Towards a complete NLO description of the nonleptonic decay rate within the LEFT/WET

Stefan Meiser

based on [Meiser, van Dyk, Virto 2411.09458] and recent work with Angel Picazo and Javier Virto

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EXCELENCIA MARÍA DE MAEZTU 04/2025-03/2031

Studienstiftung des deutschen Volkes

Motivation

Motivation: BSM interpretation of $b \to c\bar{u}q$ Puzzle

QCD factorization prediction within the Standard Model:

$$\mathcal{B}\left(\bar{B}^0 \to D^+ K^-\right) = (0.326 \pm 0.015) \cdot 10^{-3}$$
$$\mathcal{B}\left(\bar{B}_s^0 \to D_s^+ \pi^-\right) = (4.42 \pm 0.21) \cdot 10^{-3}$$

[Bordone, Gubernari, Huber, Jung, van Dyk 2007.10338]

Experimental values:

$$\mathcal{B}\left(\bar{B}^0 \to D^+ K^-\right) = (0.186 \pm 0.020) \cdot 10^{-3}$$
$$\mathcal{B}\left(\bar{B}_s^0 \to D_s^+ \pi^-\right) = (3.00 \pm 0.23) \cdot 10^{-3}$$

[PDG/LHCb/Belle/BaBar/CLEO/ARGUS]

$$\Downarrow$$

Strong tension in $\bar{B}_s^0 \to D_s^+ \pi^-$ and $\bar{B}^0 \to D^+ K^-$

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- Effective Lagrangian for $b \to c\bar{u}q$ decays ($q = d, s$):

$$\mathcal{L}^{qbcu} = \frac{4G_F}{\sqrt{2}} \sum_{i=1}^{10} \left(\mathcal{C}_i^{qbcu} \mathcal{O}_i^{qbcu} + \mathcal{C}_{i'}^{qbcu} \mathcal{O}_{i'}^{qbcu} \right) ,$$

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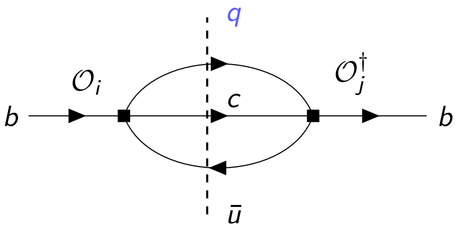
- Twenty independent operators per flavor
- Lots of parameters, but not a lot of constraints from the exclusive branching ratios
- What can we do?

Motivation: Lifetime bound in $b \to c\bar{u}q$



- Compute contribution of our $qbcu$ operators to the lifetime:

$$\Gamma_q = \Gamma_0 \, |V_{uq}^*|^2 \sum_{i,j} \left(\mathcal{C}_i^{qbcu*} \mathcal{C}_j^{qbcu} + \mathcal{C}_{i'}^{qbcu*} \mathcal{C}_{j'}^{qbcu} \right) G_{ij}$$

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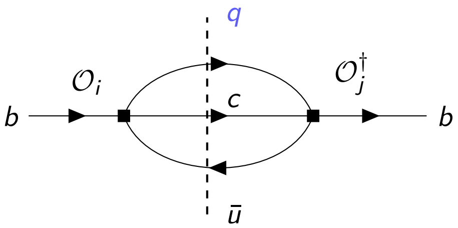

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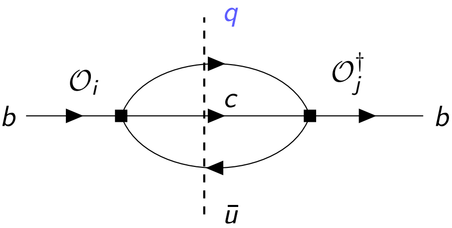

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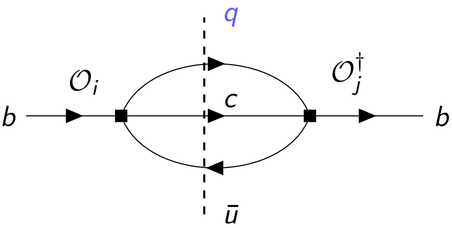

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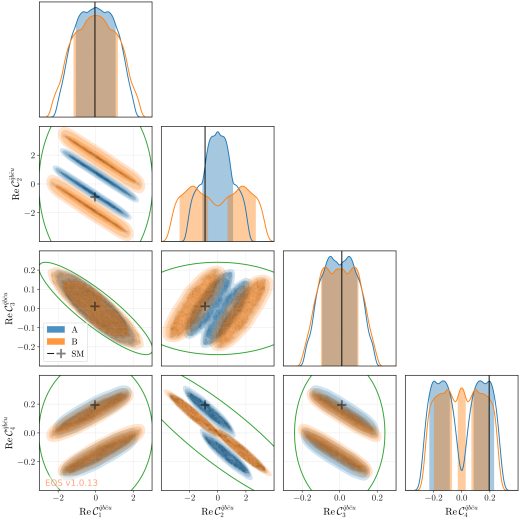
Effect of lifetime bound on WC



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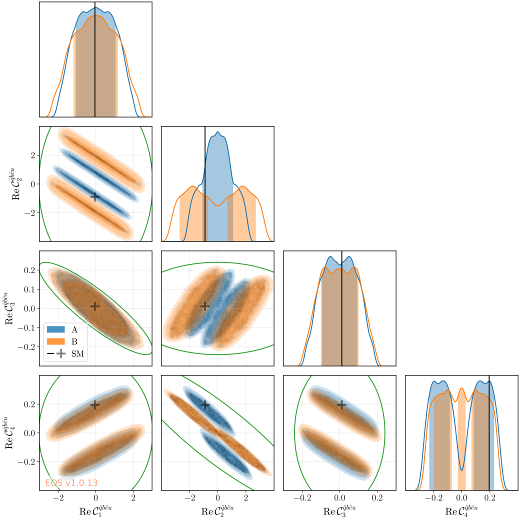

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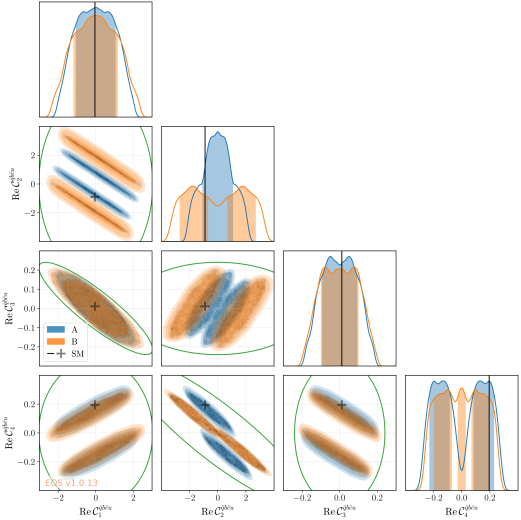

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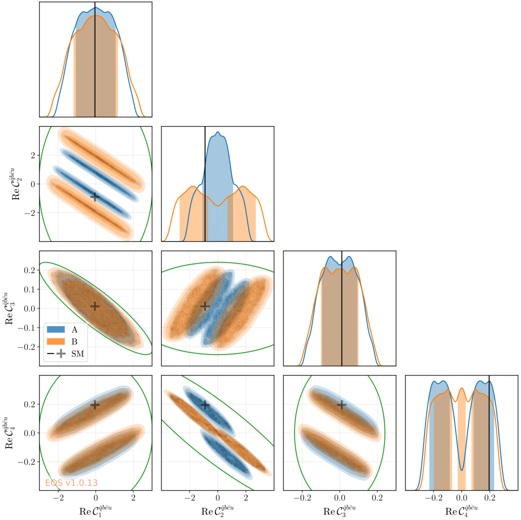

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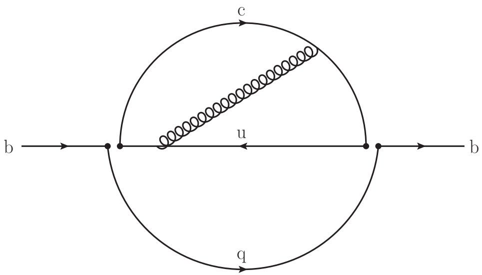
- Bounds on WC in terms of posterior distribution
- Exclusive branching fractions give some complicated structures, focus on lifetime bound
- Lifetime constraints very important for some combinations of WCs $\Rightarrow$ directions are poorly constrained by exclusive BRs

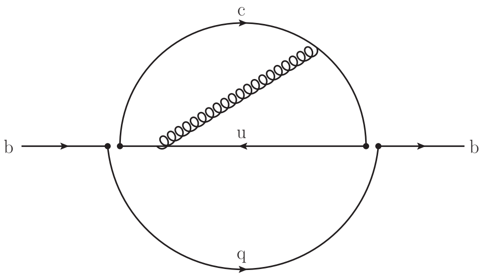
NLO corrections to the lifetime in the *qbcu* sector

NLO corrections for *qbcu*



Technical problems:

NLO corrections for *qbcu*



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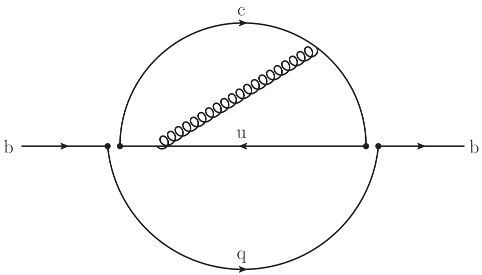

Technical problems:

- There are two Dirac traces to be computed. What to do with γ_5 in the traces?
- How to compute phase space integrals of three- and four-particle cuts efficiently?

NLO corrections: massless limit

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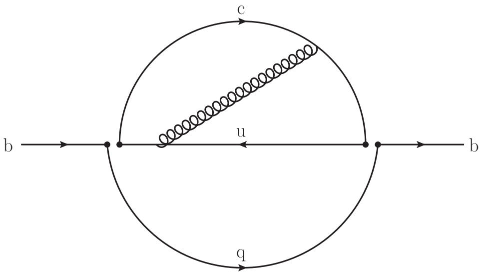
Ensure that there is only one trace containing γ_5!

NLO corrections: Getting rid of γ_5 Part 1

Solution 1: Bern basis

- Compute in a basis where γ_5 only appears in one of the currents of the four quark operators (Bern basis)

  [Aebischer, Fael, Greub, Virto 1704.06639]



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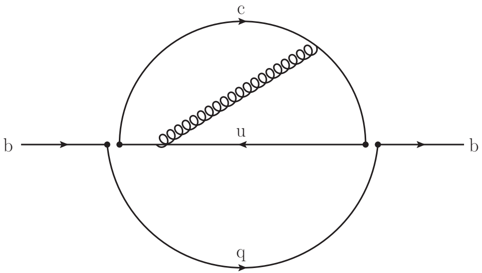
- Operators have the form $\mathcal{O}_i^{qbcu} = \left[\bar{q}\Gamma_i P_{L/R} b\right]\left[\bar{c}\Gamma_i u\right]$

  $\Rightarrow$ The trace of the *cu*-loop never has a γ_5



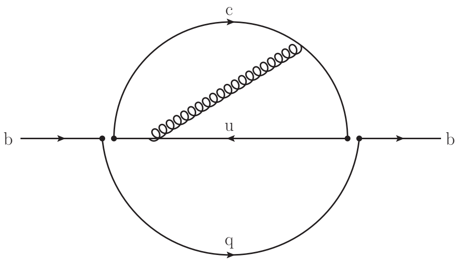
NLO corrections: Getting rid of γ_5 Part 2

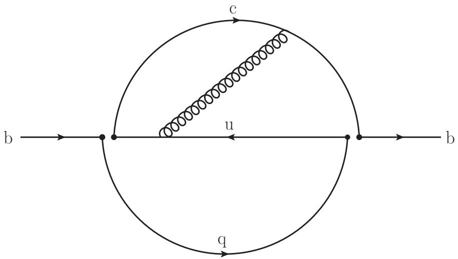
Solution 2: BMU basis with Fierz transformations [Buras, Misiak, Urban 0005183]

- Fierz transform the second operator such that there is only one trace

$$\mathcal{Q}^i = \left[\bar{q}\Gamma_i^1 b\right]\left[\bar{c}\Gamma_i^2 u\right] \to \tilde{\mathcal{Q}}^i = \left[\bar{c}\tilde{\Gamma}_i^1 b\right]\left[\bar{q}\tilde{\Gamma}_i^2 u\right]$$

$$\tilde{\Gamma} = \Gamma_0 \sum_{i,j} C_i \tilde{C}_j \tilde{G}_{ij}$$





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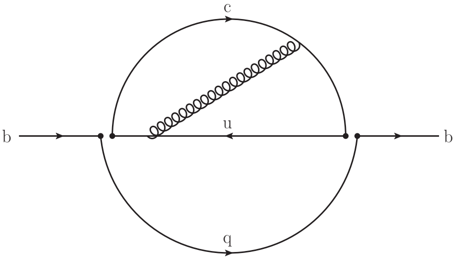
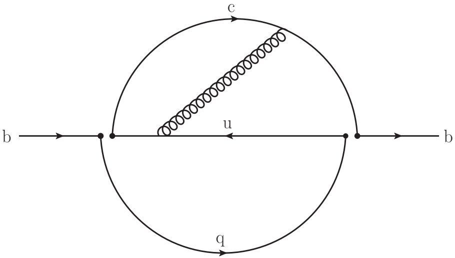
$$\tilde{\Gamma} = \Gamma_0 \sum_{i,j} C_i \tilde{C}_j \tilde{G}_{ij}$$

- But how is this related to the original rate that we wanted to compute?





Ensuring Fierz symmetry to relate Γ with $\tilde{\Gamma}$

- In SM at tree-level, it is easy to relate Γ and $\tilde{\Gamma}$:

$$\mathcal{Q}_1^{VLL} = [\bar{q}_\alpha \gamma_\mu P_L b_\beta]\,[\bar{c}_\beta \gamma^\mu P_L u_\alpha] = [\bar{c}_\alpha \gamma_\mu P_L b_\alpha]\,[\bar{q}_\beta \gamma^\mu P_L u_\beta] = \tilde{\mathcal{Q}}_2^{VLL}$$

$$\boxed{\Gamma(b \to c\bar{u}q) = \tilde{\Gamma}(b \to c\bar{u}q)\Big|_{\tilde{C}_1 \to C_2,\, \tilde{C}_2 \to C_1}}$$

[Bagan, Ball, Braun, Gosdzinsky 9408306], [Egner, Fael, Schoenwald, Steinhauser 2406.19456]

- Problems:
  1. How to generalize this to BSM?
  2. What to do if Fierz symmetry is not respected by the evanescent operators at NLO?
- Solution: *Partial change of basis*

Interlude: change of basis at NLO

- In $d = 4$, change of basis is easy:

$$\tilde{Q}_i = R_{ij} Q_j \Rightarrow \tilde{C}_i = C_j R_{ji}^{-1}$$

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$$\langle Q_i \rangle = \sum_{\ell=0}^{n} \left(\frac{\alpha_s}{4\pi} \right)^{\ell} r^{(\ell)}_{ij} \langle Q_j \rangle^{\text{tree}} \, , \qquad C_i = \sum_{\ell=0}^{n} \left(\frac{\alpha_s}{4\pi} \right)^{\ell} C^{(\ell)}_i \, .$$

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- Invariance of some (properly chosen) amplitude under a change of basis gives transformation law order by order [Gorbahn, Haisch 0411071]

$$\mathcal{A} = \tilde{\mathcal{A}} \Rightarrow \tilde{C}^{(1)}_i = C^{(1)}_j R^{-1}_{ji} + C^{(0)}_j R^{-1}_{jk} \Delta r_{ki} \,, \quad \Delta r_{ij} = (\hat{R}\, \hat{r}^{(1)} \hat{R}^{-1})_{ij} - r'^{(1)}_{ij}$$

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- Can this help us in relating Γ with $\tilde{\Gamma}$? Yes!
- Schematically, the decay rate is computed from three and four particle cuts

$$\Gamma = \Gamma_3 + \Gamma_4 \sim \int d\Pi_3 \sum |\mathcal{A}_3|^2 + \int d\Pi_4 \sum |\mathcal{A}_4|^2$$
$$\tilde{\Gamma} = \tilde{\Gamma}_3 + \tilde{\Gamma}_4 \sim \int d\Pi_3 \sum \mathcal{A}_3 \, \tilde{\mathcal{A}}_3^\dagger + \int d\Pi_4 \sum \mathcal{A}_4 \, \tilde{\mathcal{A}}_4^\dagger$$

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$$\Gamma = \Gamma_0 \sum_{i,j} C_i C_j G_{ij}$$

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We managed to avoid computing traces with γ_5 by using Fierz transformations at NLO!

Results for $qbcu$ in massless limit

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- Computed the NLO corrections in the $m_c \to 0$ limit in the Bern and the BMU basis (checked with SM results in the literature [Bagan, Ball, Braun, Gosdzinsky 9408306], [Egner, Fael, Schoenwald, Steinhauser 2406.19456])

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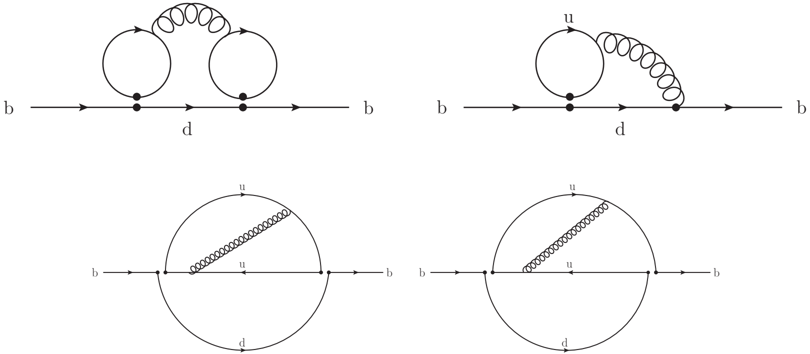
- For this to work, we did not have to choose the evanescent operators in a way that preserves Fierz symmetry at NLO
- It's all accounted for by Δr in the NLO change of basis

Massless charm quarks don't exist!

NLO corrections for three massless quarks in final state

- Three possibilities:



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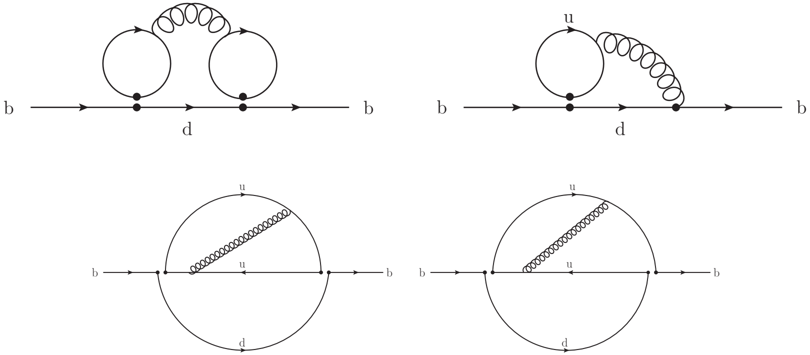

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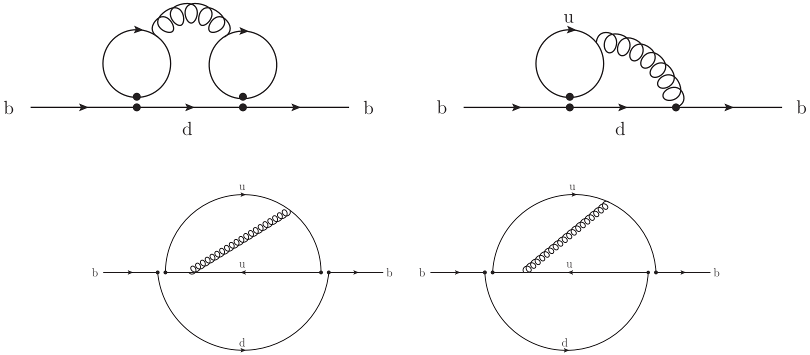

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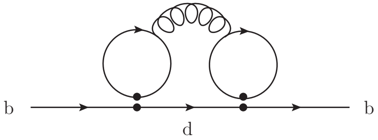
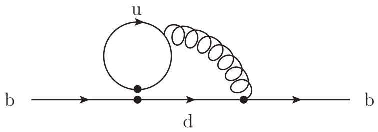
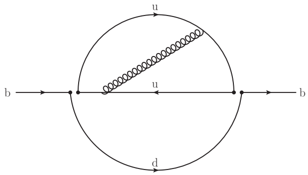
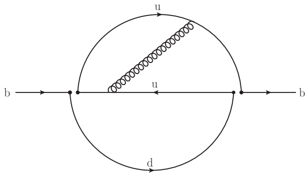
  3. Three identical flavors in the final state ($b \to q\bar{q}q$, $q = d, s$): CC, CC "crossed", penguin (closed and open), dipole



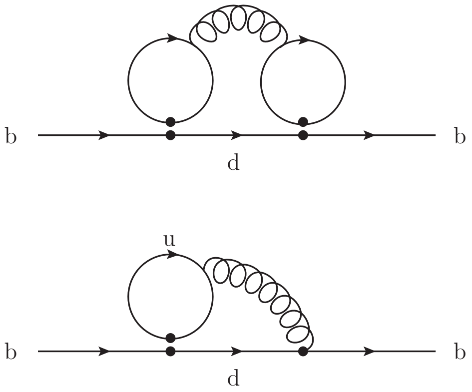






Avoiding γ_5 in Penguins (Case 1: $b \to u\bar{u}d$)

- In Bern basis no problem: the penguin loops are always closed and never carry γ_5.



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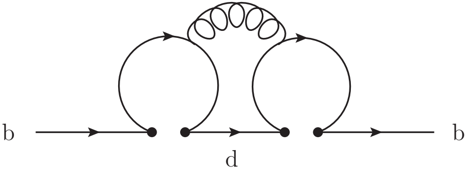

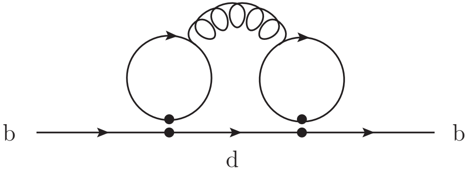

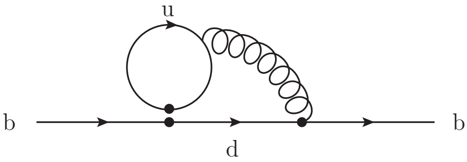

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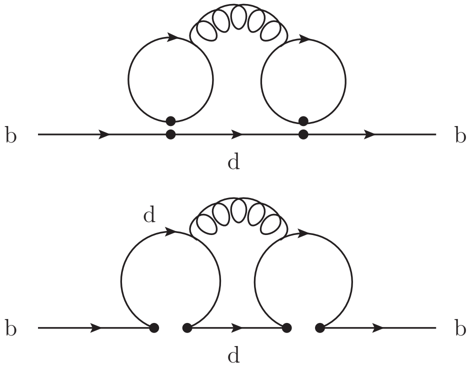
- We get open and closed penguins



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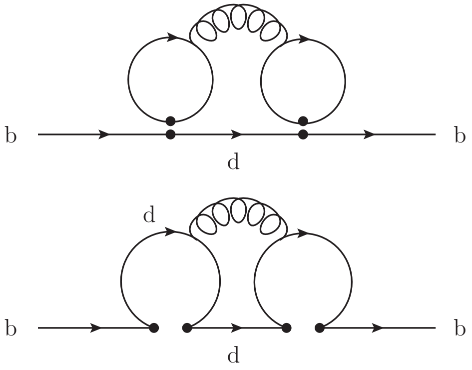

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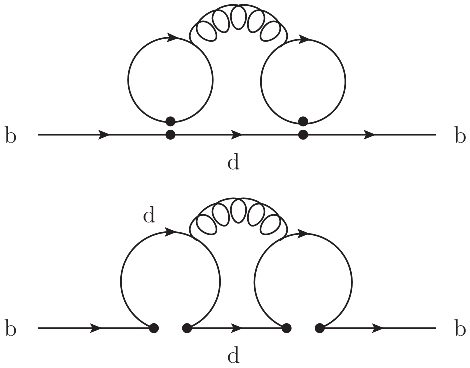

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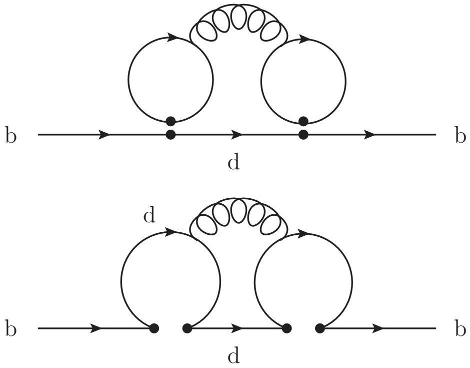

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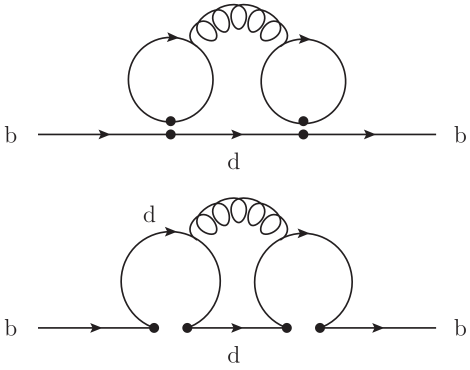
- choose singlet operators $\Rightarrow$ closed penguin loops vanish due to color



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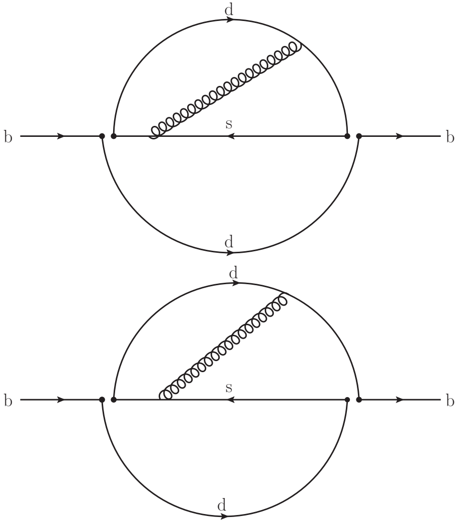

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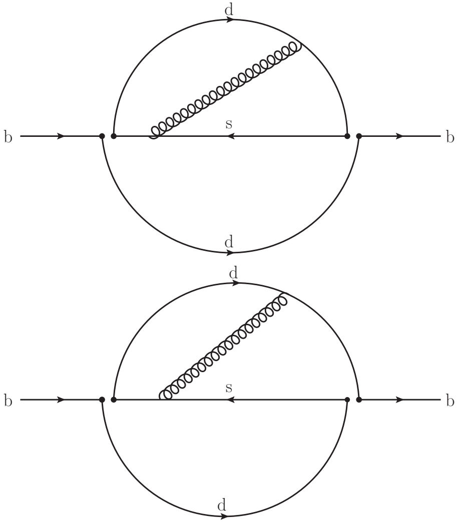

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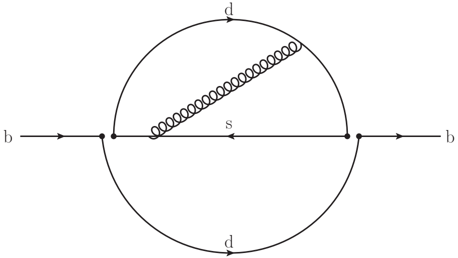

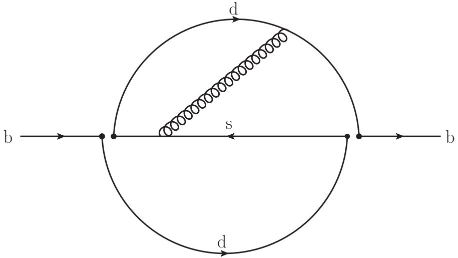

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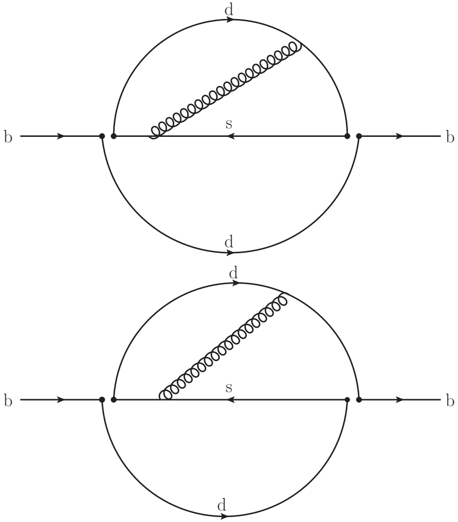
- Can we modify the change of basis to allow us to perform a change of basis only on the CC insertions while not doing anything with the CC crossed insertions?



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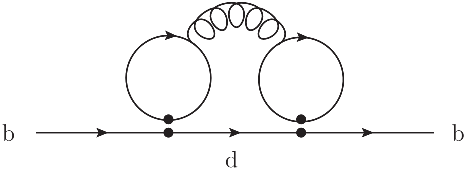

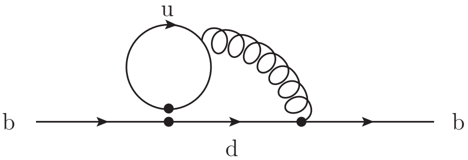

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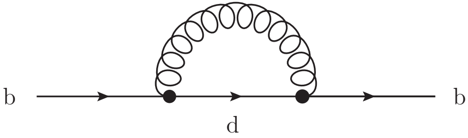

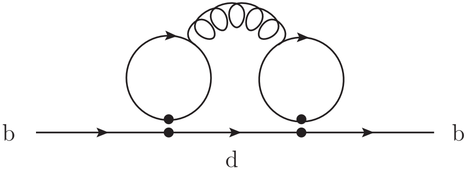

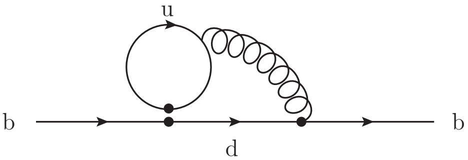

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Angel Picazo

Backup Slides

Choice of WET bases: BMU basis

$$\begin{aligned}
\mathcal{Q}_1^{VLL} &= [\bar{c}_\alpha \gamma_\mu P_L b_\beta]\,[\bar{q}_\beta \gamma^\mu P_L u_\alpha]\ , & \mathcal{Q}_2^{VLL} &= [\bar{c}_\alpha \gamma_\mu P_L b_\alpha]\,[\bar{q}_\beta \gamma^\mu P_L u_\beta]\ , \\
\mathcal{Q}_1^{SLR} &= [\bar{c}_\alpha P_L b_\beta]\,[\bar{q}_\beta P_R u_\alpha]\ , & \mathcal{Q}_2^{SLR} &= [\bar{c}_\alpha P_L b_\alpha]\,[\bar{q}_\beta P_R u_\beta]\ , \\
\mathcal{Q}_1^{VRL} &= [\bar{c}_\alpha \gamma_\mu P_R b_\beta]\,[\bar{q}_\beta \gamma^\mu P_L u_\alpha]\ , & \mathcal{Q}_2^{VRL} &= [\bar{c}_\alpha \gamma_\mu P_R b_\alpha]\,[\bar{q}_\beta \gamma^\mu P_L u_\beta]\ , \\
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\mathcal{Q}_3^{SRR} &= [\bar{c}_\alpha \sigma_{\mu\nu} P_R b_\beta]\,[\bar{q}_\beta \sigma^{\mu\nu} P_R u_\alpha]\ , & \mathcal{Q}_4^{SRR} &= [\bar{c}_\alpha \sigma_{\mu\nu} P_R b_\alpha]\,[\bar{q}_\beta \sigma^{\mu\nu} P_R u_\beta]
\end{aligned}$$

Choice of WET bases: Bern basis

$$\mathcal{O}_1^{qbcu} = [\bar{q} P_R \gamma_\mu b]\,[\bar{c}\gamma^\mu u]\,, \qquad \mathcal{O}_2^{qbcu} = \left[\bar{q} P_R \gamma_\mu T^A b\right]\left[\bar{c}\gamma^\mu T^A u\right]\,,$$

$$\mathcal{O}_3^{qbcu} = [\bar{q} P_R \gamma_{\mu\nu\rho} b]\,[\bar{c}\gamma^{\mu\nu\rho} u]\,, \qquad \mathcal{O}_4^{qbcu} = \left[\bar{q} P_R \gamma_{\mu\nu\rho} T^A b\right]\left[\bar{c}\gamma^{\mu\nu\rho} T^A u\right]\,,$$

$$\mathcal{O}_5^{qbcu} = [\bar{q} P_R b]\,[\bar{c} u]\,, \qquad \mathcal{O}_6^{qbcu} = \left[\bar{q} P_R T^A b\right]\left[\bar{c} T^A u\right]\,,$$

$$\mathcal{O}_7^{qbcu} = [\bar{q} P_R \sigma_{\mu\nu} b]\,[\bar{c}\sigma^{\mu\nu} u]\,, \qquad \mathcal{O}_8^{qbcu} = \left[\bar{q} P_R \sigma_{\mu\nu} T^A b\right]\left[\bar{c}\sigma^{\mu\nu} T^A u\right]\,,$$

$$\mathcal{O}_9^{qbcu} = [\bar{q} P_R \gamma_{\mu\nu\rho\sigma} b]\,[\bar{c}\gamma^{\mu\nu\rho\sigma} u]\,, \qquad \mathcal{O}_{10}^{qbcu} = \left[\bar{q} P_R \gamma_{\mu\nu\rho\sigma} T^A b\right]\left[\bar{c}\gamma^{\mu\nu\rho\sigma} T^A u\right]$$

Details on PS integration (changes of variables)

$$I_4 = N(d)\,4^{d-4}\int_0^1 dz\,(z\bar{z})^{d-3}\,du\,dv\,dw\,dx\;(u\bar{u})^{\frac{d-5}{2}}\;v^{d-3}(\bar{v}w\bar{w}x\bar{x})^{\frac{d-4}{2}}\;\mathcal{K}\,,$$

with

$$\hat{s}_{cu} = zvw\,,\quad \hat{s}_{cg} = \bar{z}vx\,,\quad \hat{s}_{qg} = \bar{z}\bar{v}\,, \hat{s}_{cu} = zvw\,,\quad \hat{s}_{uq} = (a^+ - a^-)u + a^-\,.$$

NLO corrections: massless limit phase space integration

- In $m_c \to 0$ limit, phase space (PS) integrals rather straightforward

$$\int_0^1 dx_i\, x_i^a \bar{x}_i^b = B(a+1, b+1)\,,$$

$$\int_0^1 dx_i\, \frac{x_i^a \bar{x}_i^b}{1-zx} = B(1+a, b+1)_2F_1(1, a+1, a+b+2, z)\,,$$

$$\int_0^1 dx\, x^a \bar{x}^b {}_2F_1(1, c+1, c+d+2; x) =$$

$$B(1+a, 1+b)_3F_2(1, 1+c, 1+a, c+d+2, a+b+2, 1)\,,$$

$$\int_0^1 dxdy \frac{x^a \bar{x}^b y^c \bar{y}^d}{1-xy} {}_2F_1(1, e, f, xy) =$$

$$\sum_{n=0}^{\infty} \frac{(1)_n (e)_n}{(f)_n} \frac{B(c+n+1, d+1) B(a+n+1, b+1)}{n!}$$

$$\times {}_3F_2(1, c+n+1, a+n+1; c+d+n+2, a+b+n+2; 1)$$