A holistic view on the heavy quark expansion

A combined $B \to X_c \ell \bar{\nu}_\ell$ and B lifetime analysis

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color meets flavor

UNIVERSITÄT BONN

Emmy Noether-Programm
DFG Deutsche Forschungsgemeinschaft

Experimental Data & Theory Calculation

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INCLUSIVE $B \to X_c l \bar{\nu}_l$: OPEN-SOURCE LIBRARY

MF, Milutin, Vos, SciPost Phys.Codeb. 55 (2025) 1

Open-source library in python: **KOLYA**

https://gitlab.com/vcb-inclusive/kolya

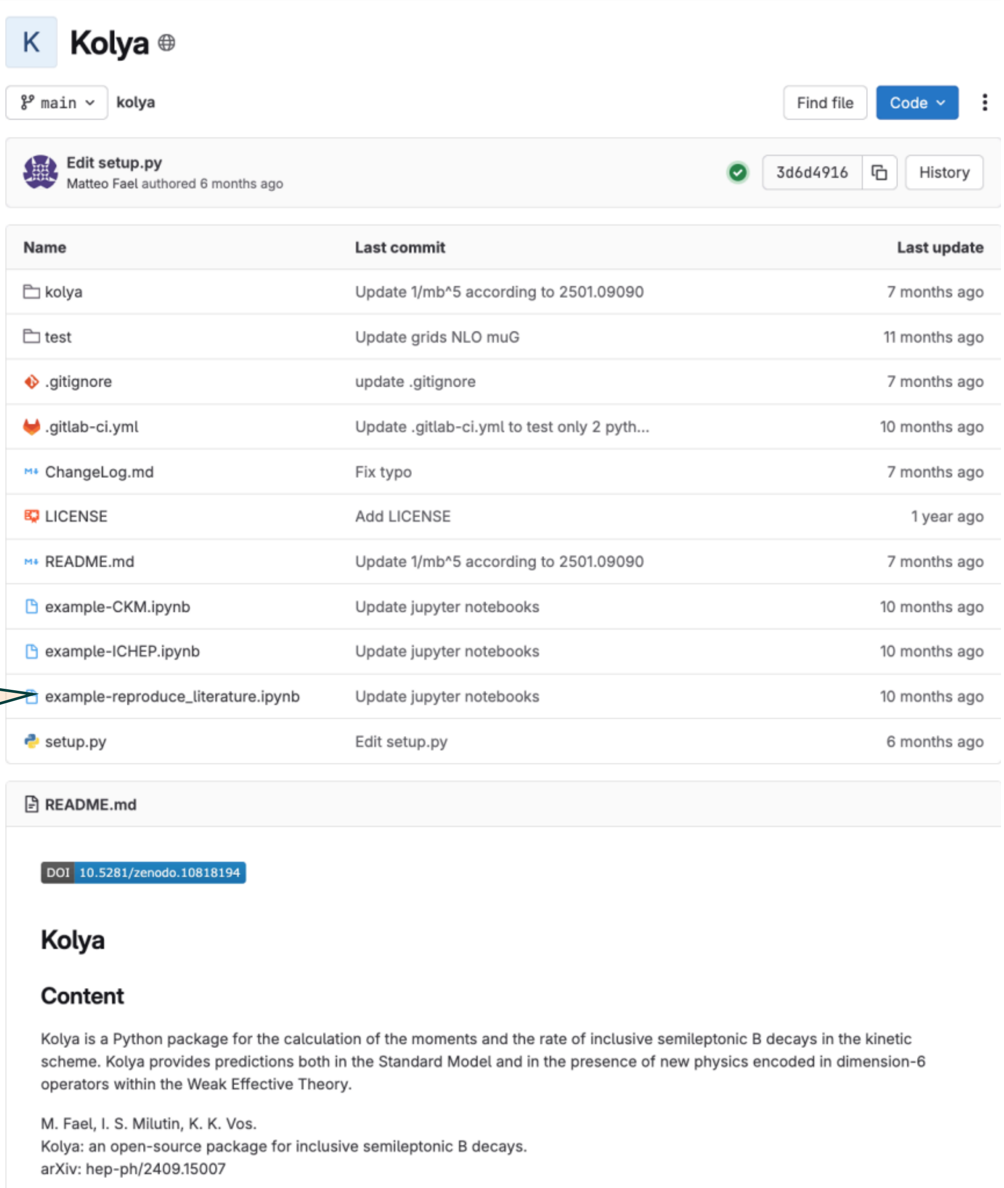
- Branching ratios & kinematic moments
- Use the kinetic scheme

Bigi, Shifman, Uraltsev, Vainshtein, *Phys.Rev.D* 56 (1997) 4017
Czarnecki, Melnikov, Uraltsev, Phys.Rev.Lett. 80 (1998) 3189
MF, Schönwald, Steinhauser, *Phys.Rev.Lett.* 125 (2020) 052003

- Interface to CRunDec for automatic α_s, $m_b^{\rm kin}$ and $\overline{m}_c$ RGE evolution

Chetyrkin, Kuhn, Steinhauser, Comput. Phys. Commun. 133 (2000) 43
Schmidt, Steinhauser, Comput. Phys. Commun. 183 (2012) 1845
Herren, Steinhauser, Comput. Phys. Commun. 224 (2018) 333

And now also B-lifetimes!
... release date pending



Name	Last commit	Last update
kolya	Update 1/mb^5 according to 2501.09090	7 months ago
test	Update grids NLO muG	11 months ago
.gitignore	update .gitignore	7 months ago
.gitlab-ci.yml	Update .gitlab-ci.yml to test only 2 pyth...	10 months ago
ChangeLog.md	Fix typo	7 months ago
LICENSE	Add LICENSE	1 year ago
README.md	Update 1/mb^5 according to 2501.09090	7 months ago
example-CKM.ipynb	Update jupyter notebooks	10 months ago
example-ICHEP.ipynb	Update jupyter notebooks	10 months ago
example-reproduce_literature.ipynb	Update jupyter notebooks	10 months ago
setup.py	Edit setup.py	6 months ago

README.md

DOI 10.5281/zenodo.10818194

Kolya

Content

Kolya is a Python package for the calculation of the moments and the rate of inclusive semileptonic B decays in the kinetic scheme. Kolya provides predictions both in the Standard Model and in the presence of new physics encoded in dimension-6 operators within the Weak Effective Theory.

M. Fael, I. S. Milutin, K. K. Vos.
Kolya: an open-source package for inclusive semileptonic B decays.
arXiv: hep-ph/2409.15007

Experimental data

Some necessary bookkeeping …

- $B \to X_c \ell \bar{\nu}_\ell$ moments
  - E_ℓ, M_X moments [Babar Collaboration Phys.Rev.D 81 (2010) 032003]
  - M_X moments [Belle Collaboration Phys.Rev.D 75 (2007) 032005]
  - E_ℓ moments [Belle Collaboration Phys.Rev.D 75 (2007) 032001]
  - M_X moments [CDF Collaboration Phys.Rev.D 71 (2005) 051103]
  - E_ℓ moments [Cleo Collaboration Phys.Rev.D 70 (2004) 032002]
  - E_ℓ, M_X moments [Delphi Collaboration Eur.Phys.J.C 45 (2006) 35-59]

Experiment	$\Delta\mathcal{B}$	$\langle E_\ell \rangle$	$\langle (E_\ell - \langle E_\ell \rangle)^2 \rangle$	$\langle (E_\ell - \langle E_\ell \rangle)^3 \rangle$
Babar	0.6, 1.2, 1.5	0.6, 0.8, 1.0, 1.2, 1.5	0.6, 1.0, 1.5	0.8, 1.2
Belle	0.6, 1.0, 1.4	1.0, 1.4	0.6, 1.4	0.8, 1.2
Belle II	–	–	–	–
CDF	–	–	–	–
Cleo	–	–	–	–
Delphi	–	0.0	0.0	0.0

Experiment	$\langle M_X^2 \rangle$	$\langle (M_X^2 - \langle M_X^2 \rangle)^2 \rangle$	$\langle (M_X^2 - \langle M_X^2 \rangle)^3 \rangle$
Babar	0.9, 1.1, 1.3, 1.5	0.8, 1.0, 1.2, 1.4	0.9, 1.3
Belle	0.7, 1.1, 1.3, 1.5	0.7, 0.9, 1.3	–
Belle II	–	–	–
CDF	0.7	0.7	–
Cleo	1.0, 1.5	1.0, 1-5	–
Delphi	0.0	0.0	0.0

Experiment	$\langle q^2 \rangle$	$\langle (q^2 - \langle q^2 \rangle)^2 \rangle$	$\langle (q^2 - \langle q^2 \rangle)^3 \rangle$
Babar	–	–	–
Belle	3.0, 4.5, 6.0, 7.5	3.0, 4.5, 6.0, 7.5	3.0, 4.5, 6.0, 7.5
Belle II	1.5, 3.0, 4.5, 6.0, 7.5	1.5, 3.0, 4.5, 6.0, 7.5	1.5, 3.0, 4.5, 6.0, 7.5
CDF	–	–	–
Cleo	–	–	–
Delphi	–	–	–

Table entries give threshold values

- Only a sub-set of the available data points are used
  - Data points are strongly correlated, and inverting existing covariance matrices runs into numerical problems
  - But: Because the data is highly correlated, the information gain of adding more data is “low”
  - We use the same moments data as [G. Finauri, P. Gambino JHEP 02 (2024) 206]
  - In addition, we use the measured partial branching ratios $\Delta\mathcal{B}$ instead of the extrapolated total branching ratio
- Nota bene: Experimental correlations between different moments are unknown!
- New Belle II measurement will tell us the importance of this missing correlation

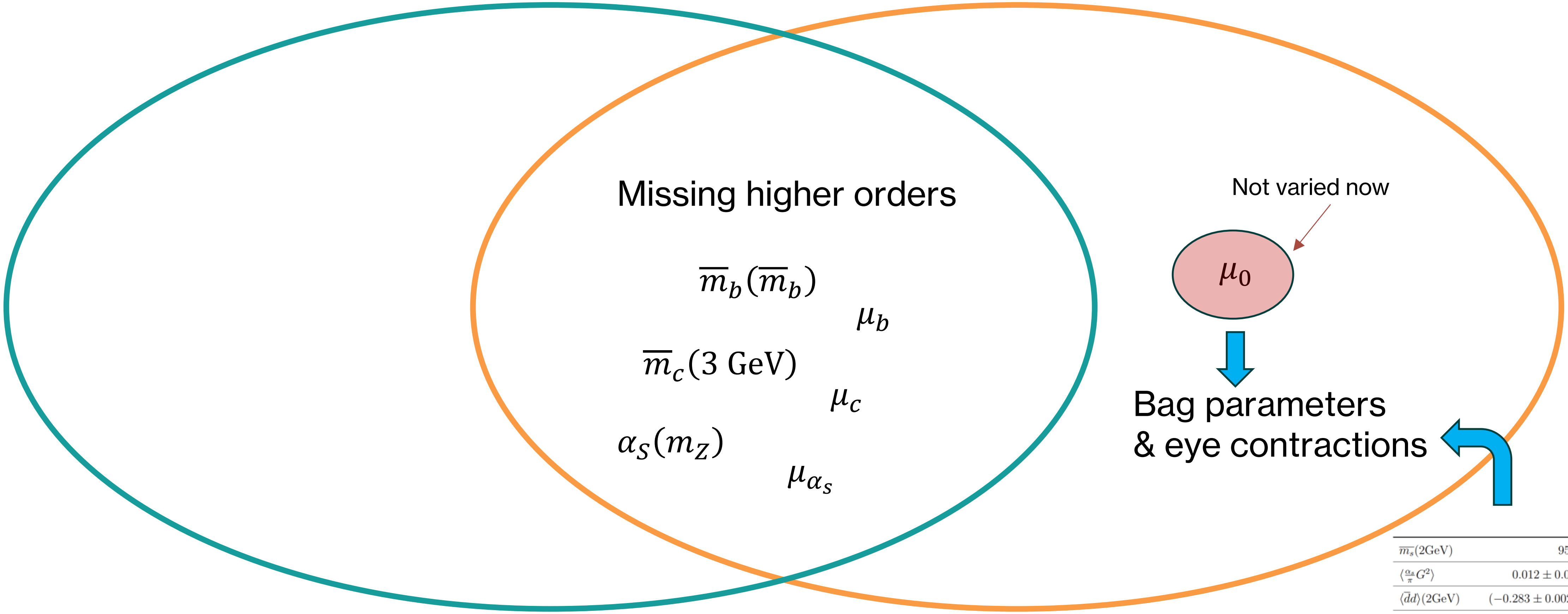
Treatment of systematics

Sources of systematics

$$\langle B_q|\tilde{O}_i^q\,|B_q\rangle = F_q^2(\mu_0)\, m_{B_q}\, \tilde{B}_i^q(\mu_0)\,,$$

$$\langle B_q|\tilde{O}_i^{q'}\,|B_q\rangle = F_q^2(\mu_0)\, m_{B_q}\, \tilde{\delta}_i^{q'q}(\mu_0)\,, \qquad q \neq q'\,,$$

$$f_{B_q} = \frac{F_q(\mu_0)}{\sqrt{m_{B_q}}}\left[1 + \frac{\alpha_s(\mu_0)}{2\pi}\left(\ln\left(\frac{m_b^2}{\mu_0^2}\right) - \frac{4}{3}\right) + \mathcal{O}\left(\frac{1}{m_b}\right)\right]$$

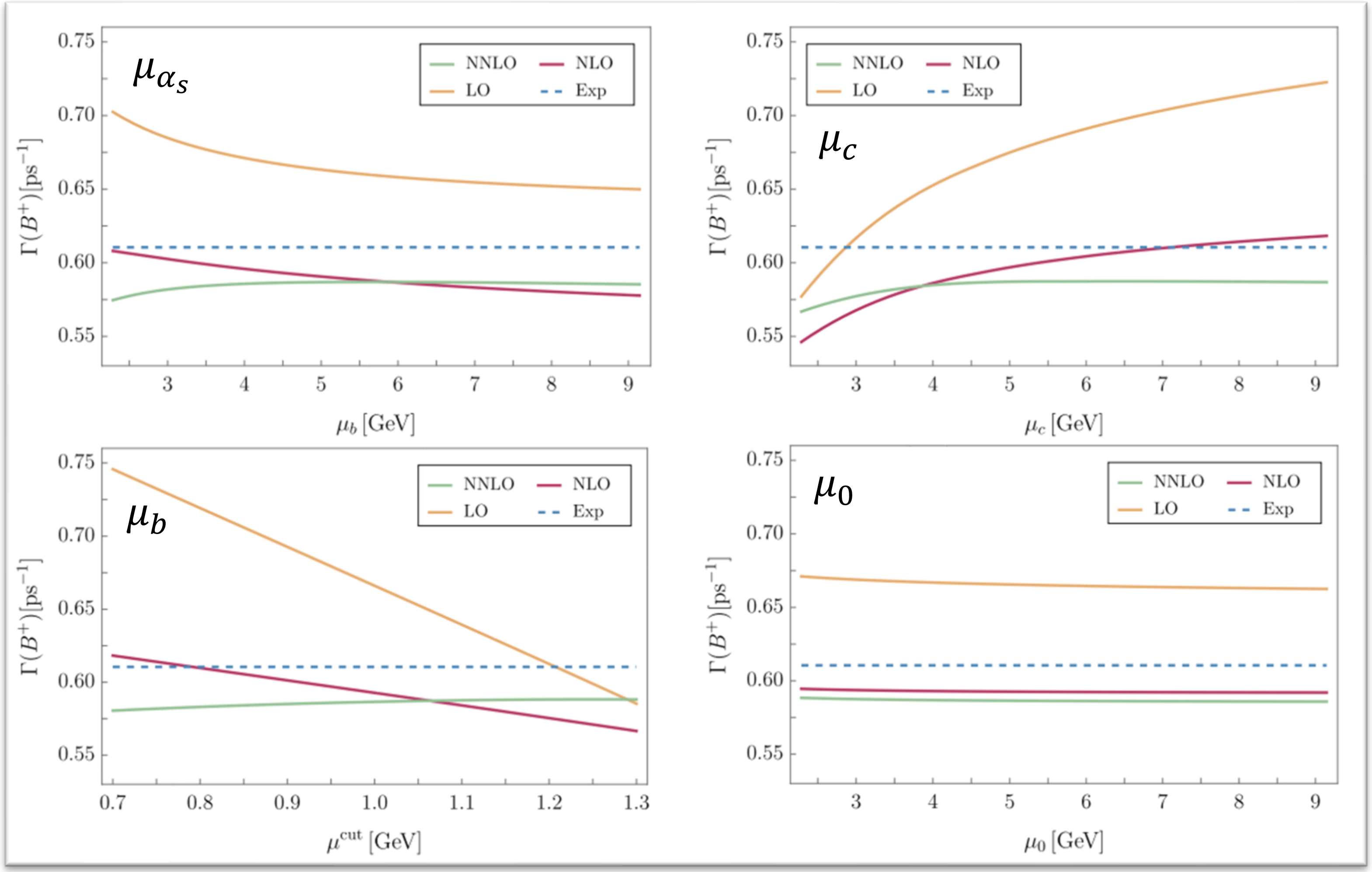


$\overline{m_s}(2\mathrm{GeV})$	95^{+9}_{-3}	MeV	[60]
$\langle \frac{\alpha_s}{\pi} G^2 \rangle$	0.012 ± 0.006	GeV^4	[61]
$\langle \bar{d}d \rangle(2\mathrm{GeV})$	$(-0.283 \pm 0.002)^3$	GeV^3	[62]
$\langle \bar{s}s \rangle(2\mathrm{GeV})$	$(-0.296 \pm 0.002)^3$	GeV^3	[63]
$\overline{m_b}(\overline{m_b})$	$4.203^{+0.016}_{-0.034}$	GeV	[64, 65]
M_Z	91.1876	GeV	[37]
$\alpha_s(M_Z)$	0.1181 ± 0.0011		[60]

Affects the inclusive $B \to X_c \ell \bar{\nu}_\ell$ moments

Affects the lifetime prediction

Sources of systematics

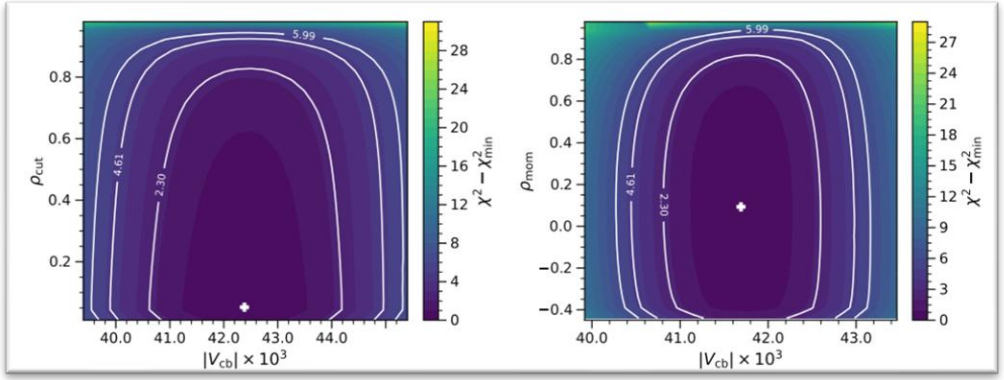


- Different naming convention!
- μ_0 only appears in the lifetime prediction
- Neglecting μ_0 should have a small impact

A quick reminder on existing approaches

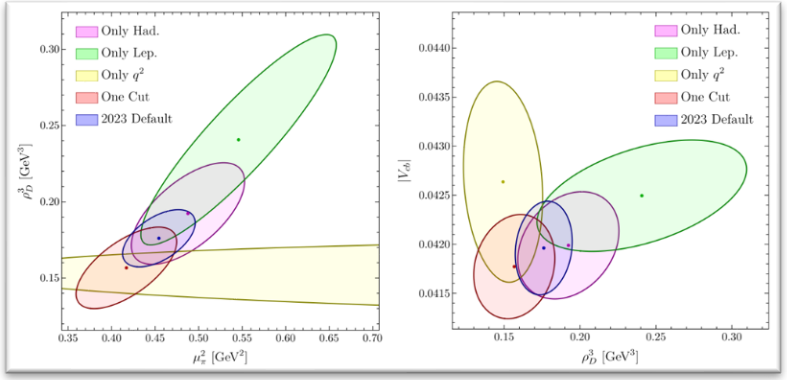
[F. Bernlochner, M. Fael, K. Olschewsky, E. Perrson, R. van Tonder, K. K. Vos, M. Welsch JHEP 10 (2022) 068]

Profile over all possible correlations and include the effect in the uncertainty



[G. Finauri, P. Gambino JHEP 02 (2024) 206]

Correlation model build from experience / intuition and include the effect in the uncertainty



- Construct a covariance matrix that captures all uncertainties, and define $C = C_{exp} + C_{theory}$ for the χ^2
- Both approaches try to capture "missing higher order" uncertainty by variations to e.g., ρ_D
- Potential caveats:
  - Non-linear dependence cannot be described with a covariance matrix
  - Assumptions for correlation model and size of uncertainty
- But: **Both models work well, and there is no indication of a bias on the result from the chosen model**

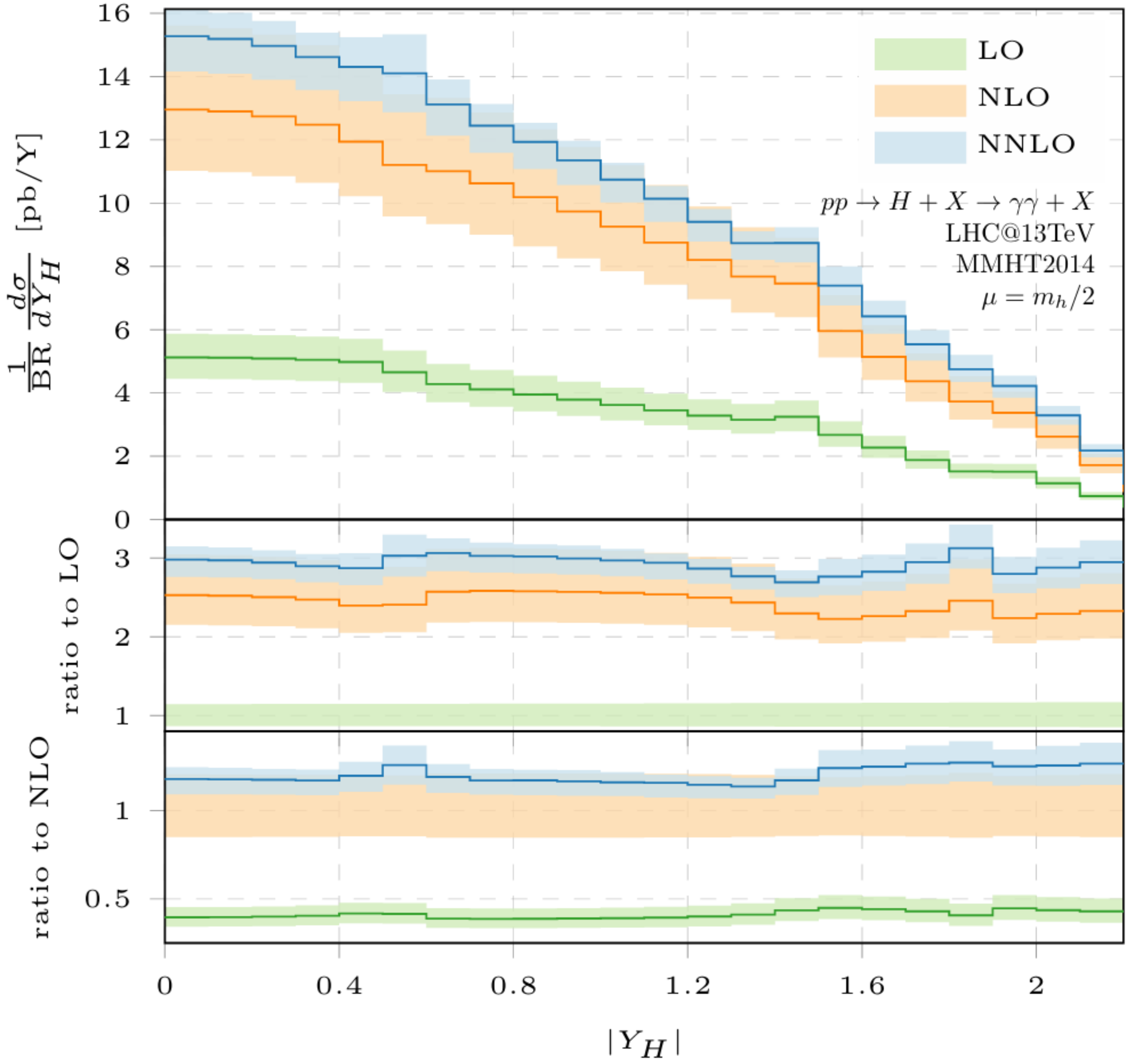
On the missing higher orders

- Provide $|V_{cb}|$, HQE parameters, etc. at fixed orders of the heavy quark expansion
- Hopefully, the parameters show converging behavior!
- One can also create a matrix of orders in HQE and the strong coupling constant

→ We extract the parameters at a **fixed order** of the heavy quark expansion and do not try to estimate the effect of missing higher orders

→ $\mathcal{O}\left(\frac{1}{m_b^4}\right)$ and $\mathcal{O}\left(\frac{1}{m_b^5}\right)$ calculations for the inclusive moments exist, but not (yet) for the lifetimes

When I think "missing higher order" I always have something like this in mind:



Classifing systematics

UNPHYSICAL

- $\mu_b = [0.75, 1.25]$ GeV
  - Smooth box constraint
- $\mu_c = [1.8, 3.2]$ GeV
  - Smooth box constraint
- $\mu_{\alpha_s} = \left[m_b^{kin}/2, 2\, m_b^{kin}\right]$
  - Smooth box constraint

The box constraints are necessary to constraint the fit to numerical stable regions, within the defined regions the constraints have no impact on the result

PHYSICAL

- $\overline{m}_b(\overline{m}_b) = 4.196(11),\ N_f = 2+1+1$
  - Gaussian constraint
- $\overline{m}_c(3\text{ GeV}) = 0.989(10), N_f = 2+1+1$
  - Gaussian constraint
- $\alpha_S(m_Z) = 0.1180(9)$
  - Gaussian constraint

BAG PARAMETERS & EYE CONTRACTIONS

- Included with Gaussian constraints
  - No correlation matrix available
  - μ_0 scale dependence not included as systematic

[M. Egner, M. Fael, A. Lenz, M. L. Piscopo, A. V. Rusov, K. Schönwald, M. Steinhauser JHEP 04 (2025) 106]

$\mu_0 = 1.5\,\text{GeV}$	$\tilde{B}_1^q$	$\tilde{B}_2^q$	$\tilde{B}_3^q$	$\tilde{B}_4^q$
$\langle B_{u,d}\vert\tilde{O}_i^{u,d}\vert B_{u,d}\rangle$	$1.0026^{+0.0246}_{-0.0221}$	$0.9982^{+0.0206}_{-0.0214}$	$-0.0057^{+0.0221}_{-0.0225}$	$-0.0014^{+0.0216}_{-0.0221}$
$\langle B_s\vert\tilde{O}_i^{s}\vert B_s\rangle$	$1.0022^{+0.0246}_{-0.0221}$	$0.9983^{+0.0246}_{-0.0221}$	$-0.0036^{+0.0265}_{-0.0270}$	$-0.0009^{+0.0259}_{-0.0265}$

$\mu_0 = 1.5\,\text{GeV}$	$\tilde{\delta}_1^{q'q}$	$\tilde{\delta}_2^{q'q}$	$\tilde{\delta}_3^{q'q}$	$\tilde{\delta}_4^{q'q}$
$\langle B_{d,u}\vert\tilde{O}_i^{u,d}\vert B_{d,u}\rangle$	$0.0026^{+0.0142}_{-0.0092}$	$-0.0018^{+0.0047}_{-0.0072}$	$-0.0004^{+0.0015}_{-0.0024}$	$0.0003^{+0.0012}_{-0.0008}$
$\langle B_s\vert\tilde{O}_i^{u,d}\vert B_s\rangle$	$0.0025^{+0.0144}_{-0.0093}$	$-0.0018^{+0.0047}_{-0.0072}$	$-0.0004^{+0.0015}_{-0.0024}$	$0.0003^{+0.0012}_{-0.0008}$
$\langle B_{d,u}\vert\tilde{O}_i^{s}\vert B_{d,u}\rangle$	$0.0023^{+0.0140}_{-0.0091}$	$-0.0017^{+0.0046}_{-0.0070}$	$-0.0004^{+0.0015}_{-0.0023}$	$0.0003^{+0.0012}_{-0.0008}$

“Novel” treatment of systematics

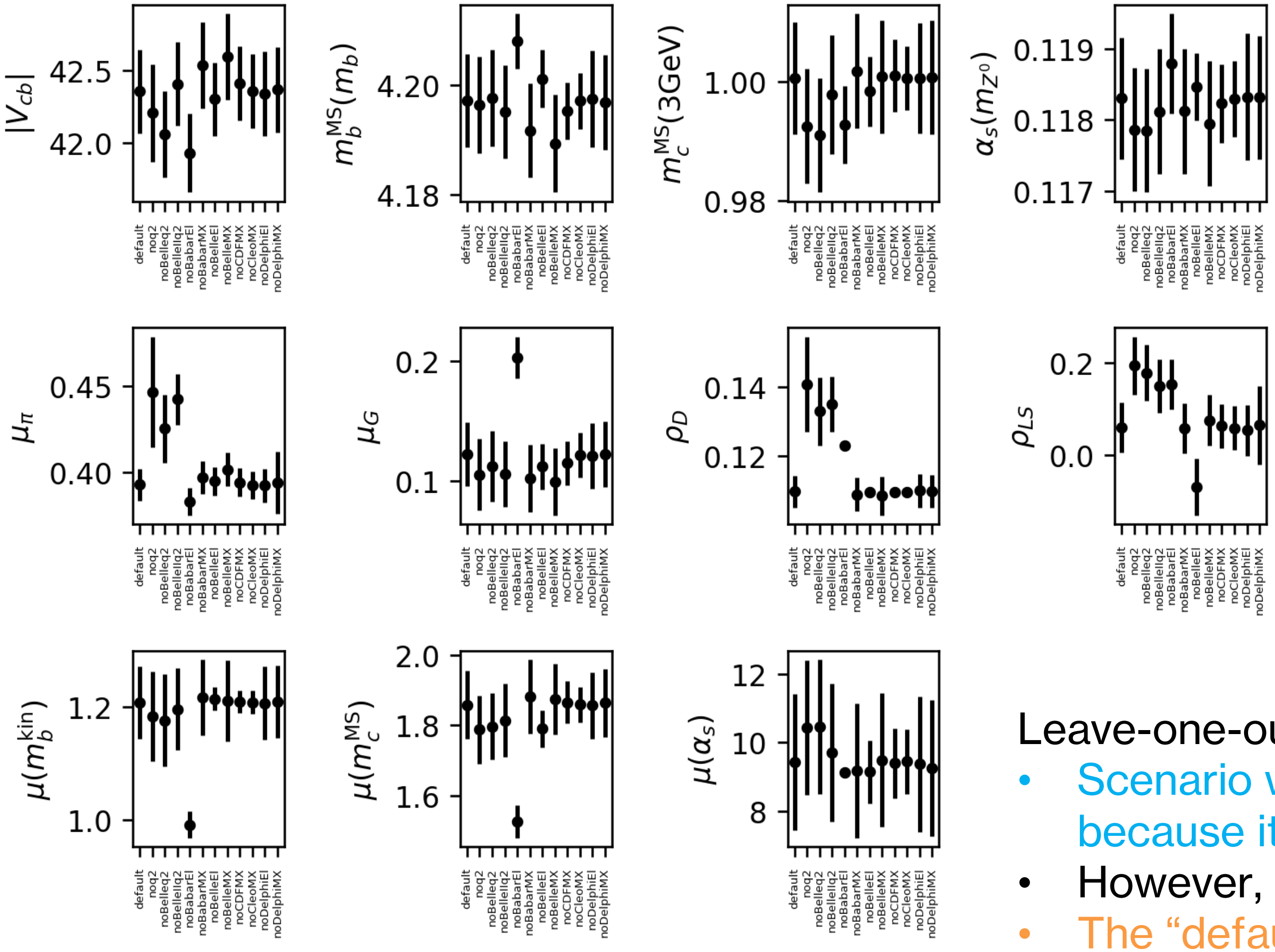
- Scale variations are a “standard tool” to estimate their uncertainty
- “Novel”: Implement the dependence on the theory parameters directly as nuisance parameters in the Likelihood instead of assuming a correlation model
- Also enables source-wise estimates of systematics
- But: the fit will choose the best-fit point, including unphysical scales
- Nota bene: Scale variations are not the final solution, see [F. Tackman JHEP 08 (2025) 098] on a thorough discussion

$$
\begin{aligned}
\chi^2 &= \chi^2\left(|V_{\rm cb}|^2, \boldsymbol{h}, \boldsymbol{\theta}, \boldsymbol{\mu}\right) \\
&= \left(\boldsymbol{m}(\boldsymbol{h}, \boldsymbol{\theta}, \boldsymbol{\mu}) - \boldsymbol{m}_{\rm meas}\right) C^{-1} \left(\boldsymbol{m}(\boldsymbol{h}, \boldsymbol{\theta}, \boldsymbol{\mu}) - \boldsymbol{m}_{\rm meas}\right)^T \\
&+ \left(\mathcal{B}_{\rm meas} - \Gamma\left(|V_{\rm cb}|, \boldsymbol{h}, \boldsymbol{\theta}, \boldsymbol{\mu}\right)\tau_B\right)^2 / \sigma_{\mathcal{B}}^2 \\
&+ \sum_{i=0}^{2} \frac{\left(\theta_i - \bar{\theta}_i\right)^2}{\sigma_{\theta_i}^2} \\
&+ \sum_{i=0}^{2} \begin{cases} 0, & \forall \mu_i \in [\bar{\mu}_i \pm \zeta\sigma_i] \\ \left(\frac{\mu_i - \bar{\mu}_i}{\kappa\sigma_i}\right)^2 - \left(\frac{\zeta}{\kappa}^2\right), & \forall \mu_i \in [\bar{\mu}_i \pm \zeta\sigma_i] \end{cases} .
\end{aligned}
$$

$$
\begin{aligned}
\boldsymbol{h} &= \left(\mu_\pi, \mu_G, \rho_D, \rho_{LS}\right) \\
\boldsymbol{\theta} &= \left(m_b^{\rm MS}(m_b), m_c^{\rm MS}(3\,\text{GeV}), \alpha_s(M_{Z^0})\right) \\
\boldsymbol{\mu} &= \left(\mu_{m_b^{\rm kin}}, \mu_{m_c^{\rm MS}}, \mu_{\alpha_s}\right)
\end{aligned}
$$

Data sanity check

A closer look to the input



	χ^2	d.o.f.	p-value	χ^2 / d.o.f.
default	120.7	73	0.0004	1.65
noq2	53.8	46	0.1997	1.17
No Babar El	90.2	63	0.0138	1.43
No Babar MX	105.7	63	0.0006	1.68
No Belle El	112.3	67	0.0004	1.68
No Belle MX	88.6	66	0.0332	1.34
No Belle q2	63.5	61	0.3898	1.04
No BelleII q2	94.2	58	0.0018	1.62
No CDF MX	117.8	71	0.0004	1.66
No Cleo MX	121.2	71	0.0002	1.71
No Delphi El	118.4	70	0.0003	1.69
No Delphi MX	120.2	70	0.0002	1.72

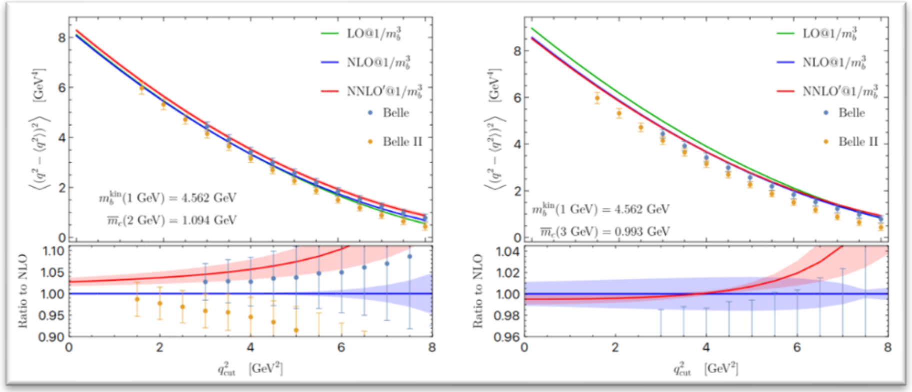
Leave-one-out validation:

- Scenario without Belle q^2 moments is our **reference** data set because it yields a good p-value
- However, excluding that data has a sizeable impact on $\mu_\pi, \rho_D, \rho_{LS}$
- The "default" selection corresponds to data sets of other existing fits to the moments

Result I: Standalone moments

Standalone moments

[F. Herren, M. Fael 2403.03976]



$\mu(m_c^{MS}) = 3$ GeV a better choice from a theory point of view

Belle q^2 excluded (Reference)

	Central	Uncertainty
$\lvert V_{cb}\rvert$	42.061	0.295
$m_b^{MS}(m_b)$	4.198	0.009
$m_c^{MS}(3\text{GeV})$	0.991	0.010
$\alpha_s(m_{Z^0})$	0.118	0.001
$\mu(m_b^{kin})$	1.176	0.082
$\mu(m_c^{MS})$	1.797	0.095
$\mu(\alpha_s)$	10.464	1.956
μ_π	0.425	0.020
μ_G	0.112	0.029
ρ_D	0.133	0.010
ρ_{LS}	0.178	0.061

$|V_{cb}|$ very precise, but no “missing higher order” uncertainty

No pull for the external inputs

- $\overline{m}_b(\overline{m}_b) = 4.196(11)$
- $\overline{m}_c(3\text{ GeV}) = 0.989(10)$
- $\alpha_S(m_Z) = 0.1180(9)$

$\mu(m_b^{kin})$ and $\mu(m_c^{MS})$ around what we expect

$\mu(\alpha_s)$ unexpectedly large, but the fit has not much sensitivity to constrain $\mu(\alpha_s)$

When we exclude NNLO and NLO power corrections, the $\mu(\alpha_s)$ best fit point is around 2 GeV

Nota bene: Parameters are extracted at reference scales, but run appropriately in the fitting procedure:

- $m_b^{MS}(m_b) \to m_b^{kin}\left(\mu_b, m_b^{MS}(m_b), m_c^{MS}(3\text{ GeV}), \alpha_s(M_{Z^0})\right)$
- $m_c^{MS}(3\text{ GeV}) \to m_c^{MS}(\mu_c)$
- $\alpha_s(M_{Z^0}) \to \alpha_s(\mu_{\alpha_s})$
- $\mu_\pi(1\text{ GeV}) \to \mu_\pi(\mu_{\alpha_s}, \mu_b)$
- $\rho_D(1\text{ GeV}) \to \rho_D(\mu_{\alpha_s}, \mu_b)$

Standalone moments

Belle q^2 excluded (Reference)

	Value	σ(total)	$\sigma(m_b^{\mathrm{MS}}(m_b))$	$\sigma(m_c^{\mathrm{MS}}(3\mathrm{GeV}))$	$\sigma(\alpha_s(m_{Z^0}))$	$\sigma(\mu(m_b^{\mathrm{kin}}))$	$\sigma(\mu(m_c^{\mathrm{MS}}))$	$\sigma(\mu(\alpha_s))$
$\lvert V_{cb}\rvert$	42.061	0.295	0.120	0.039	0.029	0.020	0.051	0.081
$m_b^{\mathrm{MS}}(m_b)$	4.198	0.009	0.009	0.002	0.000	0.003	0.002	0.000
$m_c^{\mathrm{MS}}(3\mathrm{GeV})$	0.991	0.010	0.002	0.010	0.000	0.004	0.006	0.001
$\alpha_s(m_{Z^0})$	0.118	0.001	0.000	0.000	0.001	0.000	0.000	0.000
$\mu(m_b^{\mathrm{kin}})$	1.176	0.082	0.029	0.013	0.042	0.082	0.046	0.015
$\mu(m_c^{\mathrm{MS}})$	1.797	0.095	0.010	0.060	0.014	0.064	0.095	0.022
$\mu(\alpha_s)$	10.464	1.956	0.307	0.165	0.223	0.173	0.338	1.956
μ_π	0.425	0.020	0.006	0.000	0.001	0.000	0.001	0.002
μ_G	0.112	0.029	0.016	0.009	0.005	0.002	0.005	0.003
ρ_D	0.133	0.010	0.003	0.001	0.002	0.006	0.004	0.002
ρ_{LS}	0.178	0.061	0.020	0.012	0.008	0.007	0.007	0.007

- Nuisance parameter approach allows extraction of uncertainty from individual external inputs / scale choices

Standalone moments

Belle q^2 excluded (Reference)

	Central	Uncertainty		Central	Uncertainty
$\lvert V_{cb}\rvert$	42.061	0.295	$\lvert V_{cb}\rvert$	42.356	0.287
$m_b^{\mathrm{MS}}(m_b)$	4.198	0.009	$m_b^{\mathrm{MS}}(m_b)$	4.197	0.008
$m_c^{\mathrm{MS}}(3\mathrm{GeV})$	0.991	0.010	$m_c^{\mathrm{MS}}(3\mathrm{GeV})$	1.001	0.009
$\alpha_s(m_{Z^0})$	0.118	0.001	$\alpha_s(m_{Z^0})$	0.118	0.001
$\mu(m_b^{\mathrm{kin}})$	1.176	0.082	$\mu(m_b^{\mathrm{kin}})$	1.207	0.058
$\mu(m_c^{\mathrm{MS}})$	1.797	0.095	$\mu(m_c^{\mathrm{MS}})$	1.858	0.094
$\mu(\alpha_s)$	10.464	1.956	$\mu(\alpha_s)$	9.427	1.932
μ_π	0.425	0.020	μ_π	0.393	0.009
μ_G	0.112	0.029	μ_G	0.122	0.027
ρ_D	0.133	0.010	ρ_D	0.110	0.005
ρ_{LS}	0.178	0.061	ρ_{LS}	0.059	0.055

Belle q^2 included

- Nuisance parameter approach allows extraction of uncertainty from individual external inputs / scale choices

- As seen before, including/excluding the Belle q^2 data has an impact on our result

→ *Follow PDG approach, include all data points and inflate uncertainties?*

$$\sigma = \sigma_{fit}\sqrt{\frac{\chi^2}{d.o.f}}$$

Belle q^2 included

→ $$|V_{cb}| = \left(\mathbf{42.36} \pm \mathbf{0.37}_{fit} \pm \mathbf{0.28}_{\Delta}\right) \times \mathbf{10^{-3}}$$
$$= \left(\mathbf{42.37} \pm \mathbf{0.46}\right) \times \mathbf{10^{-3}}$$

First uncertainty is the rescaled uncertainty from the fit, second uncertainty is the difference between the fit at $1/m_b^3$ and $1/m_b^4$

Comparison with other results – $|V_{cb}|$

Belle q^2 included

→ $|V_{cb}| = (42.36 \pm 0.37_{fit} \pm 0.28_{\Delta}) \times 10^{-3}$

$= (42.37 \pm 0.46) \times 10^{-3}$

Compatible result

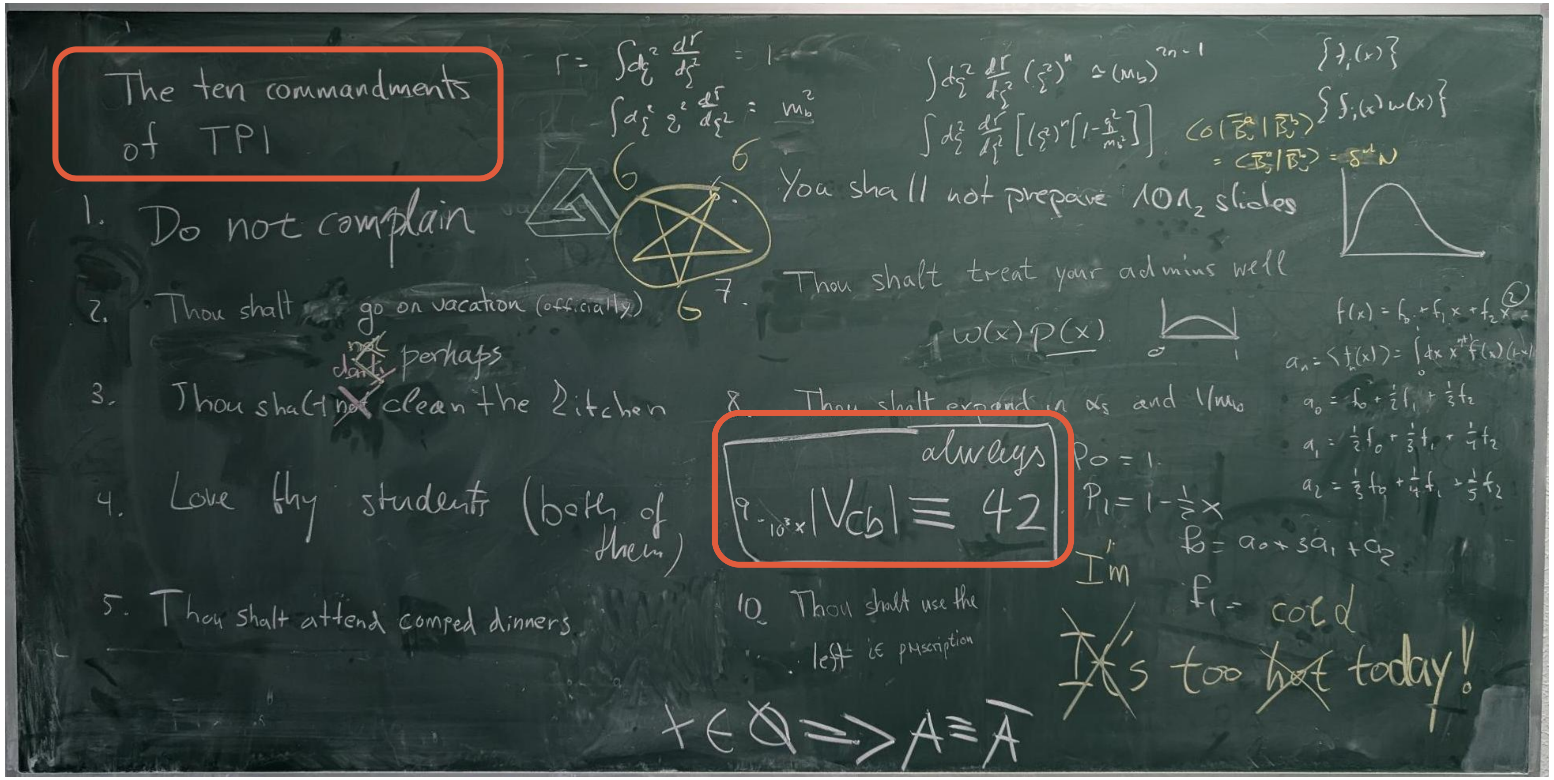
[G. Finauri, P. Gambino JHEP 02 (2024) 206]

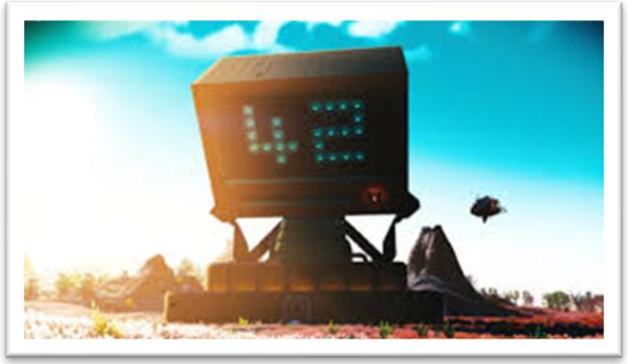
$$|V_{cb}| = (41.97 \pm 0.48) \times 10^{-3}$$

This result includes the Belle q^2 data and has an uncertainty from missing higher orders

Comparison with other results – $|V_{cb}|$

Results in line with the ten commandments…





…and from first-principles calculations

Belle q^2 included

$\rightarrow |V_{cb}| = \left(42.36 \pm 0.37_{fit} \pm 0.28_{\Delta}\right) \times 10^{-3}$

$= (42.37 \pm 0.46) \times 10^{-3}$

[G. Finauri, P. Gambino JHEP 02 (2024) 206]

$$|V_{cb}| = (41.97 \pm 0.48) \times 10^{-3}$$

This result includes the Belle q^2 data and has an uncertainty from missing higher orders

Comparison with other results – HQE parameters

Belle q^2 included

	$\lvert V_{cb}\rvert$	$m_b^{\mathrm{MS}}(m_b)$	$m_c^{\mathrm{MS}}(3\mathrm{GeV})$	$\alpha_s(m_{Z^0})$	$\mu(m_b^{\mathrm{kin}})$	$\mu(m_c^{\mathrm{MS}})$	$\mu(\alpha_s)$	μ_π	μ_G	ρ_D	ρ_{LS}
Central value	42.36	4.20	1.00	0.12	1.21	1.86	9.43	0.39	0.12	0.11	0.06
Uncertainty	0.29	0.01	0.01	0.00	0.07	0.10	1.99	0.01	0.03	0.00	0.05
	1.00	-0.42	0.12	-0.03	0.09	0.28	-0.37	0.44	-0.35	0.08	0.16
		1.00	0.26	-0.27	0.28	-0.02	0.18	-0.53	0.54	-0.49	-0.27
			1.00	0.03	0.20	0.69	-0.14	0.05	-0.34	-0.16	0.24
				1.00	0.17	0.24	0.09	-0.02	0.09	0.12	-0.05
					1.00	0.63	-0.12	0.05	-0.01	-0.81	-0.07
						1.00	-0.33	0.17	-0.27	-0.36	0.21
							1.00	-0.29	0.13	-0.37	0.09
								1.00	-0.38	0.12	0.48
									1.00	-0.09	-0.19
										1.00	0.05
											1.00

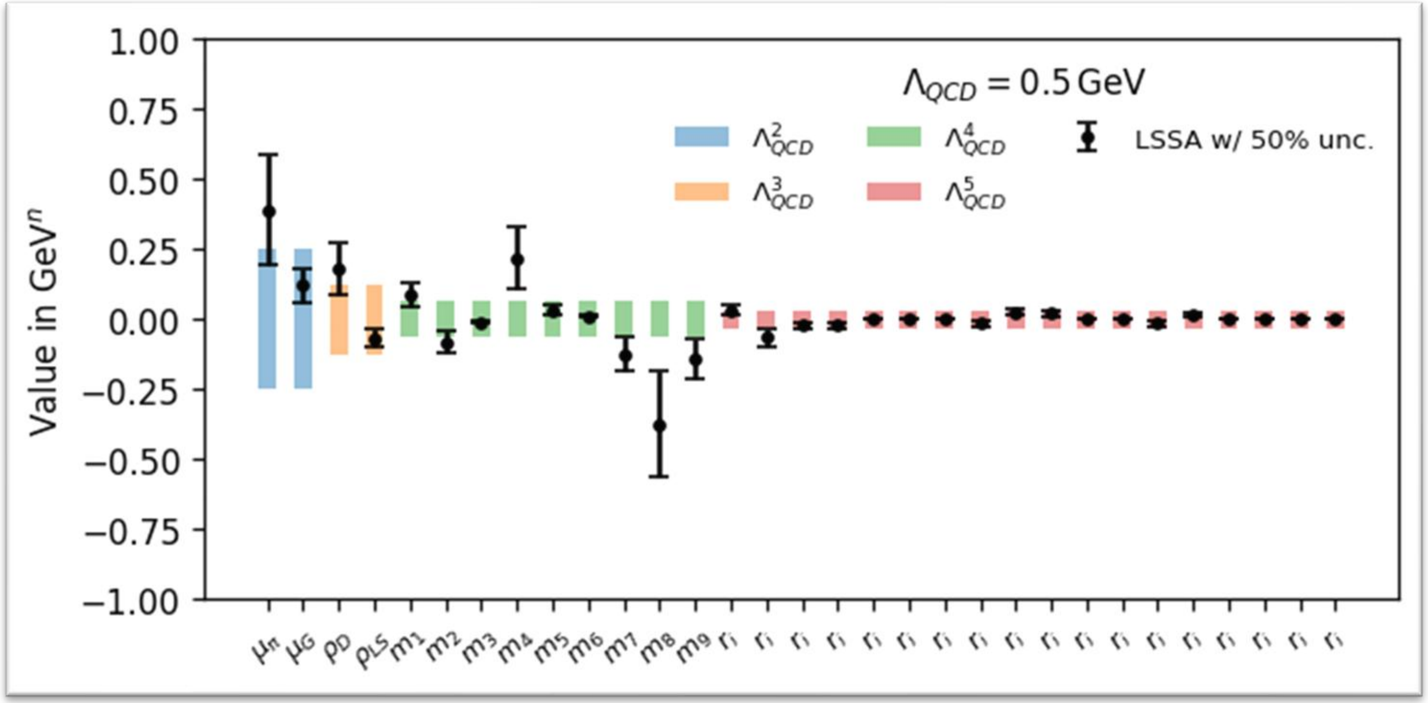
m_b^{kin}	$\overline{m}_c(2\,\mathrm{GeV})$	μ_π^2	$\mu_G^2(m_b)$	$\rho_D^3(m_b)$	ρ_{LS}^3	$\mathrm{BR}_{c\ell\nu}$	$10^3\lvert V_{cb}\rvert$
4.573	1.090	0.454	0.288	0.176	−0.113	10.63	41.97
0.012	0.010	0.043	0.049	0.019	0.090	0.15	0.48
1	0.380	-0.219	0.557	-0.013	-0.172	-0.063	-0.428
	1	0.005	-0.235	-0.051	0.083	0.030	0.071
		1	-0.083	0.537	0.241	0.140	0.335
			1	-0.247	0.010	0.007	-0.253
				1	-0.023	0.023	0.140
					1	-0.011	0.060
						1	0.696
							1

[G. Finauri, P. Gambino JHEP 02 (2024) 206]

Careful when comparing the values, difference scales are used in the fit

Back to the initial assumption

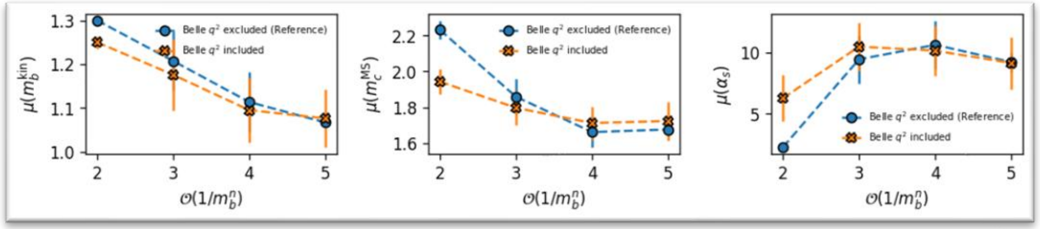
- Provide $|V_{cb}|$, HQE parameters, etc. at fixed orders of the heavy quark expansion
- Hopefully, the parameters show converging behavior!
- Higher order coefficients are constrained with $\Lambda^n_{QCD},\ \Lambda_{QCD} = 0.5$
- Alternative approach: Use LSSA predictions for higher order coefficients

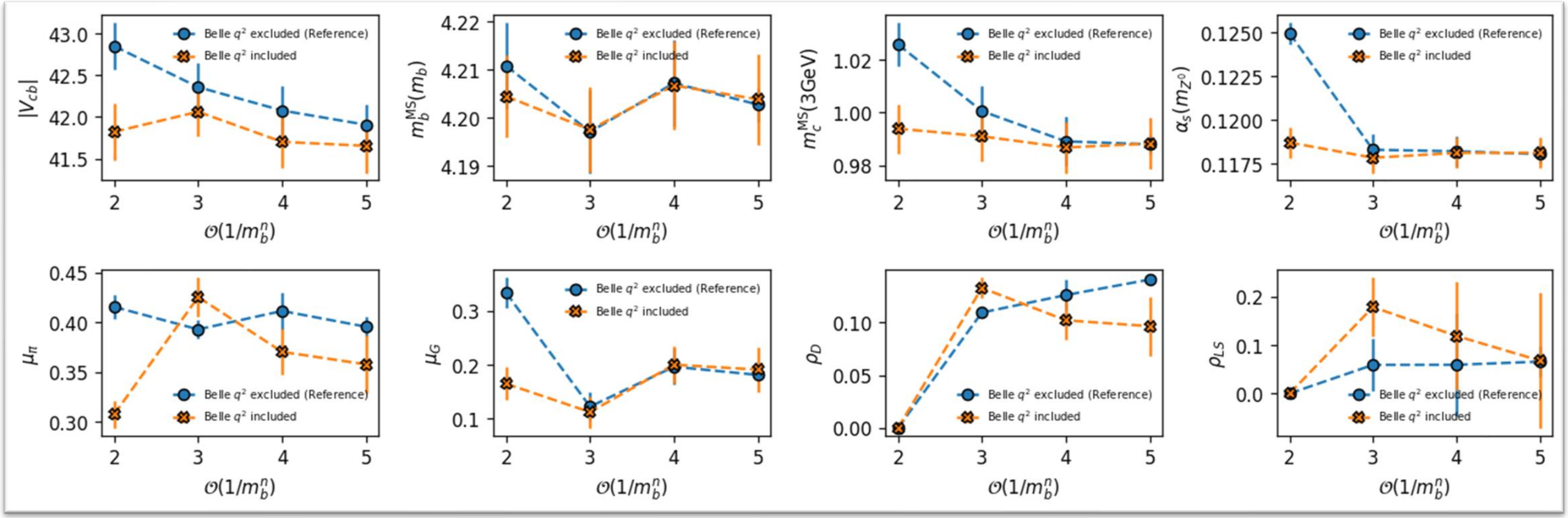


No uncertainty from inputs to LSSA propagated, but uncertainty is theory dominated

Back to the initial assumption

- Provide $|V_{cb}|$, HQE parameters, etc. at fixed orders of the heavy quark expansion
- Hopefully, the parameters show converging behavior!
- Higher order coefficients are constrained with $\Lambda^n_{QCD},\ \Lambda_{QCD} = 0.5$





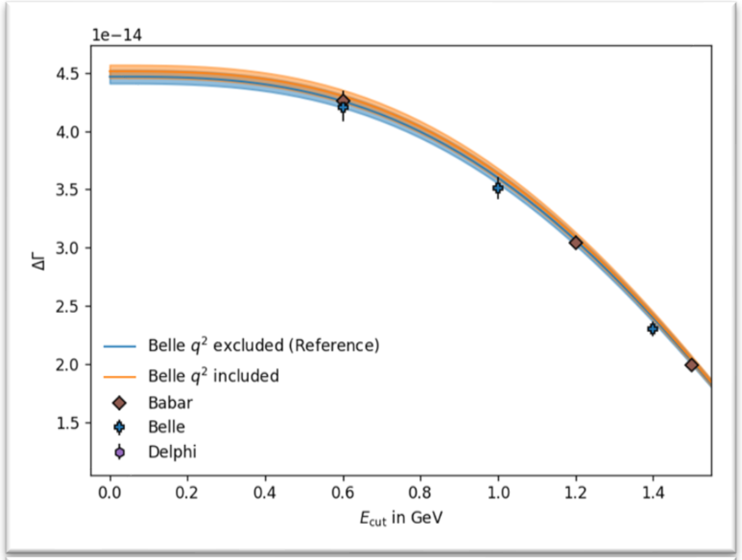
Preliminary, higher order fits now full scrutinized yet

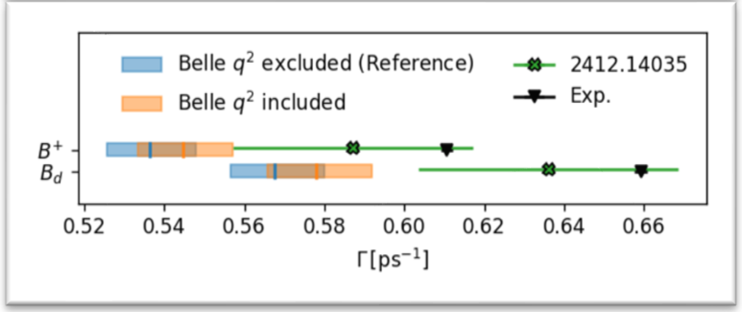
Result II: Inclusion of lifetimes

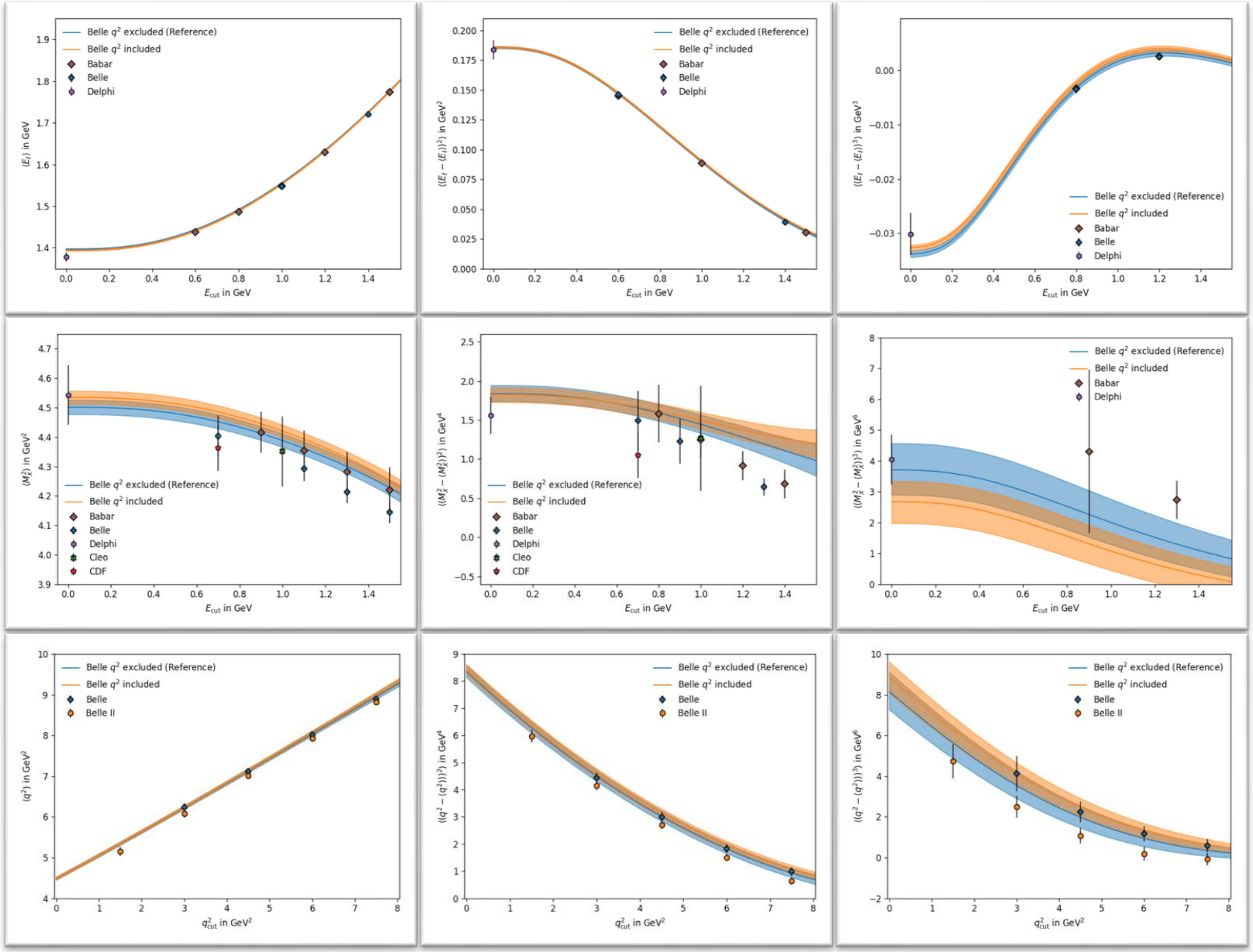
Fit result

Belle q^2 excluded (Reference)

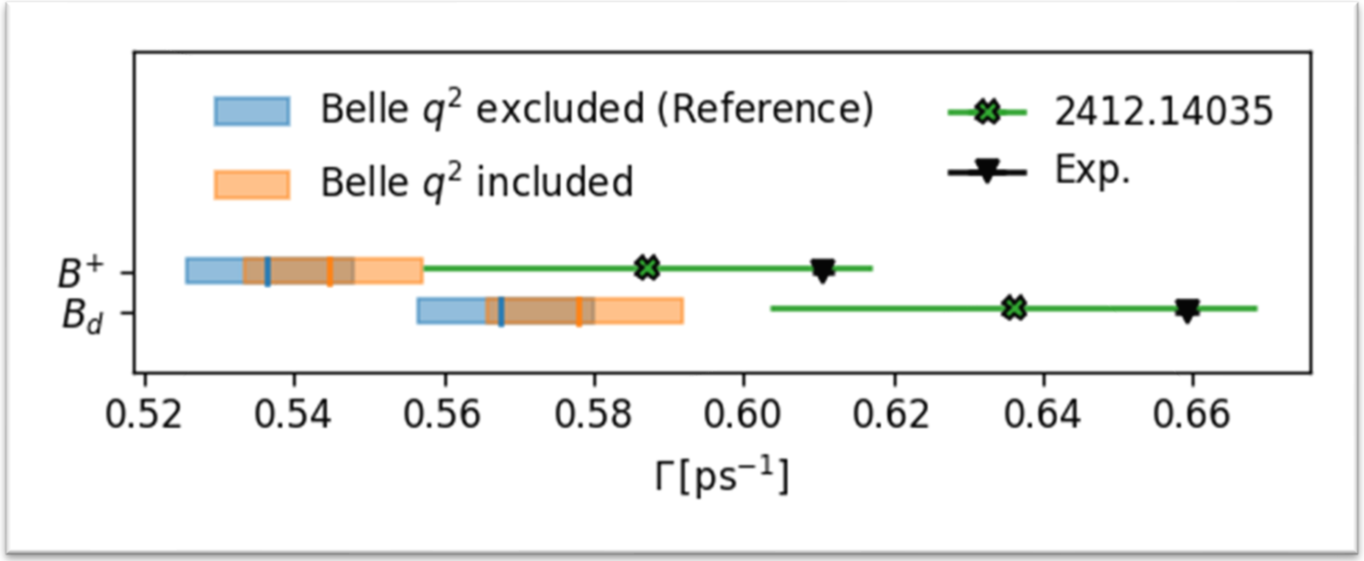
$$\frac{\chi^2}{dof} = \frac{63.5}{61} \qquad p = 0.39$$







Fit result



Consistent treatment of input parameters, unphysical scales, etc.
between the inclusive moments fits and the lifetime prediction

Fit result

	$\lvert V_{cb}\rvert$	$m_b^{\mathrm{MS}}(m_b)$	$m_c^{\mathrm{MS}}(3\mathrm{GeV})$	$\alpha_s(m_{Z^0})$	$\mu(m_b^{\mathrm{kin}})$	$\mu(m_c^{\mathrm{MS}})$	$\mu(\alpha_s)$	μ_π	μ_G	ρ_D	ρ_{LS}
Central value	42.06	4.20	0.99	0.12	1.18	1.80	10.46	0.43	0.11	0.13	0.18
Uncertainty	0.30	0.01	0.01	0.00	0.08	0.09	1.96	0.02	0.03	0.01	0.06

Belle q^2 excluded (Reference)

Inclusion of lifetimes

	$\lvert V_{cb}\rvert$	$m_b^{\mathrm{MS}}(m_b)$	$m_c^{\mathrm{MS}}(3\mathrm{GeV})$	$\alpha_s(m_{Z^0})$	$\mu(m_b^{\mathrm{kin}})$	$\mu(m_c^{\mathrm{MS}})$	$\mu(\alpha_s)$	μ_π	μ_G	ρ_D	ρ_{LS}	$\tilde{B}_1^q$	$\tilde{B}_2^q$	$\tilde{B}_3^q$	$\tilde{B}_4^q$	$\tilde{\delta}_{1,q}^{q'q}$	$\tilde{\delta}_{2,q}^{q'q}$	$\tilde{\delta}_{3,q}^{q'q}$	$\tilde{\delta}_{4,q}^{q'q}$	$\tilde{\delta}_{1,s}^{q'q}$	$\tilde{\delta}_{2,s}^{q'q}$	$\tilde{\delta}_{3,s}^{q'q}$	$\tilde{\delta}_{4,s}^{q'q}$	**BdWA**	**fBd**
Central	42.061	4.198	0.991	0.118	1.176	1.797	10.464	0.425	0.112	0.133	0.178	1.003	0.998	-0.006	-0.001	0.003	-0.002	-0.000	-0.000	0.002	-0.002	-0.000	-0.000	-0.011	0.190
Uncertainty	0.295	0.009	0.010	0.001	0.082	0.095	1.956	0.020	0.029	0.010	0.061	0.023	0.021	0.022	0.022	0.012	0.006	0.002	0.001	0.012	0.006	0.002	0.001	0.011	0.001

Correlation matrix in backup
Systematic table in backup

Same shifts observed as in standalone moments fit

	$\lvert V_{cb}\rvert$	$m_b^{\mathrm{MS}}(m_b)$	$m_c^{\mathrm{MS}}(3\mathrm{GeV})$	$\alpha_s(m_{Z^0})$	$\mu(m_b^{\mathrm{kin}})$	$\mu(m_c^{\mathrm{MS}})$	$\mu(\alpha_s)$	μ_π	μ_G	ρ_D	ρ_{LS}	$\tilde{B}_1^q$	$\tilde{B}_2^q$	$\tilde{B}_3^q$	$\tilde{B}_4^q$	$\tilde{\delta}_{1,q}^{q'q}$	$\tilde{\delta}_{2,q}^{q'q}$	$\tilde{\delta}_{3,q}^{q'q}$	$\tilde{\delta}_{4,q}^{q'q}$	$\tilde{\delta}_{1,s}^{q'q}$	$\tilde{\delta}_{2,s}^{q'q}$	$\tilde{\delta}_{3,s}^{q'q}$	$\tilde{\delta}_{4,s}^{q'q}$	**BdWA**	**fBd**
Central	42.356	4.197	1.001	0.118	1.207	1.859	9.425	0.393	0.122	0.110	0.059	1.003	0.998	-0.006	-0.001	0.003	-0.002	-0.000	-0.000	0.002	-0.002	-0.000	-0.000	-0.011	0.190
Uncertainty	0.286	0.008	0.009	0.001	0.065	0.093	1.990	0.009	0.027	0.005	0.055	0.023	0.021	0.022	0.022	0.012	0.006	0.002	0.001	0.012	0.006	0.002	0.001	0.011	0.001

Inclusion of lifetimes

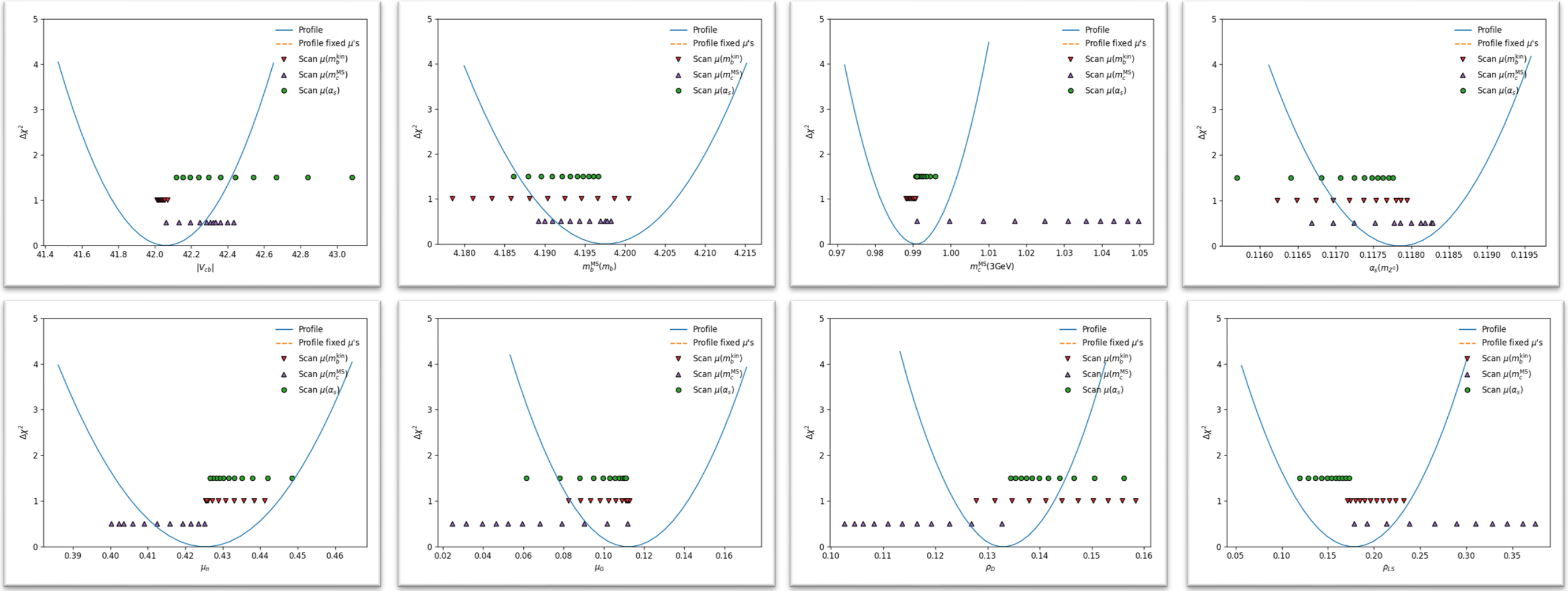
	$\lvert V_{cb}\rvert$	$m_b^{\mathrm{MS}}(m_b)$	$m_c^{\mathrm{MS}}(3\mathrm{GeV})$	$\alpha_s(m_{Z^0})$	$\mu(m_b^{\mathrm{kin}})$	$\mu(m_c^{\mathrm{MS}})$	$\mu(\alpha_s)$	μ_π	μ_G	ρ_D	ρ_{LS}
Central value	42.36	4.20	1.00	0.12	1.21	1.86	9.43	0.39	0.12	0.11	0.06
Uncertainty	0.29	0.01	0.01	0.00	0.07	0.10	1.99	0.01	0.03	0.00	0.05

Belle q^2 included

No increased precision when including the lifetime ☹

Profiles

No surprise: Scale dependent parameters show strong dependence on scale

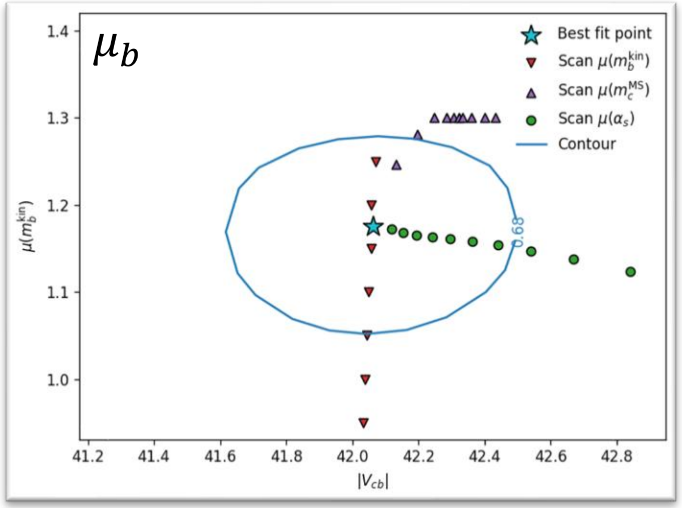


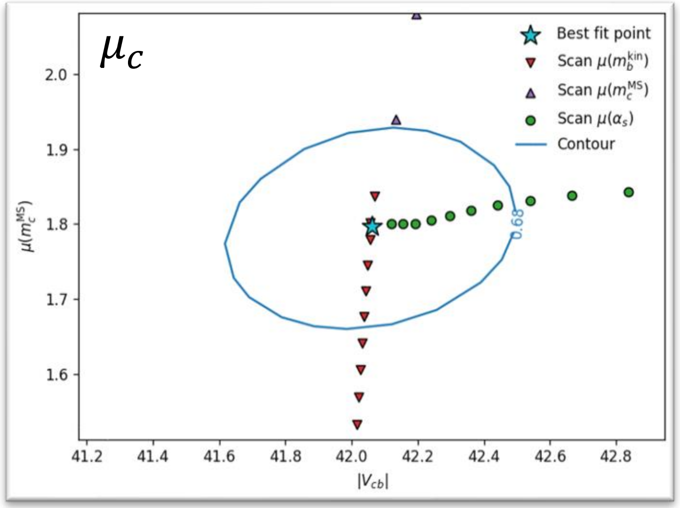
Profiles over all parameters in the Likelihood:

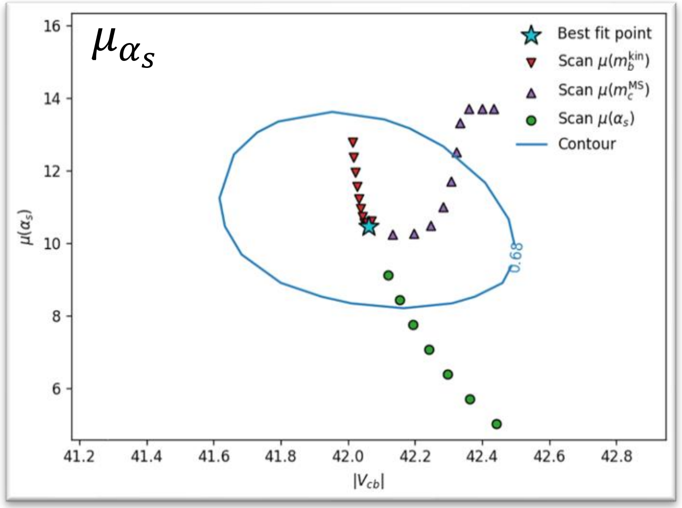
- Blue solid: 68% confidence interval
- Scan points → Variation of scales (individually) within full range and reoptimizing, the points indicate the shift of the central value for that parameter

	$\lvert V_{cb}\rvert$	$m_b^{MS}(m_b)$	$m_c^{MS}(3GeV)$	$\alpha_s(m_{Z^0})$	$\mu(m_b^{kin})$	$\mu(m_c^{MS})$	$\mu(\alpha_s)$	μ_π	μ_G	ρ_D	ρ_{LS}
Central	42.061	4.198	0.991	0.118	1.176	1.797	10.464	0.425	0.112	0.133	0.178
Uncertainty	0.295	0.009	0.010	0.001	0.082	0.095	1.956	0.020	0.029	0.010	0.061

Contours – $|V_{cb}|$ vs scales





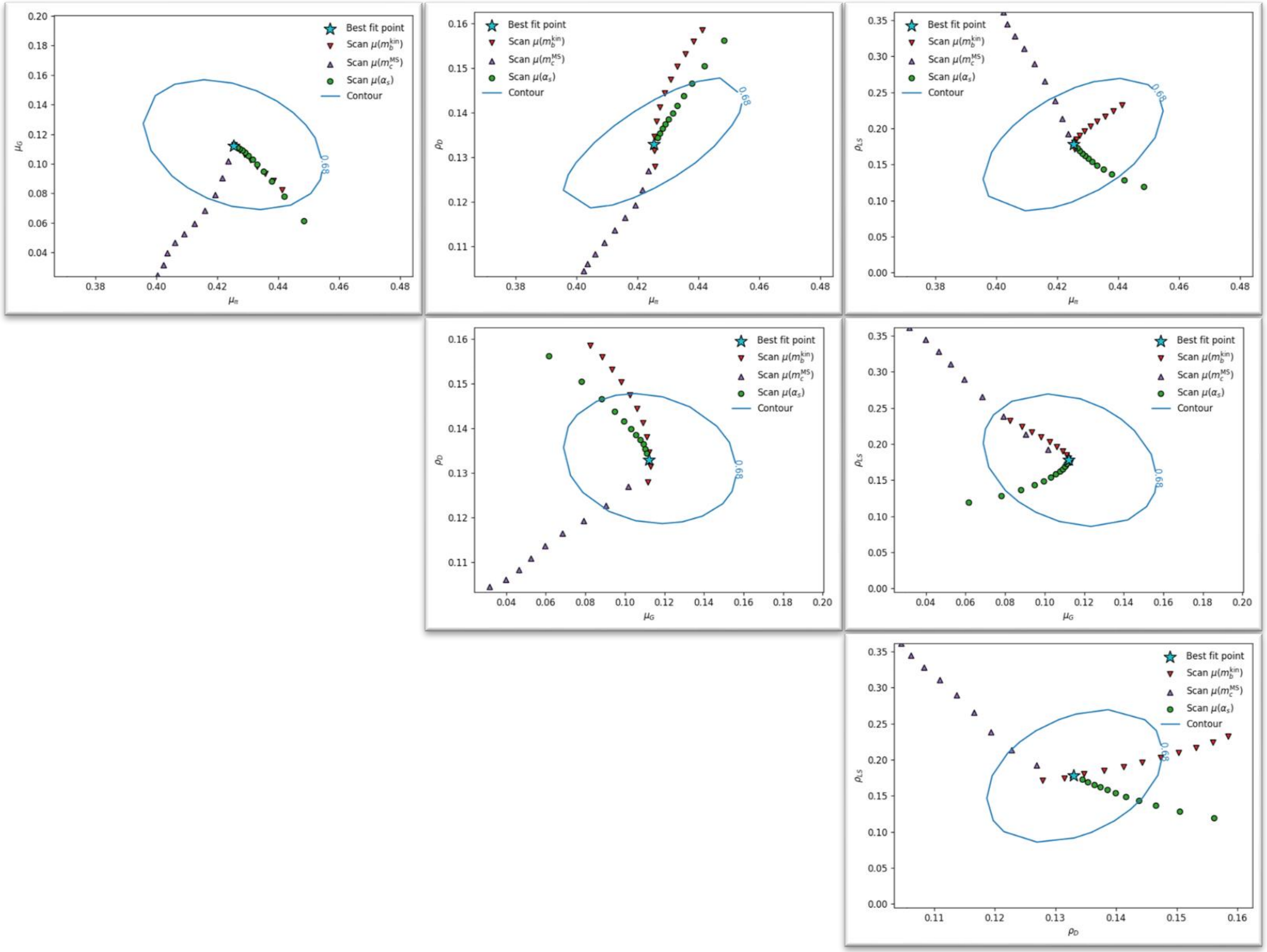


Profiles over all parameters in the Likelihood:

- Blue solid: 68% confidence interval
- Scan points → Variation of scales (individually) within full range and reoptimizing, the points indicate the shift of the central value for that parameter

	$\lvert V_{cb}\rvert$	$m_b^{MS}(m_b)$	$m_c^{MS}(3GeV)$	$\alpha_s(m_{Z^0})$	$\mu(m_b^{kin})$	$\mu(m_c^{MS})$	$\mu(\alpha_s)$	μ_π	μ_G	ρ_D	ρ_{LS}
Central	42.061	4.198	0.991	0.118	1.176	1.797	10.464	0.425	0.112	0.133	0.178
Uncertainty	0.295	0.009	0.010	0.001	0.082	0.095	1.956	0.020	0.029	0.010	0.061

Contours – HQE parameters



Summary & Conclusion

- The kolya package can calculate the inclusive $B \to X_c \ell \bar{\nu}_\ell$ moments and the B-meson lifetimes with consistent input parameter and scales (release date pending)

- First – **preliminary** - holistic HQE fit combining inclusive $B \to X_c \ell \bar{\nu}_\ell$ moments and the B-meson lifetimes
  - Developed a systematic treatment with fewer assumptions on the theory correlations
  - Allows to investigate the theory uncertainties in more detail
  - All numbers shown today should not be taken as inputs for any of your work, yet! Paper is on the way
- Need to check on the implementation of weak annihilation contribution (definition of τ_0)
- Need to address p-value when fitting the full set of experimental data (consolidate our two scenarios into one)
    - PDG approach to re-scale uncertainties, or more sophisticated approaches e.g. [1809.05778]

Backup

Fit result – Correlation matrix

	$\lvert V_{cb}\rvert$	$m_b^{\mathrm{MS}}(m_b)$	$m_c^{\mathrm{MS}}(3\mathrm{GeV})$	$\alpha_s(m_{Z^0})$	$\mu(m_b^{\mathrm{kin}})$	$\mu(m_c^{\mathrm{MS}})$	$\mu(\alpha_s)$	μ_π	μ_G	ρ_D	ρ_{LS}	$\tilde{B}_1^q$	$\tilde{B}_2^q$	$\tilde{B}_3^q$	$\tilde{B}_4^q$	$\tilde{\delta}_{1,q}^{q'q}$	$\tilde{\delta}_{2,q}^{q'q}$	$\tilde{\delta}_{3,q}^{q'q}$	$\tilde{\delta}_{4,q}^{q'q}$	$\tilde{\delta}_{1,s}^{q'q}$	$\tilde{\delta}_{2,s}^{q'q}$	$\tilde{\delta}_{3,s}^{q'q}$	$\tilde{\delta}_{4,s}^{q'q}$	**BdWA**	**fBd**
Central value	42.06	4.20	0.99	0.12	1.18	1.80	10.46	0.43	0.11	0.13	0.18	1.00	1.00	-0.01	-0.00	0.00	-0.00	-0.00	-0.00	0.00	-0.00	-0.00	-0.00	-0.01	0.19
Uncertainty	0.30	0.01	0.01	0.00	0.08	0.09	1.96	0.02	0.03	0.01	0.06	0.02	0.02	0.02	0.02	0.01	0.01	0.00	0.00	0.01	0.01	0.00	0.00	0.01	0.00
	1.00	-0.41	0.12	-0.04	0.05	0.17	-0.27	0.40	-0.32	0.18	0.17	0.00	-0.00	-0.00	-0.00	-0.00	0.00	-0.00	0.00	-0.00	-0.00	-0.00	-0.00	0.00	0.00
		1.00	0.21	-0.19	0.34	0.05	0.13	-0.32	0.55	-0.33	-0.32	-0.00	0.00	-0.00	-0.00	0.00	-0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	-0.00
			1.00	0.06	0.08	0.63	-0.05	0.02	-0.32	-0.06	0.20	0.00	-0.00	-0.00	-0.00	0.00	0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	0.00	0.00
				1.00	0.19	0.28	0.11	-0.07	0.12	-0.03	-0.09	0.00	-0.00	-0.00	-0.00	-0.00	0.00	0.00	-0.00	-0.00	-0.00	-0.00	-0.00	0.00	0.00
					1.00	0.60	-0.00	-0.02	-0.01	-0.56	-0.10	0.00	0.00	-0.00	-0.00	-0.00	0.00	0.00	-0.00	-0.00	-0.00	-0.00	-0.00	0.00	-0.00
						1.00	-0.14	-0.09	-0.22	-0.42	0.14	0.00	0.00	-0.00	-0.00	0.00	0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	0.00	-0.00
							1.00	-0.08	0.07	-0.15	0.11	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	0.00	-0.00	-0.00	-0.00	-0.00	0.00	-0.00	-0.00
								1.00	-0.31	0.72	0.51	0.00	-0.00	-0.00	-0.00	-0.00	0.00	-0.00	0.00	-0.00	-0.00	-0.00	-0.00	0.00	-0.00
									1.00	-0.14	-0.27	-0.00	0.00	-0.00	0.00	0.00	-0.00	-0.00	0.00	0.00	-0.00	-0.00	0.00	-0.00	-0.00
										1.00	0.33	0.00	-0.00	0.00	0.00	-0.00	0.00	-0.00	0.00	-0.00	-0.00	-0.00	0.00	0.00	0.00
											1.00	0.00	-0.00	-0.00	-0.00	-0.00	0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	0.00	0.00
												1.00	0.00	-0.00	0.00	0.00	0.00	0.00	-0.00	-0.00	-0.00	-0.00	-0.00	0.00	0.00
													1.00	-0.00	-0.00	0.00	-0.00	0.00	-0.00	0.00	0.00	-0.00	-0.00	-0.00	-0.00
														1.00	0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	0.00
															1.00	-0.00	-0.00	-0.00	-0.00	-0.00	0.00	-0.00	-0.00	-0.00	-0.00
																1.00	-0.00	0.00	-0.00	0.00	0.00	-0.00	-0.00	-0.00	0.00
																	1.00	-0.00	-0.00	-0.00	-0.00	0.00	-0.00	0.00	-0.00
																		1.00	-0.00	-0.00	-0.00	0.00	0.00	0.00	0.00
																			1.00	-0.00	0.00	0.00	0.00	-0.00	-0.00
																				1.00	-0.00	-0.00	-0.00	-0.00	0.00
																					1.00	-0.00	0.00	-0.00	0.00
																						1.00	-0.00	0.00	-0.00
																							1.00	-0.00	-0.00
																								1.00	-0.00
																									1.00

Belle q^2 excluded (Reference)

Fit result – Systematics table

Belle q^2 excluded (Reference)

	Value	$\sigma(\text{total})$	$\sigma(m_b^{\text{MS}}(m_b))$	$\sigma(m_c^{\text{MS}}(3\text{GeV}))$	$\sigma(\alpha_s(m_{Z^0}))$	$\sigma(\mu(m_b^{\text{kin}}))$	$\sigma(\mu(m_c^{\text{MS}}))$	$\sigma(\mu(\alpha_s))$	$\sigma(\bar{B}_1^q)$	$\sigma(\bar{B}_2^q)$	$\sigma(\bar{B}_3^q)$	$\sigma(\bar{B}_4^q)$	$\sigma(\bar{\delta}_{1,q}^{q'q})$	$\sigma(\bar{\delta}_{2,q}^{q'q})$	$\sigma(\bar{\delta}_{3,q}^{q'q})$	$\sigma(\bar{\delta}_{4,q}^{q'q})$	$\sigma(\bar{\delta}_{1,s}^{q'q})$	$\sigma(\bar{\delta}_{2,s}^{q'q})$	$\sigma(\bar{\delta}_{3,s}^{q'q})$	$\sigma(\bar{\delta}_{4,s}^{q'q})$	σ(BdWA)	σ(fBd)
$\lvert V_{cb}\rvert$	42.061	0.295	0.095	0.037	0.059	0.000	0.043	0.031	0.040	0.000	0.000	0.000	0.042	0.000	0.022	0.033	0.017	0.000	0.031	0.020	0.000	0.093
$m_b^{\text{MS}}(m_b)$	4.198	0.009	0.009	0.002	0.001	0.003	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.003	0.002	0.002	0.002	0.000	0.000	0.000	0.000	0.005
$m_c^{\text{MS}}(3\text{GeV})$	0.991	0.010	0.000	0.010	0.000	0.000	0.006	0.000	0.005	0.000	0.000	0.000	0.000	0.000	0.000	0.004	0.000	0.000	0.003	0.000	0.000	0.000
$\alpha_s(m_{Z^0})$	0.118	0.001	0.000	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
$\mu(m_b^{\text{kin}})$	1.176	0.082	0.000	0.012	0.000	0.082	0.000	0.011	0.055	0.000	0.000	0.000	0.020	0.000	0.000	0.009	0.027	0.000	0.010	0.000	0.000	0.000
$\mu(m_c^{\text{MS}})$	1.797	0.095	0.000	0.060	0.024	0.000	0.095	0.000	0.051	0.000	0.000	0.000	0.000	0.000	0.030	0.026	0.049	0.000	0.030	0.000	0.003	0.059
$\mu(\alpha_s)$	10.464	1.956	0.000	0.114	0.348	0.134	0.044	1.956	0.000	0.162	0.000	0.000	0.175	0.000	0.000	0.000	0.140	0.000	0.158	0.137	0.000	0.360
μ_π	0.425	0.020	0.006	0.001	0.007	0.000	0.001	0.000	0.000	0.000	0.001	0.000	0.000	0.000	0.000	0.001	0.000	0.000	0.001	0.000	0.000	0.010
μ_G	0.112	0.029	0.012	0.010	0.004	0.000	0.005	0.000	0.008	0.000	0.000	0.000	0.007	0.000	0.003	0.006	0.003	0.000	0.006	0.000	0.000	0.000
ρ_D	0.133	0.010	0.000	0.001	0.000	0.006	0.003	0.002	0.004	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.003	0.000	0.001	0.000	0.000	0.000
ρ_{LS}	0.178	0.061	0.016	0.012	0.000	0.000	0.000	0.000	0.010	0.000	0.000	0.000	0.010	0.000	0.000	0.008	0.000	0.000	0.007	0.000	0.000	0.000
$\bar{B}_1^q$	1.003	0.023	0.000	0.000	0.002	0.000	0.000	0.000	0.023	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.003
$\bar{B}_2^q$	0.998	0.021	0.000	0.000	0.002	0.000	0.000	0.000	0.000	0.021	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.003
$\bar{B}_3^q$	-0.006	0.022	0.000	0.000	0.002	0.000	0.000	0.000	0.000	0.000	0.022	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.003
$\bar{B}_4^q$	-0.001	0.022	0.000	0.000	0.002	0.000	0.000	0.000	0.000	0.000	0.000	0.022	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.003
$\bar{\delta}_{1,q}^{q'q}$	0.003	0.012	0.000	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.012	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.002
$\bar{\delta}_{2,q}^{q'q}$	-0.002	0.006	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.006	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001
$\bar{\delta}_{3,q}^{q'q}$	-0.000	0.002	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.002	0.000	0.000	0.000	0.000	0.000	0.000	0.000
$\bar{\delta}_{4,q}^{q'q}$	-0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000
$\bar{\delta}_{1,s}^{q'q}$	0.002	0.012	0.000	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.012	0.000	0.000	0.000	0.000	0.002
$\bar{\delta}_{2,s}^{q'q}$	-0.002	0.006	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.006	0.000	0.000	0.000	0.001
$\bar{\delta}_{3,s}^{q'q}$	-0.000	0.002	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.002	0.000	0.000	0.000
$\bar{\delta}_{4,s}^{q'q}$	-0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000	0.000
BdWA	-0.011	0.011	0.000	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.011	0.001
fBd	0.190	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001

Scale dependencies

Scale dependencies

1. $m_b^{\mathrm{MS}}(m_b) \to m_b^{\mathrm{kin}}(\mu_{m_b^{\mathrm{kin}}}, m_b^{\mathrm{MS}}(m_b), m_c^{\mathrm{MS}}(3\,\mathrm{GeV}), \alpha_s(M_{Z^0}))$, where m_b^{kin} is the bottom quark mass in the kinetic scheme at the scale $\mu_{m_b^{\mathrm{kin}}}$,
2. $m_c^{\mathrm{MS}}(3\,\mathrm{GeV}) \to m_c^{\mathrm{MS}}(\mu_{m_c^{\mathrm{MS}}})$ with $n_f = 4$ active flavors and 4-loop accuracy,
3. $\alpha_s(M_{Z^0}) \to \alpha_s(\mu_{\alpha_s})$ with $n_f = 4$ active flavors and five-loop accuracy,
4. $\mu_\pi(1\,\mathrm{GeV}) \to \mu_\pi(\mu_{\alpha_s}, \mu_b^{\mathrm{kin}})$,
5. $\rho_D(1\,\mathrm{GeV}) \to \rho_D(\mu_{\alpha_s}, \mu_b^{\mathrm{kin}})$.