\mathcal{O}^{(0,\text{Lattice})}_{\text{HQET}} \xrightarrow{\text{NP}} \mathcal{O}^{(X)}_{\text{HQET}}(\mu = t^{-1}) \xrightarrow{\text{NLO}} \mathcal{O}^{(\overline{\text{MS}})}_{\text{HQET}}(\mu = t^{-1}) \xrightarrow{\color{red}\text{LL}} \mathcal{O}^{(\overline{\text{MS}})}_{\text{HQET}}(\mu = m_b)$$

($a \ll t \ll \Lambda^{-1}_{\text{QCD}}$ window problem)

$O(\alpha_s^2)$ anomalous dimensions would be nice

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Step-scaling

$$\mathcal{O}^{(0,\mathrm{Lattice})}_{\mathrm{HQET}} \xrightarrow{\mathrm{NP}} \mathcal{O}^{(X)}_{\mathrm{HQET}}(\mu = t^{-1}) \xrightarrow{\mathrm{NLO}} \mathcal{O}^{(\overline{\mathrm{MS}})}_{\mathrm{HQET}}(\mu = t^{-1}) \xrightarrow{\color{red}{\mathrm{LL}}} \mathcal{O}^{(\overline{\mathrm{MS}})}_{\mathrm{HQET}}(\mu = m_b)$$

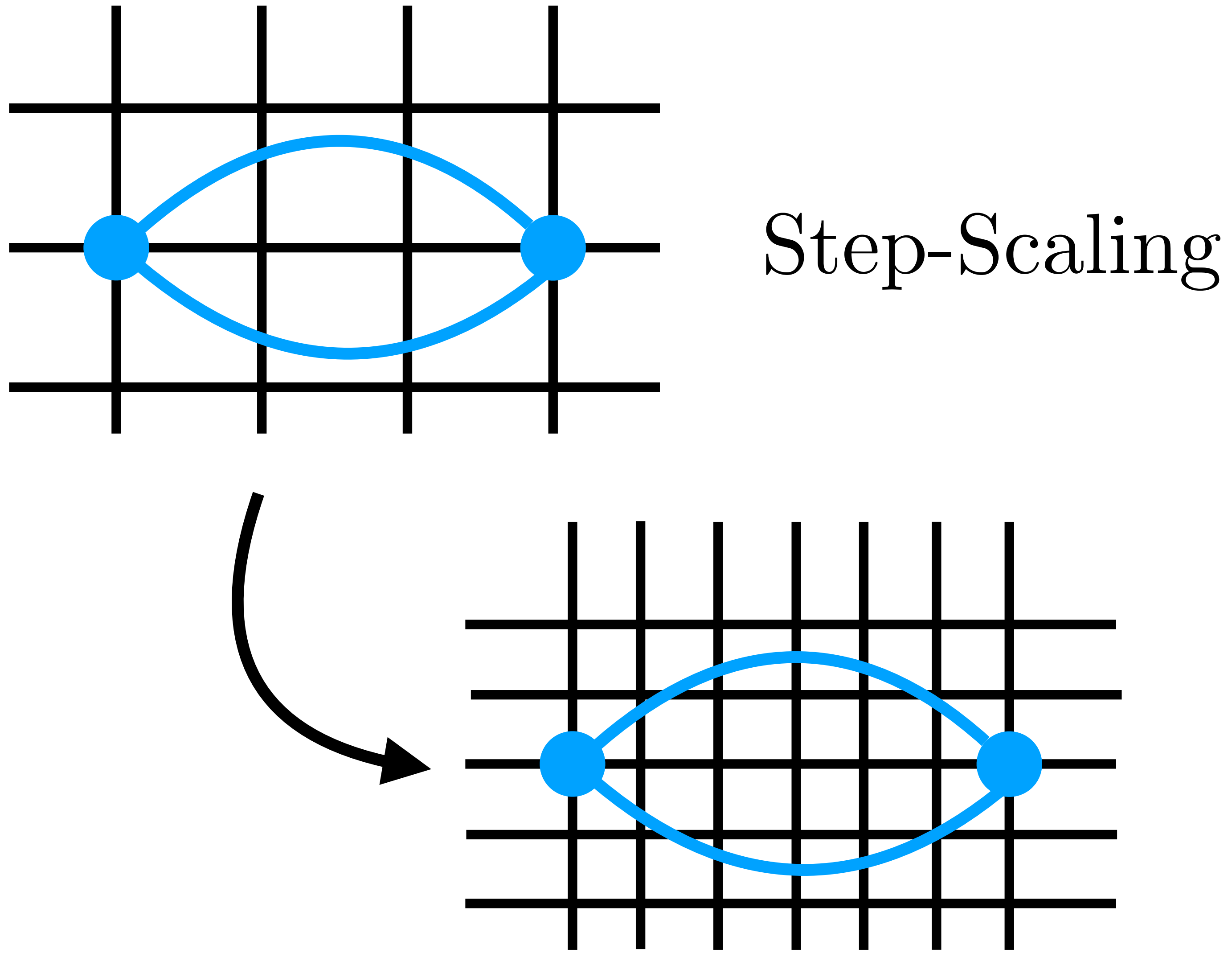
($a \ll t \ll \Lambda^{-1}_{\mathrm{QCD}}$ window problem)

$O(\alpha_s^2)$ anomalous dimensions would be nice

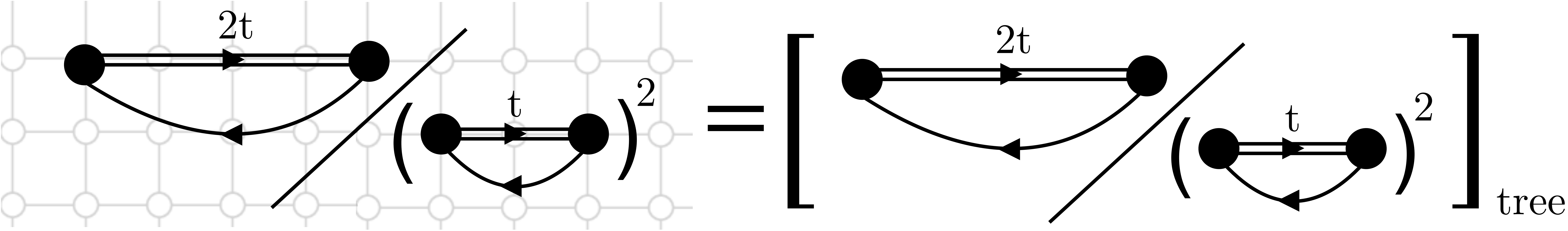
$$\mathcal{O}^{(0,\mathrm{Lattice})}_{\mathrm{HQET}} \xrightarrow{\mathrm{NP}} \mathcal{O}^{(X)}_{\mathrm{HQET}}(\mu = t_c^{-1})$$

$$\Big\downarrow \text{Step-Scaling}$$

$$\mathcal{O}^{(X)}_{\mathrm{HQET}}(\mu = t_f^{-1}) \xrightarrow{\mathrm{NLO}} \mathcal{O}^{(\overline{\mathrm{MS}})}_{\mathrm{HQET}}(\mu = t_f^{-1}) \xrightarrow{\color{red}{\mathrm{LL}}} \mathcal{O}^{(\overline{\mathrm{MS}})}_{\mathrm{HQET}}(\mu = m_b)$$



Before dim-6 operators: a study in f_B



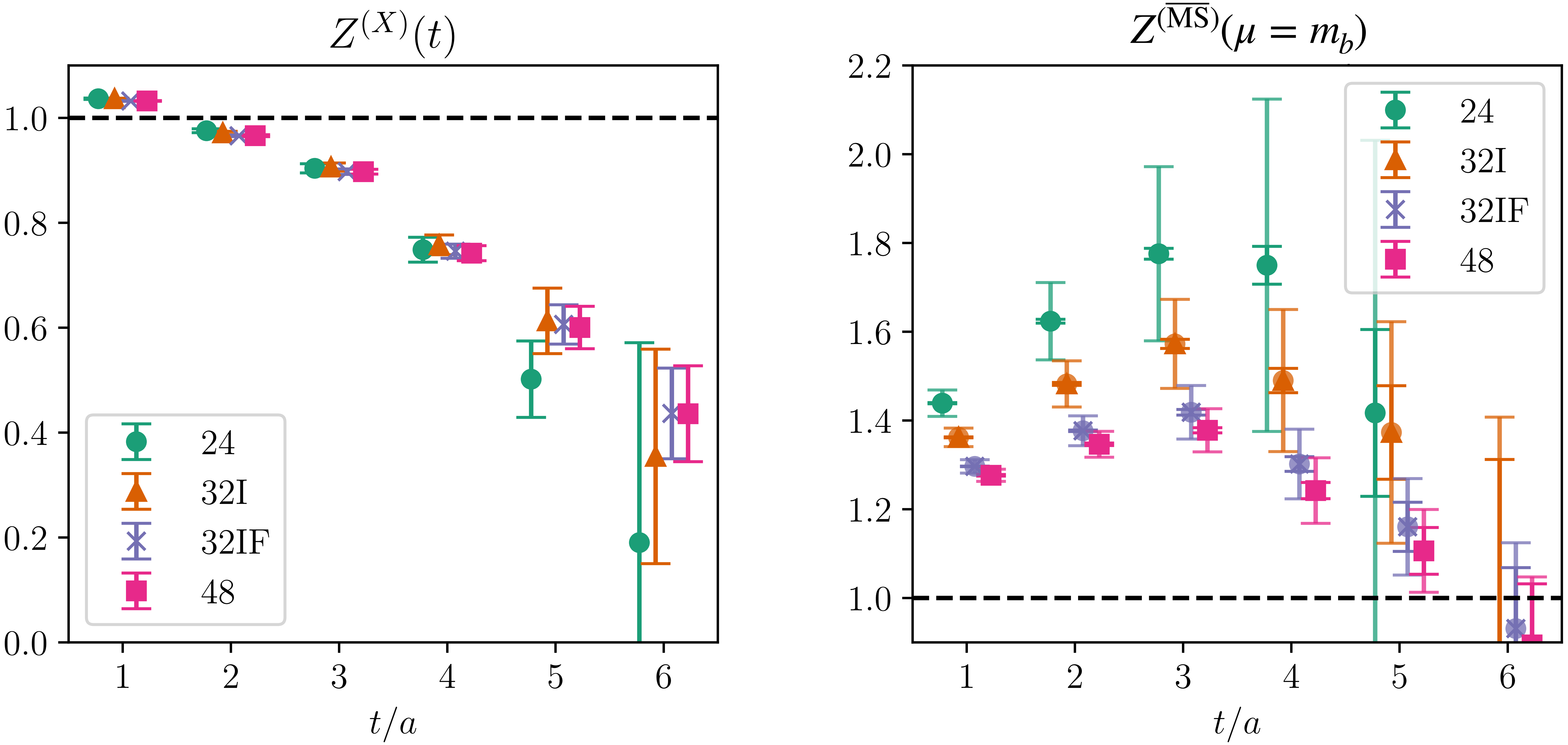
$$\mathcal{O}^{(0,\mathrm{Lattice})}_{\mathrm{HQET}} \xrightarrow{\mathrm{NP}} \mathcal{O}^{(X)}_{\mathrm{HQET}}(\mu = t_c^{-1})$$

$$\Big\downarrow \text{Step-Scaling}$$

$$\mathcal{O}^{(X)}_{\mathrm{HQET}}(\mu = t_f^{-1}) \xrightarrow{\mathrm{N^3LO}} \mathcal{O}^{(\overline{\mathrm{MS}})}_{\mathrm{HQET}}(\mu = t_f^{-1}) \xrightarrow{\mathrm{N^3LL}} \mathcal{O}^{(\overline{\mathrm{MS}})}_{\mathrm{HQET}}(\mu = m_b)$$

[Grozin, 2411.11080] [Grozin, 2311.09894]

Renormalizing without step-scaling for f_B



(inner error bars are statistical, outer error bars are due to renormalization errors)

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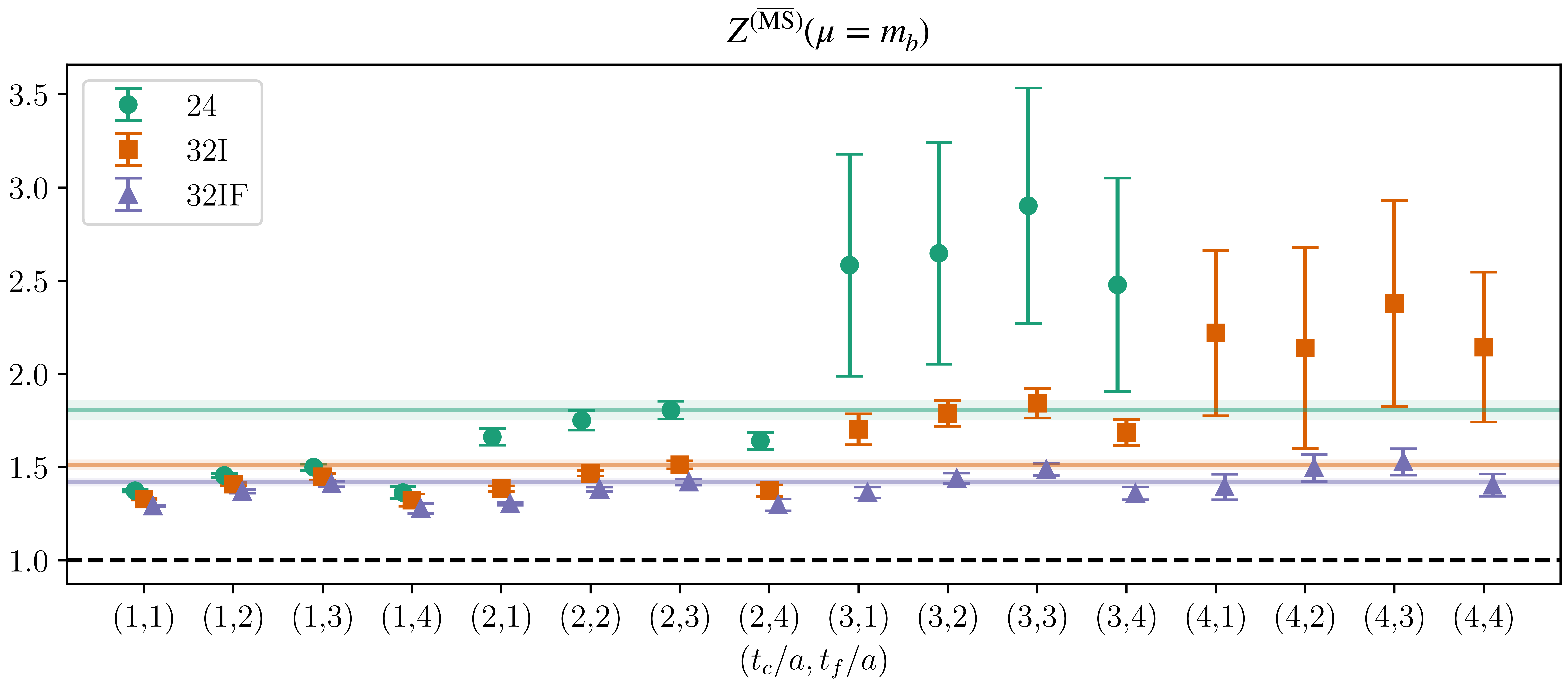
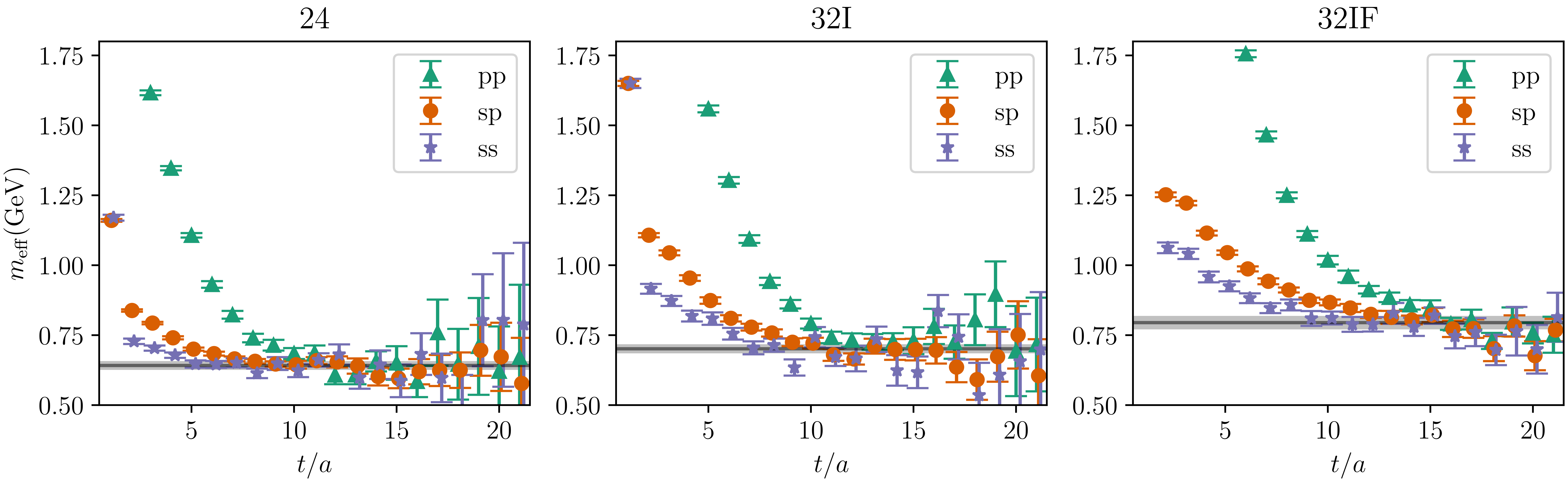
Renormalizing with step-scaling for f_B

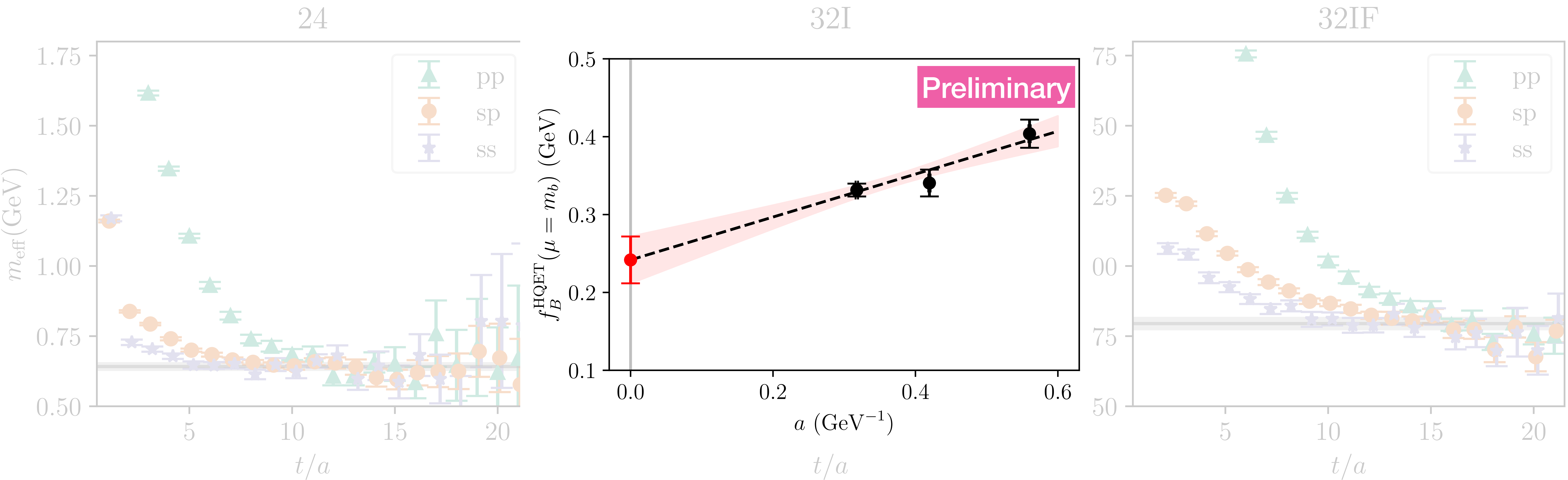


(error bars include renormalization errors)

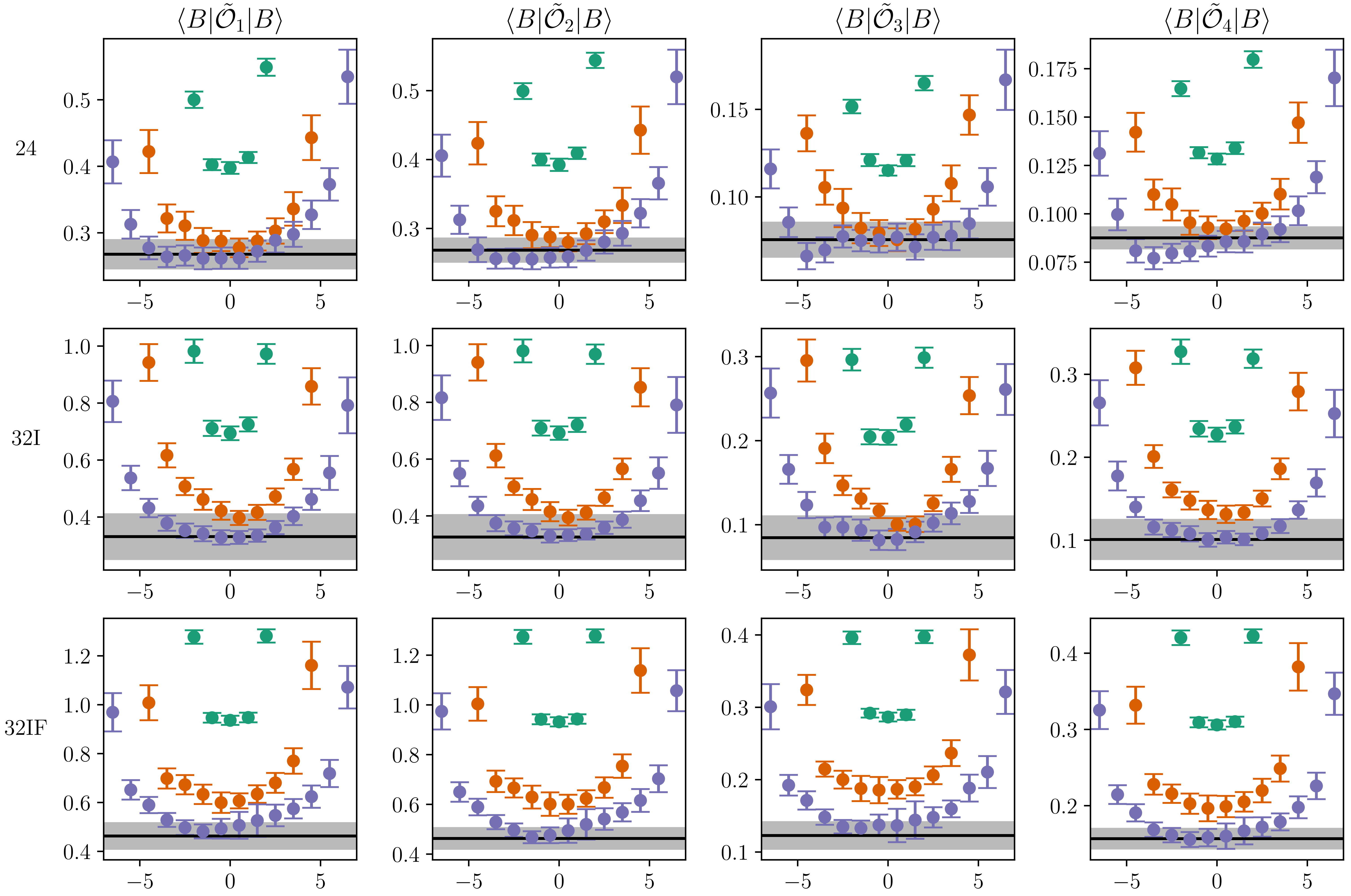
f_B continuum limit



f_B continuum limit



Ratio plots for dim-6 operators



(different basis)

$$\tilde{\mathcal{O}}_1 = (\bar{b}_i\gamma_\mu P_L q_i)(\bar{q}_j\gamma^\mu P_L b_j), \quad \tilde{\mathcal{O}}_2 = (\bar{b}_i P_L q_i)(\bar{q}_j P_R b_j),$$
$$\tilde{\mathcal{O}}_3 = (\bar{b}_i\gamma_\mu P_L q_j)(\bar{q}_j\gamma^\mu P_L b_i), \quad \tilde{\mathcal{O}}_4 = (\bar{b}_i P_L q_j)(\bar{q}_j P_R b_i),$$

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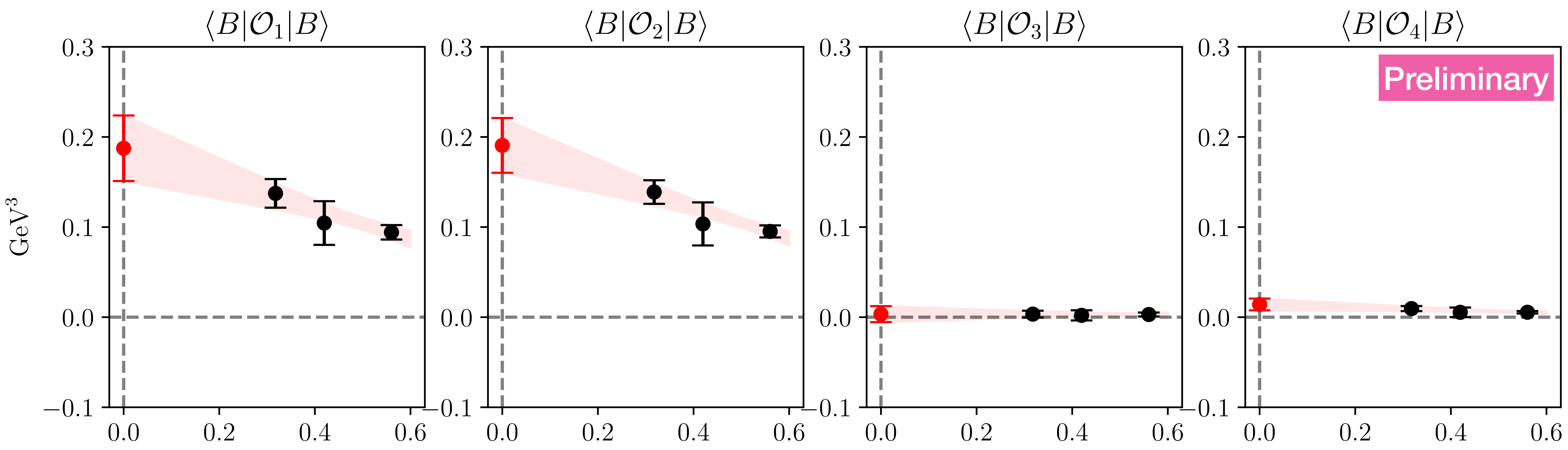
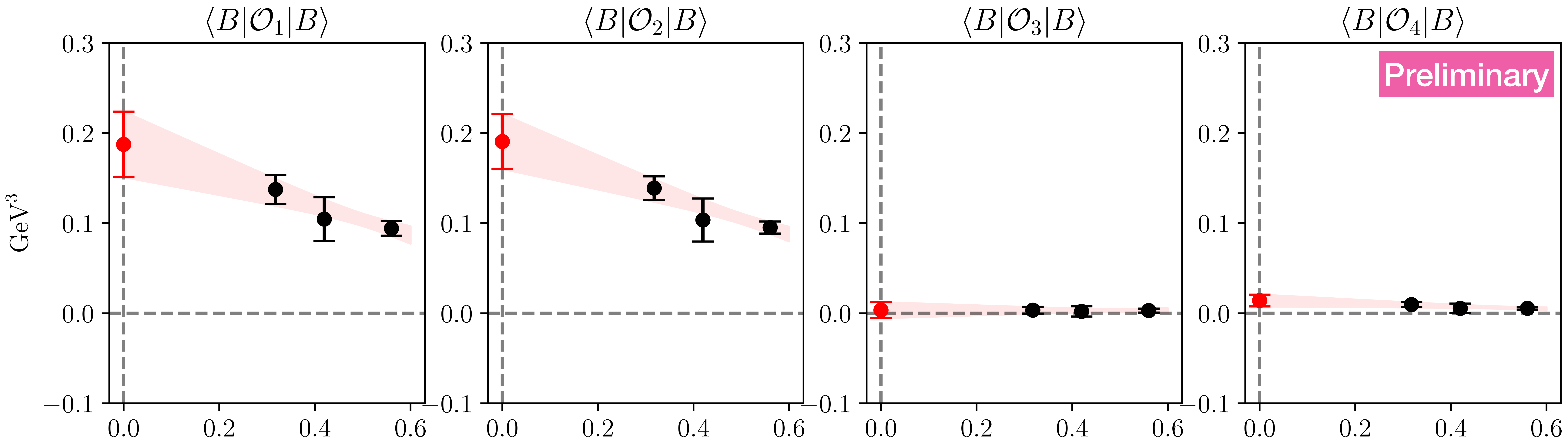


- only leading-logarithmic running of $\Delta Q = 0$ HQET operators
- No chiral extrapolation performed (results at $m_\pi \approx 350\mathrm{MeV}$)
- Error quantification on bag-parameters not performed yet

Continuum Limits



- only leading-logarithmic running of $\Delta Q = 0$ HQET operators
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Stay tuned, thank you for your attention!