Gradient-flow renormalization for lifetimes

More than a lifetime – Siegen

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in collaboration with Matthew Black, Robert Harlander, Jonas Kohnen, Antonio Rago, Andrea Shindler, Oliver Witzel

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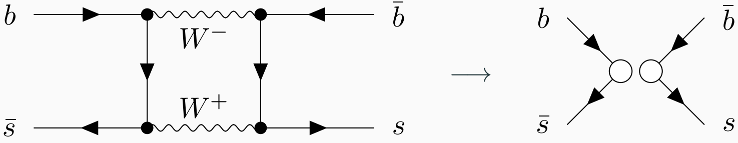
Theory in a nutshell ⇒ see Maria Laura Piscopo's talk for more details

- Tool of choice: effective theories



- + Heavy Quark Expansion (HQE) for lifetimes:

$$\Gamma(H_Q) = \Gamma_3 + \Gamma_5 \frac{\langle \mathcal{O}_5 \rangle}{m_Q^2} + \Gamma_6 \frac{\langle \mathcal{O}_6 \rangle}{m_Q^3} + \ldots + 16\pi^2 \left(\tilde{\Gamma}_6 \frac{\langle \tilde{\mathcal{O}}_6 \rangle}{m_Q^3} + \tilde{\Gamma}_7 \frac{\langle \tilde{\mathcal{O}}_7 \rangle}{m_Q^4} + \ldots \right)$$

- $\Gamma(H_Q)$ factorizes into Wilson coefficients Γ_i and matrix elements $\langle \mathcal{O}_i \rangle$
- Focus on dimension-six four-quark contributions $\tilde{\Gamma}_6$ and $\langle \tilde{\mathcal{O}}_6 \rangle$ in this talk
- Wilson coefficients $\tilde{\Gamma}_6$ can be computed perturbatively ⇒ see Francesco Moretti's talk
- Two methods for non-perturbative bag parameters $\langle \tilde{\mathcal{O}}_6 \rangle$:
  - Sum rules ⇒ see Martin Lang's talk
  - Lattice QCD ⇒ this talk as well as Matthew Black's and Joshua Lin's talks

Bag parameters on the lattice

$$\Gamma(H_Q) = \Gamma_3 + \Gamma_5 \frac{\langle \mathcal{O}_5 \rangle}{m_Q^2} + \Gamma_6 \frac{\langle \mathcal{O}_6 \rangle}{m_Q^3} + \ldots + 16\pi^2 \left(\tilde{\Gamma}_6 \frac{\langle \tilde{\mathcal{O}}_6 \rangle}{m_Q^3} + \tilde{\Gamma}_7 \frac{\langle \tilde{\mathcal{O}}_7 \rangle}{m_Q^4} + \ldots \right)$$

- While $\Gamma(H_Q)$ is scheme independent, $\tilde{\Gamma}_6$ and $\langle \tilde{\mathcal{O}}_6 \rangle$ individually are not:

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  Perturbative $\tilde{\Gamma}_6$:

  - Dimensional regularization with $D = 4 - 2\epsilon$
  - Operators mix through renormalization, also with evanescent operators (vanish in $D = 4$):

    $$\mathcal{O}^{\mathrm{R}} = Z_{\mathcal{O}\mathcal{O}} \mathcal{O} + Z_{\mathcal{O}\mathrm{E}} E$$

  - $\tilde{\Gamma}_6$ scheme dependent:
    1. Explicit dependence on μ
    2. Scheme for γ_5
    3. Choice of evanescent operators

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Lattice $\langle \tilde{\mathcal{O}}_6 \rangle$:

- Lattice spacing a as UV regulator
- Operators mix through renormalization:

$$\mathcal{O}^{\mathrm{R}} = Z_{11} \mathcal{O}_1 + Z_{12} \mathcal{O}_2$$

- Requires continuum limit $a \to 0$ afterwards
- $\langle \tilde{\mathcal{O}}_6 \rangle$ scheme dependent

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$$\Gamma(H_Q) = \Gamma_3 + \Gamma_5 \frac{\langle \mathcal{O}_5 \rangle}{m_Q^2} + \Gamma_6 \frac{\langle \mathcal{O}_6 \rangle}{m_Q^3} + \ldots + 16\pi^2 \left(\tilde{\Gamma}_6 \frac{\langle \tilde{\mathcal{O}}_6 \rangle}{m_Q^3} + \tilde{\Gamma}_7 \frac{\langle \tilde{\mathcal{O}}_7 \rangle}{m_Q^4} + \ldots \right)$$

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- $\langle \tilde{\mathcal{O}}_6 \rangle$ scheme dependent

$\Rightarrow$ Need additional calculation to match the schemes

Gradient flow

- Introduce parameter *flow time* $\tau \geq 0$ [Narayanan, Neuberger 2006; Lüscher 2009; Lüscher 2010]
- *Flowed fields* in $D+1$ dimensions obey differential *flow equations*:

Flow equations [Narayanan, Neuberger 2006; Lüscher 2010; Lüscher 2013]

$$\frac{\partial}{\partial\tau} B^a_\mu = \mathcal{D}^{ab}_\nu G^b_{\nu\mu} \quad \text{with} \quad B^a_\mu(\tau,x)\Big|_{\tau=0} = A^a_\mu(x),$$
$$\frac{\partial}{\partial\tau}\chi = \Delta\chi \quad \text{with} \quad \chi(\tau,x)|_{\tau=0} = \psi(x)$$

$$\mathcal{D}^{ab}_\mu = \delta^{ab}\partial_\mu - f^{abc}B^c_\mu, \qquad G^a_{\mu\nu} = \partial_\mu B^a_\nu - \partial_\nu B^a_\mu + f^{abc}B^b_\mu B^c_\nu,$$
$$\Delta = (\partial_\mu + B^a_\mu T^a)(\partial_\mu + B^b_\mu T^b)$$

Gradient flow schematically

- Let's simplify this a bit:

$$\frac{\partial}{\partial\tau}B^a_\mu = \mathcal{D}^{ab}_\nu G^b_{\nu\mu} \qquad \text{with} \qquad B^a_\mu(\tau,x)\big|_{\tau=0} = A^a_\mu(x)$$

$$\partial_\tau B \sim (\partial_x - g_0 B)(\partial_x B + g_0 B^2)$$

$$\sim \partial_x^2 B + g_0\partial_x B^2 - g_0^2 B^3$$

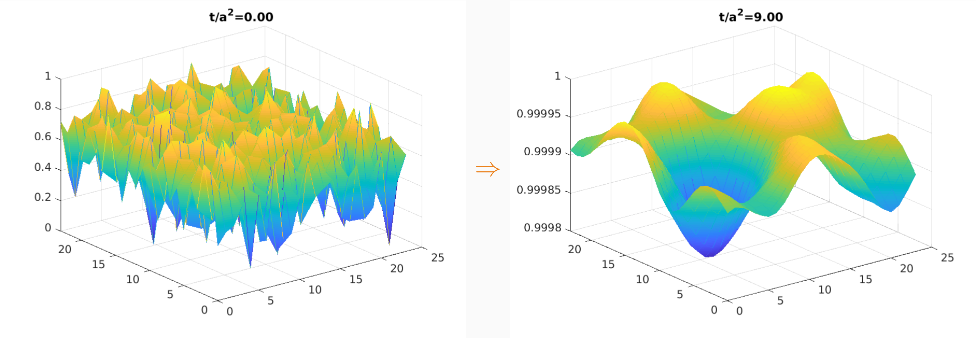
- Flow equations similar to the heat equation (thermodynamics)

$$\partial_t u(t,\vec{x}) = \alpha\Delta u(t,\vec{x}) \quad \text{with} \quad \Delta = \sum_i \partial^2_{x_i}$$

- Fields at positive flow time smeared out with smearing radius $\sqrt{8\tau}$

$\Rightarrow$ Intuition: Regulates divergences

Smearing



[Courtesy of Oliver Witzel]

Applications of the gradient flow

- Inherent smearing removing small-distance fluctuations [Narayanan, Neuberger 2006; Lüscher 2010; ...]
- Gradient-flow scale setting extremely precise and cheap [Lüscher 2010; Borsányi et al. 2012; ...]
- Composite operators do not require renormalization [Lüscher, Weisz 2011]

$\Rightarrow$ Define gradient-flow scheme which is valid both on the lattice and perturbatively:

$$\mathcal{O}_j(x) \text{ divergent} \quad \rightarrow \quad \tilde{\mathcal{O}}_j(\tau, x) \text{ finite}$$

- Consider gradient flow as renormalization group transformation [Carosso, Hasenfratz, Neil 2018; Harlander, **FL**, Neumann 2020; Hasenfratz, Monahan, Rizik, Shindler, Witzel 2022]
- Match to $\overline{\mathrm{MS}}$ scheme with small-flow-time expansion [Lüscher, Weisz 2011; Suzuki 2013; Lüscher 2013]
  - Define the energy-momentum tensor of QCD on the lattice [Suzuki 2013; Makino, Suzuki 2014; Harlander, Kluth, **FL** 2018]

    $\Rightarrow$ Applications to thermodynamics [FlowQCD since 2014]
  - Apply to electroweak Hamiltonians of flavor physics [Suzuki, Taniguchi, Suzuki, Kanaya 2020; Harlander, **FL** 2022]
- ...

Flowed operator product expansion

- Small-flow-time expansion [Lüscher, Weisz 2011] :

$$\tilde{\mathcal{O}}_i(\tau, x) = \sum_j \zeta_{ij}(\tau)\mathcal{O}_j(x) + O(\tau)$$

- Invert to express operators through flowed operators [Suzuki 2013; Lüscher 2013] :

Flowed OPE

$$\Gamma(H_Q) = \sum_i \Gamma_i \langle \mathcal{O}_i \rangle = \sum_{i,j} \Gamma_i \zeta_{ij}^{-1}(\tau)\langle \tilde{\mathcal{O}}_j(\tau)\rangle \equiv \sum_j \tilde{\Gamma}_j(\tau)\langle \tilde{\mathcal{O}}_j(\tau)\rangle$$

- $\Gamma(H_Q)$ defined in regular QCD expressed through finite flowed operators $\tilde{\mathcal{O}}_j(\tau)$
- Gradient-flow definition of $\Gamma(H_Q)$ valid both on the lattice and perturbatively

Calculation of $\Gamma(H_Q)$ in the gradient-flow scheme

$$\Gamma(H_Q) = \sum_i \Gamma_i \langle \mathcal{O}_i \rangle = \sum_{i,j} \Gamma_i \zeta_{ij}^{-1}(\tau) \langle \tilde{\mathcal{O}}_j(\tau) \rangle$$

Perturbative $\Gamma_i \cdot \zeta_{ij}^{-1}(\tau)$:

- Dimensional regularization with $D = 4 - 2\epsilon$
- Finite and scheme independent:
  1. No explicit dependence on μ
  2. No dependence on scheme for γ_5
  3. Independent of evanescent operators

Lattice $\langle \tilde{\mathcal{O}}_j(\tau) \rangle$:

- Lattice spacing a as UV regulator
- Finite for $a \to 0$
- No operator mixing

$\Rightarrow$ Gradient-flow scheme convenient for matching

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- Γ_i: Francesco Moretti's talk
- $\zeta_{ij}^{-1}(\tau)$: this talk
- $\langle \tilde{\mathcal{O}}_j(\tau) \rangle$: Matthew Black's talk

Gradient-flow Lagrangian

- Write Lagrangian for the gradient flow as [Lüscher, Weisz 2011; Lüscher 2013]

$$\mathcal{L} = \mathcal{L}_{\text{QCD}} + \mathcal{L}_B + \mathcal{L}_\chi,$$

$$\mathcal{L}_{\text{QCD}} = \frac{1}{4g^2} F^a_{\mu\nu} F^a_{\mu\nu} + \sum_{f=1}^{n_\mathrm{f}} \bar{\psi}_f (\not{D}^{\mathrm{F}} + m_f) \psi_f + \ldots$$

- Construct flowed Lagrangian using Lagrange multiplier fields $L^a_\mu(\tau, x)$ and $\lambda_f(\tau, x)$:

$$\mathcal{L}_B = -2 \int_0^\infty \mathrm{d}\tau \, \mathrm{Tr} \left[L^a_\mu T^a \left(\partial_\tau B^b_\mu T^b - \mathcal{D}^{bc}_\nu G^c_{\nu\mu} T^b \right) \right], \qquad \partial_\tau B^a_\mu = \mathcal{D}^{ab}_\nu G^b_{\nu\mu}$$

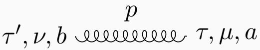
$$\mathcal{L}_\chi = \sum_{f=1}^{n_\mathrm{f}} \int_0^\infty \mathrm{d}\tau \left(\bar{\lambda}_f \left(\partial_\tau - \Delta \right) \chi_f + \bar{\chi}_f \left(\overleftarrow{\partial_\tau} - \overleftarrow{\Delta} \right) \lambda_f \right), \qquad \partial_\tau \chi = \Delta \chi, \quad \partial_\tau \bar{\chi} = \bar{\chi} \overleftarrow{\Delta}$$

$\Rightarrow$ Flow equations automatically satisfied

$\Rightarrow$ QCD Feynman rules + gradient-flow Feynman rules (complete list in [Artz, Harlander, **FL**, Neumann, Prausa 2019])

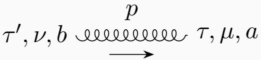
Gradient-flow Feynman rules

- Flowed propagators



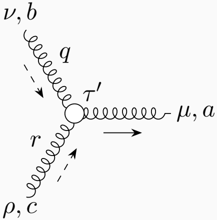
$$= \delta^{ab}\,\frac{1}{p^2}\,\delta_{\mu\nu}\, \mathrm{e}^{-(\tau+\tau')p^2}$$

- Flow lines



$$= \delta^{ab}\,\theta(t-s)\,\delta_{\mu\nu}\, \mathrm{e}^{-(\tau-\tau')p^2}$$

- Flow vertices



$$= -\mathrm{i}gf^{abc}\int_0^\infty \mathrm{d}\tau' \left(\delta_{\nu\rho}(r-q)_\mu + 2\delta_{\mu\nu}q_\rho - 2\delta_{\mu\rho}r_\nu\right)$$

$\Rightarrow$ Can integrate into standard tool chains for perturbative calculations

Operator basis for $\Delta Q = 0$ lifetime differences

$$\Gamma(H_Q) = \sum_i \Gamma_i \langle \mathcal{O}_i \rangle = \sum_{i,j} \Gamma_i \zeta_{ij}^{-1}(\tau) \langle \tilde{\mathcal{O}}_j(\tau) \rangle$$

- Define finite flowed operators:

$$\begin{aligned}
\mathcal{O}_1 &= (\bar{Q}\gamma_\mu(1-\gamma_5)q)(\bar{q}\gamma_\mu(1-\gamma_5)Q) && \Rightarrow && \tilde{\mathcal{O}}_1 = (\bar{\tilde{Q}}\gamma_\mu(1-\gamma_5)\tilde{q})(\bar{\tilde{q}}\gamma_\mu(1-\gamma_5)\tilde{Q}) \\
\mathcal{O}_2 &= (\bar{Q}(1-\gamma_5)q)(\bar{q}(1+\gamma_5)Q) && \Rightarrow && \tilde{\mathcal{O}}_2 = (\bar{\tilde{Q}}(1-\gamma_5)\tilde{q})(\bar{\tilde{q}}(1+\gamma_5)\tilde{Q}) \\
T_1 &= (\bar{Q}\gamma_\mu(1-\gamma_5)T^A q)(\bar{q}\gamma_\mu(1-\gamma_5)T^A Q) && \Rightarrow && \tilde{T}_1 = (\bar{\tilde{Q}}\gamma_\mu(1-\gamma_5)T^A\tilde{q})(\bar{\tilde{q}}\gamma_\mu(1-\gamma_5)T^A\tilde{Q}) \\
T_2 &= (\bar{Q}(1-\gamma_5)T^A q)(\bar{q}\gamma_\mu(1+\gamma_5)T^A Q) && \Rightarrow && \tilde{T}_2 = (\bar{\tilde{Q}}(1-\gamma_5)T^A\tilde{q})(\bar{\tilde{q}}\gamma_\mu(1+\gamma_5)T^A\tilde{Q})
\end{aligned}$$

- Reminder: $\tilde{\mathcal{O}}_i(\tau)$ do not require renormalization
- Compute matching matrix $\zeta_{ij}(\tau)$:

$$\tilde{\mathcal{O}}_i(\tau, x) = \sum_j \zeta_{ij}(\tau)\mathcal{O}_j(x)$$

Method of projectors

- Define projectors [Gorishny, Larin, Tkachov 1983; Gorishny, Larin 1987]

$$P_k[\mathcal{O}_i] \equiv D_k \langle 0|\mathcal{O}_i|k\rangle \stackrel{!}{=} \delta_{ik} + O(\alpha_\mathrm{s})$$

- Apply to small flow-time expansion:

$$P_k[\tilde{\mathcal{O}}_i(\tau)] = \sum_j \zeta_{ij}(\tau) P_k[\mathcal{O}_j]$$

- $\zeta_{ij}(\tau)$ only depend on τ

$\Rightarrow$ Set all other scales to zero

$\Rightarrow$ No perturbative corrections to $P_k[\mathcal{O}_j]$, because all loop integrals are scaleless

"Master formula"

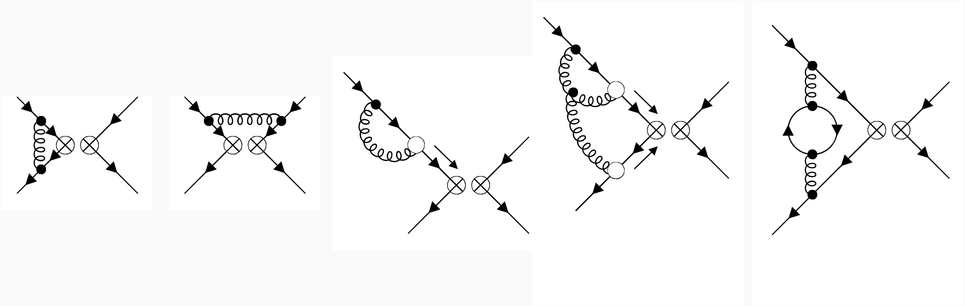
$$\zeta_{ij}(\tau) = P_j[\tilde{\mathcal{O}}_i(\tau)]\Big|_{p=m=0}$$

Calculation of ζ_{ij}^{-1}

- Projector for $\mathcal{O}_1 = (\bar{Q}\gamma_\mu(1-\gamma_5)q)(\bar{q}\gamma_\mu(1-\gamma_5)Q)$:

$$P_1[\mathcal{O}] = \frac{1}{16N_c^2}\,\mathrm{Tr}_{\mathrm{line}\,1}\mathrm{Tr}_{\mathrm{line}\,2}\,\langle 0|\big(Q(1-\gamma_5)\gamma_\nu\bar{q}\big)\big(q(1-\gamma_5)\gamma_\nu\bar{Q}\big)\mathcal{O}|0\rangle\Big|_{p=m=0}$$

- Sample diagrams:



Automatized calculation

- `qgraf` [Nogueira 1991] : Generate Feynman diagrams
- `q2e`/`tapir` [Gerlach, Herren, Lang 2022] and `exp` [Harlander, Seidensticker, Steinhauser 1998; Seidensticker 1999] : Assign diagrams to topologies and prepare `FORM` code
- `FORM` [Vermaseren 2000; Kuipers, Ueda, Vermaseren, Vollinga 2013] : Insert Feynman rules, perform tensor reduction, Dirac traces, and color algebra [van Ritbergen, Schellekens, Vermaseren 1998]
- Generate system of equations employing integration-by-parts-like relations [Tkachov 1981; Chetyrkin, Tkachov 1981] with in-house `Mathematica` code
- `Kira` [Maierhöfer, Usovitsch, Uwer 2017; Klappert, **FL**, Maierhöfer, Usovitsch 2020; **FL**, Usovitsch, Wu 2025] $\oplus$ `FireFly` [Klappert, **FL** 2019; Klappert, Klein, **FL** 2020] : Solve system to express all integrals through six master integrals with Laporta algorithm [Laporta 2000]
- Master integrals computed in [Harlander, Kluth, **FL** 2018]

Results for the matching matrix ζ^{-1}

$$\zeta^{-1}(\tau) = \mathbb{1} + \frac{\alpha_s}{\pi} \begin{pmatrix} 0 & 0 & -\frac{11}{4} - \frac{3}{2} L_{\mu\tau} & 0 \\ 0 & \frac{5}{3} + 2L_{\mu\tau} & 0 & \frac{1}{2} \\ -\frac{11}{18} - \frac{1}{3} L_{\mu\tau} & 0 & \frac{11}{12} + \frac{1}{2} L_{\mu\tau} & 0 \\ 0 & \frac{1}{9} & 0 & \frac{9}{8} - \frac{1}{4} L_{\mu\tau} \end{pmatrix} + \left(\frac{\alpha_s}{\pi}\right)^2 \Big(\dots \Big) + \mathcal{O}(\alpha_s^3)$$

- Normalized by non-singlet axial currents [Borgulat, Harlander, Kohnen, **FL** 2023]
- $\alpha_s = \alpha_s(\mu)$ renormalized in $\overline{\text{MS}}$ scheme
- $L_{\mu\tau} = \ln 2\mu^2\tau + \gamma_E$
- Set $N_c = 3$, $T_R = \frac{1}{2}$

$\Rightarrow$ Ready to be used in Matthew Black's talk

Running along the flow time

- Wilson coefficients and bag parameters depend on renormalization scale μ in $\overline{\text{MS}}$ scheme
- Running determined by anomalous dimension $\gamma(\mu)$:

$$\mu^2 \frac{\mathrm{d}\mathcal{O}(\mu)}{\mathrm{d}\mu^2} = -\gamma(\mu)\mathcal{O}(\mu)$$

- Similarly, dependence on flow time τ in gradient-flow scheme
- Running determined by flowed anomalous dimension [Harlander, **FL**, Neumann 2020] :

$$\tau\partial_\tau \tilde{\mathcal{O}}(\tau) = \tilde{\gamma}(\tau)\tilde{\mathcal{O}}(\tau)$$

with

$$\tilde{\gamma}(\tau) = (\tau\partial_\tau\zeta(\tau))\zeta^{-1}(\tau)$$

⇒ Can evolve the flowed bag parameters along the flow time to stabilize them

Summary and outlook

- Combining perturbative Wilson coefficients and non-perturbative bag parameters from lattice QCD nontrivial
- Gradient-flow scheme convenient because it is valid both on the lattice and perturbatively
- Computed the matching matrix for lifetime differences through NNLO in QCD
- Ready for bag parameters $\Rightarrow$ Matthew Black's talk
- And for checking renormalization scheme independence with Wilson coefficients

  $\Rightarrow$ Francesco Moretti's talk

Gradient flow

- Introduce parameter *flow time* $\tau \geq 0$ [Narayanan, Neuberger 2006; Lüscher 2009; Lüscher 2010]
- *Flowed fields* in $D+1$ dimensions obey differential *flow equations*:

$$\partial_\tau \Phi(\tau, x) = -\left.\frac{\delta S[\phi(x)]}{\delta \phi(x)}\right|_{\Phi(\tau,x)} \sim D_x \Phi(\tau, x) \quad \text{with} \quad \Phi(\tau, x)|_{\tau=0} = \phi(x)$$

- Flow equation drives flowed fields to minimum of action
- Flow equation similar to the heat equation (thermodynamics)

$$\partial_t u(t, \vec{x}) = \alpha \Delta u(t, \vec{x}) \quad \text{with} \quad \Delta = \sum_i \partial_{x_i}^2$$

- Fields at positive flow time smeared out with smearing radius $\sqrt{8t}$

$\Rightarrow$ Intuition: Regulates divergences