

$|V_{cb}|$ and $|V_{ub}|$ at Belle II

Munira Khan on behalf of the Belle II collaboration



Belle II detector

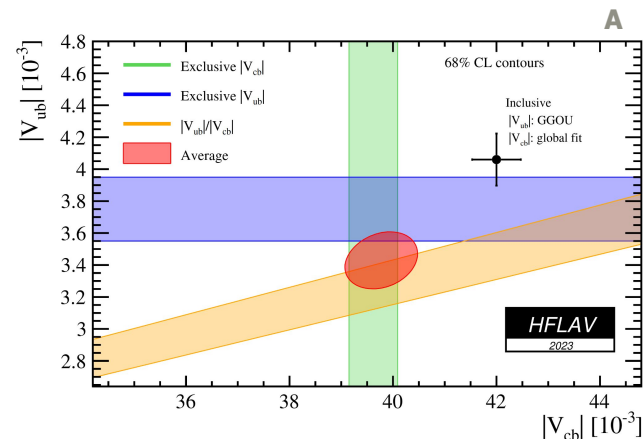
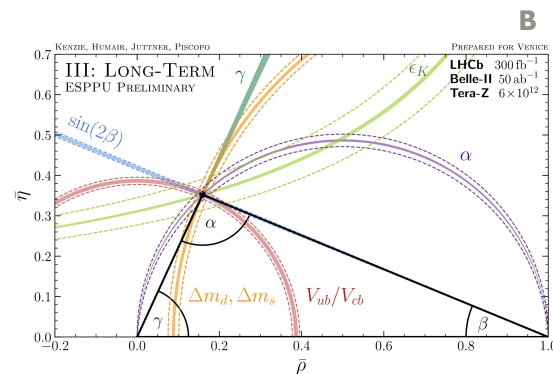
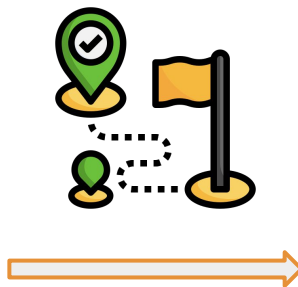
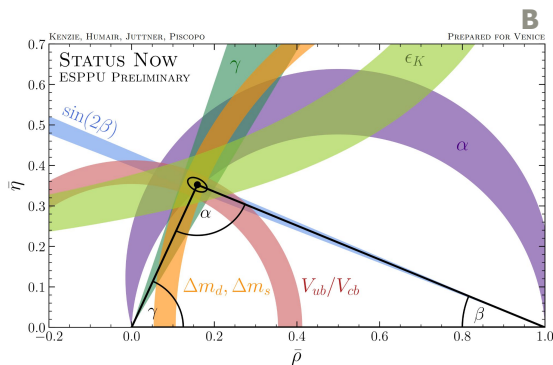
SuperKEKB
(HER + LER)



CKM matrix elements

- Precision measurement of CKM matrix elements are important inputs to SM and enable ways to test it
- Reducing the tension of inclusive vs exclusive determinations serves the scope of ESPP

→ Measurements of (semi-)leptonic decays at Belle II



Experimental site

SuperKEKB

B-factory experiment in Tsukuba, Japan

Asymmetric e^+e^- collider at $Y(4S)$ resonance

Luminosity record of $5.1 \times 10^{34} \text{ cm}^{-2} \text{ s}^{-1}$

Run 1 (2019 – 2022): 424 fb^{-1} of data, 365 fb^{-1} at $Y(4S)$

Run 2 (2024 – ongoing): 150 fb^{-1} of data

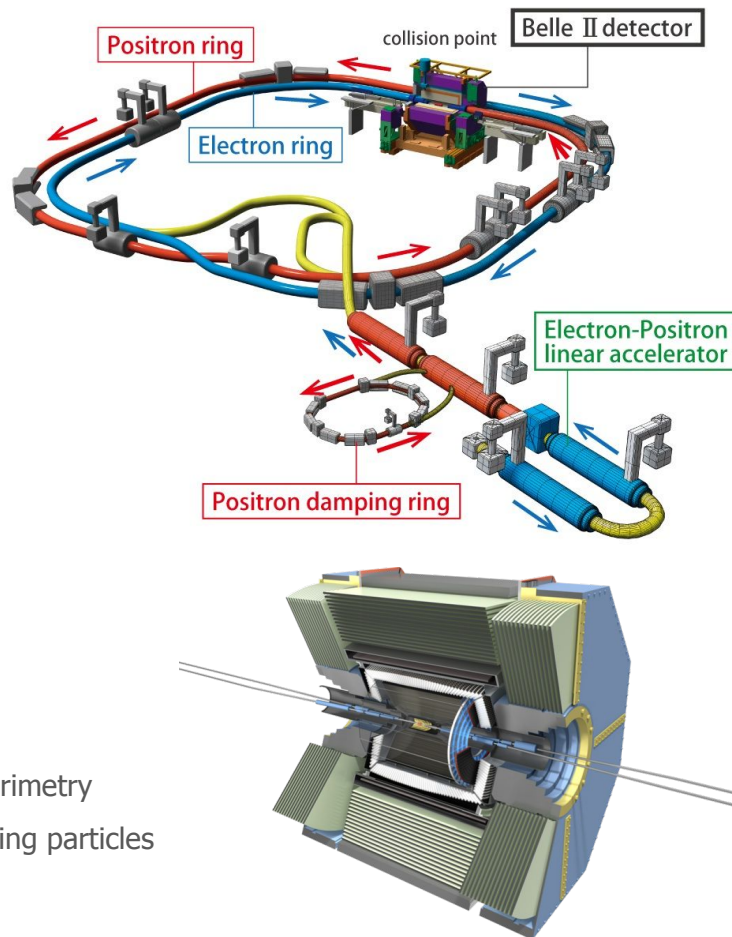
$\sigma(b\bar{b}) \sim 1.1 \text{ nb}$ BB events

$\sigma(q\bar{q}) \sim 3.7 \text{ nb}$ ($q = u, d, s, c$) continuum events

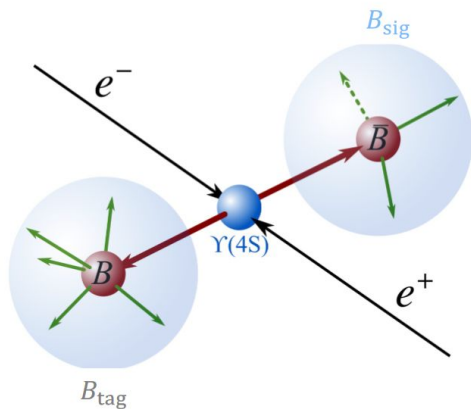
Belle II

- Hermetic & dedicated sub-detectors for tracking, PID and calorimetry
- Well-suited to inclusive measurements and analyses with missing particles

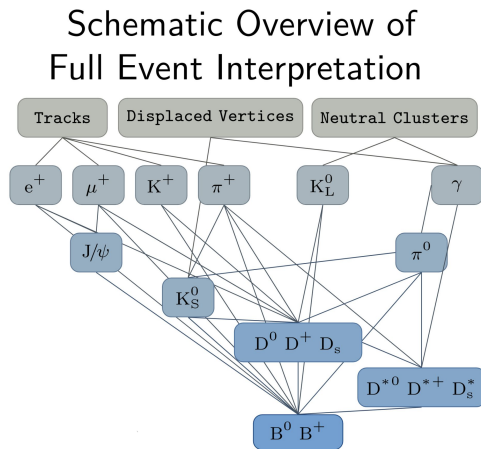
munira.khan@uni-bonn.de



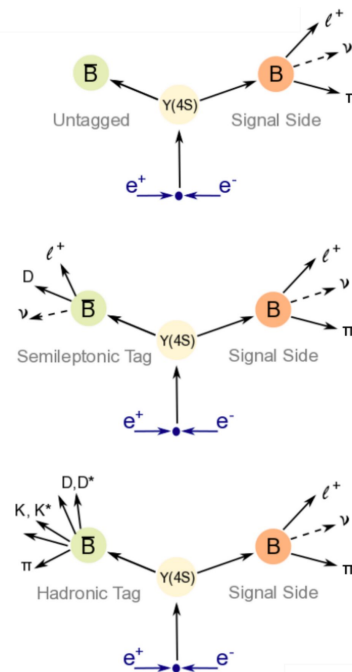
Events kinematics and tagging at Belle II



$$e^+e^- \rightarrow \Upsilon(4S) \rightarrow B_{\text{sig}}B_{\text{tag}}$$

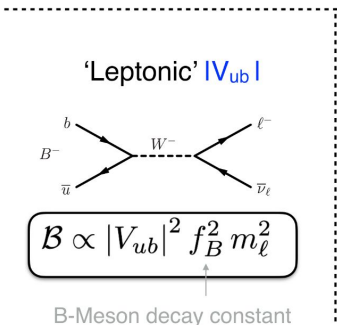
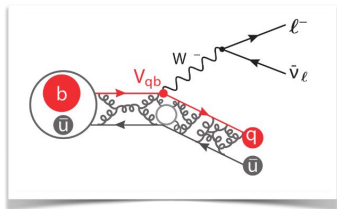


Comput Softw Big Sci 3, 6 (2019)



Strategy for event tagging

Inclusive vs Exclusive $|V_{qb}|$ determinations



Inclusive $|V_{ub}|$

$$\bar{B} \rightarrow X_u \ell \bar{\nu}_\ell$$

+ Fermi Motion / Shape Function

Inclusive $|V_{cb}|$

$$\bar{B} \rightarrow X_c \ell \bar{\nu}_\ell$$

Operator Product Expansion

$$\mathcal{B} = |V_{qb}|^2 \left[\Gamma(b \rightarrow q \ell \bar{\nu}_\ell) + 1/m_{c,b} + \alpha_s + \dots \right]$$

Exclusive $|V_{ub}|$

$$\bar{B} \rightarrow \pi \ell \bar{\nu}_\ell, \Lambda_b \rightarrow p \mu \bar{\nu}_\mu$$

Exclusive $|V_{cb}|$

$$\bar{B} \rightarrow D \ell \bar{\nu}_\ell, \bar{B} \rightarrow D^* \ell \bar{\nu}_\ell$$

$$\mathcal{B} \propto |V_{cb}|^2 f^2$$

Form Factors

$$\langle B | H_\mu | P \rangle = (p + p')_\mu f_+$$

→ Long standing discrepancy between inclusive and exclusive determinations

Measurements at Belle II

$|V_{cb}|$ Exclusive

- $B \rightarrow D \ell \nu$ (untagged)

arXiv:2506.15256

- $B \rightarrow D^* \ell \nu$ (untagged)

Phys.Rev.D 108 (2023) 9, 9

$$|V_{cb}| = (40.13 \pm 0.27 \pm 0.93 \pm 0.58) \times 10^{-3}$$

$|V_{cb}|$ Inclusive

- $B \rightarrow X_c \ell \nu$ (q^2 moments, had. tagged)

Phys.Rev.D 107 (2023) 7, 072002

J. High Energy Phys. 10 (2022) 068

- $B \rightarrow X_c \ell \nu$ (M_x moments, had. tagged)

arXiv:2009.04493

munira.khan@uni-bonn.de

$|V_{ub}|$ Exclusive

- $B^0 \rightarrow \pi^- \ell^+ \nu_\ell$ and $B^+ \rightarrow \rho^0 \ell^+ \nu_\ell$ (untagged)

Phys.Rev.D 111 (2025) 11, 112009

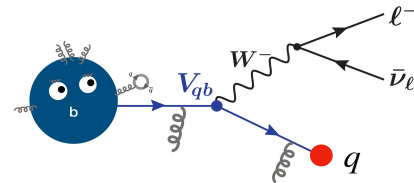
- $B \rightarrow \tau \nu$

arXiv:2502.04885

$|V_{ub}|$ Inclusive

- $B \rightarrow X_u \ell \nu$ partial branching fractions (had. tagged)

Paper in preparation



Exclusive $|V_{cb}|$

Determination of $|V_{cb}|$ using $B \rightarrow D\ell\nu_\ell$ decays
at Belle II

$B \rightarrow D\ell\nu$ at Belle II

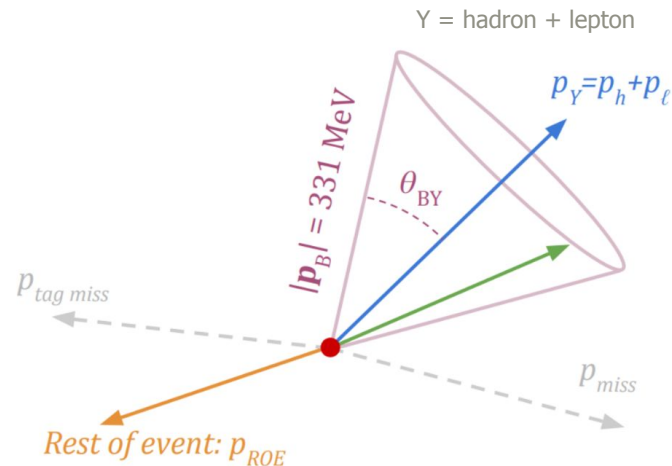
Advantages to $B \rightarrow D\ell\nu$: Depends only on one form factor
and one main background ($B \rightarrow D^*\ell\nu$ feed-down)

Differential measurement of using $w = \frac{m_B^2 + m_D^2 - q^2}{2m_B m_D}$

through $\cos\theta_{BY} = \frac{2 E_{\text{Beam}} E_Y - m_B^2 - m_Y^2}{2|\vec{p}_B||\vec{p}_Y|}$

Access B momentum direction via

- Calculating $w = v_B \cdot v_D$ for each potential direction of B momentum
- Calculating weights $u = \frac{1}{2}(1 - \hat{p}_B^* \cdot \hat{p}_{\text{ROE}}^*) \sin^2 \theta_B^*$



$B \rightarrow D\ell\nu$ at Belle II

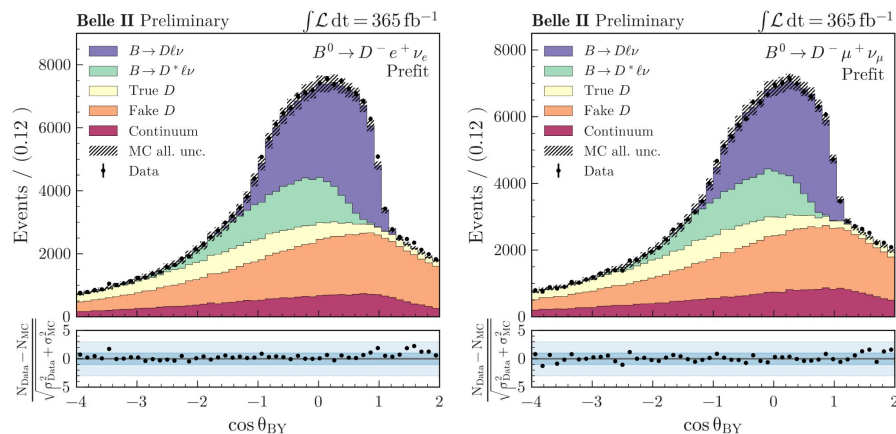
Reconstruct: $D^0 \rightarrow K^- \pi^+$ & $D^+ \rightarrow K^- \pi^+ \pi^+$ and lepton of matching charge (e, μ)

D^* veto and selections to reduce other backgrounds

Several control samples validated and correction factors derived from some of them

Signal expected in $\cos\theta_{BY} = [-1, 1]$

$$\cos\theta_{BY} = \frac{2 E_{\text{Beam}} E_Y - m_B^2 - m_Y^2}{2|\vec{p}_B||\vec{p}_Y|}$$

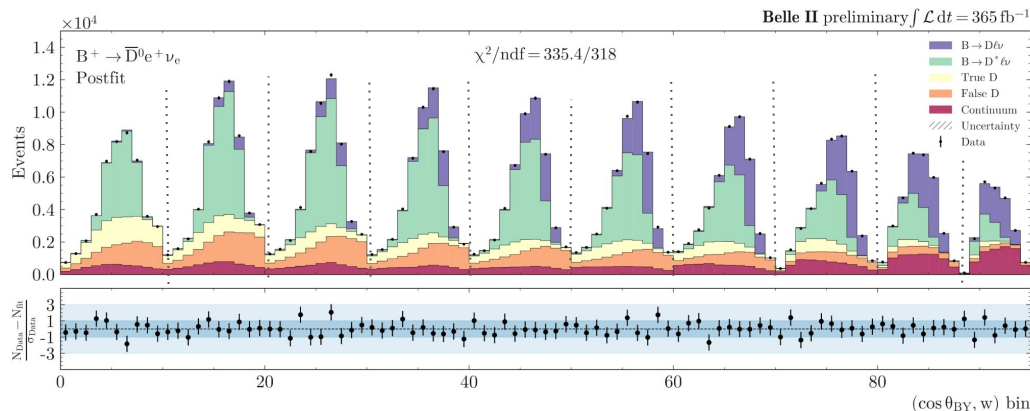


Signal extraction fit

Maximum likelihood fit in 10 equal-width bins of $\cos\theta_{BY}$ and 10 bins of w

Five components:

- $B \rightarrow D\ell\nu$ signal
- $B \rightarrow D^*\ell\nu$ feed-down
- Correctly reconstructed D
- Mis-reconstructed D
- $ee \rightarrow$ Lighter quarks and tau pairs



Bin-to-bin migration effects are accounted for by constructing signal templates based on generated values of w

$$\mathcal{B}(B^0 \rightarrow D^- \ell^+ \nu_\ell) = (2.06 \pm 0.05 (\text{stat.}) \pm 0.10 (\text{sys.}))\%$$

$$\mathcal{B}(B^+ \rightarrow \bar{D}^0 \ell^+ \nu_\ell) = (2.31 \pm 0.04 (\text{stat.}) \pm 0.09 (\text{sys.}))\%$$

$|V_{cb}|$ extraction and results

$$\underbrace{\frac{d\Gamma(B \rightarrow \bar{D}\ell^+\nu_\ell)}{dw}}_{\propto \text{signal yield}} \propto (w-1)^{3/2} \underbrace{\eta_{EW}^2 (1 + \delta_C^{+,0})}_{\text{QED correction}} \underbrace{\mathcal{G}^2(w)}_{\text{form factor}} |V_{cb}|^2$$

$$w = \frac{m_B^2 + m_D^2 - q^2}{2m_B m_D} \quad q^2 = (p_B - p_D)^2$$

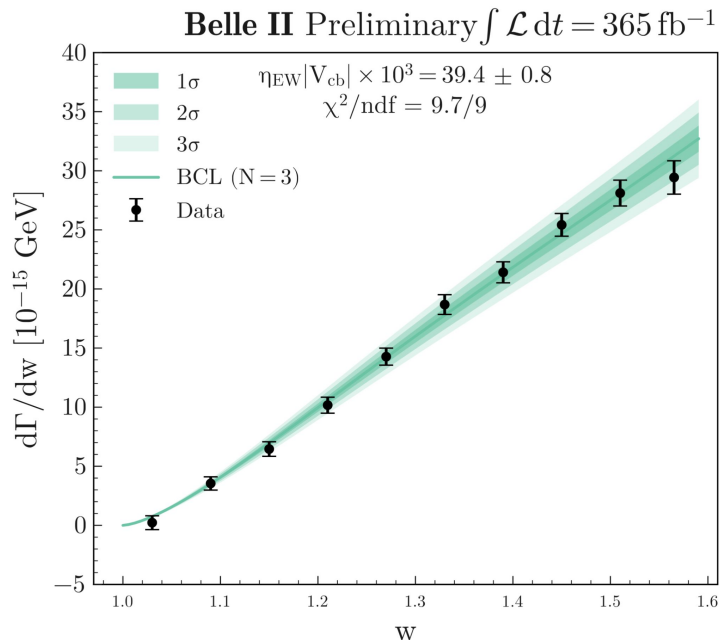
Two form factor models:

→ BCL (Bourrely, Caprini, Lellouch) parameterization [PRD 79, 013008 (2009)]

→ CLN (Caprini, Lellouch, Neubert) parameterization [Nucl. Phys. B 530, 153 (1998)]

$$|V_{cb}| = (39.2 \pm 0.4 \pm 0.6 \pm 0.5) \times 10^{-3}$$

The most precise measurement to date using $B \rightarrow D\ell\nu$!



Exclusive $|V_{ub}|$

Determination of $|V_{ub}|$ from simultaneous measurements of untagged $B^0 \rightarrow \pi^- \ell^+ \nu_\ell$ and $B^+ \rightarrow \rho^0 \ell^+ \nu_\ell$ decays

Analysis setup

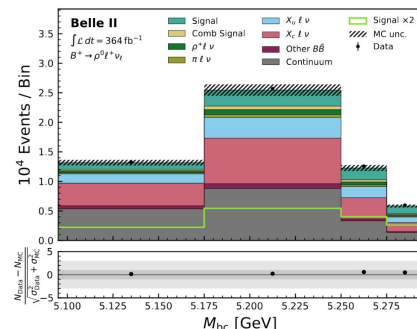
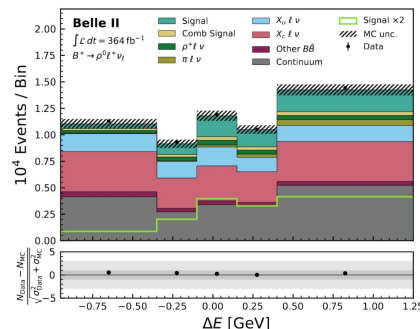
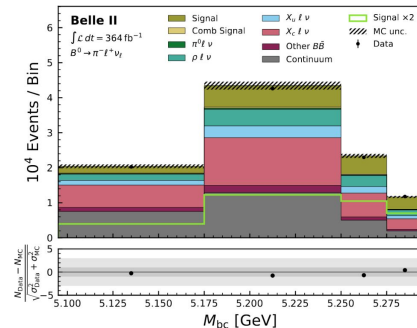
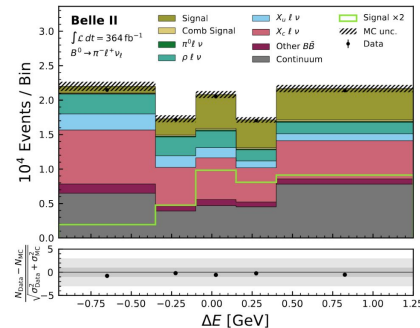
Signal components:

- True Signal
- Combinatorial Signal
- Isospin-conjugate signal
- Cross-feed signal

Background suppression:

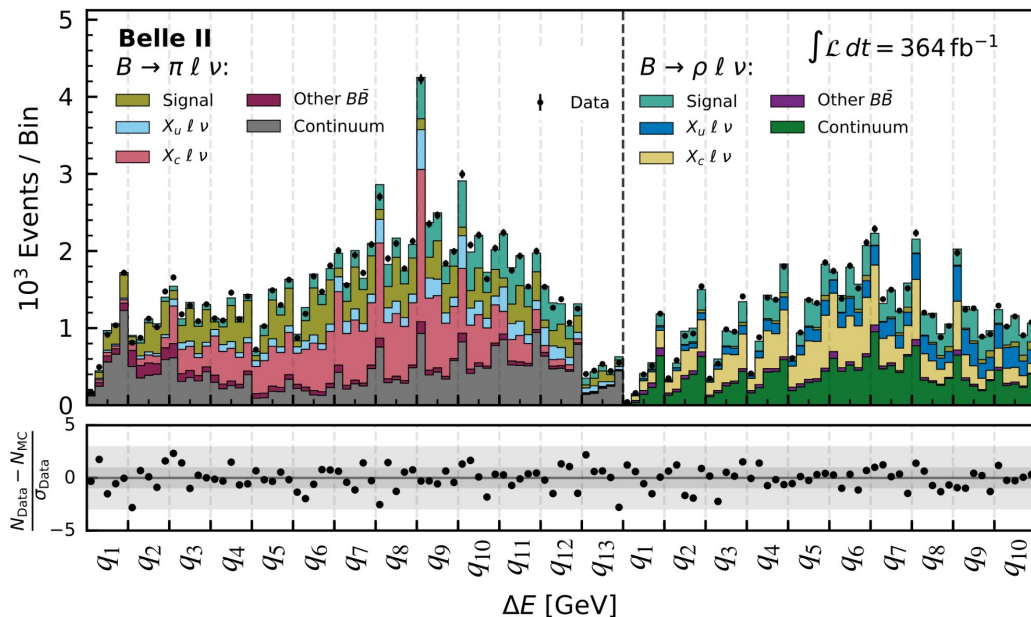
BDTs for BB and continuum suppression
trained wrt. on signal mode and q^2 bin
 $\rightarrow 2 \times (13 + 11) = 48$ BDTs

Variables of interest: ΔE , M_{bc} , q^2



Fit results

Simultaneous 3D fit in 460 bins: $(13+10)$ of q^2 x 5 of ΔE x 4 of M_{bc}



Signal yields are obtained in bins of true q^2

Total branching fraction as sum of partial ones obtained from fitted yields and efficiencies

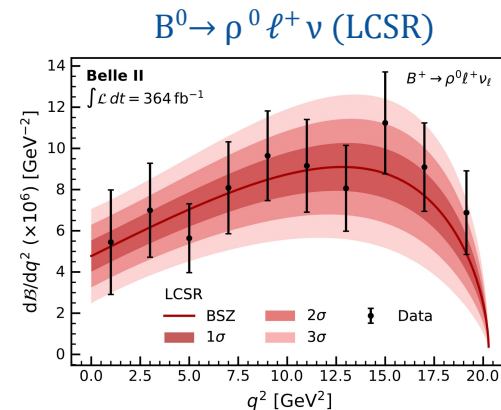
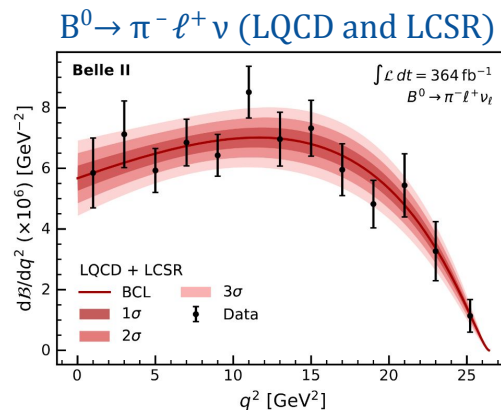
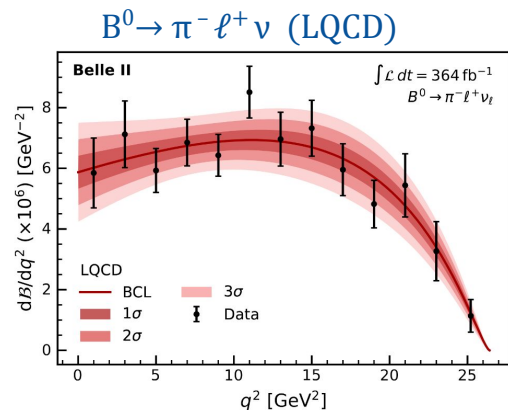
$$\mathcal{B}(B^0 \rightarrow \pi^- \ell^+ \nu_\ell) = (1.516 \pm 0.042 \pm 0.059) \times 10^{-4}$$

$$\mathcal{B}(B^+ \rightarrow \rho^0 \ell^+ \nu_\ell) = (1.625 \pm 0.079 \pm 0.180) \times 10^{-4}$$

→ Consistent with SM

Determination of $|V_{ub}|$

Estimate $|V_{ub}|$ by minimizing $\chi^2 = \sum_{i,j=1}^N (\Delta B_i - \Delta \Gamma_i \tau) C_{ij}^{-1} (\Delta B_j - \Delta \Gamma_j \tau) + \chi_{theo}^2$



LQCD only

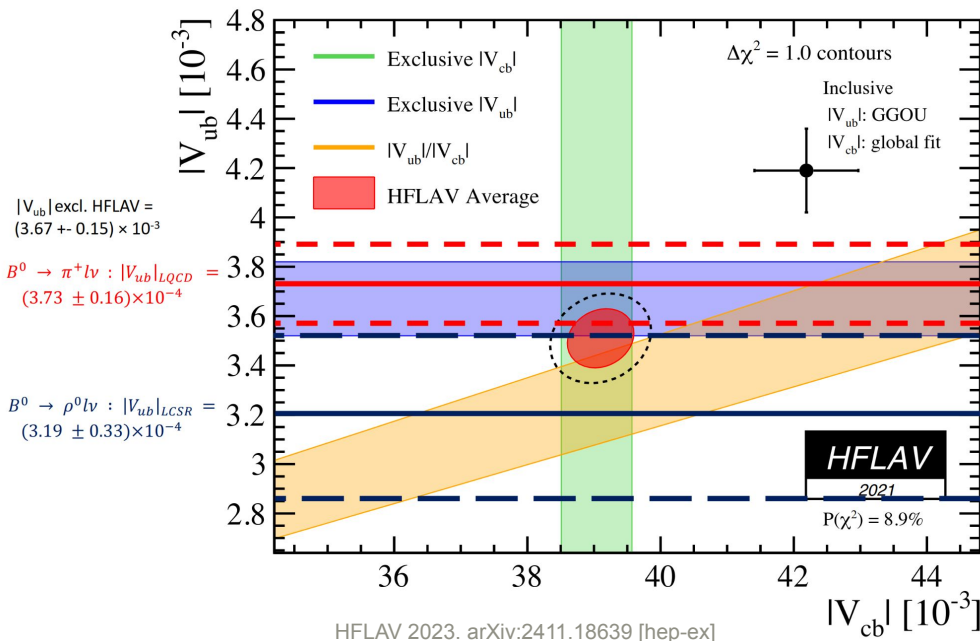
$$|V_{ub}|_{B \rightarrow \pi \ell \nu_\ell} = (3.93 \pm 0.09 \pm 0.13 \pm 0.19) \times 10^{-3}$$

LQCD + LCSR

$$|V_{ub}|_{B \rightarrow \pi \ell \nu_\ell} = (3.73 \pm 0.07 \pm 0.07 \pm 0.16) \times 10^{-3}$$

$$|V_{ub}|_{B \rightarrow \rho \ell \nu_\ell} = (3.19 \pm 0.12 \pm 0.17 \pm 0.26) \times 10^{-3}$$

Exclusive $|V_{ub}|$ result



Separate $|V_{ub}|$ extraction fits for π and ρ mode

Different theory calculations:

- only LQCD constraints
- LQCD + LCSR

Experimentally limited by size of off-resonance dataset and description of the non-resonant $B \rightarrow X_u \ell \nu$ backgrounds

Inclusive $|V_{cb}|$

Measurement of Lepton Mass Squared Moments in $B \rightarrow X_c \ell \nu$ decays with the Belle II Experiment

Inclusive $|V_{cb}|$

- Theory uncertainty on incl. $|V_{cb}|$ associated to truncation of the HQE and perturbative expansion

→ Measuring moments allows reduction of this uncertainty

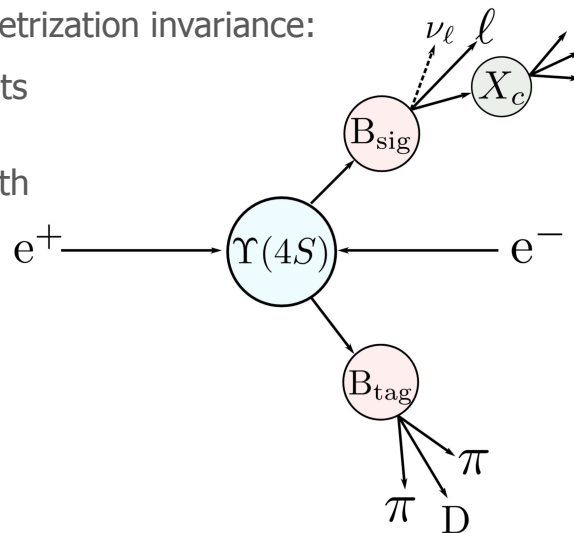
- Reduced number of parameters by exploiting reparametrization invariance: not allowed for lepton-energy or hadron-mass moments

→ Novel method to determine $|V_{cb}|$ from q^2 moments with reduced number of parameters in the fit

[JHEP 02 (2019), 177]

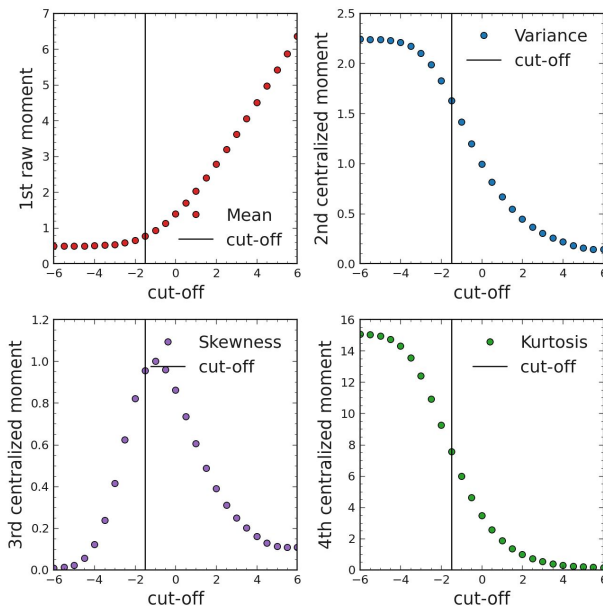
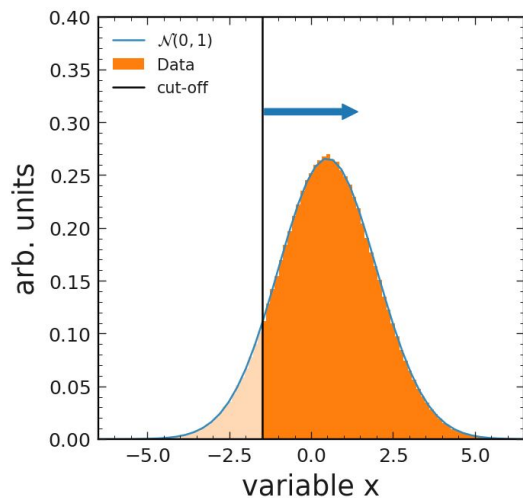
→ First result of q^2 moments from Belle II at 62 fb^{-1}

$$q^2 \equiv (p_\ell + p_\nu)^2 = (p_B - p_X)^2$$



Inclusive $|V_{cb}|$

Moments measured w.r.t. various lower thresholds in the distribution



$$\mu_n = \int_{-\infty}^{\infty} (x - c)^n f(x) dx$$

Raw moment: $c = 0$

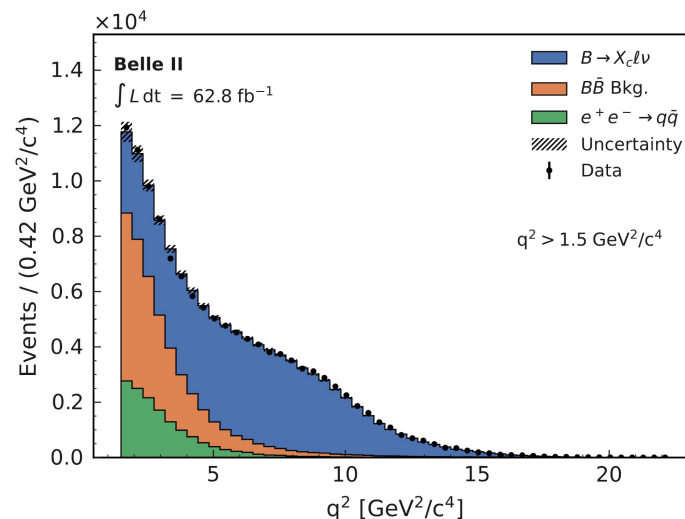
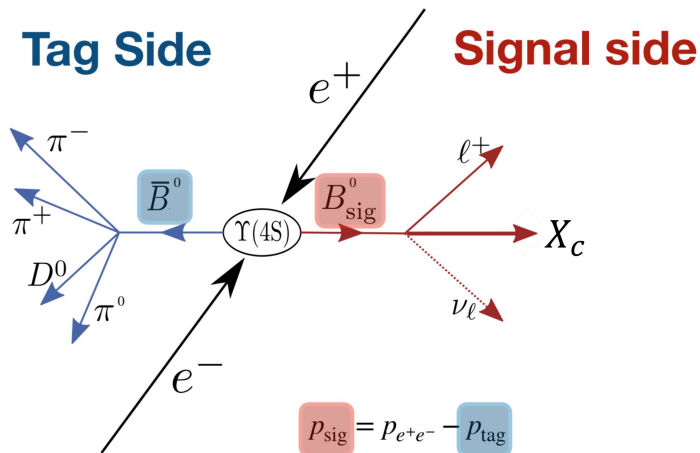
Central moment: $c = \text{Mean}$

Reconstruction strategy

- Employ hadronic tagging to obtain full event kinematic information
- Kinematic fit to improve resolution in M_X and q^2

Variables of interest:

M_X, q^2



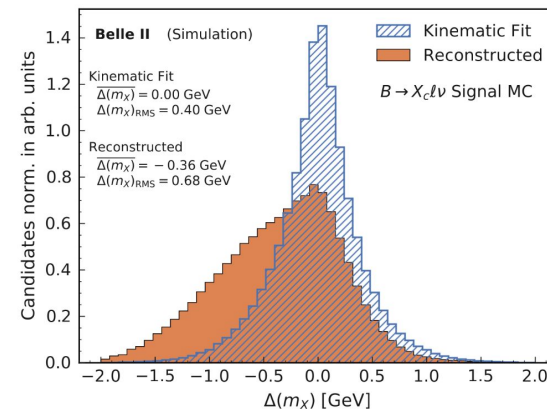
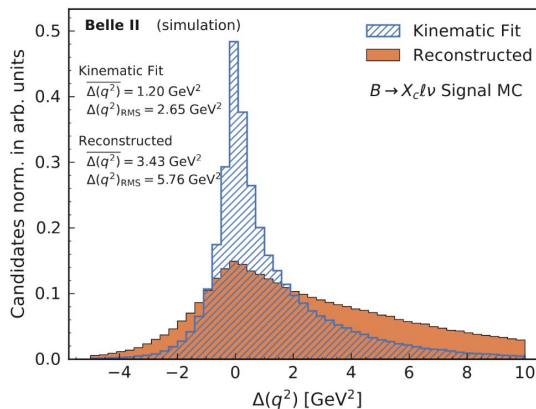
Kinematic Fit

Not sufficient resolution and bias for M_X and q^2 after reconstruction

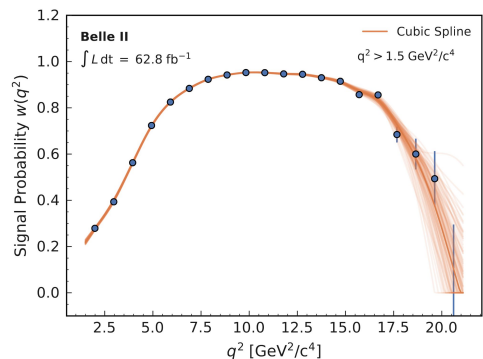
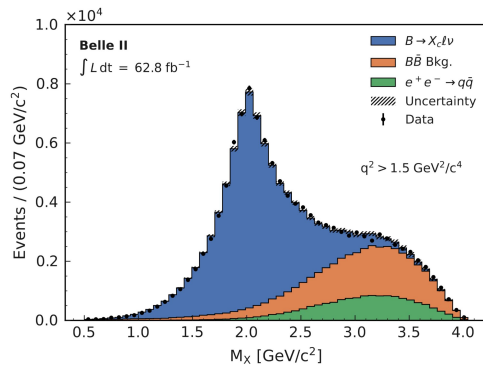
Use kinematic constraints:

- B meson mass at 5.279 GeV
- No missing momentum
- Positive M_X^2

✓ Improved resolution in M_X and q^2



Background subtraction fit



Event-wise master formula

$$\langle q^{2n} \rangle = \frac{\sum_{i=0}^{N_{data}} w(q_i^2) \times q_{calib,i}^{2n}}{\sum_{j=0}^{N_{data}} w(q_j^2)} \times C_{calib} \times C_{gen}$$

Fit to M_X to extract Signal and Background normalization

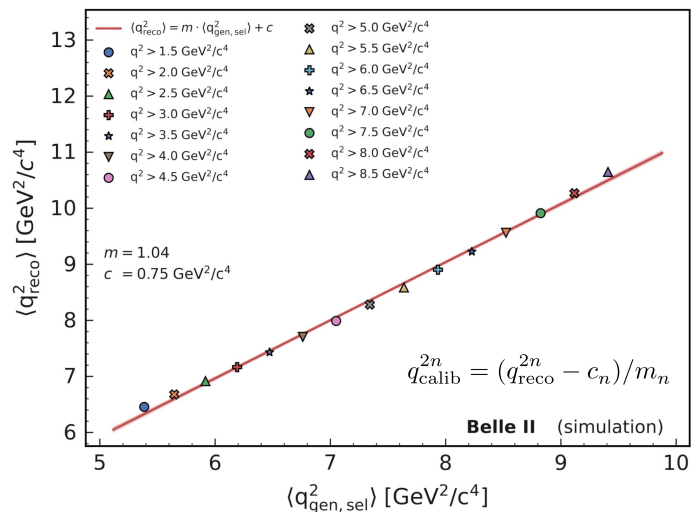
- Free floating signal-yield
- bin-wise nuisance parameters
- One fit for each lower threshold cut in q^2

→ Bin-wise signal probability

Cubic spline interpolation to obtain continuous signal probability functions

Calibration procedure

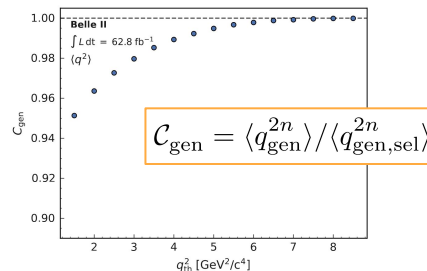
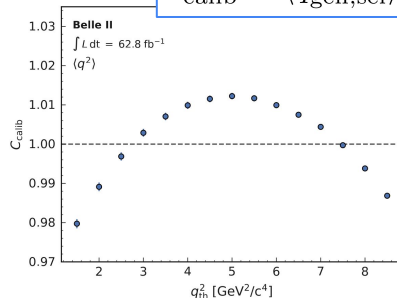
- Linear fit on MC to reconstructed moments vs generator-level moments
- For each moment & threshold cut, constant factors C_{calib} and C_{gen} correct further bias



Event-wise master formula

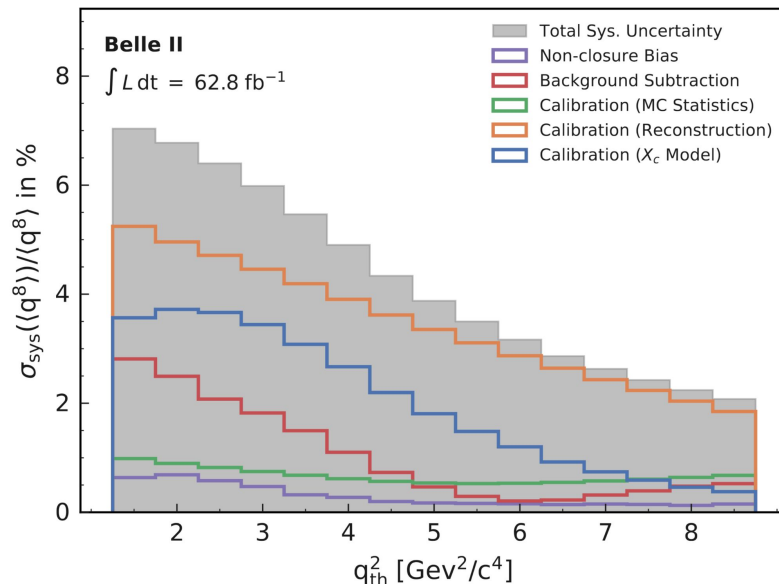
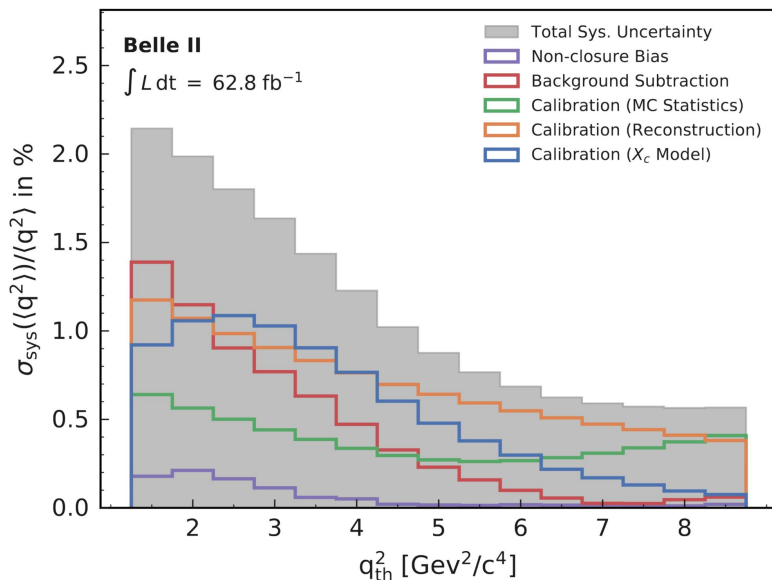
$$\langle q^{2n} \rangle = \frac{\sum_{i=0}^{N_{\text{data}}} w(q_i^2) \times q_{\text{calib},i}^{2n}}{\sum_{j=0}^{N_{\text{data}}} w(q_j^2)} \times C_{\text{calib}} \times C_{\text{gen}}$$

$$C_{\text{calib}} = \langle q_{\text{gen, sel}}^{2n} \rangle / \langle q_{\text{calib}}^{2n} \rangle$$



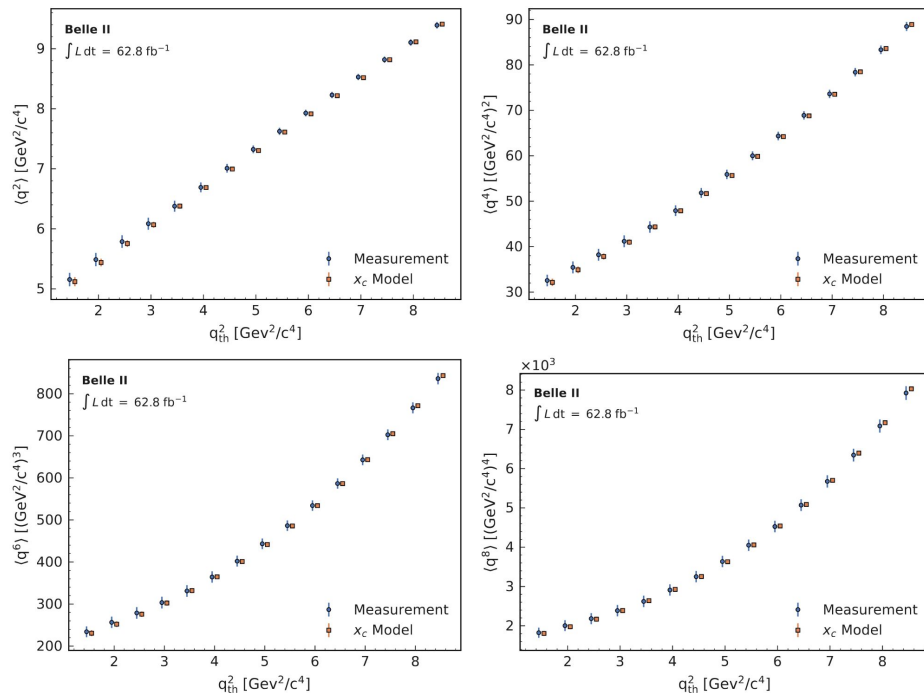
Systematic uncertainties

Systematics dependant on lower threshold cut and moment of q^2



Results

Published in:
Phys.Rev.D 107 (2023) 7, 072002



Extracted moments can be used in
fit to determine value of $|V_{cb}|$
(Extracted $|V_{cb}|$ value combined with Belle result)

$$|V_{cb}| = (41.70 \pm 0.69) \times 10^{-3}$$

[J. High Energy Phys. 10 \(2022\) 068](#)

Inclusive $|V_{ub}|$

Measurement of inclusive $B \rightarrow X_u \ell \nu$ partial branching fractions and $|V_{ub}|$ at Belle II

Measurement outline

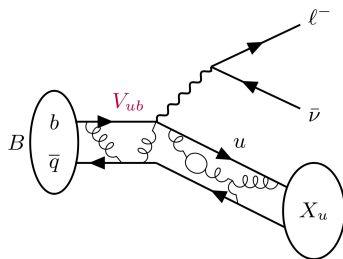
Kinematic selections break inclusivity requirement of the HQE and predictions of differential decay rates become necessary

→ Predictions rely on effective description of the b-quark motion inside the meson

→ Shape functions

Three phase space regions

(87%, 57% and 32% of full available phase space)

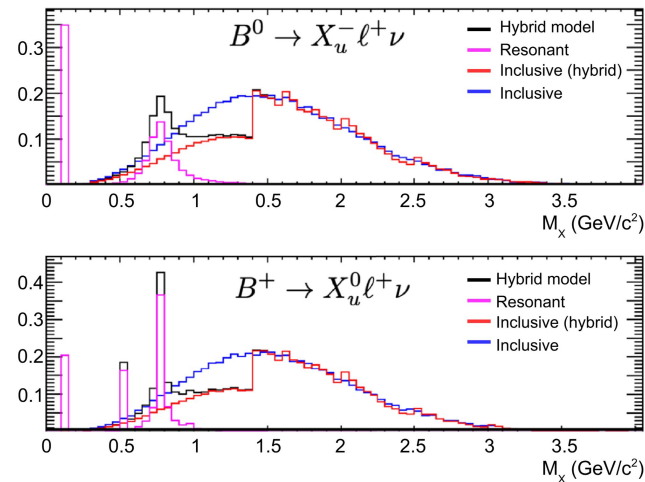


Hybrid model approach to combine resonant and non-resonant modelling

3D re-weighting in E_ℓ^B, M_X, q^2

$$w_{ijk} = \frac{\Delta \mathcal{B}_{ijk}^{\text{inc}} - \Delta \mathcal{B}_{ijk}^{\text{exc}}}{\Delta \mathcal{B}_{ijk}^{\text{inc}}}$$

Phys. J. C 80, 966 (2020)



Selections and modelling corrections

Hadronic tagging and inclusive reconstruction of X

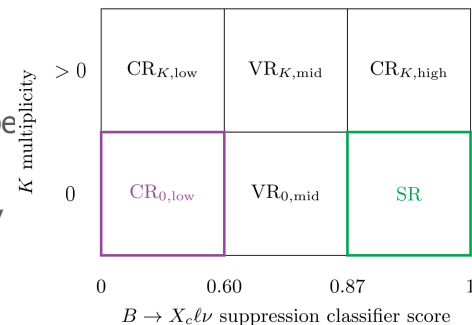
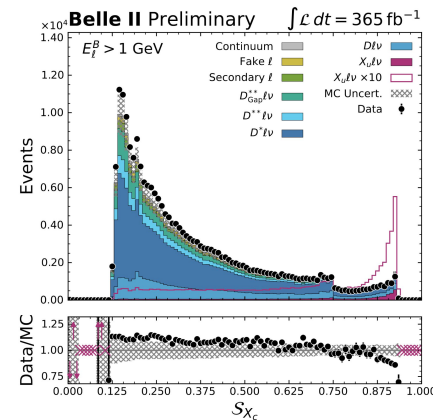
Variables of interest: E_ℓ^B , $M_{X'}$, q^2

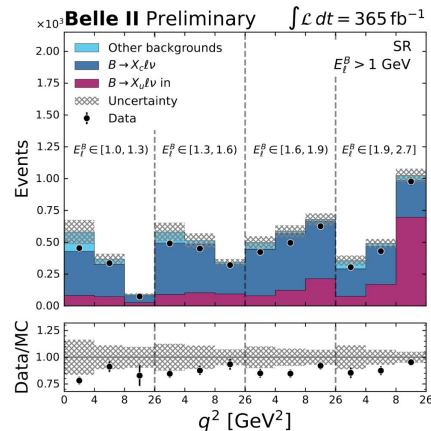
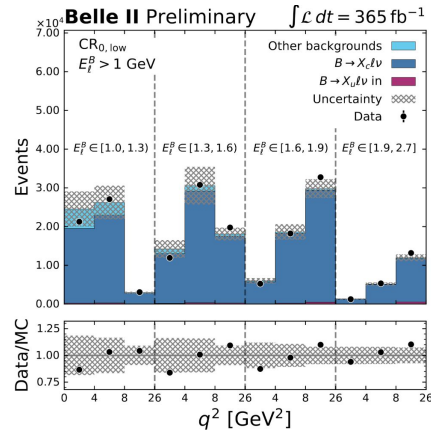
Background suppression

- BDT-based corrections to continuum shape and normalization
- ML-algorithm trained on nine kinematic variables to suppress $B \rightarrow X_c \ell \nu$ background

Modelling corrections to $B \rightarrow X_c \ell \nu$

- mismodelling could not be attributed to a single source: normalization and shape correction are corrected separately: normalization from ABCD method
- Shape correction is directly implemented into the signal extraction procedure by simultaneously fitting $CR_{0,low}$ ($B \rightarrow X_c \ell \nu$ enriched region)





Signal Extraction

Simultaneous fit in signal region (SR) and control region (CR)

2D fit in E_ℓ^B and q^2

Three templates:

Signal $B \rightarrow X_u \ell \nu$, $B \rightarrow X_c \ell \nu$ and other bkg.

Different phase-space selections

Unconstrained $B \rightarrow X_u \ell \nu$ normalization factor in SR (fixed in CR)

→ Accounts for $B \rightarrow X_c \ell \nu$ mismodelling and correlations

Phase space selections	Acceptance
$E_\ell^B > 1.0 \text{ GeV}$	87%
$E_\ell^B > 1.0 \text{ GeV}$ $M_X < 1.7 \text{ GeV}$	57%
$E_\ell^B > 1.0 \text{ GeV}$ $M_X < 1.7 \text{ GeV}$ $q^2 > 8 \text{ GeV}^2$	31%

$|V_{ub}|$ determination

$$|V_{ub}| = \sqrt{\frac{\Delta\mathcal{B}(B \rightarrow X_u \ell \nu)}{\tau_B \Delta\Gamma(B \rightarrow X_u \ell \nu)}}$$

Three theoretical frameworks:

- BLNP [PRD 72, 073006 (2005)]
- Dressed Gluon Expansion [JHEP 10, 058 (2007)]
- Gambino, Giordano, Ossola and Uraltsev [JHEP 01, 097 (2006)]

Result in most inclusive phase space region (GGOU):

$$\Delta\mathcal{B}(B \rightarrow X_u \ell \nu) = (1.54 \pm 0.08 \pm 0.12) \times 10^{-3}$$

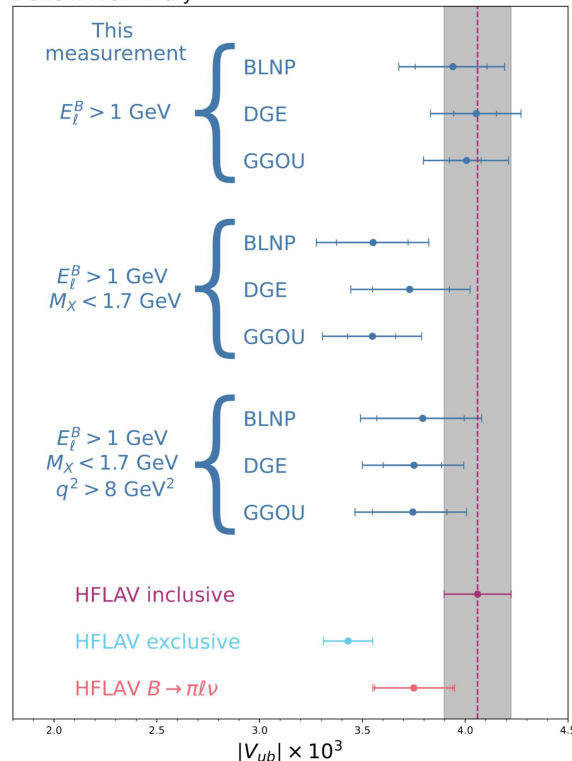
$$|V_{ub}| = (4.01 \pm 0.11 \pm 0.16_{-0.08}^{+0.07}) \times 10^{-3}$$

→ Consistent with inclusive HFLAV average, exceeds exclusive average

→ Competitive with Belle thanks to better tagging algorithm and more modern background suppression methods

→ Leading systematics: $B \rightarrow X_u \ell \nu$ modelling and corrections on $B \rightarrow X_c \ell \nu$ modelling

Belle II Preliminary



Exclusive $|V_{ub}|$

Measurement of $B \rightarrow \tau \nu$ branching fraction
with a hadronic tagging method at Belle II

Analysis setup

Clean in theory, experimentally challenging

Helicity suppressed in SM

Sensitive to NP contributions (e.g. charged Higgs)

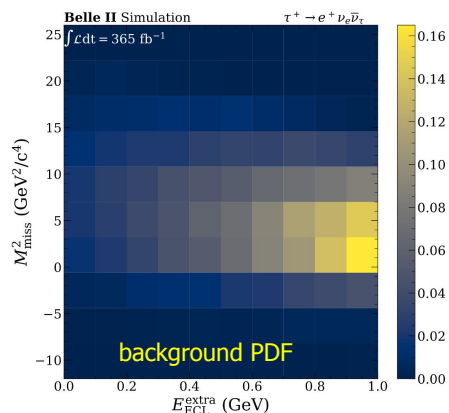
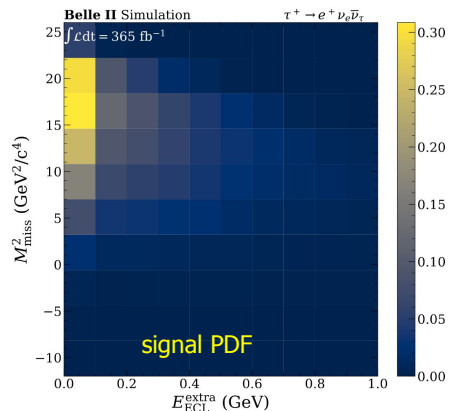
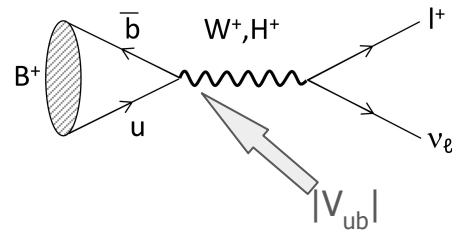
Fully reconstruct Btag and analyse simultaneously for four tau decay modes:

$\tau \rightarrow e \nu \nu$, $\tau \rightarrow \mu \nu \nu$, $\tau \rightarrow \pi \nu$, $\tau \rightarrow \rho \nu$ (72% of tau decay modes)

Reconstructing the Rest of the Event (ROE):

→ Sum of cleaned-up energy in calorimeter: $E_{\text{ECL}}^{\text{extra}}$

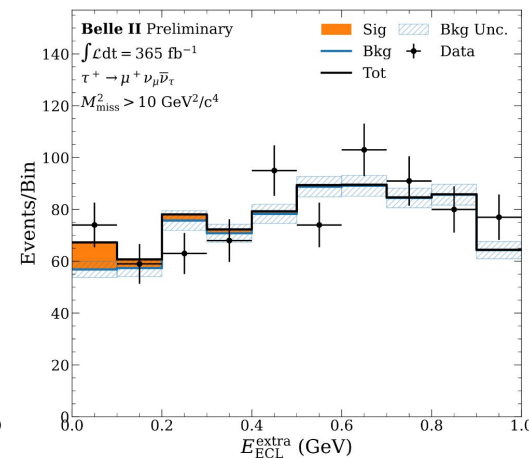
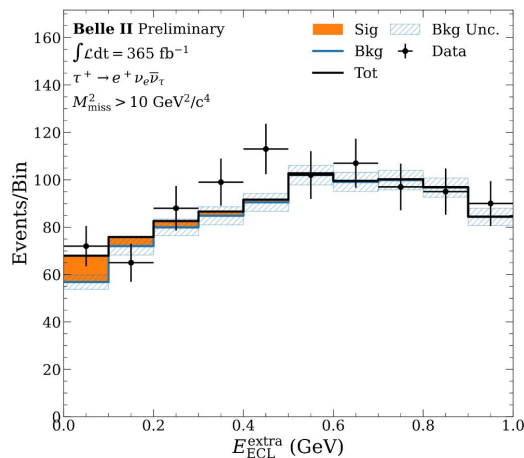
→ Missing mass of undetected particles: M_{miss}^2



Signal extraction procedure

Suppress continuum background with two separate BDTs for hadronic and lepton decay modes

Signal yield extracted from simultaneous 2D binned likelihood fit on $E_{\text{ECL}}^{\text{extra}}$ and M_{miss}^2 to all four channels



Exclusive $|V_{ub}|$ result

Branching fraction is measured with 3σ significance

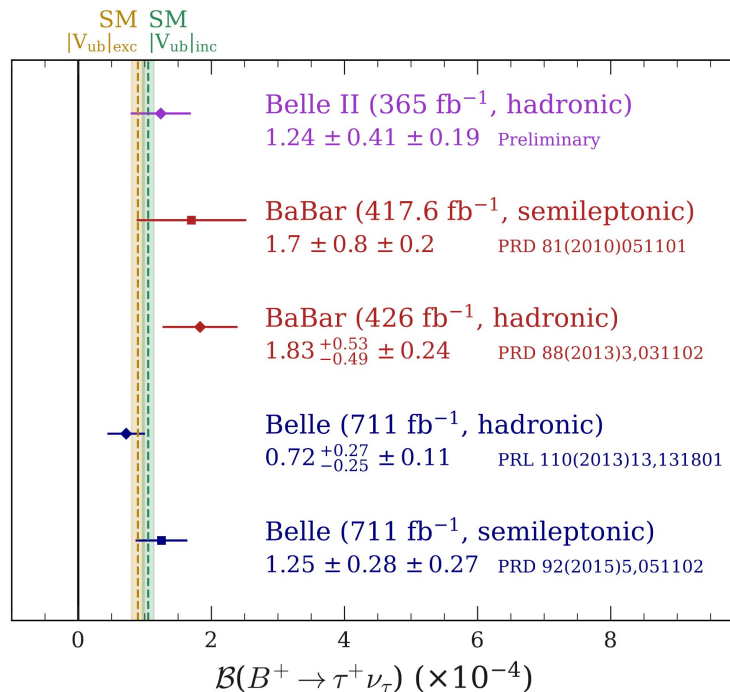
$$\mathcal{B}(B^+ \rightarrow \tau^+ \nu_\tau) = [1.24 \pm 0.41(\text{stat.}) \pm 0.19(\text{syst.})] \times 10^{-4}$$

→ Consistent with current world average and SM prediction

$$|V_{ub}|_{B^+ \rightarrow \tau^+ \nu_\tau} = [4.41^{+0.74}_{-0.89}] \times 10^{-3}$$

Uncertainty competitive with previous analysis despite lower statistics

→ Improved tagging algorithm and optimized selection



Summary and Outlook

Summary and Outlook

Several results from Belle II were presented

Results on moments at Belle II:

- q^2 moments (Phys.Rev.D 107 (2023) 7, 072002)
- M_x moments (arXiv:2009.04493)

Many ongoing measurements on moments:

- First simultaneous measurement of E_ℓ^B , q^2 and M_x moments in $B \rightarrow X_c \ell \nu$ decays
- q^2 and M_x moments in $B_s \rightarrow X_c \ell \nu$ decays

$|V_{cb}|$ Exclusive

- $B \rightarrow D \ell \nu$ (untagged)

arXiv:2506.15256

$$|V_{cb}| = (39.2 \pm 0.4 \pm 0.6 \pm 0.5) \times 10^{-3}$$

- $B \rightarrow D^* \ell \nu$ (untagged)

Phys.Rev.D 108 (2023) 9, 9

$$|V_{cb}| = (40.13 \pm 0.27 \pm 0.93 \pm 0.58) \times 10^{-3}$$

$|V_{cb}|$ Inclusive

- $B \rightarrow X_c \ell \nu$ (q^2 moments, had. tagged)

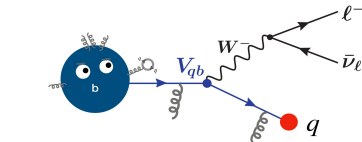
Phys.Rev.D 107 (2023) 7, 072002

J. High Energy Phys. 10 (2022) 068

$$|V_{cb}| = (41.70 \pm 0.69) \times 10^{-3}$$

- $B \rightarrow X_c \ell \nu$ (M_x moments, had. tagged)

arXiv:2009.04493



$|V_{ub}|$ Exclusive

- $B^0 \rightarrow \pi^- \ell^+ \nu_\ell$ and $B^+ \rightarrow \rho^0 \ell^+ \nu_\ell$ (untagged)

Phys.Rev.D 111 (2025) 11, 112009

- $B \rightarrow \tau \nu$

arXiv:2502.04885

$$|V_{ub}| = (3.73 \pm 0.07 \pm 0.07 \pm 0.16) \times 10^{-3}$$

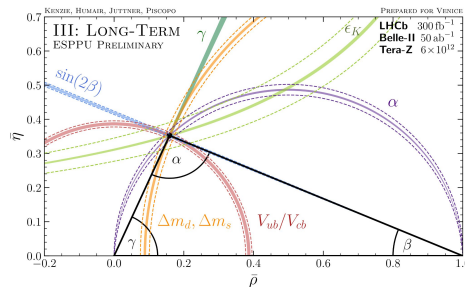
$$|V_{ub}| = (4.41 \pm 0.89) \times 10^{-3}$$

$|V_{ub}|$ Inclusive

- $B \rightarrow X_u \ell \nu$ partial branching fractions (had. tagged)

Paper in preparation

$$|V_{ub}| = (4.01 \pm 0.11 \pm 0.16) \times 10^{-3}$$



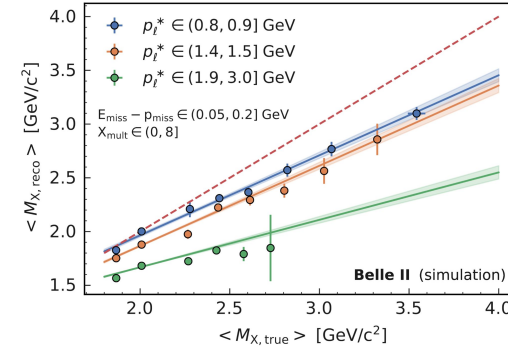
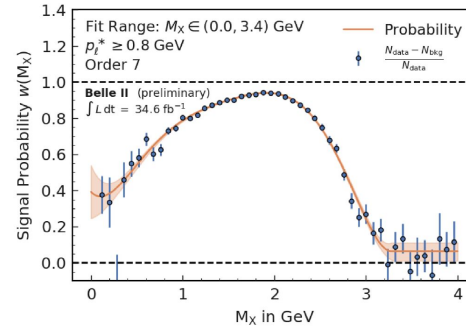
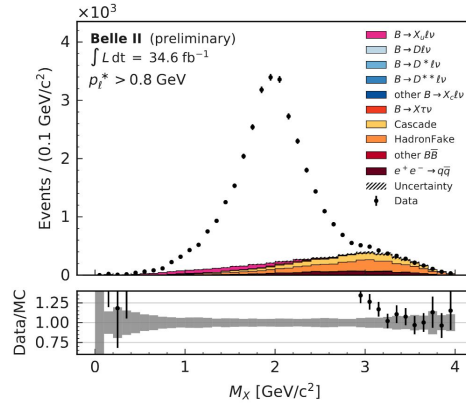


Backup

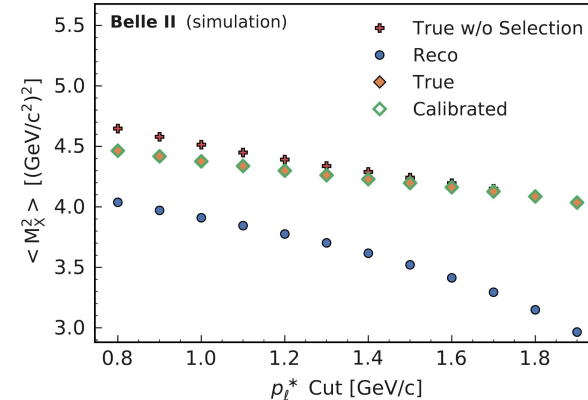
Inclusive $|V_{cb}|$

Measurement of Hadronic Mass Moments in $B \rightarrow X_c \ell \nu$ decays at Belle II

Background subtraction and calibration



$$\langle M_X^n \rangle = \frac{\sum_i w_i(M_X) M_{X, \text{calib}}^n}{\sum_i w_i(M_X)} \times \mathcal{C}_{\text{calib}} \times \mathcal{C}_{\text{true}}$$



Results

Consistent with
previous results but
non-competitive result
at 32.6 fb^{-1}

