Inclusive semileptonic fits

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= More than a lifetime =

Why inclusive decays?

- Set up OPE and heavy quark expansion
- Well established framework
- Extract important CKM parameters $|V_{cb}|, |V_{ub}|$ (and $|V_{cs}$?)
- Extract power corrections from data
- Cross check of exclusive decays

Inclusive $B \to X_c$ decays

Inclusive Decays [Recap]

Inclusive $B \to X_c \ell \nu$: Heavy Quark Expansion (HQE)

- b quark mass is large compared to Λ_{QCD}
- Setting up the HQE: momentum of b quark: $p_b = m_b v + k$, expand in $k \sim iD$
- Optical Theorem $\to$ (local) Operator Product Expansion (OPE)

$$d\Gamma = d\Gamma_0 + \frac{d\Gamma_1}{m_b} + \frac{d\Gamma_2}{m_b^2} + \ldots \qquad d\Gamma_i = \sum_k C_i^{(k)} \left\langle B | O_i^{(k)} | B \right\rangle$$

- $C_i^{(k)}$ perturbative Wilson coefficients
- $\langle B | \ldots | B \rangle$ non-perturbative matrix elements $\to$ string of iD
- operators contain chains of covariant derivatives

HQE elements: $\langle B | \mathcal{O}_i^{(n)} | B \rangle = \langle B | \bar{b}_v (iD_\mu) \ldots (iD_{\mu_n}) b_v | B \rangle$

- Extracted from data

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- Extracted from data
- Ideas for the lattice Juetner et al. [202305.14092]

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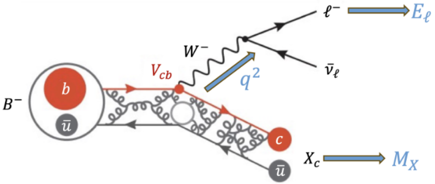
- Extracted from data

- Γ_2 :μ_π^2 and μ_G^2 at $1/m_b^2$
- Γ_3 :ρ_D^3 and ρ_{LS}^3 at $1/m_b^3$
- Many more at $1/m_b^{4,5}$ Mannel, Turczyk, Uraltsev, JHEP 1010 (2011) 109

Moments of the spectrum

BABAR, PRD 68 (2004) 111104; BABAR, PRD 81 (2010) 032003; Belle, PRD 75 (2007) 032005. Pic from M. Fael

Non-perturbative matrix elements obtained from moments of differential rate



$$\langle O^n \rangle_{\rm cut} = \frac{\int_{\rm cut} dO\, O^n \frac{d\Gamma}{dO}}{\int_{\rm cut} dO \frac{d\Gamma}{dO}}$$

$M_X^2 = (p_B - q)^2, E_\ell = v_B \cdot p_\ell$ and $q^2 = (p_\nu + p_\ell)^2$

hadronic mass, lepton energy and q^2 moments

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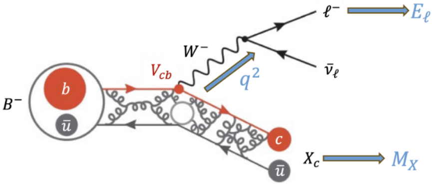

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hadronic mass, lepton energy and q^2 moments

- Different phase space cuts give additional (correlated) observables
- $\mu_\pi^2, \mu_G^2, \rho_D^3 + \ldots \to$ extracted from data $\to$ total rate $\to |V_{cb}|$

Moments of the spectrum

Moments and total rate are double expansion in α_s and HQE parameters

$$
\begin{aligned}
L_i =& \frac{1}{\Gamma_0} \int_{E_l \geq E_{\mathrm{cut}}} \mathrm{d}E_l \, \mathrm{d}q_0 \, \mathrm{d}q^2 \, (E_l)^i \, \frac{\mathrm{d}^3\Gamma}{\mathrm{d}q^2 \, \mathrm{d}q_0 \, \mathrm{d}E_l} \\
=& (m_b)^i \Bigg[L_i^{(0)} + L_i^{(1)} \frac{\alpha_s(\mu_s)}{\pi} + L_i^{(2)} \left(\frac{\alpha_s(\mu_s)}{\pi} \right)^2 + \frac{\mu_\pi^2}{m_b^2} \left(L_{i,\pi}^{(0)} + L_{i,\pi}^{(1)} \frac{\alpha_s(\mu_s)}{\pi} \right) \\
& + \frac{\mu_G^2(\mu_b)}{m_b^2} \left(L_{i,G}^{(0)} + L_{i,G}^{(1)} \frac{\alpha_s(\mu_s)}{\pi} \right) + \frac{\rho_D^3(\mu_b)}{m_b^3} \left(L_{i,D}^{(0)} + L_{i,D}^{(1)} \frac{\alpha_s(\mu_s)}{\pi} \right) \\
& + \frac{\rho_{LS}^3(\mu_b)}{m_b^3} \left(L_{i,LS}^{(0)} + L_{i,LS}^{(1)} \frac{\alpha_s(\mu_s)}{\pi} \right) + O\left(\frac{1}{m_b^4} \right) \Bigg] ,
\end{aligned}
$$

State-of-the-art in inclusive $b \to c$

Jezabek, Kuhn, NPB 314 (1989) 1; Melnikov, PLB 666 (2008) 336; Pak, Czarnecki, PRD 78 (2008) 114015; Becher, Boos, Lunghi, JHEP 0712 (2007) 062; Alberti, Gambino, Nandi, JHEP 1401 (2014) 147; Mannel, Pivovarov, Rosenthal, PLB 741 (2015) 290; Fael, Schonwald, Steinhauser, Phys Rev. D 104 (2021) 016003; Fael, Schonwald, Steinhauser, Phys Rev. Lett. 125 (2020) 052003; Fael, Schonwald, Steinhauser, Phys Rev. D 103 (2021) 014005,

$$\Gamma \propto |V_{cb}|^2 m_b^5 \left[\Gamma_0 + \Gamma_0^{(1)} \frac{\alpha_s}{\pi} + \Gamma_0^{(2)} \left(\frac{\alpha_s}{\pi}\right)^2 + \Gamma_0^{(3)} \left(\frac{\alpha_s}{\pi}\right)^3 + \frac{\mu_\pi^2}{m_b^2} \left(\Gamma^{(\pi,0)} + \frac{\alpha_s}{\pi} \Gamma^{(\pi,1)} \right) \right.$$
$$\left. + \frac{\mu_G^2}{m_b^2} \left(\Gamma^{(G,0)} + \frac{\alpha_s}{\pi} \Gamma^{(G,1)} \right) + \frac{\rho_D^3}{m_b^3} \left(\Gamma^{(D,0)} + \Gamma_0^{(1)} \left(\frac{\alpha_s}{\pi}\right) \right) + \mathcal{O}\left(\frac{1}{m_b^4}\right) + \cdots \right)$$

- Include terms up to $1/m_b^4$* see also Gambino, Healey, Turczyk [2016]
- α_s^3 to total rate and kinetic mass Fael, Schonwald, Steinhauser [2020, 2021]
- $\alpha_s \rho_D^3$ for total rate Mannel, Pivovarov [2020]
- Kinetic mass scheme 1411.6560,1107.3100; hep-ph/0401063

E_ℓ, M_X moments:

$$|V_{cb}|_{\rm incl}^{\rm BCG} = (42.16 \pm 0.51) \times 10^{-3}$$

q^2 moments*:

$$|V_{cb}|_{\rm incl}^{q^2} = (41.69 \pm 0.63) \times 10^{-3}$$

Gambino, Schwanda, PRD 89 (2014) 014022;
Alberti, Gambino et al, PRL 114 (2015) 061802;
Bordone, Capdevila, Gambino, Phys.Lett.B 822 (2021) 136679; Bernlochner, Welsch, Fael, Olschewsky, Persson, van Tonder, KKV [2205.10274]

The advantage of q^2 moments

Mannel, KKV, JHEP 1806 (2018) 115; Fael, Mannel, KKV, JHEP 02 (2019) 177, Mannel, Milutin, KKV [2311.1200]

- Standard lepton energy and hadronic mass moments are not RPI quantities
- Only RPI moments are q^2 moments
- Determinations from Belle and Belle II availabe Phys. Rev. D 104, 112011 (2022), Phys. Rev. D 107, 072002 (2023)

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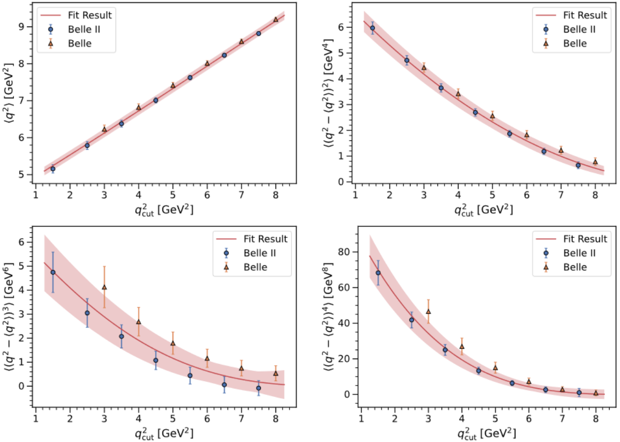
Quirks:

- Setting up the HQE: momentum of b quark: $p_b = m_b v + k$, expand in $k \sim iD$
- Choice of v not unique: Reparametrization invariance (RPI)
- links different orders in $1/m_b \to$ reduction of parameters
- up to $1/m_b^4$: 8 parameters (previous 13)
- Fit up $1/m_b^4$ without assumptions possible?

q^2 moments

Bernlochner, Welsch, Fael, Olschewsky, Persson, van Tonder, KKV [2205.10274]



Centralized moments as function of q^2_{cut}

q^2 moments only analysis

Bernlochner, Welsch, Fael, Olschewsky, Persson, van Tonder, KKV [2205.10274]

$$|V_{cb}|^{q^2}_{\rm incl} = (41.69 \pm 0.27|_{\mathcal{B}} \pm 0.31|_{\Gamma} \pm 0.18|_{\rm exp.} \pm 0.17|_{\rm theo} \pm 0.34|_{\rm const.}) \times 10^{-3}$$

- First extraction using q^2 moments with $1/m_b^4$ terms
- NNLO corrections to moments not included
- Higher order coefficients important to check convergence of the HQE

$$r_E^4 = (0.02 \pm 0.34) \cdot 10^{-1}\mathrm{GeV}^4 \quad r_G^4 = (-0.21 \pm 0.69)\mathrm{GeV}^4$$

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- Inputs for $B \to X_u \ell\nu$, B lifetimes Talk Markus! and $B \to X_s \ell\ell$
- Agreement with BCG extraction (differs due to branching ratio inputs)

  Bordone,Capdevila, Gambino [2021]

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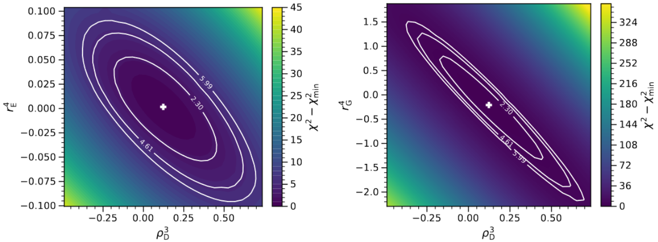

First combined fit

Gambino, Finauri [2310.20324]

- Includes terms up to $1/m_b^3$
- Calculation of BLM α_s^2 corrections to q^2 moments
- Includes QED corrections to the lepton moments Bigi, Bordone, Gambino, Haisch, Piccione, JHEP 11 (2023) 163

Combined E_ℓ, M_X, q^2 moments:

$$|V_{cb}|^{\rm GF}_{\rm incl,all} = (41.95 \pm 0.27|_{\rm exp} \pm 0.31|_{\rm th} \pm 0.25|_\Gamma) \times 10^{-3} = (41.95 \pm 0.48) \times 10^{-3}$$

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- Agrees with previous determinations, reduced uncertainty
- Agrees with determination from three points at fixed cut

First combined Fit

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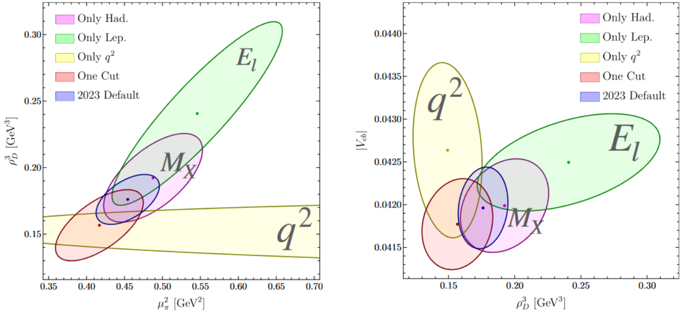
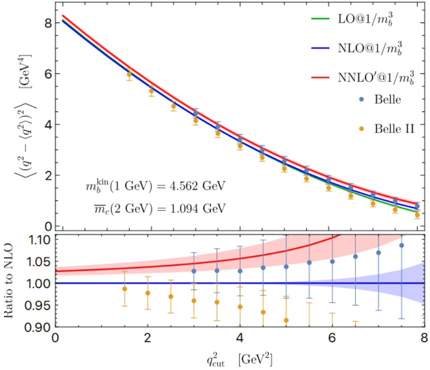
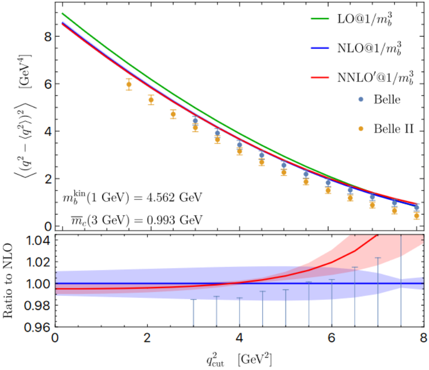


- Complementary between different measurements
- Extracted $\rho_D^3 = (0.176 \pm 0.019)$ GeV^3 (compare to our $\rho_D^3 = (0.12 \pm 0.20)$ GeV^3)
- Important input for lifetimes and $B \to X_u$, $B \to X_s$

Full NNLO corrections to q^2 momemts

Herren, Fael [2403.03976]





$\overline{m_c}(2\ \mathrm{GeV})$ not ideal choice

$\overline{m_c}(3\ \mathrm{GeV})$ better

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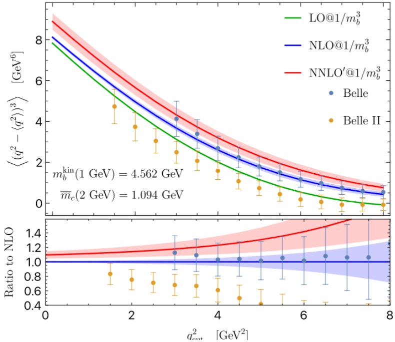

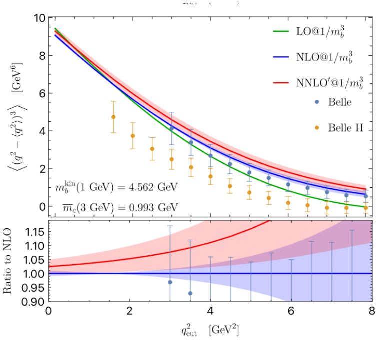

NNLO effects mainly re-absorbed in the fit into a shift of ρ_D^3, r_E^4 and r_G^4.
No major shift in $|V_{cb}|$.

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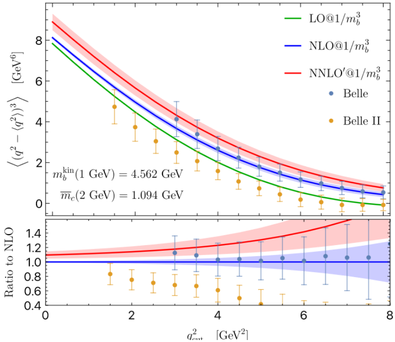

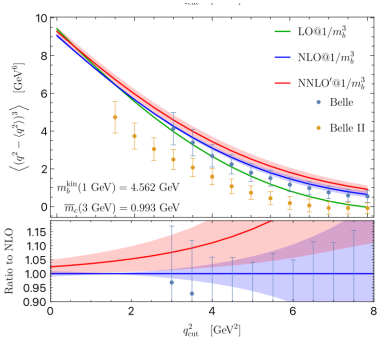

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Full combined analysis and updated q^2 fits in progress!

Inclusive decays: The Kolya package

Kolya package, Fael, Mulatin, KKV [2409.15007]

Open source Python package:
https://gitlab.com/vcb-inclusive/kolya

- HQE predictions for several observables:
  - Centralized $\langle E_\ell \rangle$ moments
  - Centralized $\langle q^2 \rangle$ moments
  - Centralized $\langle M_X^2 \rangle$ moments
  - Total rate + branching ratio with kinematic cut

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Features:

- Includes power corrections up to $1/m_b^5$ Mannel, Milutin, KKV [2311.1200]
- Employs kinetic scheme for m_b and $\overline{\mathrm{MS}}$ for m_c
- Interface with CRunDec for automatic RGE evolution Chetyrkin, Kuhn, Steinhauser, Smidth, Herren

QCD corrections in Kolya

Nir, Pak, Downlin, Egner, Fael, Steinhauser, Gambino, Manohar, Block, Becher, Alberti, Mannel, Turczyk, Dassinger, Schonwald.

$\Gamma_{\rm sl}$	tree	α_s	α_s^2	α_s^3
Partonic				['11]
μ_π^2, μ_G^2		['06, '12,'13,'15]		
ρ_D^3, ρ_{LS}^3		['21]		
$1/m_b^4, 1/m_b^5$	['06,'10,'18,'23]			
$q_n(q^2_{\rm cut})$	tree	α_s	α_s^2	
Partonic			['24]	
μ_G^2, μ_π^2		['12,'13]		
ρ_D^3, ρ_{LS}		['21]		
$1/m_b^4, 1/m_b^5$	['18, '23]			
$\ell_n(E_{\rm cut}), h_n(E_{\rm cut})$	tree	α_s	$\alpha_s^2\beta_0$	α_s^2
Partonic			['05]	['08]*
μ_G^2, μ_π^2		['07,'13]		
ρ_D^3				
$1/m_b^4, 1/m_b^5$	['06, '10,'23]			

- *only known for fixed m_c/m_b and lepton energy cuts

New Physics in Semileptonic Inclusive Decays

Fael, Rahimi, KKV [2208.04282]

$P_{L(R)} = 1/2\,(1 \mp \gamma_5)$ and $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$

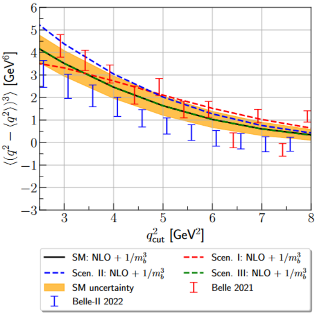
$$\mathcal{H}_{\text{eff}} = \frac{4 G_F V_{cb}}{\sqrt{2}} \left[\left(1 + C_{V_L}\right) O_{V_L} + \sum_{i = V_R, S_L, S_R, T} C_i\, O_i \right],$$

$$\begin{aligned} O_{V_{L(R)}} &= \left(\bar{c}\gamma_\mu P_{L(R)} b\right) \left(\bar{\ell}\gamma^\mu P_L \nu_\ell\right) , \\ O_{S_{L(R)}} &= \left(\bar{c} P_{L(R)} b\right) \left(\bar{\ell} P_L \nu_\ell\right) \\ O_T &= \left(\bar{c}\, \sigma_{\mu\nu} P_L b\right) \left(\bar{\ell}\, \sigma^{\mu\nu} P_L \nu_\ell\right) . \end{aligned}$$

$$\begin{aligned} \langle \mathcal{M} \rangle = {} & \xi_{\text{SM}} + |C_{V_R}|^2\, \xi_{\text{NP}}^{\langle V_R, V_R \rangle} + |C_{S_L}|^2\, \xi_{\text{NP}}^{\langle S_L, S_L \rangle} + |C_{S_R}|^2\, \xi_{\text{NP}}^{\langle S_R, S_R \rangle} + |C_T|^2\, \xi_{\text{NP}}^{\langle T, T \rangle} \\ & + \text{Re}((C_{V_L} - 1) C_{V_R}^*)\, \xi_{\text{NP}}^{\langle V_L, V_R \rangle} + \text{Re}(C_{S_L} C_{S_R}^*)\, \xi_{\text{NP}}^{\langle S_L, S_R \rangle} + \text{Re}(C_{S_L} C_T^*)\, \xi_{\text{NP}}^{\langle S_L, T \rangle} \\ & + \text{Re}(C_{S_R} C_T^*)\, \xi_{\text{NP}}^{\langle S_R, T \rangle} , \end{aligned}$$

New Physics?

Fael, Rahimi, KKV [2208.04282]



- NP would also influence the moments of the spectrum
- Requires a simultaneous fit of hadronic parameters and NP

Inclusive $|V_{cb}|$?

Bernlochner, Welsch, Fael, Olschewsky, Persson, van Tonder, KKV [2205.10274], Bordone, Capdevila, Gambino [2107.00604], Gambino, Schwanda [2014], Finauri, Gambino [2023]Prim, Milutin, Fael, Bernlochner, KKV [in progress]

Use Kolya + Experimental measurements $\rightarrow$ $|V_{cb}|$ and HQE parameters

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Use Kolya + Experimental measurements $\rightarrow$ $|V_{cb}|$ and HQE parameters
Challenge: How to deal with theoretical uncertainties?

Theoretical uncertainties

Bernlochner, Welsch, Fael, Olschewsky, Persson, van Tonder, KKV [2205.10274], Bordone, Capdevila, Gambino [2107.00604], Gambino, Schwanda [2014], Finauri, Gambino [2023]

- Vary scale of α_s to account for perturbative corrections
- Vary m_b^{kin} and scale of charm μ_c

Theoretical uncertainties

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"Known" uncertainties

- Vary scale of α_s to account for perturbative corrections
- Vary $m_b^{\rm kin}$ and scale of charm μ_c

Missing higher order HQE parameters

- Vary ρ_D^3 by 30% to account for missing HQE parameters
- Vary μ_G^2 by 20% to account for missing $\alpha_s\times$ HQE parameters
- Assume theoretical correlations!
  - Strong correlations between different cuts, no correlation between different models see Gambino, Schwanda, Finauri
  - Flexible correlation added as nuisance parameter see Bernlochner, Fael, KKV

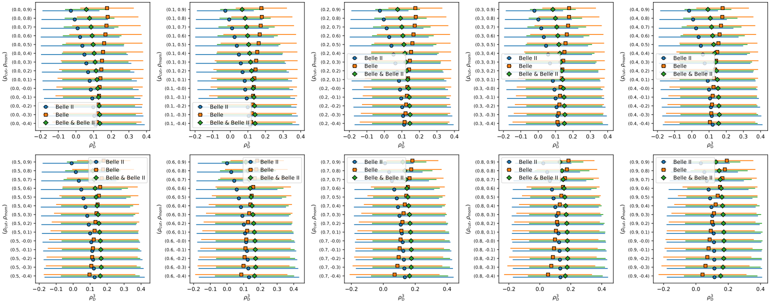
What about theory correlations?

Bernlochner, Welsch, Fael, Olschewsky, Persson, van Tonder, KKV [2205.10274]

- Flexible correlations between moments $\rho_{\rm mom}$ and different cuts $\rho_{\rm cut}$

$$\rho_n[q_n(q_A^2) - q_n(q_B^2)] = \rho_{\rm cut}^x \qquad x = \frac{|q_A^2 - q_B^2|}{0.5\mathrm{GeV}^2}$$

- Included by adding a penalty term to the χ^2
- Scan over large range of values + add as nuisance parameters in fit
- V_{cb} stable w.r.t. theory correlations



What about theory correlations?

Bernlochner, Welsch, Fael, Olschewsky, Persson, van Tonder, KKV [2205.10274]

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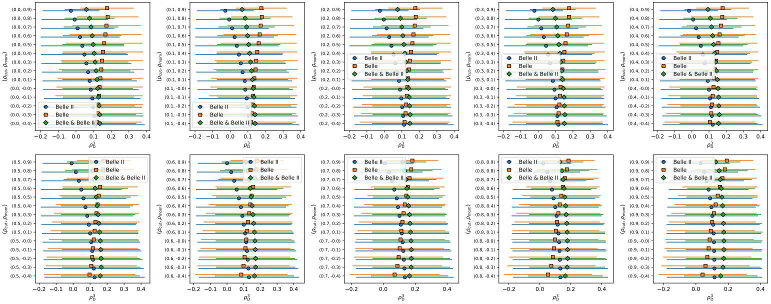
- Included by adding a penalty term to the χ^2
- Scan over large range of values + add as nuisance parameters in fit
- V_{cb} uncertainty includes large range of correlations



Theoretical uncertainties

Bernlochner, Welsch, Fael, Olschewsky, Persson, van Tonder, KKV [2205.10274], Bordone, Capdevila, Gambino [2107.00604], Gambino, Schwanda [2014], Finauri, Gambino [2023]

"Known" uncertainties

- Vary scale of α_s to account for perturbative corrections
- Vary $m_b^{\rm kin}$ and scale of charm μ_c

Missing higher order HQE parameters

- Vary ρ_D^3 by 30% to account for missing HQE parameters
- Vary μ_G^2 by 20% to account for missing $\alpha_s\times$ HQE parameters
- Assume theoretical correlations!
  - Strong correlations between different cuts, no correlation between different models see Gambino, Schwanda, Finauri
  - Flexible correlation added as nuisance parameter see Bernlochner, Fael, KKV

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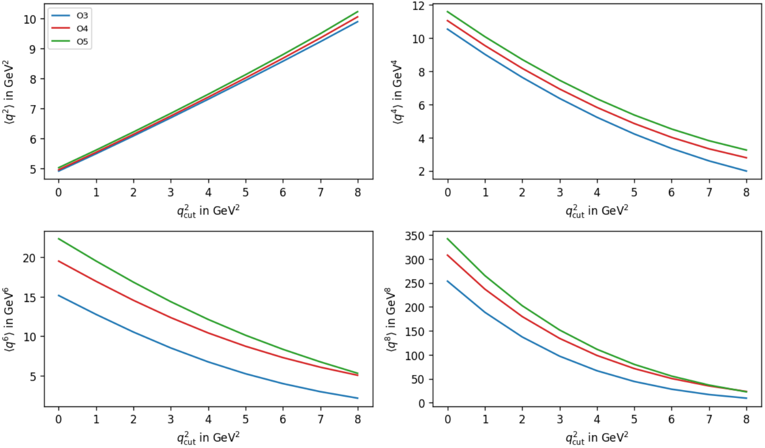
  - Flexible correlation added as nuisance parameter see Bernlochner, Fael, KKV

Can we use the $1/m_b^{4,5}$ corrections to better access the theory uncertainty?

Effect of higher-order corrections

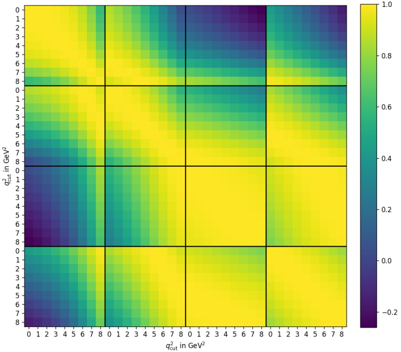
Kolya



Using LLSA inputs for the HQE parameters and NNLO corrections

Theoretical correlations

Preliminary! Prim, Milutin, Fael, Bernlochner, KKV [in progress]



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- Correlations between different moments!

Theoretical correlations

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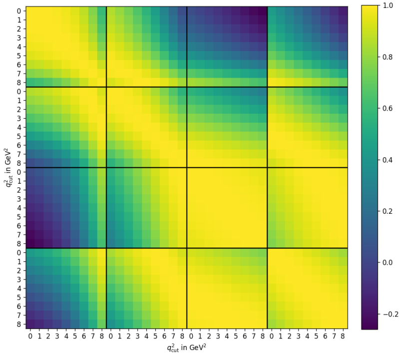
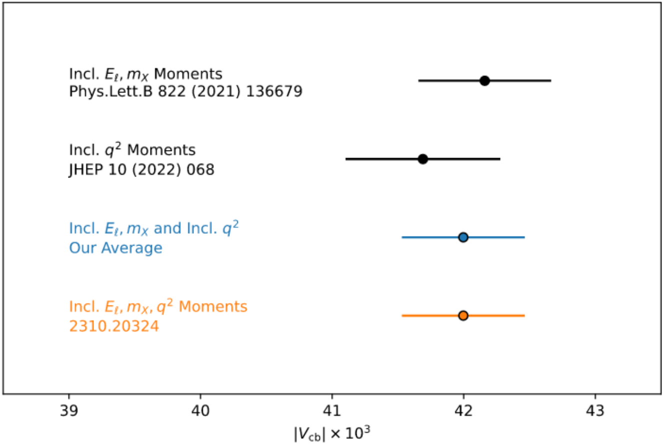


- Large correlations between different cuts
- Correlations between different moments!
- Full analysis tomorrow by Markus

Summary of $|V_{cb}|$ inclusive

Fael, Prim, KKV, Eur. Phys. J. Spec. Top. (2024). https://doi.org/10.1140/epjs/s11734-024-01090-w

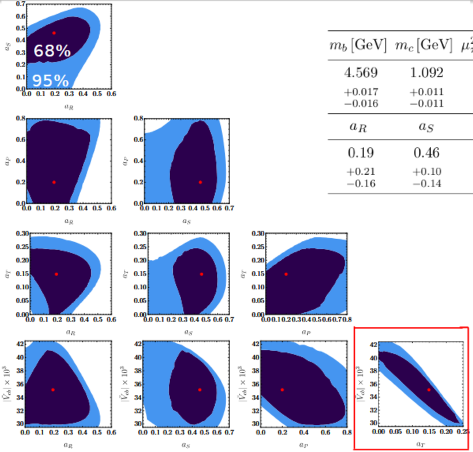


- Need new branching ratio measurements!

What about NP?

Finauri, Carvunis, Gambino, Jung, Mächler[2507.22123]; slide from CKM 2025

Fit Result with Generic NP



m_b [GeV]	m_c [GeV]	μ_π^2 [GeV^2]	μ_G^2 [GeV^2]	ρ_D^3 [GeV^3]	ρ_{LS}^3 [GeV^3]	$\mathrm{BR}_{c\ell\bar{\nu}}$ [%]
4.569	1.092	0.454	0.342	0.195	−0.126	10.72
$^{+0.017}_{-0.016}$	$^{+0.011}_{-0.011}$	$^{+0.049}_{-0.050}$	$^{+0.065}_{-0.067}$	$^{+0.027}_{-0.029}$	$^{+0.096}_{-0.104}$	$^{+0.16}_{-0.16}$
a_R	a_S	a_P	a_T	$\cos(\delta_R)$	$\cos(\delta_{ST})$	$\cos(\delta_{PT})$
0.19	0.46	0.20	0.15	0.99	−0.94	0.91
$^{+0.21}_{-0.16}$	$^{+0.10}_{-0.14}$	$^{+0.43}_{-0.20}$	$^{+0.07}_{-0.11}$	$^{+0.01}_{-0.81}$	$^{+0.92}_{-0.06}$	$^{+0.09}_{-1.91}$

first global fit for $\bar{B} \to X_c \ell \bar{\nu}$ with generic NP

- $\chi^2_{\text{min}}/\text{d.o.f.} = 36.04/67$
- SM fit: $\chi^2_{\text{min}}/\text{d.o.f.} = 42.58/74$
- first row SM compatible
- strong correlation in $a_T - |\tilde{V}_{cb}|$
- $\boxed{|\tilde{V}_{cb}| = (35.1^{+4.2}_{-3.6}) \cdot 10^{-3}}$
- competitive with exclusive
- no preference for NP

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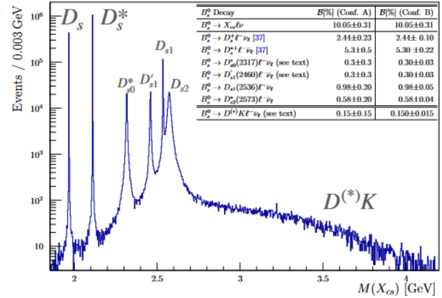
Other Inclusive Decays

De Cian, Feliks, Rotondo, KKV [2312.05147]. Pic from M. Fael

First study of the possiblities using sum-over-exclusive technique



B_s^0 Decay	$\mathcal{B}$[%] (Conf. A)	$\mathcal{B}$[%] (Conf. B)
$B_s^0 \to X_{cs}\ell\nu$	10.05±0.31	10.05±0.31
$B_s^0 \to D_s^+\ell^-\nu_\ell$ [37]	2.44±0.23	2.44± 0.10
$B_s^0 \to D_s^{*+}\ell^-\bar{\nu}_\ell$ [37]	5.3±0.5	5.30 ±0.22
$B_s^0 \to D_{s0}^*(2317)\ell^-\nu_\ell$ (see text)	0.3±0.3	0.30±0.03
$B_s^0 \to D'_{s1}(2460)\ell^-\bar{\nu}_\ell$ (see text)	0.3±0.3	0.30±0.03
$B_s^0 \to D_{s1}(2536)\ell^-\nu_\ell$	0.98±0.20	0.98±0.05
$B_s^0 \to D_{s2}^*(2573)\ell^-\bar{\nu}_\ell$	0.58±0.20	0.58±0.04
$B_s^0 \to D^{(*)}K\ell^-\nu_\ell$ (see text)	0.15±0.15	0.150±0.015

- B_s spectrum well-separated
- Only M_X^2 moments available
- Study $SU(3)$ breaking of HQE

Inclusive B_s decays?

De Cian, Feliks, Rotondo, KKV [2312.05147]. Pic from M. Fael

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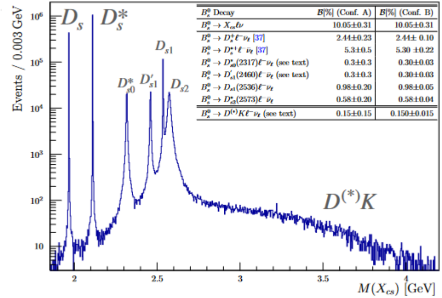

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- B_s spectrum well-separated
- Only M_X^2 moments available
- Study $SU(3)$ breaking of HQE

- Improve knowledge D_s^{**} states
- Understand non-resonant contribution
- V_{cs} extraction requires Branching ratio from Belle II!

Outlook for Kolya

Plans to expand Kolya version 1 with:

- QED effects Bigi, Bordone, Gambino, Haisch, Piccione [2308.02849]
- Exact results for NNLO corrections to E_ℓ and M_X moments
- RGE of HQE parameters
- NLO corrections to HQE parameters for E_ℓ and M_X moments

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And additional observables:

- Forward-backward asymmetry
- $R_X = \Gamma_{B\to X_c\tau\bar{\nu}_\tau}/\Gamma_{B\to X_c l\bar{\nu}_l}$
- Lifetimes Talk by Markus
- Predictions for the decay into charmless final states $B \to X_u l\bar{\nu}_l$
- Inclusive D decays Mannel, Fael, KKV [1910.05234]

Any other suggestions?

Outlook: fits in inclusive decays

- First analysis of q^2 moments [done]
- Full analysis of all moments [done by Gambino and Finauri]
- Full analysis of all moments using new fit setup [in progress]

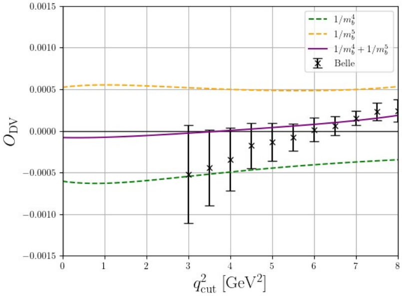
How to handle missing higher orders and theory uncertainties?

Outlook: power-correction sensitive observables

Mannel, Milutin, Verkade, KKV [2407.01473]; Belle collaboration [2109.01685]

$$\bar{q}_i = C_i^{(0)} + \frac{\mu_G^2}{m_b^2} C_i^{(2)} + \frac{\tilde{\rho}_D^3}{m_b^3} C_i^{(3)} + R_i \ ,$$

- R_i contains higher order terms
- We can then construct an observable only sensitive to higher $k+1$ powers $O_{\rm DV}^{(k)} \sim \Lambda^{k+1}/m_n^{k+1}$
- First study using $O_{\rm DV}^{(3)} = \xi_1 \frac{\bar{q}_1}{m_b^2} + \xi_2 \frac{\bar{q}_2}{m_b^4} + \xi_3 \frac{\bar{q}_3}{m_b^6} + \xi_4 \frac{\bar{q}_4}{m_b^8}$



More than a lifetime?

Definitely enough to do!

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Definitely enough to do!

Close collaboration between theory and experiment necessary!

Backup

Theory guidance to include power corrections

Lowest State Saturation Approximation (LSSA)

$$\langle B|O_1 O_2|B\rangle = \sum_n \langle B|O_1|n\rangle\langle n|O_2|B\rangle$$

$$\rho_D^3 = \varepsilon\mu_\pi^2, \qquad \rho_{LS}^3 = -\varepsilon\mu_G^2, \qquad \varepsilon \sim 0.4 \text{ GeV}$$

Mannel, Turczyk, Uraltsev JHEP 1011 (2010) 109; Heinonen, Mannel, NPB 889 (2014) 46

- Gives central values but uncertainty challenging to estimate

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- Gives central values but uncertainty challenging to estimate
- $\mathcal{O}(1/m_b^4, 1/m_b^5)$ can then be included in fit Healey, Turczyk, Gambino, PLB 763 (2016) 60
    - LSSA estimated as priors (60% gaussian uncertainty)
    - -0.25% shift on $|V_{cb}|$ due to power corrections

New Physics?

Finauri, Carvunis, Gambino, Jung, Mächler[2507.22123]; slide from CKM 2025

Why $|V_{cb}|$ so Low?

Blind direction between a_T^2 term and $a_T a_S$, $a_T a_P$
for: $a_{S,P} \sim 3a_T$, $\cos\delta_{ST} \simeq -1$, $\cos\delta_{PT} \simeq 1$

	$10 \times \Gamma/\Gamma_0$
ζ_{SM}	$6.580 - 0.461_{\rm pow} - 0.560_{\alpha_s} - 0.009_{\alpha_s/m_b^2} - 0.034_{\alpha_s/m_b^3} - 0.105_{\alpha_s^2} + 0.003_{\alpha_s^3} + 0.058_{\alpha_{\rm em}}$
ζ_{RR}	$6.580 - 0.461_{\rm pow} - 0.560_{\alpha_s}$
ζ_{LR}	$-4.280 + 0.634_{\rm pow} + 0.464_{\alpha_s}$
ζ_{TT}	$78.964 - 7.865_{\rm pow}$
ζ_{SS}	$5.430 + 0.240_{\rm pow}$
ζ_{PP}	$1.150 - 0.118_{\rm pow}$

interference terms not present in Γ
- new observables could resolve the issue!

good candidate: A_{FB} [Herren 2022][Fael, Rahimi, Vos 2022]
- exclusive decays as well

	L_1 $[10^{-2}\ \text{GeV}]$
ξ_{SM}	$157.276 - 1.676_{\rm pow} - 0.312_{\alpha_s} - 0.463_{\frac{\alpha_s}{m_b^2}} + 0.084_{\alpha_s^2}$
ξ_{RR}	$-9.760 + 1.114_{\rm pow} + 0.208_{\alpha_s}$
ξ_{LR}	$-0.368 + 0.606_{\rm pow} - 0.197_{\alpha_s}$
ξ_{LR^2}	$-0.247 + 0.427_{\rm pow} - 0.128_{\alpha_s}$
ξ_{TT}	$-78.076 + 9.286_{\rm pow}$
ξ_{SS}	$0.184 + 3.006_{\rm pow}$
ξ_{PP}	$-0.184 + 0.191_{\rm pow}$
ξ_{ST}	$-19.519 - 2.147_{\rm pow}$
ξ_{PT}	$19.519 - 0.025_{\rm pow}$

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Kolya: Installation and features

Fael, Milutin, KKV [2409.15007], https://gitlab.com/vcb-inclusive/kolya.git

$: git clone https://gitlab.com/vcb-inclusive/kolya.git
$: cd kolya
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```

- Several example Jupyter notebooks available.
- Several cross-checks with literature performed
- Default up to $1/m_b^3$. Higher orders included via flagmb4$=1$ and flagmb5$=1$
- Implemented both "RPI" and "historical" (perp) basis
- LLSA predictions for the HQE elements implemented

Kolya: Installation and features

Fael, Milutin, KKV [2409.15007], https://gitlab.com/vcb-inclusive/kolya.git

- Implemented both "RPI" and "historical" (perp) basis
- LLSA predictions for the HQE elements implemented
- Gives fast "predictions" for moments and total rate

The classes `parameters.LSSA_HQE_parameters` and `parameters.LSSA_HQE_parameters_RPI` store the same HQE parameters as `parameters.HQE_parameters` and `parameters.HQE_parameters_RPI` in the "perp" and RPI basis respectively up to $1/m_b^5$. They are initialized to values predicted by the 'lowest-lying state saturation ansatz' (LSSA).

In [14]:

```
hqe_perp = kolya.parameters.LSSA_HQE_parameters()
hqe_RPI = kolya.parameters.LSSA_HQE_parameters_RPI()

print(r'LSSA prediction for rhoD in the perp basis: ', hqe_perp.rhoD)
print('LSSA prediction for m1 in the perp basis: ', hqe_perp.m1, '\n')

print('LSSA prediction for rhoD in the RPI basis: ', hqe_RPI.rhoD)
print('LSSA prediction for rEtilde in the RPI basis: ', hqe_RPI.rEtilde)
```

Out [14]:

```
LSSA prediction for rhoD in the perp basis:  0.231
LSSA prediction for m1 in the perp basis:  0.126

LSSA prediction for rhoD in the RPI basis:  0.205
LSSA prediction for rEtilde in the RPI basis:  0.098
```

Kolya: Installation and features

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Total Rate

We define the total rate as

$$\Gamma_{\rm sl} = \frac{G_F^2 (m_b^{\rm kin})^5}{192\pi^3} |V_{cb}|^2 X$$

The coefficients X is a function of the quark masses, α_s, the HQE parameters and the Wilson coefficients. It is evaluated by the function `X_Gamma_KIN_MS(par, hqe, wc)`

```
[5]: hqe = kolya.parameters.HQE_parameters(
         muG = 0.306,
         rhoD = 0.185,
         rhoLS = -0.13,
         mupi = 0.477,
     )
     wc  = kolya.parameters.WCoefficients()
     kolya.TotalRate.X_Gamma_KIN_MS(par,hqe,wc)
```

```
[5]: 0.539225163728085
```

The branching ratio is given by the function `BranchingRatio_KIN_MS(Vcb,par,hqe,wc)`

```
[6]: Vcb = 42.2e-2
     kolya.TotalRate.BranchingRatio_KIN_MS(Vcb,par,hqe,wc)
```

```
[6]: 10.555834162102016
```

Kolya: Installation and features

Fael, Milutin, KKV [2409.15007], https://gitlab.com/vcb-inclusive/kolya.git. Herren, Fael [2403.03976]

```
>>> kolya.Q2moments.moment_1_KIN_MS(8.0, par, hqe, wc, flag_DEBUG=1)
Q2moment n. 1 LO =  9.148659808170105
Q2moment n. 1 NLO = api * -1.319532010835962
Q2moment n. 1 NNLO = api^2 * -9.616956902561078
Q2moment n. 1 NLO pw = api * -0.7873907726673756
Q2moment n. 1 NNLO from NLO pw = api^2 * 8.39048437244325
```

- Includes NNLO corrections
- NNLO and NLO to power corrections can be turned off