Constraining inverse moment of B-meson distribution amplitude using Lattice QCD data

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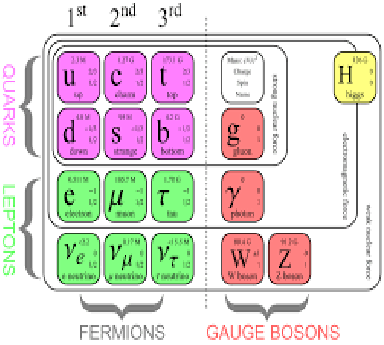
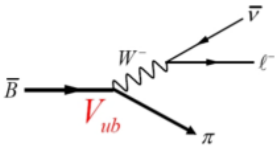
The Standard Model

- Theory describing the three fundamental forces of nature namely, the strong, weak and electromagnetic forces.
- The gauge symmetry of the SM: $SU(3)_c \times SU(2)_L \times U(1)_Y$.
- Not the complete theory of nature.
- Presence of dark matter
- Origin of non-zero neutrino masses
- Matter-antimatter asymmetry
- Accelerating expansion of the universe



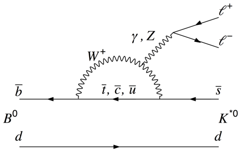
Different transitions

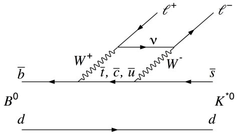
Charged current transitions



- Clean extractions of the CKM matrix elements.
- Ability of charged-current (CC) weak interactions to probe new physics is sharply limited.

Neutral current transitions





- Potential probes of physics at high energy scales.

Form Factors

- Hadronic matrix elements $\langle h^{'}|\bar{q}^{'}\Gamma q|h\rangle$ parameterized in terms of appropriate form factors.
- Non-perturbative quantities and serve as dominant sources of uncertainties in the theoretical predictions of observables.
- Shape of the decay rate distribution from the shape of the corresponding form-factors in the whole q^2 region.
- Most precise estimates from Lattice QCD (in high q^2 region).
- Light-cone sum rules and QCD factorization/soft-collinear effective theory used in the low q^2 regime where Lattice QCD results aren't supposed to be reliable.
- LCSR computations employing B-meson light-cone distribution amplitudes (LCDAs) depend on λ_B, which is currently highly uncertain.

Correlation function

- B-to-vacuum correlator -

$$\mathcal{F}^{\mu\nu}(q,k) = i\int \mathrm{d}^4x\, e^{ik\cdot x}\, \left\langle 0\left|\mathcal{T}\{J^{\nu}_{int}(x), J^{\mu}_{weak}(0)\}\right|\bar{B}_{q_2}(q+k)\right\rangle$$

$$J^{\nu}_{int} = \bar{q}_2(x)\gamma^{\nu}\gamma_5 q_1(x), \qquad J^{\mu}_{weak}(0) \equiv \bar{q}_1(0)\Gamma^{\mu}_{w} b(0)$$

- The correlator related to form factor via the dispersion relation -

$$\mathcal{F}^{\mu\nu}_{\text{had}}(q,k) = \frac{\langle 0\left|\bar{q}_2\gamma^{\nu}\gamma_5 q_1\right|P(k)\rangle \left\langle P(k)\left|\bar{q}_1\Gamma^{\mu}_{w} b\right|\bar{B}(q+k)\right\rangle}{m_P^2 - k^2} + \dots\,,$$

$$\langle 0\left|\bar{q}_2\gamma^{\nu}\gamma_5 q_1\right|P(k)\rangle = ik^{\nu} f_P\,.$$

- The form factors -

$$\langle P(k)|\bar{q}_1\gamma^{\mu} b|B(p)\rangle = \left[(p+k)^{\mu} - \frac{m_B^2 - m_P^2}{q^2}q^{\mu}\right] f_{+} + \frac{m_B^2 - m_P^2}{q^2}q^{\mu}\, f_0$$

$$\langle P(k)|\bar{q}_1\sigma^{\mu\nu}\, q_{\nu} b|B(p)\rangle = \frac{i f_T}{m_B + m_P}\,\left[q^2\,(p+k)^{\mu} - (m_B^2 - m_P^2)\, q^{\mu}\right]$$

Correlation function

- The correlation function -

$$\mathcal{F}^{\mu\nu}_{\rm OPE}(q,k) = \int d^4x\, e^{ik\cdot x} \int \frac{d^4l}{(2\pi)^4} e^{-il\cdot x} [\gamma^\nu \gamma_5 \frac{\not{l} + m_{q_1}}{m_{q_1}^2 - l^2} \Gamma^\mu_w]_{\alpha\beta}$$
$$\langle 0 | \bar{q}_2^\alpha(x) b^\beta(0) | \bar{B}(q+k) \rangle$$

- $\langle 0 | \bar{q}_2^\alpha(x) b^\beta(0) | \bar{B}(q+k) \rangle \longrightarrow \phi_+, \bar{\phi}, g_+, \bar{g}$
- Different twist light-cone distribution amplitudes (LCDAs) -

$$\bar{\phi}(w) = \int_0^w d\eta (\phi_+(\eta) - \phi_-(\eta))\,, \tag{1}$$
$$\bar{g}(w) = \int_0^w d\eta (g_+(\eta) - g_-(\eta))\,, \tag{2}$$

- The main input involved in parameterizing the LCDAs -

$$\lambda_B = \Big(\int_0^\infty dw \frac{\phi_+(w)}{w} \Big)^{-1} .$$

Models considered

- The Exponential Model -

  Based on combining the regime of low momentum of quarks and gluons with an exponential suppression at large momentum.

- The Local Duality Model -

  Based on the duality assumption to match the B-meson state with the perturbative spectral density integrated over the duality region.

- Two-particle B-meson LCDAs $\sim (2w_0 - w)^p$.

Estimates of λ_B

- QCD sum rule technique $\rightarrow$ the precise estimate of this parameter is not achievable.
- Measurements of the photoleptonic decay $B \rightarrow \gamma \ell \nu \rightarrow$ most accurate estimates of λ_B may be possible.
- Experimentally quite challenging $\rightarrow$ Measurement limited by low signal yield $\rightarrow$ lower limit on $\lambda_B > 0.24$ GeV at 90% confidence interval (CI) [Belle, arXiv: 1810.12976].
- Indirect extractions -
  - LCSR prediction with B-meson LCDA of $B \rightarrow \pi$ form factor (at zero momentum transfer) matched with the QCD sum rule prediction of the same form factor computed using pion distribution amplitude.
  - Comparison between LCSR and SCET calculations of $B \rightarrow \gamma$ form factors.
- Different models of the B-meson LCDAs provide different estimates $\rightarrow$ λ_B highly model-dependent.

Estimates of λ_B

Approach	λ_B (MeV)	Details
QCD sum rule	460 ± 110 [Braun et al. hep-ph/0309330] 383 ± 153 [Khodjamirian et al. 2008.03935]	
$B \to \pi$ [Wang et al. 1506.00667]	354^{+38}_{-30} 368^{+42}_{-32} 389^{+35}_{-28} 303^{+35}_{-26}	Model-I Model-II Model-III Model-IV
$B \to \gamma$ [Janowski et al. 2106.13616]	365 ± 60 310 ± 60 415 ± 60	Model-I Model-II Model-III
$B \to \rho$ [Gao et al. 1907.11092]	343^{+64}_{-79} 370^{+69}_{-86}	Exponential Model Local Duality Model

Estimates of λ_B

- A complementary approach is vital.
- Use of the recent developments in Lattice QCD form factor calculations.
- Form factors for the $B \to K\ell\ell$ decay obtained for the first time across the full physical range of momentum transfer using the highly improved staggered quark formalism for all valence quarks on eight ensembles of gluon field configurations. [HPQCD, 2207.12468]
- Use of finer lattices $\to$ generate data points in the kinematic region close to $q^2 = 0 \to$ allowing larger momenta to be imparted to the daughter meson.
- In this analysis, we incorporate the values of the form factors at $q^2 = 0$ and express them in the framework of B-meson LCSRs, employing the two-particle LCDAs.

Analysis

- The χ^2- statistic -

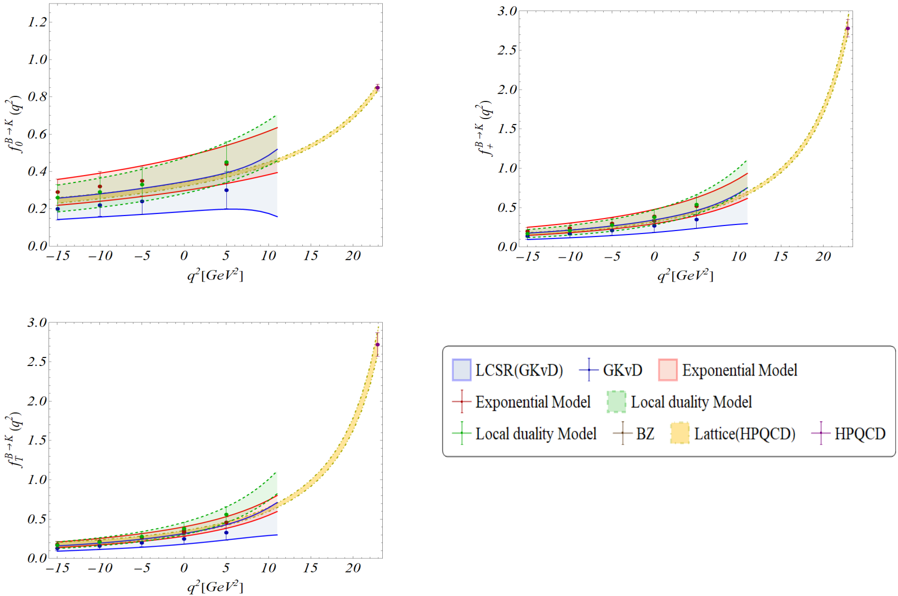
$$\chi^2 = \sum_{i,j} (O_i^{\text{lattice}} - O_i^{\text{theo}}). Cov_{ij}^{-1}. (O_j^{\text{lattice}} - O_j^{\text{theo}}) + \chi^2_{\text{nuis}} .$$

- The nuisance parameters λ_E^2, λ_H^2, s_0 and M^2 in χ^2_{nuis}.
- The parameters λ_E^2, λ_H^2 and s_0 follow Gaussian distributions, whereas M^2 follows a uniform distribution.
- The values of λ_B -

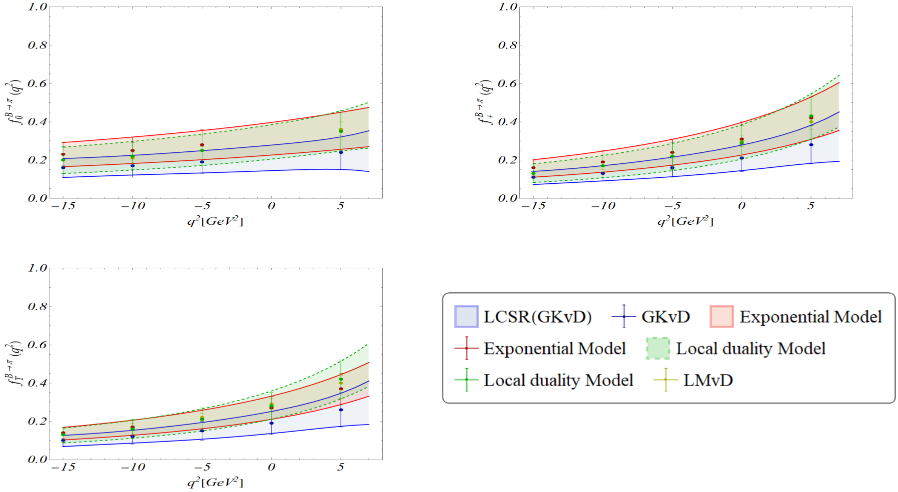
$$\lambda_B(1\,\text{GeV}) = 338^{+68}_{-9}\,\text{MeV} \quad (\text{Exponential Model}),$$
$$\lambda_B(1\,\text{GeV}) = 472^{+110}_{-41}\text{MeV} \quad (\text{Local Duality Model}).$$

- In agreement with earlier QCD sum rule estimates within a $\pm 1\sigma$ uncertainty.

Variation of form factors for the $B \to K$ channel

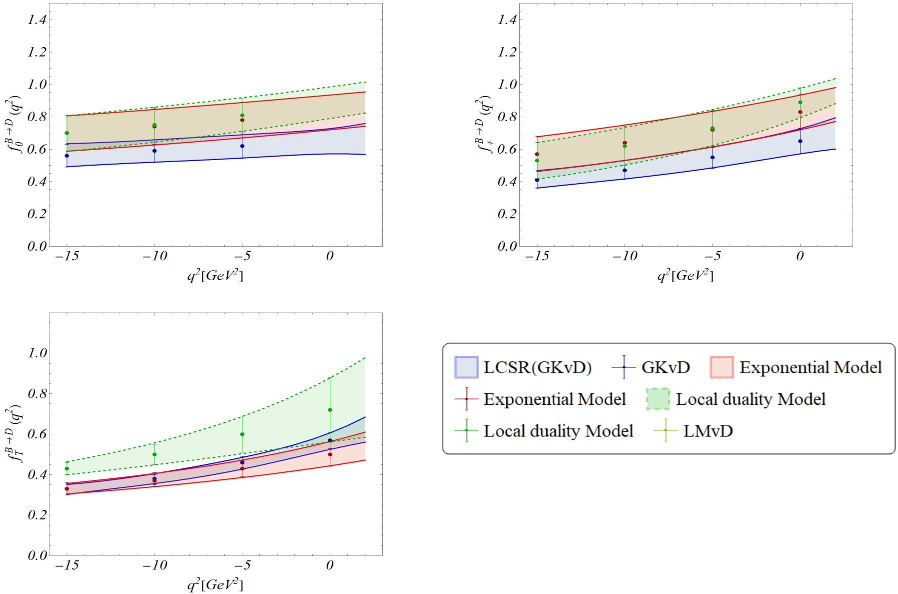


Variation of form factors for the $B \to \pi$ channel



- Exponential Model: f_T at $q^2 = -10\,\mathrm{GeV}^2$
- Local Duality Model: f_0 at $q^2 = -5\,\mathrm{GeV}^2$ and f_+ at $q^2 = -10, -5\,\mathrm{GeV}^2$.

Variation of form factors for the $B \to D$ channel



Branching fraction of $B \to \gamma \ell \nu$

- In addition to λ_B, also influenced by the logarithmic moments of the leading-twist B-meson LCDA ϕ_+.
- The obtained branching fraction parameterized as a function of λ_B for various minimum photon energy thresholds: $E_\gamma > 1,\ 1.5,\ 2\,\mathrm{GeV}$.
- For the Exponential Model,

$$\mathcal{B}(B \to \gamma \ell \nu) = \begin{cases} (0.89 \pm 0.10) \times 10^{-6} & (E_\gamma > 1.5 \text{ GeV}), \\ (0.34 \pm 0.02) \times 10^{-6} & (E_\gamma > 2.0 \text{ GeV})\,, \end{cases}$$

- For the Local Duality Model,

$$\mathcal{B}(B \to \gamma \ell \nu) = \begin{cases} (0.36 \pm 0.03) \times 10^{-6} & (E_\gamma > 1.5 \text{ GeV}), \\ (0.14 \pm 0.01) \times 10^{-6} & (E_\gamma > 2.0 \text{ GeV})\,. \end{cases}$$

- These predictions satisfy experimental upper limit of 3×10^{-6} at 90% CI.

Conclusions

- Constrained the first inverse moment of the B-meson LCDA, λ_B using the recent Lattice QCD results for the $B \to K$ form factors at zero momentum transfer ($q^2 = 0$) from the HPQCD collaboration.
- Our estimated values are $\lambda_B = 338^{+68}_{-9}$ MeV (472^{+110}_{-41} MeV) using the Exponential Model (Local Duality Model), exhibiting agreement with earlier QCD sum rule estimates within a $\pm 1\sigma$ uncertainty.
- Uncertainty improvement achieved with the Exponential Model is nearly a factor of two compared to previous results.
- The form factors obtained exhibit a high level of consistency with the previous analyses, except for the form factor f_T in the Local Duality Model for the $B \to K$ and $B \to D$ channels in specific q^2 regions.
- The branching fraction for $B \to \gamma \ell \nu$ mode satisfies the experimental upper limit provided by the Belle collaboration.

Thank you!

Back-up (Exponential model)

$$\Phi_+(\omega) = \frac{\omega}{\lambda_B^2} e^{-\omega/\lambda_B}, \tag{3}$$

$$\Phi_-(\omega) = \frac{1}{\lambda_B} e^{-\omega/\lambda_B} \tag{4}$$

$$- \frac{\lambda_E^2 - \lambda_H^2}{18\lambda_B^5} \left(2\lambda_B^2 - 4\omega\lambda_B + \omega^2\right) e^{-\omega/\lambda_B}\,, \tag{5}$$

$$g_+(\omega) = -\frac{\lambda_E^2}{6\lambda_B^2} \left\{ (\omega - 2\lambda_B)\mathrm{Ei}\left(-\frac{\omega}{\lambda_B}\right) + (\omega + 2\lambda_B) e^{-\omega/\lambda_B} \right.$$

$$\left. \times \left(\ln\frac{\omega}{\lambda_B} + \gamma_E\right) - 2\omega e^{-\omega/\lambda_B} \right\} \tag{6}$$

$$+ \frac{e^{-\omega/\lambda_B}}{2\lambda_B}\omega^2 \left\{ 1 - \frac{1}{36\lambda_B^2}(\lambda_E^2 - \lambda_H^2) \right\}, \tag{7}$$

$$g_-^{WW}(\omega) = \frac{3\omega}{4}\, e^{-\omega/\lambda_B}\,. \tag{8}$$

Back-up (Local duality model)

$$\phi_+(\omega,\mu_0) = \frac{3}{4\omega_0^3}\omega\,(2\omega_0-\omega)\,\theta(2\omega_0-\omega)\,, \tag{9}$$

$$\phi_-(\omega,\mu_0) = \frac{1}{8\omega_0^3}\left[3(2\omega_0-\omega)^2 - \frac{10(\lambda_E^2-\lambda_H^2)}{3\omega_0^2}\times \right. \tag{10}$$

$$\left.\left(3\omega^2 - 6\omega\omega_0 + 2\omega_0^2\right)\right]\theta(2\omega_0-\omega), \tag{11}$$

$$g_+(\omega,\mu_0) = \frac{5\theta(2\omega_0-\omega)}{384\omega_0^5}\Big\{\omega(2\omega_0-\omega)\Big[8\lambda_E^2(\omega^2-4\omega\omega_0+2\omega_0^2) \tag{12}$$

$$+\,\omega(2\omega_0-\omega)(2\lambda_H^2+9\omega_0^2)\Big] + 4\lambda_E^2\Big[16\omega_0^3(\omega_0-\omega)$$

$$\times \ln\left(1-\frac{\omega}{2\omega_0}\right) + \omega^3(4\omega_0-\omega)\,\ln\left(\frac{2\omega_0}{\omega}-1\right)\Big]\Big\}, \tag{13}$$