Mixing: theory

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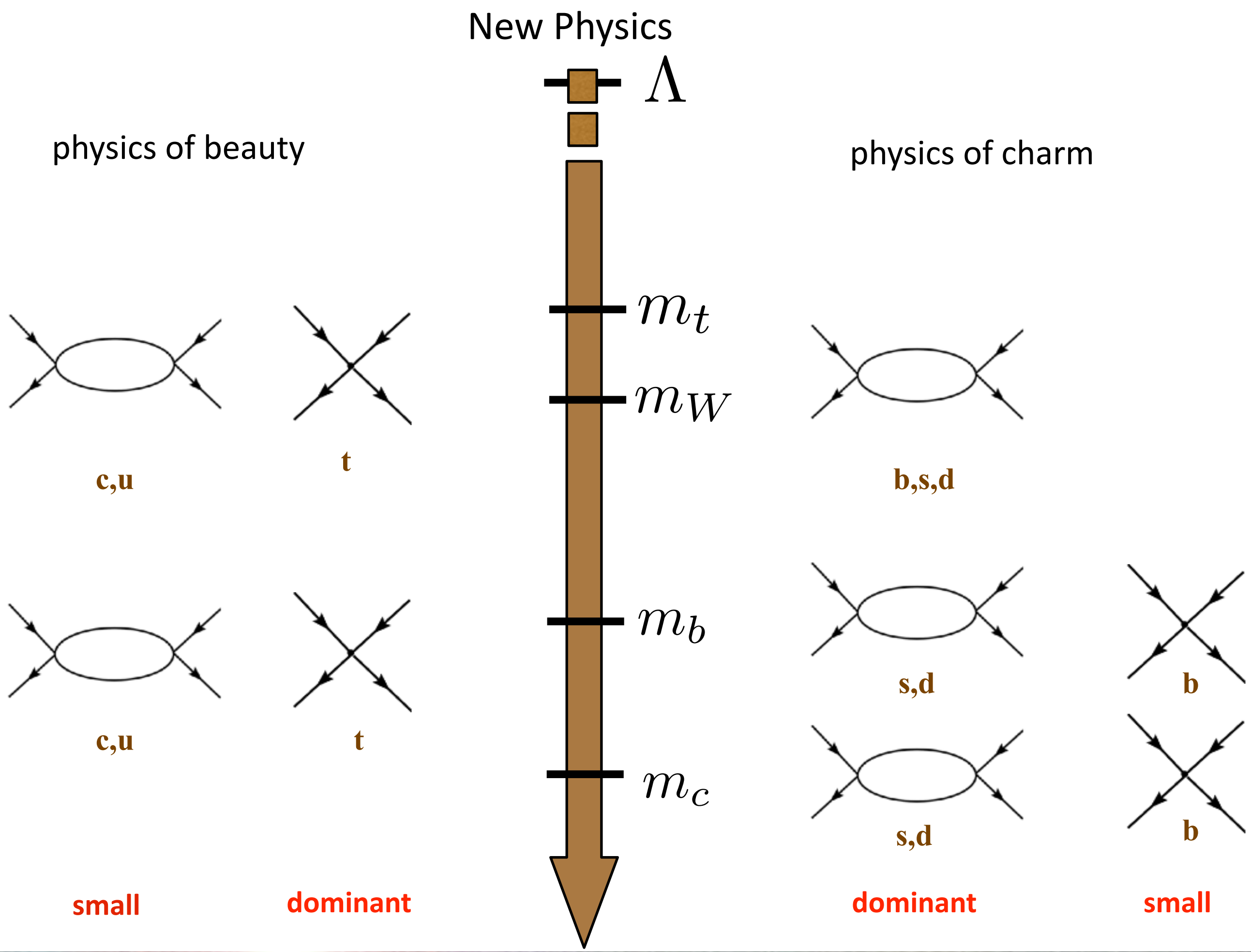
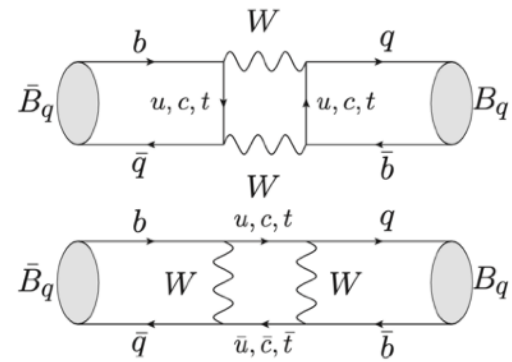
- Main goal: to understand physics at the most fundamental scale
  - it is important to understand relevant energy scales for the problem at hand



- $\Delta b = 2$ transitions: study time development of a B-system

$$i\frac{d}{dt}|B_q(t)\rangle = \left(M - \frac{i}{2}\Gamma\right)|B_q(t)\rangle$$

$$\langle B_q^0|\mathcal{H}|\overline{B_q^0}\rangle = M_{12} - \frac{i}{2}\Gamma_{12} \qquad \langle \overline{B_q^0}|\mathcal{H}|B_q^0\rangle = M_{12}^* - \frac{i}{2}\Gamma_{12}^*$$



  – diagonalize the Hamiltonian to get the mass eigenstates

$$|B_{L,H}\rangle = p|B_q^0\rangle \pm q|\overline{B_q^0}\rangle$$

$$|B_q^0(t)\rangle = \quad g_+(t)|B_q^0\rangle + \frac{q}{p}g_-(t)|\overline{B_q^0}\rangle$$

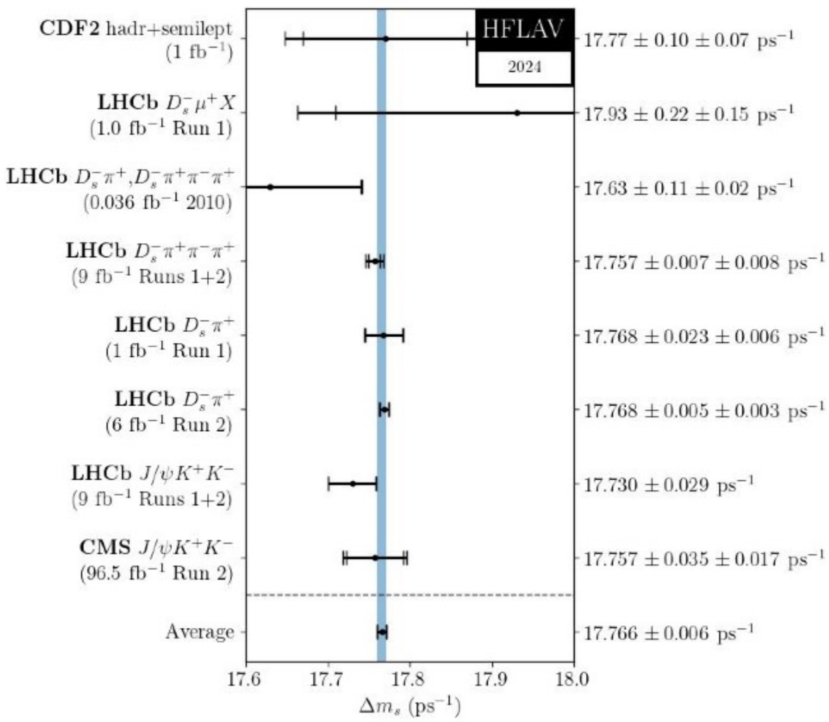
$$|\overline{B_q^0}(t)\rangle = \frac{p}{q}g_+(t)|B_q^0\rangle + \quad g_-(t)|\overline{B_q^0}\rangle$$

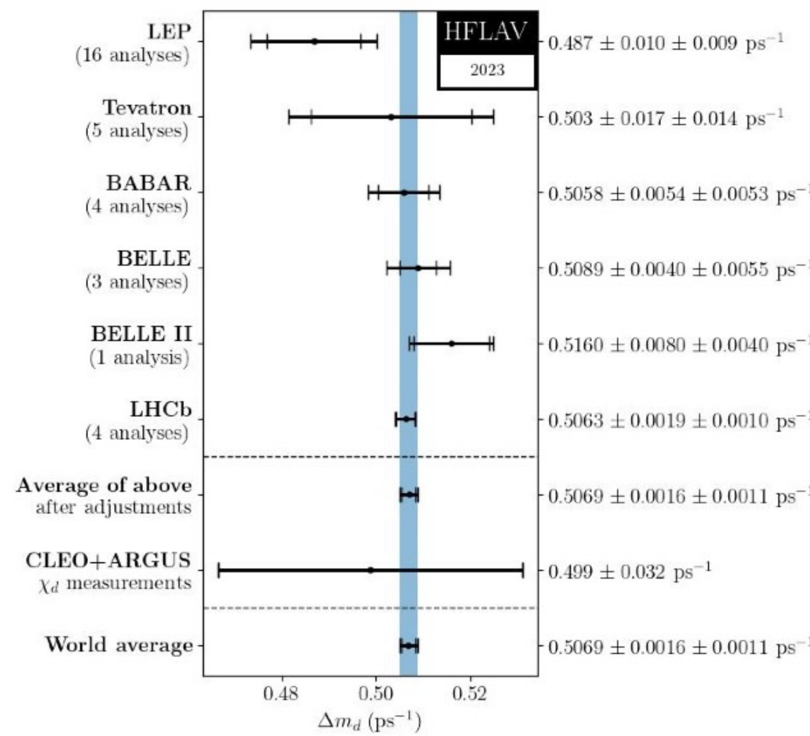
with

$$\begin{aligned} g_+(t) &= e^{-imt}\,e^{-\Gamma t/2}\left[\ \cosh\frac{\Delta\Gamma\,t}{4}\cos\frac{\Delta M\,t}{2} - i\sinh\frac{\Delta\Gamma\,t}{4}\sin\frac{\Delta M\,t}{2}\right], \\ g_-(t) &= e^{-imt}\,e^{-\Gamma t/2}\left[-\sinh\frac{\Delta\Gamma\,t}{4}\cos\frac{\Delta M\,t}{2} + i\cosh\frac{\Delta\Gamma\,t}{4}\sin\frac{\Delta M\,t}{2}\right]. \end{aligned}$$

  – … and define the mass and lifetime differences $x_q = \dfrac{m_H - m_L}{\Gamma_q}, \quad y_q = \dfrac{m_L - m_H}{2\Gamma_q}$

- The experimental values for the mass differences have been measured by many experiments



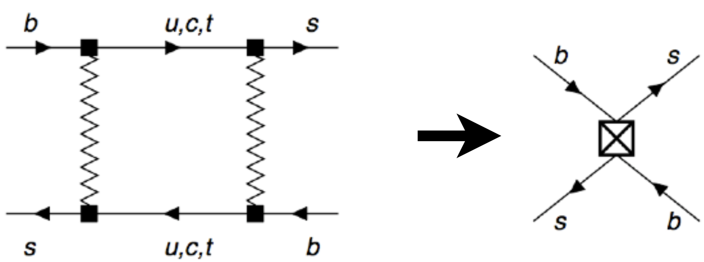


$$\Delta m_s^{\rm exp} = (17.765 \pm 0.006)\ \mathrm{ps}^{-1}$$
$$x_s^{\rm exp} = 26.93 \pm 0.10$$

$$\Delta m_d^{\rm exp} = (0.5069 \pm 0.0019)\ \mathrm{ps}^{-1}$$
$$x_d^{\rm exp} = 0.7697 \pm 0.0035$$

- Both Δm_q and $\Delta\Gamma_q$ can be computed in the $m_b \to \infty$ limit:

For Δm_q:



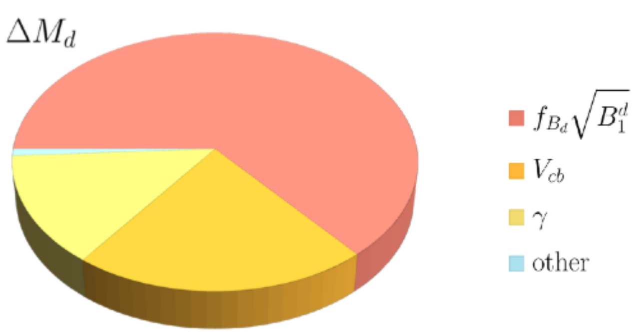
$$\tilde{Q}^q_{6,1} = (\bar{q}^\alpha \gamma_\mu (1-\gamma_5) b^\alpha)(\bar{q}^\beta \gamma^\mu (1-\gamma_5) b^\beta)$$

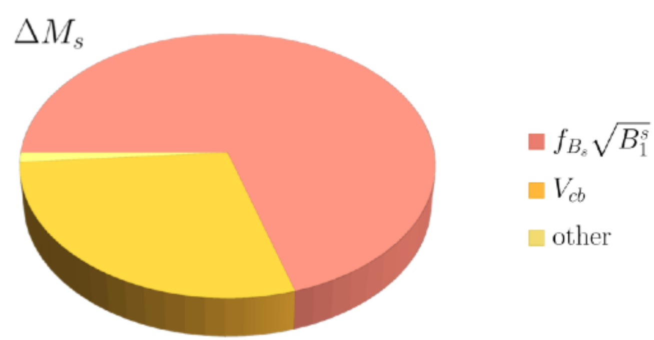
$$\langle \tilde{Q}^q_{6,1} \rangle \equiv \langle \bar{B}_q | \tilde{Q}^q_{6,1} | B_q \rangle = \frac{8}{3} M^2_{B_q} f^2_{B_q} B^q_1(\mu)$$

$$M^q_{12} = \frac{G_F^2}{12\pi^2} \lambda^2_{tq} M^2_W S_0(x_t) \hat{\eta}_B f^2_{B_q} M_{B_q} B^q_1$$

A.Buras, M.Jamin, P. Weisz

Lattice QCD		
$\langle \tilde{Q}_{6,1-5} \rangle_{B_{d,s}}$	2016	FNAL-MILC [270]
$\langle \tilde{Q}_{6,1-5} \rangle_{B_{d,s}}$	2019	HPQCD [271]
HQET sum rule		
$\langle \tilde{Q}_{6,1} \rangle_{B_d}$	2017	Grozin, Klein, Mannel, Pivovarov [272]
$\langle \tilde{Q}_{6,1-5} \rangle_{B_d}$	2017	Kirk, Lenz, Rauh [246]
$\langle \tilde{Q}_{6,1-5} \rangle_{B_s}$	2019	King, Lenz, Rauh [273]





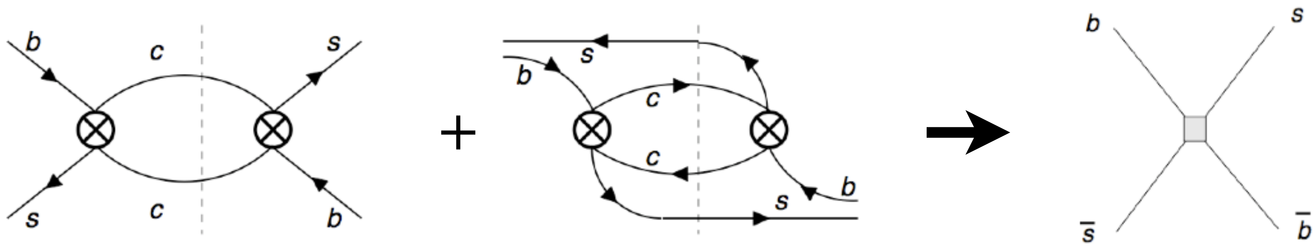
$$\Delta m_d^{\rm th} = (0.535 \pm 0.021)\,{\rm ps}^{-1} \qquad \Delta m_s^{\rm th} = (18.23 \pm 0.63)\,{\rm ps}^{-1}$$

J. Albrecht, F. Bernlochner, A. Lenz, A. Rusov

B-mixing parameters: theoretical predictions

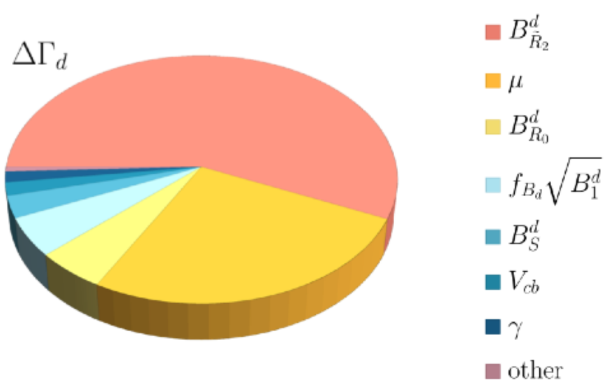
- Both Δm_q and $\Delta\Gamma_q$ can be computed in the $m_b \to \infty$ limit:

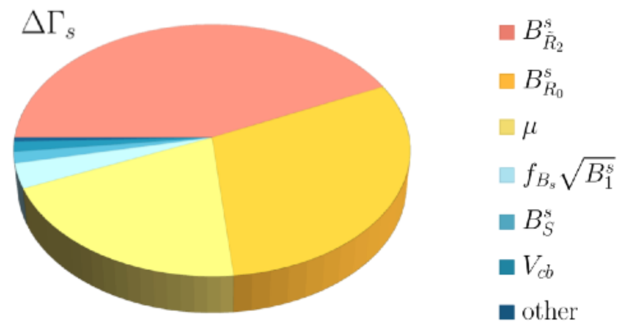
For $\Delta\Gamma_q$:



$$\Gamma_{21}(B_s) = \sum_k \frac{C_k(\mu)}{m_b^k} \langle B_s | \mathcal{O}_k^{\Delta B=2}(\mu) | B_s \rangle.$$

A. Lenz, U. Nierste





$$\Delta\Gamma_d^{\rm th} = (2.7 \pm 0.4) \times 10^{-3} {\rm ps}^{-1}$$

$$\Delta\Gamma_s^{\rm th} = (9.1 \pm 1.5) \times 10^{-2} {\rm ps}^{-1}$$

J. Albrecht, F. Bernlochner, A. Lenz, A. Rusov

- $\Delta\Gamma$s: a calculation yields:

$$\Gamma_{21}(B_s) = - \frac{G_F^2 m_b^2}{12\pi(2M_{B_s})}(V_{cb}^* V_{cs})^2 \left[[F(z)+P(z)] \langle Q \rangle + [F_S(z)+P_S(z)] \langle Q_S \rangle + \delta_{1/m} + \delta_{1/m^2} \right]$$

WC (incl. pQCD corr): Beneke et al, Ciuchini et al

★ ... with operators

$$Q = (\bar b_i s_i)_{V-A} (\bar b_j s_j)_{V-A}, \ Q_S = (\bar b_i s_i)_{S-P} (\bar b_j s_j)_{S-P}$$
$$\tilde Q = (\bar b_i s_j)_{V-A} (\bar b_j s_i)_{V-A}, \ \tilde Q_S = (\bar b_i s_j)_{S-P} (\bar b_j s_i)_{S-P}$$

$$\langle Q \rangle = 2\frac{1+N_c}{N_c} f_{B_s}^2 M_{B_s}^2 B$$
$$\langle Q_S \rangle = \frac{1-2N_c}{N_c} \frac{M_{B_s}^4}{(m_b+m_s)^2} f_{B_s}^2 B_S$$

★ ... so the result (up to $1/m_b^2$) is:

$$\begin{aligned} \Delta\Gamma_{B_s} = & \Big[0.0005B + 0.1732B_s + 0.0024B_1 - 0.0237B_2 - 0.0024B_3 - 0.0436B_4 \\ + & \ 2\times10^{-5}\alpha_1 + 4\times10^{-5}\alpha_2 + 4\times10^{-5}\alpha_3 + 0.0009\alpha_4 - 0.0007\alpha_5 \\ + & \ 0.0002\beta_1 - 0.0002\beta_2 + 6\times10^{-5}\beta_3 - 6\times10^{-5}\beta_4 - 1\times10^{-5}\beta_5 \\ - & \ 1\times10^{-5}\beta_6 + 1\times10^{-5}\beta_7 + 1\times10^{-5}\beta_8 \Big] \quad (\mathrm{ps}^{-1}). \end{aligned}$$

A.Badin, F. Gabbiani, A.A.P.
Phys. Lett. B653, 230 (2007)

- $\Delta c = 2$ transitions: study time development of a D-system

$$i\frac{d}{dt}|D(t)\rangle = \left(M - \frac{i}{2}\Gamma\right)|D(t)\rangle$$

$$\langle D^0|\mathcal{H}|\overline{D^0}\rangle = M_{12} - \frac{i}{2}\Gamma_{12} \qquad \langle \overline{D^0}|\mathcal{H}|D^0\rangle = M_{12}^* - \frac{i}{2}\Gamma_{12}^*$$

  – diagonalize the Hamiltonian and define the mass an lifetime differences

$$x_D = \frac{M_2 - M_1}{\Gamma_D}, \; y_D = \frac{\Gamma_2 - \Gamma_1}{2\Gamma_D}$$

  – … which can be calculated as real and imaginary parts of a correlation function

$$y_{\rm D} = \frac{1}{2M_{\rm D}\Gamma_{\rm D}} \,{\rm Im}\, \langle \overline{D^0}|\, i \int {\rm d}^4x\, T\left\{\mathcal{H}_w^{|\Delta C|=1}(x)\, \mathcal{H}_w^{|\Delta C|=1}(0)\right\}|D^0\rangle$$

bi-local time-ordered product

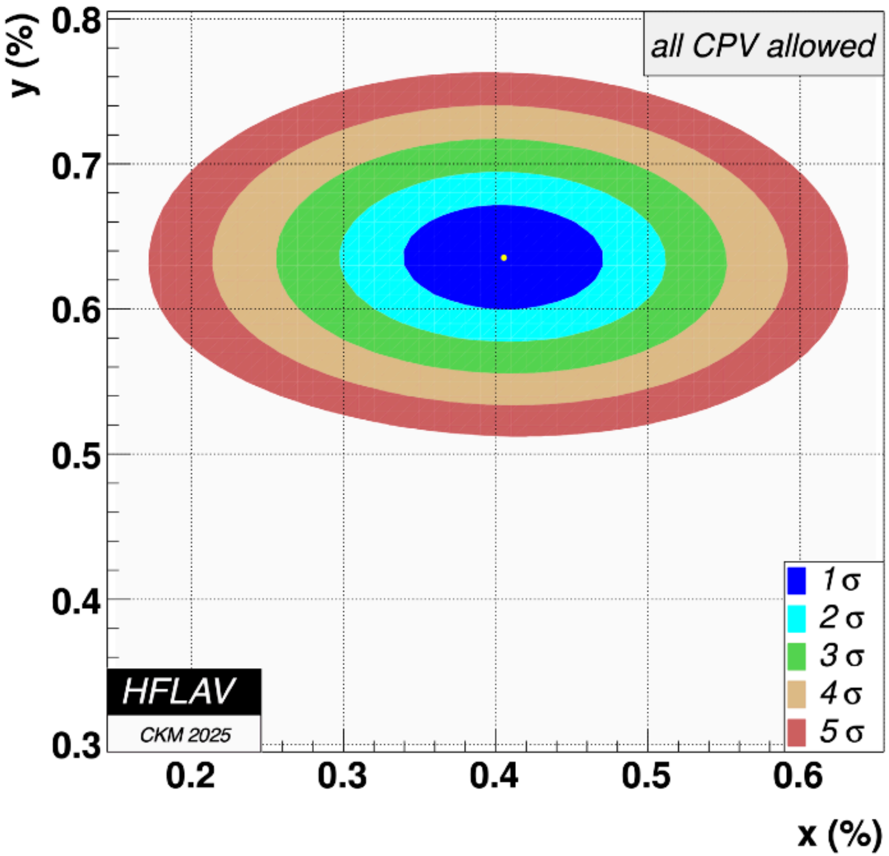
$$x_{\rm D} = \frac{1}{2M_{\rm D}\Gamma_{\rm D}} \,{\rm Re}\, \left[2\langle \overline{D^0}|H^{|\Delta C|=2}\,|D^0\rangle + \langle \overline{D^0}|\, i \int {\rm d}^4x\, T\left\{\mathcal{H}_w^{|\Delta C|=1}(x)\, \mathcal{H}_w^{|\Delta C|=1}(0)\right\}|D^0\rangle\right]$$

local operator
(b-quark, NP, …): small?

bi-local time-ordered product

D-mixing: experimental measurements

- The experimental values for the mixing parameters have been measured



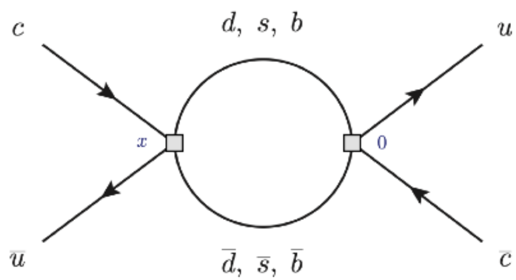
$$x_D^{\rm exp} = (0.405 \pm 0.043)\%$$
$$y_D^{\rm exp} = (0.636 \pm 0.024)\%$$

Note that $x_D \leq y_D$ in charm, whereas in beauty it is always $x_q \gg y_q$

- Lowest order contribution: dimension-6 operators

E. Golowich and A.A.P.
Phys. Lett. B625 (2005) 53

$$x_D^{(6)} = -\frac{G_F^2}{2}\frac{m_c^2}{2M_D\Gamma_D}\xi_s^2 x_s^4 \frac{1}{2\pi^2}\left[C_1^2\langle O_{V-A}\rangle + 2\left(C_1^2 - 2C_1C_2 - 3C_2^2\right)\langle O_{S-P}\rangle\right]$$



with the operators
$$O_{V-A} = \left(\bar{c}\gamma^\mu(1-\gamma^5)u\right)\left(\bar{c}\gamma_\mu(1-\gamma^5)u\right),$$
$$O_{S-P} = \left(\bar{c}\left(1-\gamma^5\right)u\right)\left(\bar{c}\left(1-\gamma^5\right)u\right).$$

$$\langle\overline{D^0}|O_{V-A}|D^0\rangle = \frac{8}{3}f_D^2M_D^2B_{\rm D} \quad \text{and} \quad \langle\overline{D^0}|O_{S-P}|D^0\rangle = -\frac{5}{3}f_D^2M_D^2\overline{B}_{\rm D}^{(S)}$$

- at the lowest order in $1/m_c$, x_D and y_D depend on high powers of $x_s = m_s/m_c$

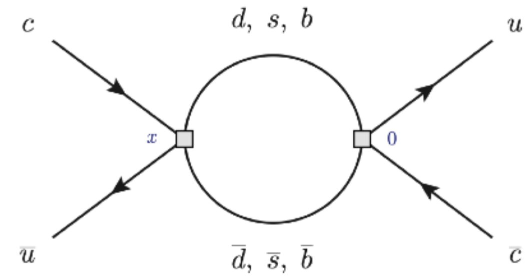
$$x_D^{(6)} = -\frac{G_F^2m_c^2f_D^2M_D}{3\pi^2\Gamma_D}\ \xi_s^2\ \boxed{x_s^4}\ \left[C_1^2B_{\rm D} - \frac{5}{4}(C_2^2 - 2C_1C_2 - 3C_2^2)\overline{B}_{\rm D}^{(S)}\right]$$

$$y_D^{(6)} = -\frac{G_F^2m_c^2f_D^2M_D}{3\pi\Gamma_D}\ \xi_s^2\ \boxed{x_s^6}\ \left(C_1^2 - 2C_1C_2 - 3C_2^2\right)\left[B_{\rm D} - \frac{5}{2}\overline{B}_{\rm D}^{(S)}\right]$$

Numerically,
$$x_D^{LO} \equiv x_D^{(6)} = (0.37 \pm 0.02)\times 10^{-5}$$
$$x_D^{exp} = (0.407 \pm 0.044)\%$$

What is going on?

- Lowest order contribution: dimension-6 operators

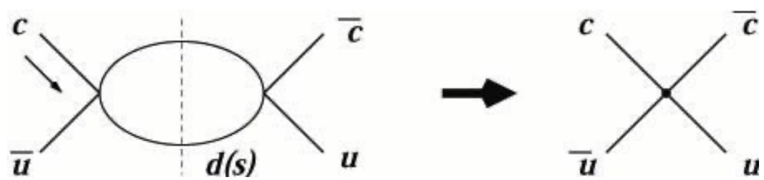


- Note that $1/m_c$ is not small, while factors of m_s make the result small
  - keep $V_{ub} \neq 0$, so the leading SU(3)-breaking contribution is suppressed by $\lambda_b^2 \sim \lambda^{10}$ (small)
  - perturbative QCD corrections? Should the renormalization scales for the $s\bar{s}$, $d\bar{d}$, and $s\bar{d}/d\bar{s}$ intermediate scales be the same?

$$\Gamma_{12} = \sum_{q_1 q_2 = ss, sd, dd} \Gamma_3^{q_1 q_2}(\mu_1^{q_1 q_2}, \mu_2^{q_1 q_2}) \langle Q \rangle (\mu_2^{q_1 q_2}) \frac{1}{m_c^3} + \ldots$$

A. Lenz, M. L. Piscopo, C. Vlahos
Phys. Rev. D 102, 093002 (2020)

- In the heavy-quark limit $m_c \to \infty$ we have $m_c \gg \sum m_{\text{intermediate quarks}}$, so $E_{\text{released}} \sim m_c$
  - the situation is similar to B-physics, where it is "short-distance" dominated
  - one can consistently compute pQCD and 1/m corrections



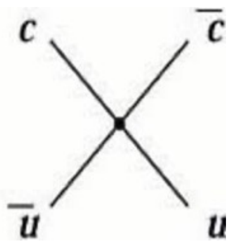
- But wait, m_c is NOT infinitely large! What happens for finite m_c???
  - what other phenomena can change chirality of an s-quark?

H. Georgi, …
I. Bigi, N. Uraltsev

M. Bobrowski, A. Lenz, J. Riedl,
J. Rohrwild, JHEP 1003 (2010) 009

Guestimate: $x \sim y \sim 10^{-3}$?

- What should we expect from the general arguments: group theory?
  - note that $\langle D^0 | H_w H_w | \overline{D}^0 \rangle \;\Rightarrow\; \langle 0 | D H_w H_w D | 0 \rangle$ is an $SU(3)_F$ singlet



with $D \to D_i$ that belongs to **3** of $SU(3)_F$

$H_w = H_k^{ij}$ is $(\bar{q}_i c)(\bar{q}_j q_k)$ or $3 \times \bar{3} \times \bar{3} = \overline{15} + 6 + \bar{3} + \bar{3}$

  - the quark operators can be written in terms of their $SU(3)_F$ properties

$$O_6 = (\bar{s}c)(\bar{u}d) - (\bar{u}c)(\bar{s}d) + s_1(\bar{d}c)(\bar{u}d) - s_1(\bar{u}c)(\bar{d}d) - s_1(\bar{s}c)(\bar{u}s) + s_1(\bar{u}c)(\bar{s}s) - s_1^2(\bar{d}c)(\bar{u}s) + s_1^2(\bar{u}c)(\bar{d}s)$$

$$O_{\overline{15}} = (\bar{s}c)(\bar{u}d) + (\bar{u}c)(\bar{s}d) + s_1(\bar{d}c)(\bar{u}d) + s_1(\bar{u}c)(\bar{d}d) - s_1(\bar{s}c)(\bar{u}s) - s_1(\bar{u}c)(\bar{s}s) - s_1^2(\bar{d}c)(\bar{u}s) - s_1^2(\bar{u}c)(\bar{d}s)$$

  - introduce SU(3) breaking via $M_j^i = \mathrm{diag}(m_u, m_d, m_s) \to 1 + 8$
  - all non-zero matrix elements built out of D_i, H_k^{ij} and M_j^i must be $SU(3)_F$ singlets

- What should we expect from the general arguments: group theory?
  - note that $\langle D^0 | H_w H_w | \overline{D}^0 \rangle \;\Rightarrow\; \langle 0 | D H_w H_w D | 0 \rangle$ is an $SU(3)_F$ singlet

    also note that $D_i D_j$ is symmetric $\Rightarrow$belongs to 6 of $SU(3)_F$

    then, the product $H_w H_w \Rightarrow O_{\overline{60}} + O_{42} + O_{15'}$
  - there is no $\bar{6}$ in the decomposition of $H_w H_w \Rightarrow$ no $SU(3)_F$ singlet can be formed

    D-mixing is prohibited by $SU(3)_F$ symmetry
  - consider a single insertion of $M^i_j \;\Rightarrow D_6 M$ transforms as $6 \times 8 = 24 + \overline{15} + 6 + \overline{3}$

    $\Rightarrow$ still no $SU(3)_F$ singlet can be formed

    no D-mixing at the first order of $SU(3)_F$ breaking
  - consider double insertion of $M^i_j \;\Rightarrow D_6 MM$ transforms as

$$6 \times (8 \times 8)_S = (\boxed{60 + \overline{42}} + 24 + \overline{15} + \overline{15} + 6) + (24 + 15 + 6 + \overline{3}) + 6$$

D-mixing occurs at the <u>second</u> order of $SU(3)_F$ breaking

Falk, Grossman, Ligeti, AAP, PRD65, 054034, (2002)

Aside: how to flip chirality in QCD

- Chirality flip plays a very important role in D-mixing
  - it seems like the "culprit" in $x_D^{\exp} \gg x_D^{\text{theory}}$ is the small value of m_s
- Are there any other ways to flip quark's chirality in QCD?
  - magnetic moment of quarks $\langle \bar{q}\sigma G q\rangle = \langle \overline{q_R}\sigma G q_L\rangle + \langle \overline{q_L}\sigma G q_R\rangle$
  - the vacuum of QCD is not empty; it is filled with quark-antiquark pairs that form chiral condensate $\langle \bar{q}q\rangle \neq 0$

H. Georgi, …
I. Bigi, N. Uraltsev

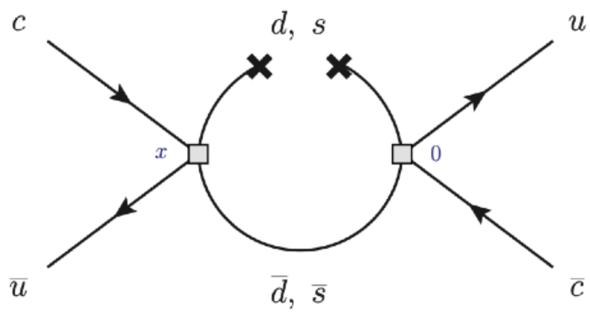
Operator Product Expansion: if $\langle \bar{s}s\rangle_0/m_c^3 \gg x_s = m_s/m_c$ then some higher order terms in $1/m_c$ would be (numerically) more important than the leading ones!

Let's find those terms

- Next-to-lowest order contribution (in m_s): dimension-9 operators
  - recall that we only look pick the terms with lower powers of m_s

$$T^{(9)}_{qq'} = \frac{iG_F^2}{2} \int d^4x \left(\bar{c}^i(x)\Gamma_\mu \overline{q'^j(x)\right)\left(\bar{q}^k(x)\Gamma^\mu u^l(x)\right)\left(\bar{c}^m(0)\Gamma_\nu q^n(0)\right)\left(\overline{q}'^p}(0)\Gamma^\nu u^q(0)\right).$$

  - take matrix elements, to evaluate them assume factorization ansatze



$$x_D^{(9)} \sim \sum_{q=s,d} C_q \langle D^0| \left(\bar{c}\Gamma_1 u\right)\left(\bar{q}q\right)\left(\bar{c}\Gamma_2 u\right)|\overline{D}^0\rangle$$

$$\langle D^0| \left(\bar{c}\Gamma_1 u\right)\left(\bar{q}q\right)\left(\bar{c}\Gamma_2 u\right)|\overline{D}^0\rangle \sim \langle \bar{q}q\rangle \langle D^0| \left(\bar{c}\Gamma_1 u\right)\left(\bar{c}\Gamma_2 u\right)|\overline{D}^0\rangle$$

  - drop terms proportional to $x_d = m_d/m_c$

$$x_D^{(9)} = \frac{G_F^2}{3}\frac{\xi_s^2 m_c^2}{2M_D\Gamma_D}\frac{\langle \bar{s}s\rangle_0}{m_c^3}\boxed{x_s^3}\left[\left(C_1^2 - 2C_1C_2 - 3C_2^2\right)\left(\langle O_{V-A}\rangle + 4\langle O_{S-P}\rangle\right)\right]$$

We traded more powers of $1/m_c$ for fewer powers of x_s

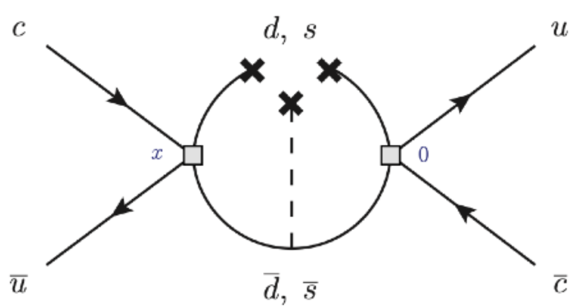
This contribution should be numerically more important, provided that $\langle \bar{s}s\rangle_0/m_c^3 \gg x_s$

- Next-to-lowest order contribution (in m_s): dimension-11 operators
  - recall that we only look pick the terms with lower powers of m_s, expand the propoagator

$$S(x,0) = S_0(x,0) + S_1(x,0) + \dots$$

$$T^{(9)}_{qq'} = \frac{iG_F^2}{2} \int d^4x \left(\overline{c}^i(x)\Gamma_\mu q'^j(x)\right)\left(\overline{q}^k(x)\Gamma^\mu u^l(x)\right)\left(\overline{c}^m(0)\Gamma_\nu q^n(0)\right)\left(\overline{q'}^p(0)\Gamma^\nu u^q(0)\right)$$

  - the expansion of nonlocal product also contains the mixed quark gluon condensate
    - it can also flip chirality $\langle \overline{q}\sigma G q\rangle = \langle \overline{q_R}\sigma G q_L\rangle + \langle \overline{q_L}\sigma G q_R\rangle$



where $\langle \overline{q} i g\sigma G q\rangle \equiv \langle \overline{q} i g \sigma^{\mu\nu} G^a_{\mu\nu} \frac{\lambda^a}{2} q\rangle,$

$$x_D^{(11)} \sim \sum_{q=s,d} C_{\overline{q}Gq} \langle D^0|\left(\overline{c}\Gamma_1 u\right)\left(\overline{q}\sigma G q\right)\left(\overline{c}\Gamma_2 u\right)|\overline{D}^0\rangle$$

$$\sim \langle \overline{q}\sigma G q\rangle \sum_{q=s,d} C_{\overline{q}Gq} \langle D^0|\left(\overline{c}\Gamma_1 u\right)\left(\overline{c}\Gamma_2 u\right)|\overline{D}^0\rangle$$

L. Dulibic, B. Melic, AAP, arXiv:2508.16337 [hep-ph]

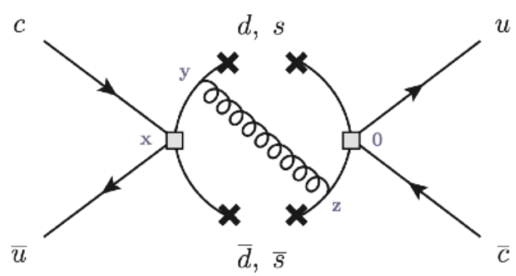
  - we can see that it is also proportional to the terms with fewer powers of m_s

$$x_D^{(11)} = \frac{G_F^2}{2}\frac{\xi_s^2 m_c^2}{2M_D\Gamma_D}\left(\frac{\lambda_q^2}{m_c^2}\frac{\langle \overline{s}s\rangle_0}{m_c^3}\boxed{x_s^3} + \frac{8}{9}\pi\alpha_s^{NP}\frac{\langle \overline{s}s\rangle_0^2 - \langle \overline{d}d\rangle_0^2}{m_c^6}\boxed{x_s^2}\right)\frac{1}{6}\left(C_1^2\langle O_{V-A}\rangle + 8\langle O_{S-P}\rangle(C_1^2 + 4C_1C_2 + 6C_2^2)\right)$$

We traded more powers of $1/m_c$ for fewer powers of x_s

- Next-to-next-to-lowest order contribution (in m_s): dimension-12 operators
  - recall that we only look pick the terms with lower powers of m_s

$$T_{4q} = i \int d^4x\, d^4y\, d^4z\, (-ig)^2 \left(\overline{q}_1^m(y)\gamma^\rho (T^a)^{mn} S_{q_1}^{ni}(y,x)\Gamma^\mu c^j(x)\right)\left(\overline{u}^k(x)\Gamma_\mu q_3^l(x)\right)$$
$$\times\, S_G(y,z)\left(\overline{u}^q(0)\Gamma_\nu q_2^r(0)\right)\left(\overline{q}_4^o(z)\gamma_\rho (T^a)^{op} S_{q_4}^{ps}(z,0)\Gamma^\nu c^t(0)\right)$$



Note: an example!

Note: as the value of the four-quark condensate is noit known well, employ double-factorization:

(1) factorize four quark condensate from the operator

(2) factorize four-quark condensate into a product of two two-quark condensate

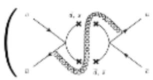
  - we can see that it is, again, proportional to the terms with fewer powers of m_s

Numerically largest?

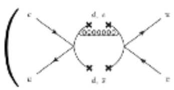
$$x_D^{(12)} = \frac{G_F^2}{2}\frac{\xi_s^2 m_c^2}{2M_D\Gamma_D}\frac{\langle \overline{s}s\rangle_0^2}{m_c^6}\pi\alpha_s(\mu)\boxed{x_s^2}$$
$$\times \frac{1}{108}\Big[3(-9C_1^2 + 172C_1C_2 + 411C_2^2)\langle O_{V-A}\rangle$$
$$+\; 8(13C_1^2 + 452C_1C_2 + 495C_2^2)\langle O_{S-P}\rangle\Big]$$

Higher-order operators: dimension-12

- Expectation: the fewest powers of x_s from dimension-12 operators

$x_D^{(12)}$ ()

$$\approx -2\times10^{-7} = \frac{G_F^2}{4}\xi^2\frac{m_c^2}{2\Gamma_D M_D}\frac{4\pi\alpha_s}{108}\frac{\langle\bar{s}s\rangle_0^2}{m_c^6}x_s^2 \times\left[\left(7C_1^2-10C_1C_2-15C_2^2\right)\langle O_{V-A}\rangle+24(C_1-3C_2)(C_1+C_2)\langle O_{S-P}\rangle\right].$$

$x_D^{(12)}$ ()

$$\approx +2\times10^{-6} = -\frac{G_F^2}{2}\xi^2\frac{m_c^2}{2\Gamma_D M_D}\frac{4\pi\alpha_s}{108}\frac{\langle\bar{s}s\rangle_0^2}{m_c^6}x_s^2 \times\left[\left(27C_1^2-112C_1C_2-168C_2^2\right)\langle O_{V-A}\rangle + 8\left(22C_1^2-42C_1C_2+63C_2^2\right)\langle O_{S-P}\rangle\right],$$

$x_D^{(12)}$ ()

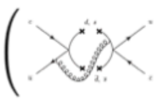
$$\approx -7\times10^{-7} = \frac{7G_F^2}{4}\xi^2\frac{m_c^2}{2\Gamma_D M_D}\frac{4\pi\alpha_s}{108}\frac{\langle\bar{s}s\rangle_0^2}{m_c^6}x_s^2 \times\left[-7C_1C_2\langle O_{V-A}\rangle+8\left(2C_1^2-C_1C_2\right)\langle O_{S-P}\rangle\right],$$

$x_D^{(12)}$ ()

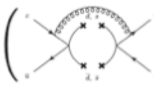
$$\approx +2\times10^{-7} = \frac{G_F^2}{2}\xi^2\frac{m_c^2}{2\Gamma_D M_D}\frac{4\pi\alpha_s}{108}\frac{\langle\bar{s}s\rangle_0^2}{m_c^6}x_s^2 \times\left[\left(3C_1^2+C_1C_2\right)\langle O_{V-A}\rangle+28\left(C_1^2+3C_1C_2\right)\langle O_{S-P}\rangle\right],$$

$x_D^{(12)}$ ()

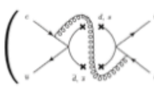
$$\approx -6\times10^{-7} = \frac{G_F^2}{2}\xi^2\frac{m_c^2}{2\Gamma_D M_D}\frac{4\pi\alpha_s}{108}\frac{\langle\bar{s}s\rangle_0^2}{m_c^6}x_s^2 \times\left[C_1\left(16C_1-C_2\right)\langle O_{V-A}\rangle+24C_1\left(2C_1-C_2\right)\langle O_{S-P}\rangle\right],$$

$x_D^{(12)}$ ()

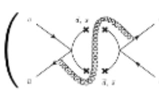
$$\approx -3\times10^{-9} = \frac{G_F^2}{8}\xi^2\frac{m_c^2}{2\Gamma_D M_D}\frac{4\pi\alpha_s}{108}\frac{\langle\bar{s}s\rangle_0^2}{m_c^6}x_s^2 \times\left[-C_1\left(7C_1+59C_2\right)\langle O_{V-A}\rangle+24C_1\left(C_1+C_2\right)\langle O_{S-P}\rangle\right],$$

$x_D^{(12)}$ ()

$$\approx -2\times10^{-7} = \frac{G_F^2}{4}\xi^2\frac{m_c^2}{2\Gamma_D M_D}\frac{4\pi\alpha_s}{108}\frac{\langle\bar{s}s\rangle_0^2}{m_c^6}x_s^2 \times\left[\left(16C_1^2+14C_1C_2+21C_2^2\right)\langle O_{V-A}\rangle+24\left(2C_1^2+2C_1C_2+3C_2^2\right)\langle O_{S-P}\rangle\right]$$

$x_D^{(12)}$ ()

$$\approx +2\times10^{-7} = -\frac{G_F^2}{8}\xi^2\frac{m_c^2}{2\Gamma_D M_D}\frac{4\pi\alpha_s}{108}\frac{\langle\bar{s}s\rangle_0^2}{m_c^6}x_s^2 \times\left[\left(19C_1^2-18C_1C_2-27C_2^2\right)\langle O_{V-A}\rangle+56(C_1-3C_2)(C_1+C_2)\langle O_{S-P}\rangle\right],$$

$x_D^{(12)}$ ()

$$\approx -2\times10^{-7} = \frac{G_F^2}{4}\xi^2\frac{m_c^2}{2\Gamma_D M_D}\frac{4\pi\alpha_s}{108}\frac{\langle\bar{s}s\rangle_0^2}{m_c^6}x_s^2 \times\left[\left(7C_1^2-10C_1C_2-15C_2^2\right)\langle O_{V-A}\rangle+24(C_1-3C_2)(C_1+C_2)\langle O_{S-P}\rangle\right].$$

  - nine unique diagrams, the rest can be related to the diagrams above

Results: putting all together

- What should we expect from this calculation?
  - recall that we only look pick the terms with fewest powers of m_s at each order

Contribution	Result for x_D $[10^{-5}]$
$x_D^{(9)}$	0.99 ± 0.13
$x_D^{(11)}$	0.051 ± 0.009
$x_D^{(12)}$	0.24 ± 0.06
Total $= x_D^{\text{cond}}$	1.27 ± 0.18
x_D^{LO}	0.37 ± 0.02
x_D^{NLO} [14]	-0.45 ± 0.02

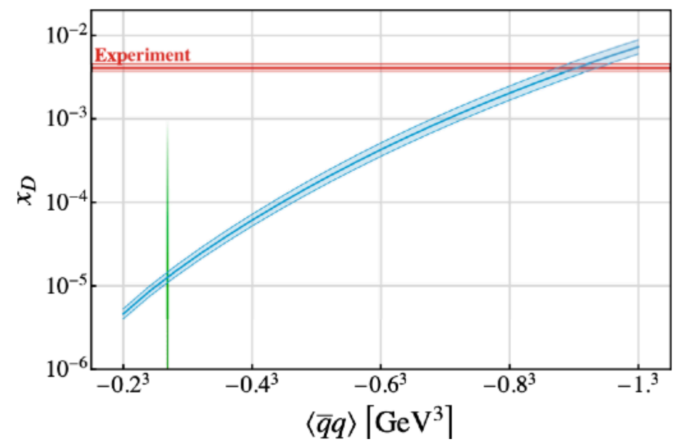
$$x_D^{exp} = (0.407 \pm 0.044)\%$$

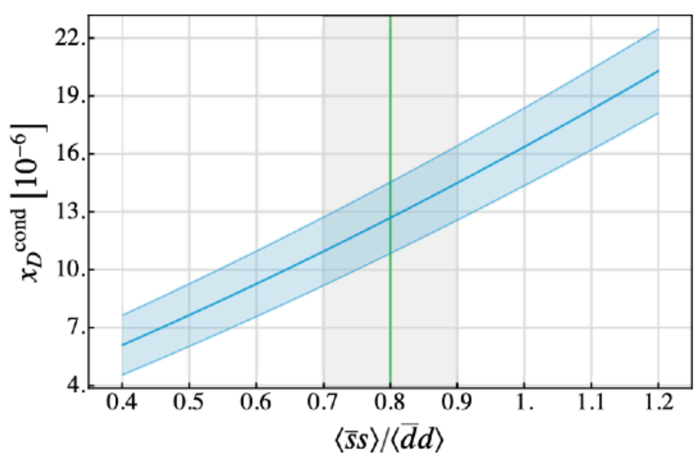
  - theoretical errors are evaluated by adding parameter uncertainties in quadrature

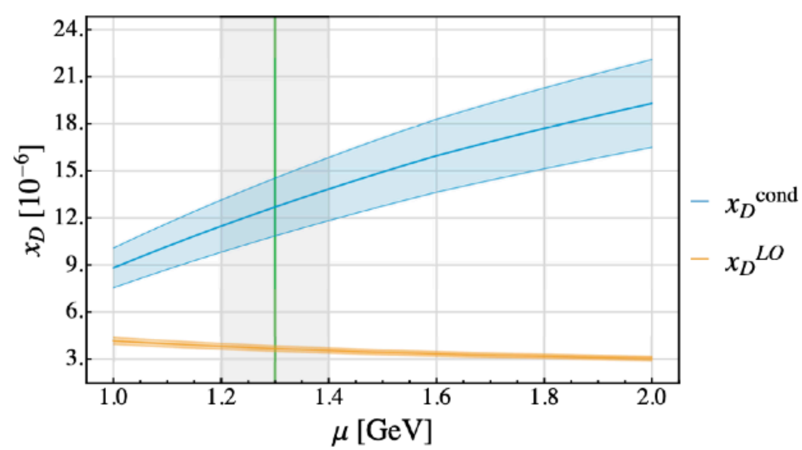
    we checked that other schemes do not chnage the result much
  - the results are smaller than expected before as it is no longer the case that $\langle \bar{s}s \rangle_0 / m_c^3 \gg x_s$
  - it seems that theoretical predictions are two orders of magnitude smaller than exp, which might be due to the use factorization approximation, parameter values, or (even) a possible New Physics contribution

- We can evaluate the uncertainties of the final result
  - note that the value of the quark condensate $\langle \bar{s}s\rangle_0 \ll 1$ GeV³

Parameter	Numerical value
f_D [42]	$0.212\,\mathrm{GeV}$
$\langle O_{V-A}\rangle(3\,\mathrm{GeV})$ [43]	$(0.322 \pm 0.022)\ \mathrm{GeV}^4$
$\langle O_{S-P}\rangle(3\,\mathrm{GeV})$ [43]	$(-0.624 \pm 0.007)\ \mathrm{GeV}^4$
$\langle \bar{q}q\rangle_0(1.3\,\mathrm{GeV})$	$(-268.5 \pm 1.3\,\mathrm{MeV})^3$







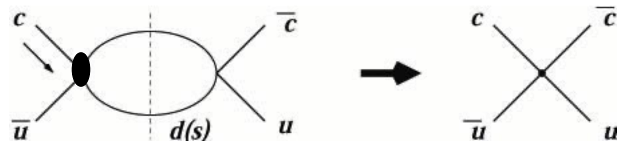
New Physics and the lifetime difference

- (Heavy) New Physics contributions are local at $\mu = m_c$

$$\left(M - \frac{i}{2}\Gamma\right)_{ij} = m_D^{(0)}\delta_{ij} + \frac{1}{2m_D}\left\langle D_i^0 \left| H_W^{\Delta C=2} \right| D_j^0 \right\rangle + \frac{1}{2m_D}\sum_I \frac{\left\langle D_i^0 \left| H_W^{\Delta C=1} \right| I \right\rangle \left\langle I \left| H_W^{\Delta C=1} \right| D_j^0 \right\rangle}{m_D^2 - m_I^2 + i\varepsilon}$$

- There is a (theoretical) limit where y is dominated by New Physics

Amplitude $A_n = \left\langle D^0 \left| \left(H_{SM}^{\Delta C=1} + H_{NP}^{\Delta C=1}\right) \right| n \right\rangle \equiv A_n^{SM} + A_n^{NP}$



Suppose $\left|A_n^{NP}\right| / \left|A_n^{SM}\right| \sim O(\text{exp. uncertainty}) \leq 10\%$

Example: $y = \frac{1}{2\Gamma}\sum_n \rho_n \left(\bar{A}_n^{SM} + \bar{A}_n^{NP}\right)\left(A_n^{SM} + A_n^{NP}\right) \approx \frac{1}{2\Gamma}\sum_n \rho_n \bar{A}_n^{SM} A_n^{SM} + \boxed{\frac{1}{2\Gamma}\sum_n \rho_n \left(\bar{A}_n^{SM} A_n^{NP} + \bar{A}_n^{NP} A_n^{SM}\right)}$

phase space

Zero in the SU(3) limit

Falk, Grossman, Ligeti, and A.A.P.
Phys.Rev. D65, 054034, 2002
2nd order effect!!!

Can be significant?

E.Golowich, S. Pakvasa and A.A.P.
Phys. Rev. Lett. 98, 181801, 2007

- New Physics in $\Delta B = 2$ transitions: study time development
  - neutral baryons can mix if baryon number is broken: $\mathcal{L} = m_n \bar{n} n + \frac{\delta m}{2} n^T \mathcal{C} n$

$$i \frac{\partial}{\partial t} \begin{bmatrix} n \\ \bar{n} \end{bmatrix} = \mathcal{M} \begin{bmatrix} n \\ \bar{n} \end{bmatrix}, \ \mathcal{M}_B = \begin{pmatrix} m_n - \vec{\mu}_n \cdot \vec{B} - i\Gamma/2 & \delta m \\ \delta m & m_n + \vec{\mu}_n \cdot \vec{B} - i\Gamma/2 \end{pmatrix}.$$

  - diagonalize the Hamiltonian and look for the oscillations

$$P(n \to \bar{n})(t) = \sin^2(2\theta) \sin^2 \left[(\Delta E)t/2\right]^2 e^{-\Gamma t}$$

$$= \frac{(\delta m)^2}{(\vec{\mu}_n \cdot \vec{B})^2 + (\delta m)^2} \sin^2 \left[\left(\sqrt{\left(\vec{\mu}_n \cdot \vec{B}\right)^2 + (\delta m)^2} \right) t \right]^2 e^{-\Gamma t}$$

  - theoretical expectations: the Lagrangian $\mathcal{L}_{\text{eff}} \sim -\frac{1}{\Lambda^5} (dud)^2$

$$\delta m = \langle n | \int d^3 x \mathcal{H}_{\text{eff}} | \bar{n} \rangle \simeq \Lambda_{\text{QCD}}^6 / \Lambda^5$$

This implies that one pays a steep price of Λ^{-10} suppression for probing high scales Λ!

Can the BNV scale be different for heavy quarks? Maybe… Can one measure $\Lambda_b \overline{\Lambda_b}$ oscillations? Probably not…

- Computation of charm mixing amplitudes is a difficult task
  - no obvious model-independent/perturbative technique
  - SU(3)/flavor flow fits need theory input/better exp data
  - need better theory calculation in "inclusive" techniques
- Why is that?
  - no dominant heavy dof, as in beauty decays
  - charm quark is not very heavy
  - "exclucive" techniques need to fit over large number of decays, AND cannot use current experimental data on D-decays: cancellations complicate predictions
  - "inclusive" techniques depend on large number of matrix elements; factorization might affect the predictions, AND $\langle \bar{s}s \rangle_0 / m_c^3 \gg x_s$ is not true
  - upcoming lattice calculation?

L. Dulibic, B. Melic, AAP, arXiv:2508.16337 [hep-ph]

M. Di Carlo, F. Erben and M. T. Hansen

- Philosophy: if one fits all possible exclusive nonleptonic decays of the D^0 and use it to predict the lifetime in mixing, does it constitute a prediction?

``Charm physics''
A.A.P., *Eur. Phys. J. ST* 233 (2024) 2, 439-456
D. Friday, E. Gersabeck, A. Lenz, M.L. Piscopo, 2506.15584

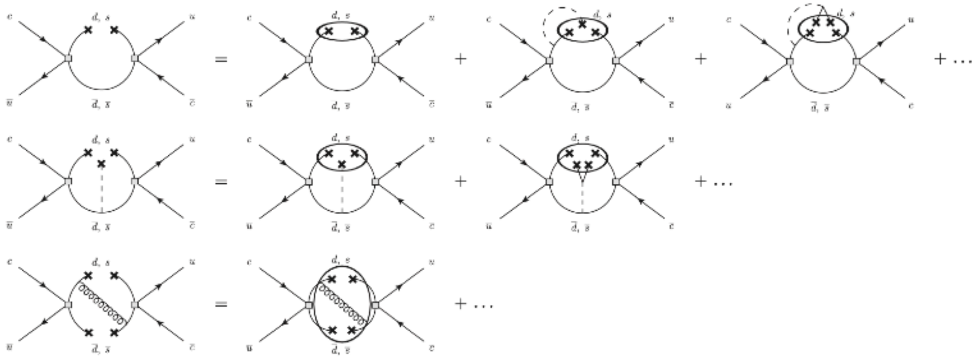
Things to take home

➤ More philosophy: mass difference in $D^0\overline{D^0}$ mixing

Theory ✗ **Experiment** ✗ Not a very interesting case…	**Theory** ✗ **Experiment** ✓ SM wins again?
Theory ✓ **Experiment** ✗ SM wins again!	**Theory** ✓ **Experiment** ✓ New Physics!


EXTRA!

Expansions



$$S_1(x,0)^{jp} = -\frac{i}{8\pi^2}(\widetilde{G}_{\alpha\beta})^{jp} x^\alpha \gamma^\beta \gamma^5 \frac{m_{q'}}{\sqrt{-x^2}} K_1(m_{q'}\sqrt{-x^2})$$
$$+ \frac{1}{16\pi^2}(G_{\alpha\beta})^{jp}\sigma^{\alpha\beta} m_{q'} K_0(m_{q'}\sqrt{-x^2}).$$

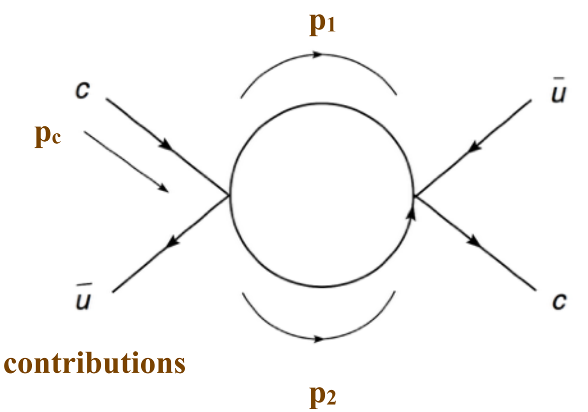
$$\langle \overline{q}^i(x) q^j(0)\rangle = \delta^{ij}\frac{\langle \overline{q}q\rangle_0}{12}\left[\delta_{\alpha\beta}\left(1+\frac{x^2}{8}\lambda_q^2+\ldots\right)\right.$$
$$\left. + i\not{x}_{\beta\alpha}\left(\frac{m}{4}+\frac{x^2}{4}\left(\frac{m\lambda_q^2}{12}-\frac{2}{81}\pi\alpha_s^{NP}\frac{\langle\overline{q}q\rangle_0^2}{\langle\overline{q}q\rangle_0}\right)+\ldots\right)\right]$$

$$\langle \overline{q}^a_\alpha(x) G^{cd}_{\mu\nu}(0) q^b_\beta(0)\rangle = \frac{1}{384}\left(\delta^{bd}\delta^{ac}-\frac{1}{3}\delta^{cd}\delta^{ab}\right) \qquad (4$$
$$\times\left[\left(\sigma_{\mu\nu}+m\left(i\sigma_{\mu\nu}\not{x}+\gamma_\mu x_\nu-\gamma_\nu x_\mu\right)\right)_{\beta\alpha}\lambda_q^2\langle\overline{q}q\rangle_0+\frac{i}{2}\left(\not{x}\sigma_{\mu\nu}\right)_{\beta\alpha}\frac{16}{9}\pi\alpha_s^{\rm NP}\langle\overline{q}q\rangle_0^2+\ldots\right]$$

Inclusive approach to mixing: quark-hadron duality

★ How can one tell that a process is dominated by long-distance or short-distance?

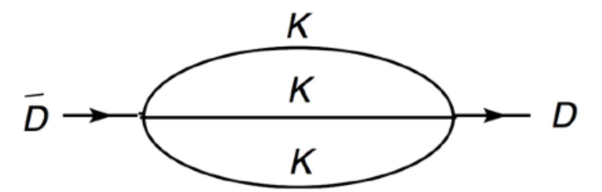
★ Let's look at how the momentum is routed in a leading-order diagram
- injected momentum is $p_c \sim m_c$
- thus, $p_1 \sim p_2 \sim m_c/2 \sim O(\Lambda_{QCD})$?



Still OK with OPE, signals large nonperturbative contributions

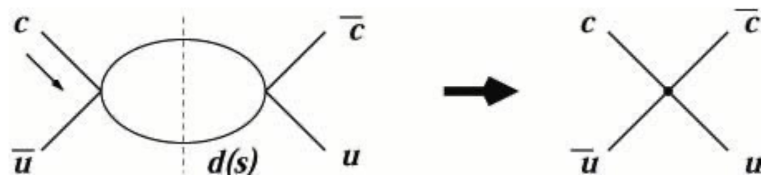
★ For a particular example of the lifetime difference, have hadronic intermediate states

-let's use an example of KKK intermediate state
- in this example, $E_{released} \sim m_D - 3\ m_K \sim O(\Lambda_{QCD})$



★ Similar threshold effects exist in B-mixing calculations
- but $m_b \gg \sum m_{intermediate\ quarks}$, so $E_{released} \sim m_b$ (almost) always
- quark-hadron duality takes care of the rest!

- Lowest order contribution: dimension-6 operators



… as can be seen from a "straightforward computation"…

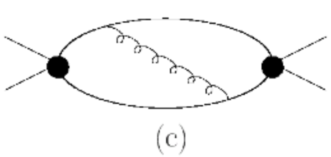
$$y_{\rm LO}^{(z^3)} = \frac{G_F^2 m_c^2 f_D^2 M_D}{3\pi\Gamma_D}\, \xi_s^2\, z^3 \left(C_2^2 - 2C_1C_2 - 3C_1^2\right)\left[B_{\rm D} - \frac{5}{2}\overline{B}_{\rm D}^{(S)}\right] \propto m_s^6\ \Lambda^{-6}$$

$$x_{\rm LO}^{(z^2)} = \frac{G_F^2 m_c^2 f_D^2 M_D}{3\pi^2\Gamma_D}\, \xi_s^2\, z^2 \left[C_2^2 B_{\rm D} - \frac{5}{4}\left(C_2^2 - 2C_1C_2 - 3C_1^2\right)\overline{B}_{\rm D}^{(S)}\right] \propto m_s^4\ \Lambda^{-4}$$

… $x_{LO} \gg y_{LO}$!!!

with $\left\langle D^0 \left| \overline{u}\,\Gamma_\mu c\ \overline{u}\Gamma^\mu c \right| D^0 \right\rangle = \frac{1+N_C}{N_C}\frac{4F_D^2 m_D^2}{2\,m_D} B_D$, *etc.*

Notice, however, that at NLO in QCD $(x_{NLO}, y_{NLO}) \gg (x_{LO}, y_{LO})$:



Example of NLO contribution

$$y_{\rm NLO}^{(2)} = \frac{G_F^2 m_c^2 f_D^2 M_D}{3\pi\Gamma_D}\, \xi_s^2\, \frac{\alpha_s}{4\pi}\, z^2 \Bigg(B_{\rm D}\left[-\left(\frac{77}{6} - \frac{8\pi^2}{9}\right) C_2^2 + 14\ C_1C_2 + 8\ C_1^2\right] - \frac{5}{2}\overline{B}_{\rm D}^{(S)}\left[\left(\frac{8\pi^2}{9} - \frac{25}{3}\right) C_2^2 + 20\ C_1C_2 + 32\ C_1^2\right]\Bigg) ,$$

$x_{NLO} \sim y_{NLO}$!

Similar for x (trust me!)

E. Golowich and A.A.P.
Phys. Lett. B625 (2005) 53

Exclusive approach to mixing: use data?

★ LD calculation: saturate the correlator by hadronic states, e.g.

$$y = \frac{1}{2\Gamma} \sum_n \rho_n \left[\langle D^0 | H_W^{\Delta C=1} | n \rangle \langle n | H_W^{\Delta C=1} | \overline{D}^0 \rangle + \langle \overline{D}^0 | H_W^{\Delta C=1} | n \rangle \langle n | H_W^{\Delta C=1} | D^0 \rangle \right]$$

… with n being all states to which D^0 and $\overline{D}^0$ can decay. Consider ππ, πK, KK intermediate states as an example…

J. Donoghue et. al.
L. Wolfenstein
P. Colangelo et. al.

H.Y. Cheng and C. Chiang

$$\begin{aligned} y_2 \quad &= \quad Br(D^0 \to K^+K^-) + Br(D^0 \to \pi^+\pi^-) \\ &- \quad 2\cos\delta \sqrt{Br(D^0 \to K^+\pi^-) Br(D^0 \to \pi^+K^-)} \end{aligned}$$

cancellation expected

If every Br is known up to O(1%) ➡ the result is expected to be O(1%)!

The result here is a series of large numbers with alternating signs, <u>SU(3) forces 0</u>

If experimental data on Br is used, are we <u>only sensitive to exit. uncertainties</u>?

★ Need to "repackage" the analysis: look at complete multiplet contribution

$$y = \sum_{F_R} y_{F,R}\, Br(D^0 \to F_R) = \sum_{F_R} y_{F,R} \frac{1}{\Gamma} \sum_{n \in F_R} \Gamma(D^0 \to n)$$

Falk, Grossman, Ligeti, Nir. A.A.P.
Phys.Rev. D69, 114021, 2004
Falk, Grossman, Ligeti, and A.A.P.
Phys.Rev. D65, 054034, 2002

Exclusive approach to mixing: use data!

★ What if we insist on using experimental data anyway?

★ Ex., one can employ Factorizaton-Assisted Topological Amplitudes

in units of 10^{-3}

Modes	$\mathcal{B}$(exp)	$\mathcal{B}$(FAT)	Modes	$\mathcal{B}$(exp)	$\mathcal{B}$(FAT)	Modes	$\mathcal{B}$(exp)	$\mathcal{B}$(FAT)
$\pi^0\overline{K}^0$	24.0 ± 0.8	24.2 ± 0.8	$\pi^0\overline{K}^{*0}$	37.5 ± 2.9	35.9 ± 2.2	$\overline{K}^0\rho^0$	$12.8^{+1.4}_{-1.6}$	13.5 ± 1.4
π^+K^-	39.3 ± 0.4	39.2 ± 0.4	π^+K^{*-}	54.3 ± 4.4	62.5 ± 2.7	$K^-\rho^+$	111.0 ± 9.0	105.0 ± 5.2
$\eta\overline{K}^0$	9.70 ± 0.6	9.6 ± 0.6	$\eta\overline{K}^{*0}$	9.6 ± 3.0	6.1 ± 1.0	$\overline{K}^0\omega$	22.2 ± 1.2	22.3 ± 1.1
$\eta'\overline{K}^0$	19.0 ± 1.0	19.5 ± 1.0	$\eta'\overline{K}^{*0}$	< 1.10	0.19 ± 0.01	$\overline{K}^0\phi$	$8.47^{+0.66}_{-0.34}$	8.2 ± 0.6
$\pi^+\pi^-$	1.421 ± 0.025	1.44 ± 0.02	$\pi^+\rho^-$	5.09 ± 0.34	4.5 ± 0.2	$\pi^-\rho^+$	10.0 ± 0.6	9.2 ± 0.3
K^+K^-	4.01 ± 0.07	4.05 ± 0.07	K^+K^{*-}	1.62 ± 0.15	1.8 ± 0.1	K^-K^{*+}	4.50 ± 0.30	4.3 ± 0.2
$K^0\overline{K}^0$	0.36 ± 0.08	0.29 ± 0.07	$K^0\overline{K}^{*0}$	0.18 ± 0.04	0.19 ± 0.03	$\overline{K}^0K^{*0}$	0.21 ± 0.04	0.19 ± 0.03
$\pi^0\eta$	0.69 ± 0.07	0.74 ± 0.03	$\eta\rho^0$		1.4 ± 0.2	$\pi^0\omega$	0.117 ± 0.035	0.10 ± 0.03
$\pi^0\eta'$	0.91 ± 0.14	1.08 ± 0.05	$\eta'\rho^0$		0.25 ± 0.01	$\pi^0\phi$	1.35 ± 0.10	1.4 ± 0.1
$\eta\eta$	1.70 ± 0.20	1.86 ± 0.06	$\eta\omega$	2.21 ± 0.23	2.0 ± 0.1	$\eta\phi$	0.14 ± 0.05	0.18 ± 0.04
$\eta\eta'$	1.07 ± 0.26	1.05 ± 0.08	$\eta'\omega$		0.044 ± 0.004			
$\pi^0\pi^0$	0.826 ± 0.035	0.78 ± 0.03	$\pi^0\rho^0$	3.82 ± 0.29	4.1 ± 0.2			
π^0K^0		0.069 ± 0.002	π^0K^{*0}		0.103 ± 0.006	$K^0\rho^0$		0.039 ± 0.004
π^-K^+	0.133 ± 0.009	0.133 ± 0.001	π^-K^{*+}	$0.345^{+0.180}_{-0.102}$	0.40 ± 0.02	$K^+\rho^-$		0.144 ± 0.009
ηK^0		0.027 ± 0.002	ηK^{*0}		0.017 ± 0.003	$K^0\omega$		0.064 ± 0.003
$\eta'K^0$		0.056 ± 0.003	$\eta'K^{*0}$		0.00055 ± 0.00004	$K^0\phi$		0.024 ± 0.002

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★ … but it appears to yield a smaller result, $y_{PP+PV} = (0.21 \pm 0.07)\%$,