Current experimental status for selected $B^0_{(s)} \to K^{(*)0}\bar{K}^{(*)0}$ observables

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Possible new physics links between non-leptonic and semi-leptonic B-decays

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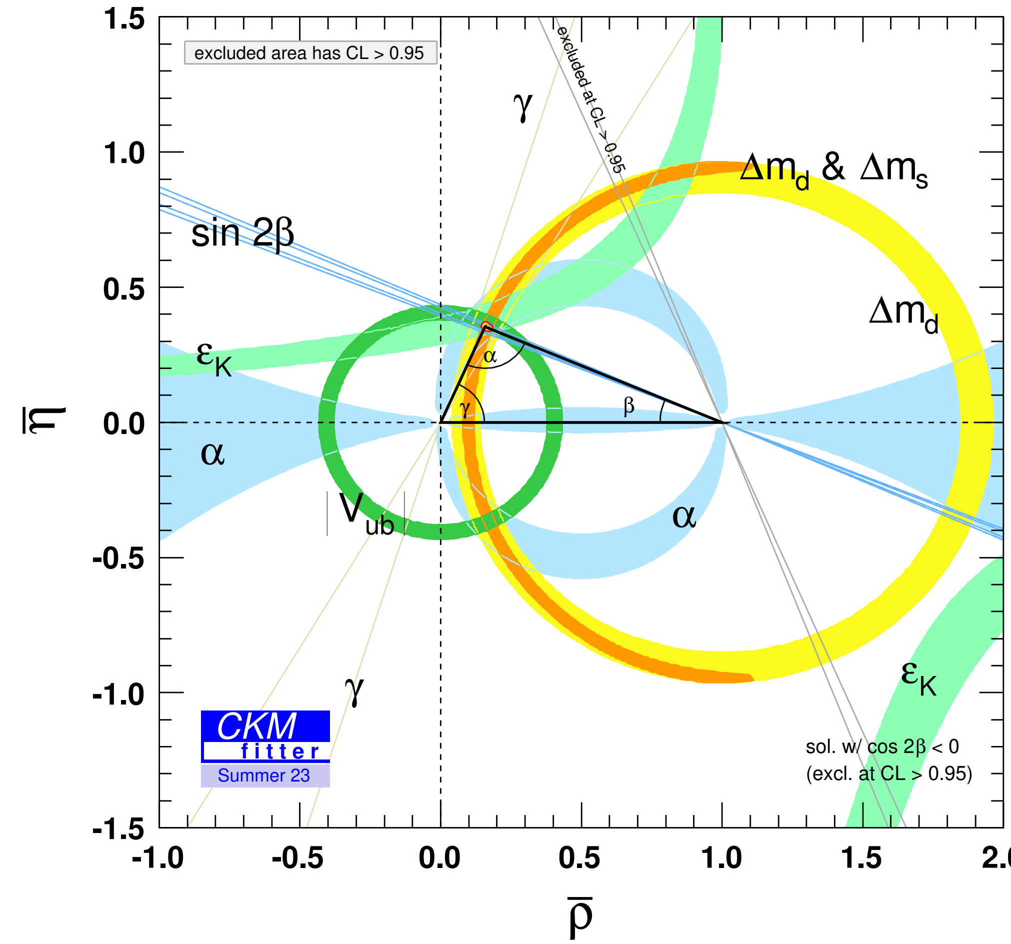
Beyond the Flavour Anomalies V
Siegen, Germany, April 10th, 2024

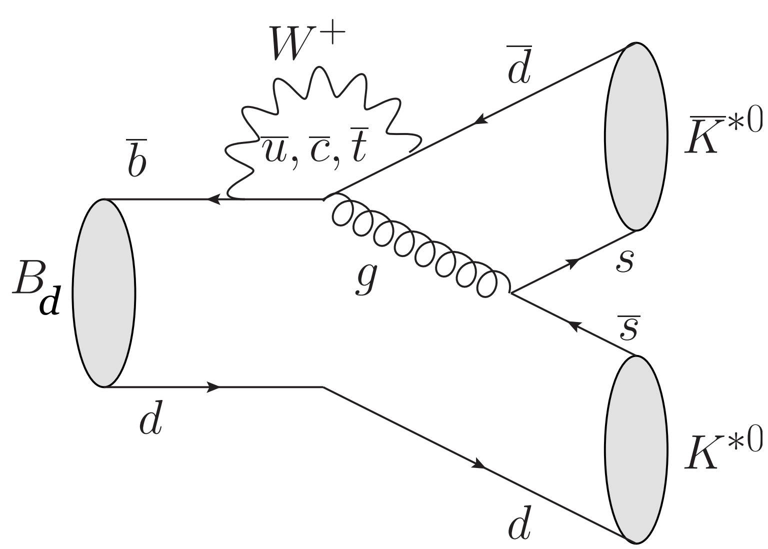
[Based on: Lizana, Matias, BAS, 2306.09178]

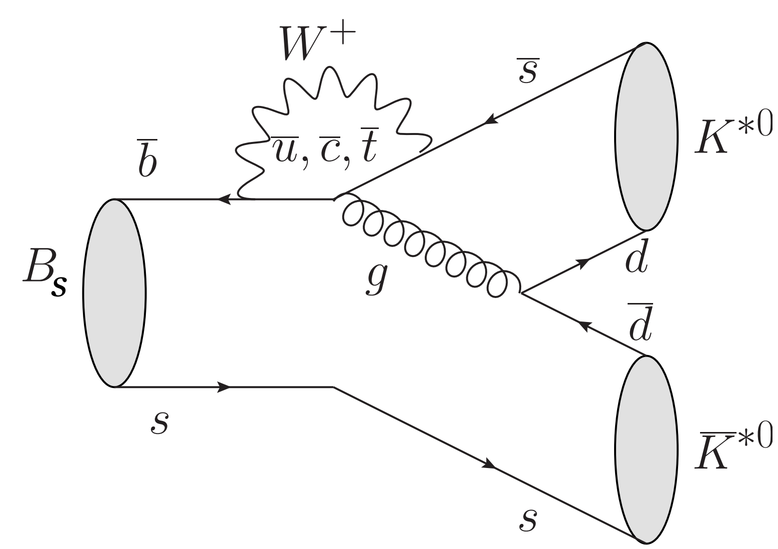
Introduction

- FCNC $B^0_{(s)} \to K^{(*)0}\bar{K}^{(*)0}$ are loop suppressed in the SM:
    - Offer sensitivity to potential NP contributions
    - Access measurements of CPV phases β, β_s via gluonic penguin diagrams



- Leading order SM diagrams are connected by U-spin symmetry → interesting property to exploit when computing SM predictions





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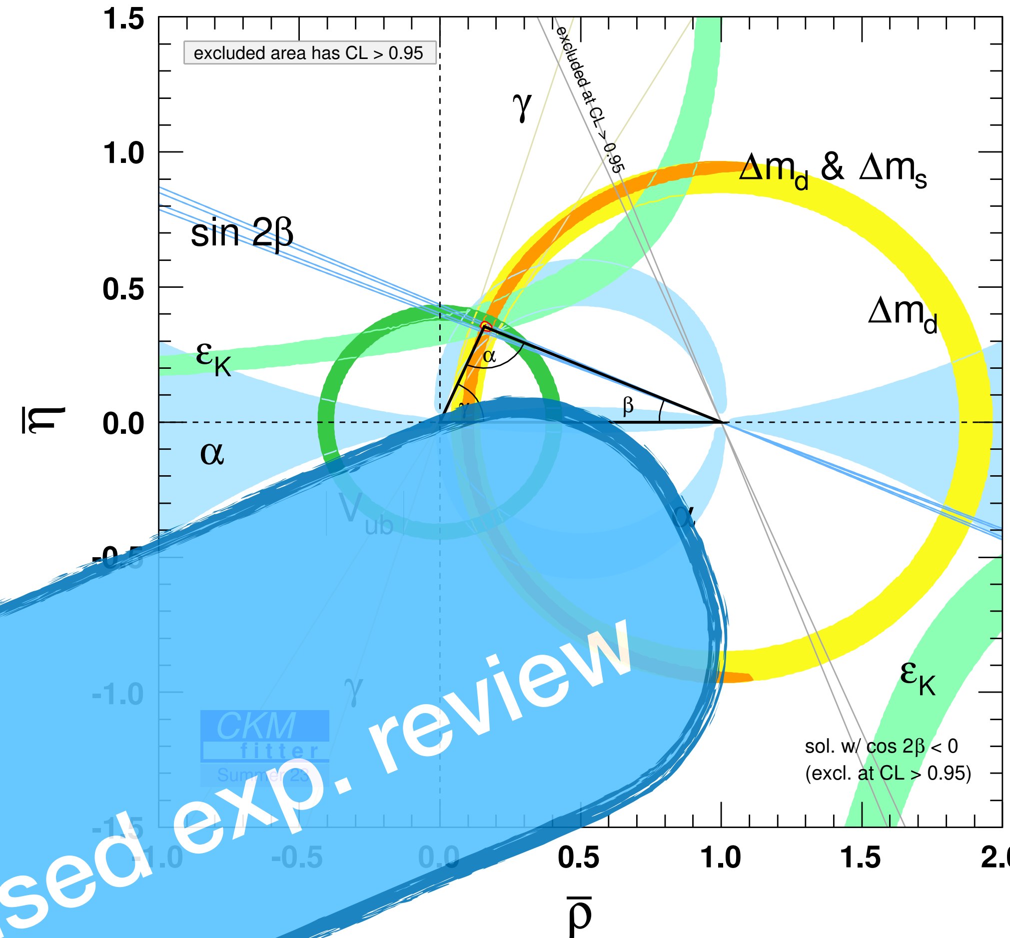

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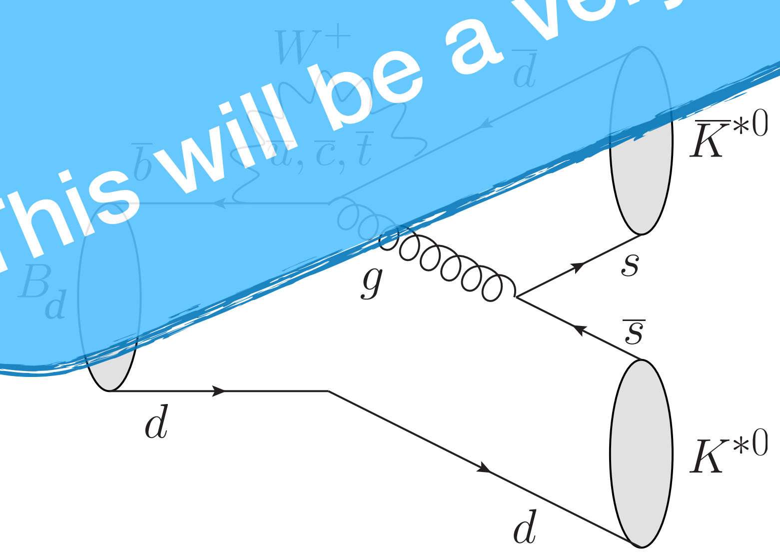
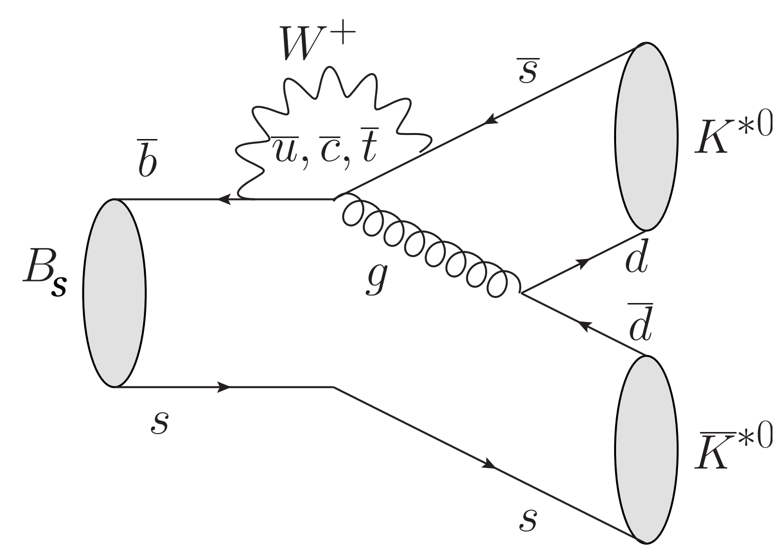
! Disclaimer !
This will be a very LHCb focused exp. review





$B \to K^{(*)0}\bar{K}^{(*)0}$ measurements

Level of difficulty, statistical power needed

- Integrated branching ratios

- At LHCb branching ratios are accessed relatively to control modes with same FS to cancel systematic uncertainties (detection effs., $\sigma_{b\bar{b}}$, L ...)

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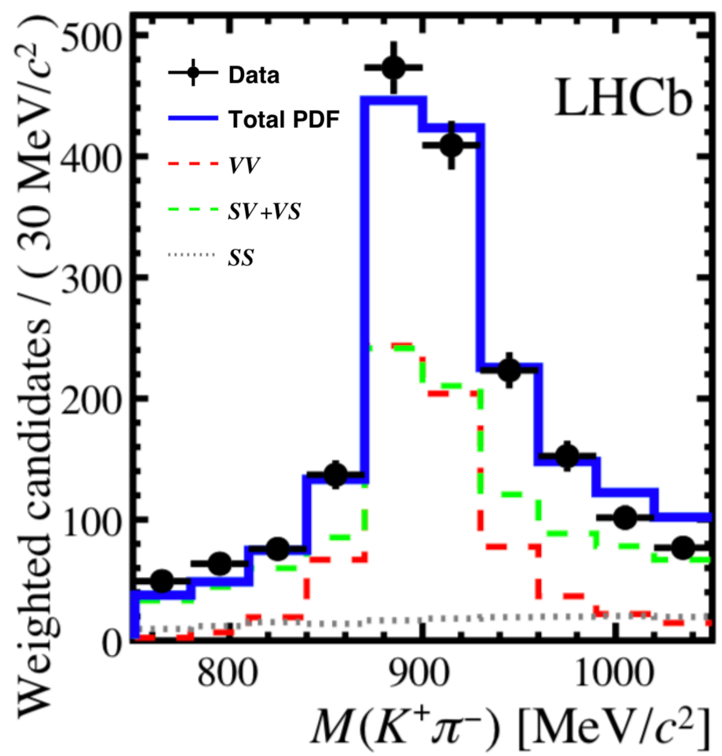
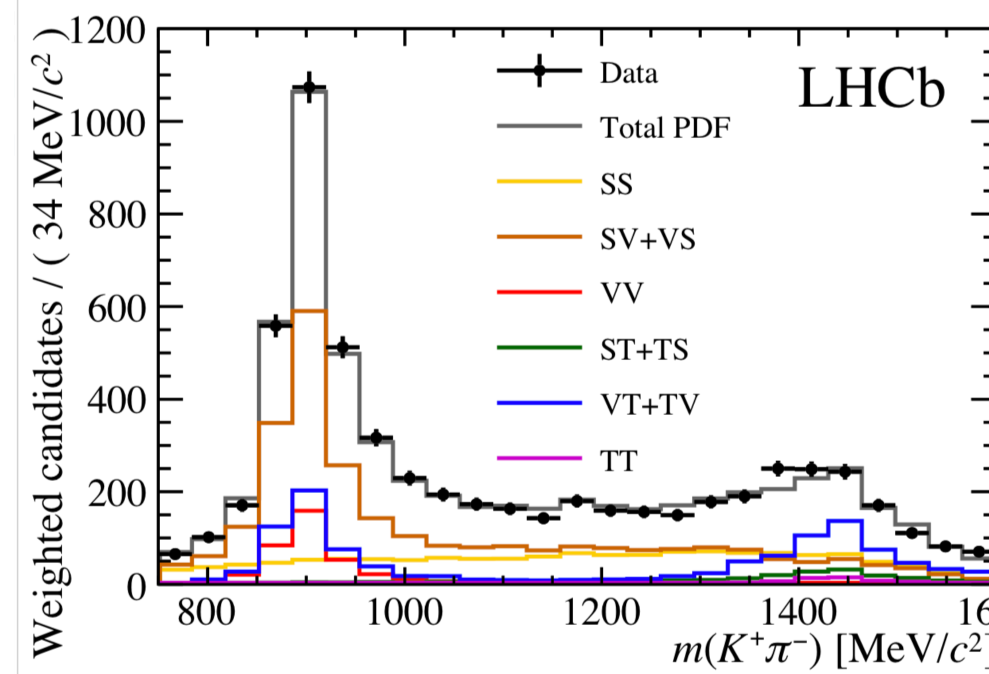
- $K^{*0}(892) \to K^{\pm}\pi^{\mp}$ FS are selected in $m_{K\pi}$ window →presence of broad $K^{*0}(800), K^{*0}(1430)$ and NR contributions to be included





[JHEP 07 (2019) 032] [LHCb: JHEP 03 (2018) 140]

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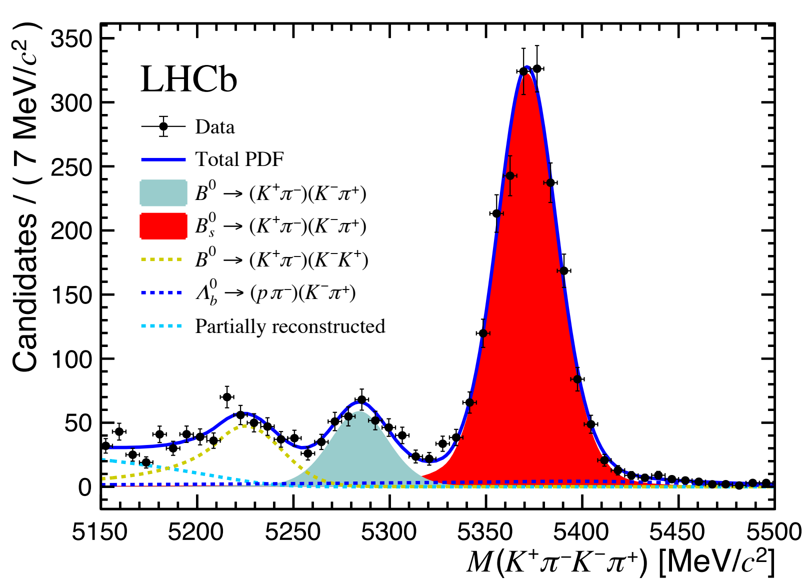
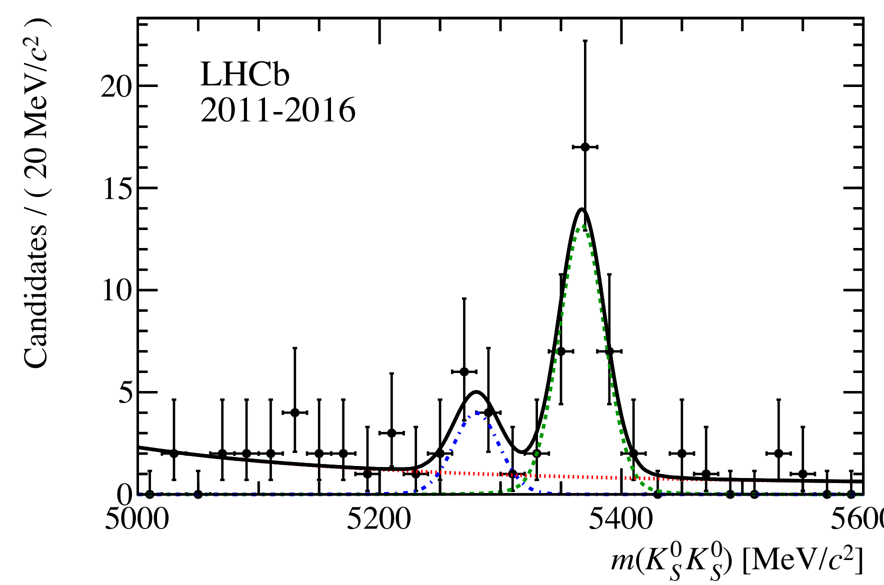
- K^0 are reconstructed as $K_S \to \pi^{\pm}\pi^{\mp}$ which are relatively long lived particles, roughly 2/3 of them decay outside of the VELO acceptance→Poor B momentum resolution and reconstruction efficiency



[JHEP 07 (2019) 032]



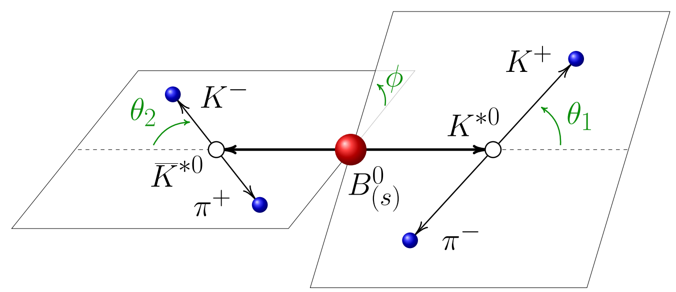
[PRD 102 (2020) 1, 012011]

$B \to K^{(*)0}\bar{K}^{(*)0}$ measurements

Level of difficulty, statistical power needed

- Integrated branching ratios
- Angular analyses

- Study of differential distributions of $B \to K^{(*)0}\bar{K}^{(*)0}$ decays offer access to rich angular structure



$$\frac{d\Gamma}{d\Omega dm_1 dm_2} \propto |\sum_i A_i g_i(m_1, m_2, \Omega)|^2$$

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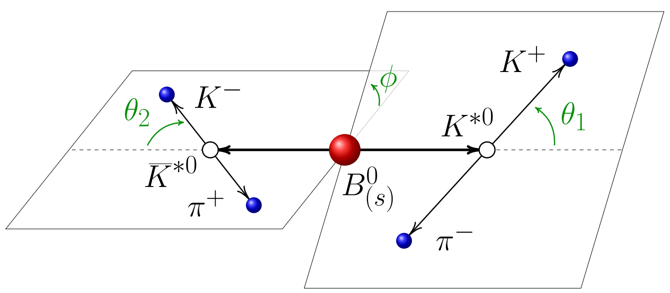

$$\frac{d\Gamma}{d\Omega dm_1 dm_2} \propto \left|\sum_i A_i g_i(m_1, m_2, \Omega)\right|^2$$

$P \to VV:$ 3 polarisation amplitudes, $A_0, A_\perp, A_\parallel$, extract their magnitude and phases together with their relative fractions

$$f_{L,\parallel,\perp} = \frac{|A_{0,\parallel,\perp}|^2}{|A_0|^2 + |A_\parallel|^2 + |A_\perp|^2}$$

$P \to PP, P \to VP:$ 1 polarisation amplitude

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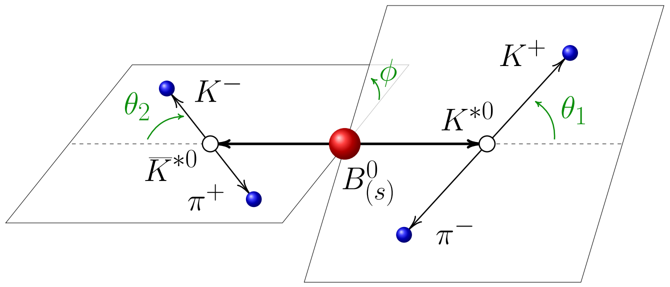

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- Require control of the warping effects due to angular acceptance, mass requirements and detector resolution
- Careful treatment of efficiency dependence on decay model and parametrisation

$B \to K^{(*)0}\bar{K}^{(*)0}$ measurements

Level of difficulty, statistical power needed

- Integrated branching ratios
- Angular analyses
- Time integrated CP asymmetries
- Time dependent CP asymmetries

- Flavour untagged angular analyses allow to access CP observables: [PRD.88.016007]
  - Triple product asymmetries involving products of the kind:
    $$\vec{q} \cdot (\vec{\varepsilon}_1 \times \vec{\varepsilon}_2)$$
  - (In)direct CP asymmetries accessible for polarisation amplitudes combinations where one of the A's is CP odd
    $$Re[A_h A^*_{h'} + \eta_h \eta_{h'} \bar{A}_h \bar{A}^*_{h'}]$$

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- Flavour tagged analyses allow to access to TD CP violation and measurement of the CKM angles β, β_s
  - Requires decay careful time acceptance modelling
  - Involves flavour tagging of the B at production

$B \to K^{(*)0}\bar{K}^{(*)0}$ measurements

Today's focus

Level of difficulty, statistical power needed

- Integrated branching ratios
- Angular analyses
- Time integrated CP asymmetries
- Time dependent CP asymmetries

- Flavour untagged angular analyses allow to access CP observables: [PRD.88.016007]
  - Triple product asymmetries
  - (In)direct CP asymmetries accessible as well
- Flavour tagged analyses allow to access to TD CP violation and measurement of the CKM angles β, β_s
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$B \to K^{*0}\bar{K}^{*0}$ at LHCb

- The heavy-quark limit implies the polarisation hierarchy $f_L \gg f_{\parallel,\perp}$ in $B_{(s)} \to K^{*0}\bar{K}^{0*}$ decays, with QCDF predicting [Nucl.Phys.B774:64-101,2007] :

$$f_L^{B_0} = 0.69^{+0.16}_{-0.20} \qquad f_L^{B_s} = 0.72^{+0.16}_{-0.21}$$

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t

[LHCb: Phys.Lett.B 709 (2012) 50]

[LHCb: JHEP 07 (2015) 166]

[LHCb: JHEP 03 (2018) 140]

[LHCb: JHEP 07 (2019) 032]

First observation of $B_s \to K^{*0}\bar{K}^{0*}$ with 35pb^{-1} of data

- Anomalously low value of $f_L^{B_s}$

$$f_L^{B_s} = 0.31 \pm 0.12(\text{stat}) \pm 0.04(\text{syst})$$

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TI CP asymmetries in $B_s \to K^{*0}\bar{K}^{0*}$ with $1\mathrm{fb}^{-1}$ of data

- Within uncertainties consistent with no CP violation
- Low value of $f_L^{B_s}$ confirmed

$$f_L^{B_s} = 0.201 \pm 0.057(\mathrm{stat}) \pm 0.040(\mathrm{syst})$$

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TD CP asymmetries in $B_s \to K^{*0}\bar{K}^{0*}$ with 3fb^{-1} of data

- First measurement of the CP-violating phase

$$\phi_s^{s\bar{s}} = -0.10 \pm 0.13(\text{stat}) \pm 0.14(\text{syst})$$

- Low value of $f_L^{B_s}$ confirmed

$$f_L^{B_s} = 0.208 \pm 0.032(\text{stat}) \pm 0.046(\text{syst})$$

$B \to K^{*0}\bar{K}^{*0}$ at LHCb

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[LHCb: JHEP 07 (2019) 032]

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Angular analysis of both B^0 and B_s with $3\mathrm{fb}^{-1}$ of data,

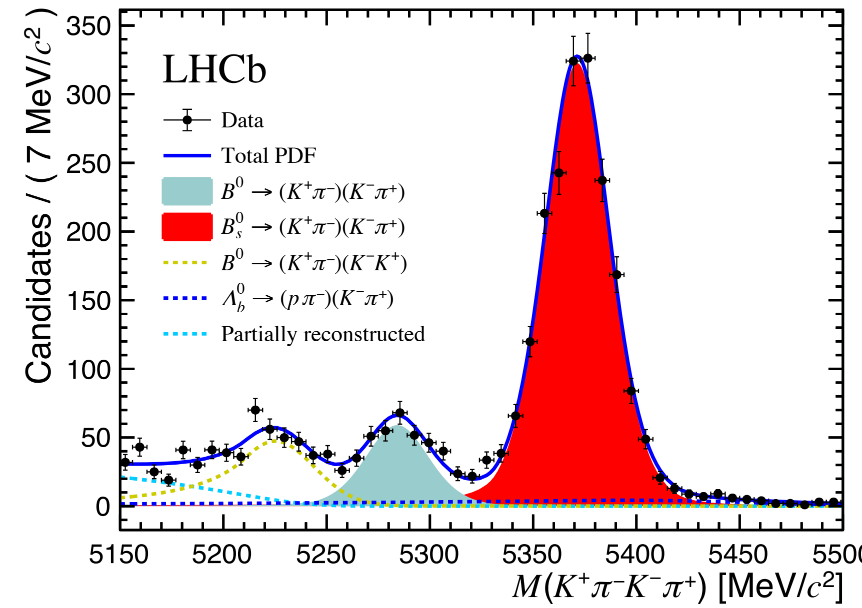
- $f_L^{B^0}$ well compatible with SM prediction!

$$f_L^{B^0} = 0.724 \pm 0.051(\mathrm{stat}) \pm 0.016(\mathrm{syst})$$

Amplitude analysis of $B_{(s)} \to K^{*0}\bar{K}^{0*}$ decays

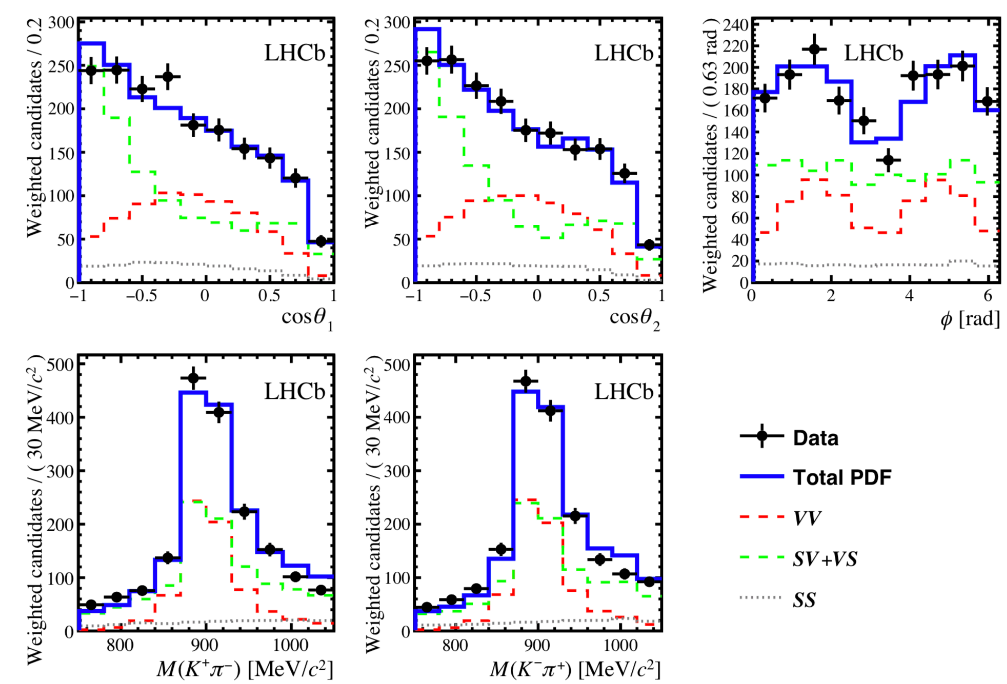
Analysis strategy, in brief:

- sWeight the 4-body invariant mass to disentangle B^0 from B_s contributions and remove backgrounds
- Perform an amplitude analysis to measure amplitude magnitudes and relative phases



- Account for both vector and scalar components, total of 6 amplitudes:

	A_i	η_i	$g_i(m_1, m_2, \theta_1, \theta_2, \phi)$
VV	A_0	1	$\cos\theta_1 \cos\theta_2 \mathcal{M}_1(m_1)\mathcal{M}_1(m_2)$
	$A_\parallel$	1	$\frac{1}{\sqrt{2}} \sin\theta_1 \sin\theta_2 \cos\phi \mathcal{M}_1(m_1)\mathcal{M}_1(m_2)$
	$A_\perp$	-1	$\frac{i}{\sqrt{2}} \sin\theta_1 \sin\theta_2 \sin\phi \mathcal{M}_1(m_1)\mathcal{M}_1(m_2)$
PV,VP	A_S^+	-1	$-\frac{1}{\sqrt{6}}(\cos\theta_1 \mathcal{M}_1(m_1)\mathcal{M}_0(m_2) - \cos\theta_2 \mathcal{M}_0(m_1)\mathcal{M}_1(m_2))$
	A_S^-	1	$-\frac{1}{\sqrt{6}}(\cos\theta_1 \mathcal{M}_1(m_1)\mathcal{M}_0(m_2) + \cos\theta_2 \mathcal{M}_0(m_1)\mathcal{M}_1(m_2))$
PP	A_{SS}	1	$-\frac{1}{3}\mathcal{M}_0(m_1)\mathcal{M}_0(m_2)$



Amplitude analysis of $B_{(s)} \to K^{*0}\bar{K}^{0*}$ decays

Results with Run 1 data:

Parameter	$B^0 \to K^{*0}\overline{K}^{*0}$	$B^0_s \to K^{*0}\overline{K}^{*0}$
f_L	$0.724 \pm 0.051 \pm 0.016$	$0.240 \pm 0.031 \pm 0.025$
S–wave fraction	$0.408 \pm 0.050 \pm 0.017$	$0.694 \pm 0.016 \pm 0.010$

$$\frac{\mathcal{B}(B^0 \to K^{*0}\bar{K}^{*0})}{\mathcal{B}(B_s \to K^{*0}\bar{K}^{*0})} = [7.58 \pm 0.57(\text{stat}) \pm 0.25(\text{syst}) \pm 0.16(f_s/f_d)]\%$$

Combine with previous $B_s \to K^{*0}\bar{K}^{0*}$ Br to get:

$$\mathcal{B}(B^0 \to K^{*0}\overline{K}^{*0}) = (8.0 \pm 0.9\,(\text{stat}) \pm 0.4\,(\text{syst})) \times 10^{-7}$$

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Results with Run 1 data:

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C

As pointed out in [JHEP06(2023)108] U-spin symmetry can be exploited to reduce theoretical uncertainties building ratios of longitudinally polarised branching ratios:

$$L_{K^*\bar{K}^*} = \rho(m_{K^*}, m_{\bar{K}^*}) \frac{f_L^{B_s}}{f_L^{B_d}} \frac{\mathcal{B}(B_s \to K^{*0}\bar{K}^{*0})}{\mathcal{B}(B_d \to K^{*0}\bar{K}^{*0})}$$

$$L^{\mathrm{SM}}_{K^*\bar{K}^*} = 19.53^{+9.14}_{-6.64} \quad \xleftrightarrow{2.6\,\sigma} \quad L^{\mathrm{exp}}_{K^*\bar{K}^*} = 4.43 \pm 0.92$$

Main sources of uncertainties are form $B \to K^*$ factors (currently LCSR from [Barucha, Straub, Zwicky] and affected by LCDA end point singularities, chance for discussion?

Amplitude analysis of $B_{(s)} \to K^{*0}\bar{K}^{0*}$ decays

An update using full Run1+Run2 is underway!

- Using naive luminosity scaling the measurement on $f_L^{B_s}$ is going to be systematically dominated:
  - Major contributions to syst. budget are from simulation sample size and S-wave mass model
  - Reduce the latter by decomposing the amplitudes using angular momentum eigenfunctions rather than helicity basis
  - Ensure translatability of the results between the two bases

Decay mode	
Parameter	f_L
Bias data-simulation	0.004
Fit method	0.001
Kinematic acceptance	0.011
Resolution	0.002
P–wave mass model	0.001
S–wave mass model	0.021
Differences data-simulation	0.002
Background subtraction	0.000
Peaking backgrounds	0.003
Time acceptance	0.008
Total systematic unc.	0.025

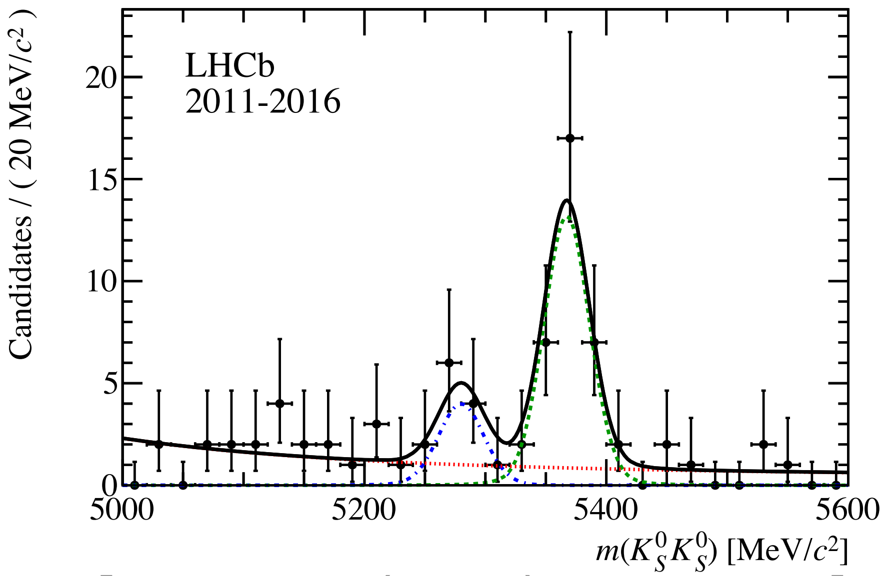
[JHEP 07 (2019) 032]

Currently in advanced state, inclusive Br measurements are almost finalised, angular fitter is well understood, soon to enter WG circulation

Br measurement of $B_{(s)} \to K^0\bar{K}^0$

Analysis strategy:

- Measure yields and efficiencies for the signal and normalisation modes to get the Br
- No amp analysis needed



[PRD 102 (2020) 1, 012011]

Results using Run1+2015,2016 data:

$$\mathcal{B}(B^0_s \to K^0_S K^0_S) = [8.3 \pm 1.6\,(\text{stat}) \pm 0.9\,(\text{syst}) \pm 0.8\,(\text{norm}) \pm 0.3\,(f_s/f_d)] \times 10^{-6},$$

$B^0 \to \phi K_S$

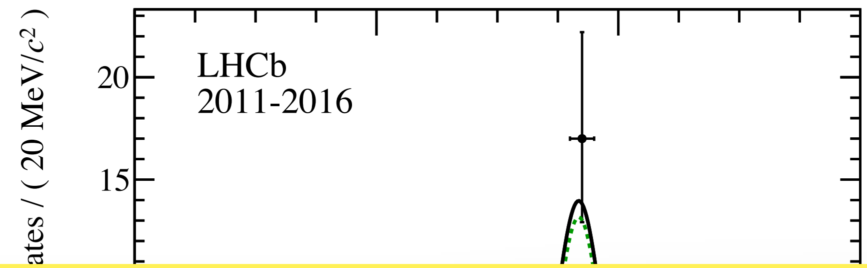
If $B_s \to K_S\bar{K}_S$ normalisation is chosen:

$$\frac{\mathcal{B}(B^0 \to K^0_S K^0_S)}{\mathcal{B}(B^0_s \to K^0_S K^0_S)} = [7.5 \pm 3.1\,(\text{stat}) \pm 0.5\,(\text{syst}) \pm 0.3\,(f_s/f_d)] \times 10^{-2}.$$

- Benefit from large systematic uncertainties cancellation in the ratio of Br yielding a statistically dominated result

Br measurement of $B_{(s)} \to K^0 \bar{K}^0$

Analysis strategy:



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As pointed out in [JHEP06(2023)108] U-spin symmetry can be exploited to reduce theoretical uncertainties building ratios of branching ratios:

$$L_{K\bar{K}} = \rho(m_{K^0}, m_{\bar{K}^0}) \frac{\mathcal{B}(B_s \to K^0 \bar{K}^0)}{\mathcal{B}(B_d \to K^0 \bar{K}^0)}$$

$$L_{K\bar{K}}^{\mathrm{SM}} = 26.00^{+3.88}_{-3.59} \quad \xleftrightarrow{2.4\,\sigma} \quad L_{K\bar{K}}^{\mathrm{exp}} = 14.58 \pm 3.37$$

Interesting coherent deviation in this mode as well, is this a systematic long distance effect or does this have short distance origin? (FF uncertainties lower in this case)

yielding a statistically dominated result

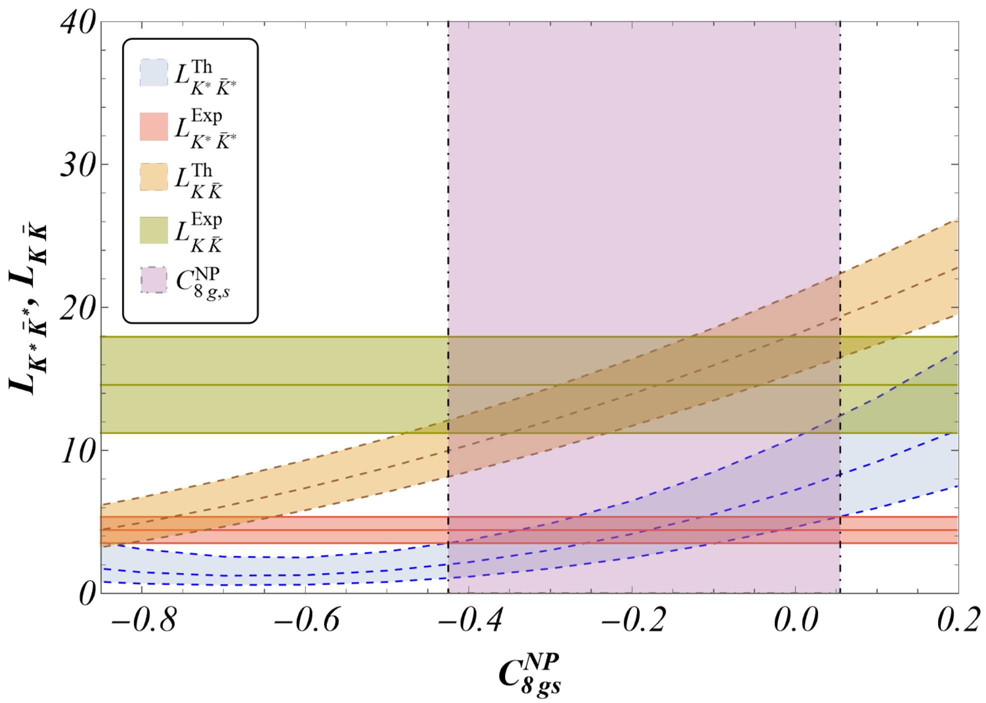
A coherent NP explanation?

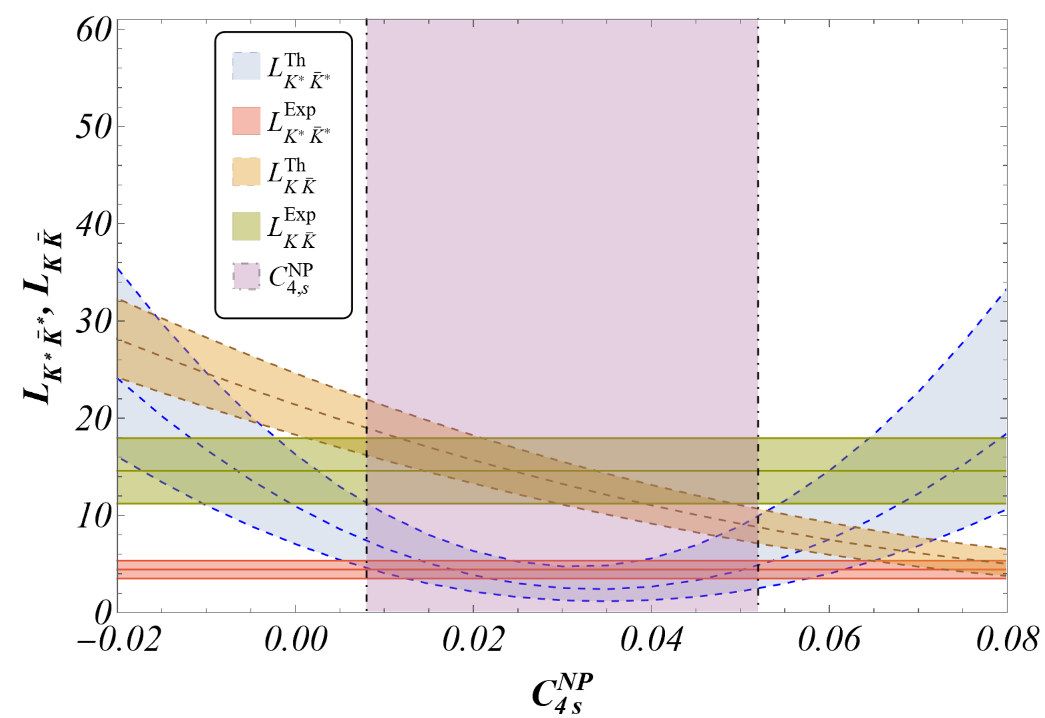
[JHEP06(2023)108]

Local and non-local contributions in $b \to q\bar{q}s$ transitions are separated via an effective hamiltonian

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{p=c,u} \lambda_p^{(q)} \Big(\mathcal{C}_{1s}^p Q_{1s}^p + \mathcal{C}_{2s}^p Q_{2s}^p + \sum_{i=3\ldots10} \mathcal{C}_{is} Q_{is} + \mathcal{C}_{7\gamma s} Q_{7\gamma s} + \mathcal{C}_{8gs} Q_{8gs} \Big)$$

- Express L observables as function of the WCs and constrain potential NP contribution



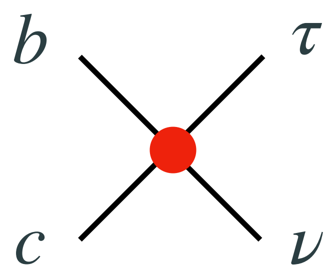


Beyond the Flavor Anomalies

- Three experimental "curiosities" of interest for this talk

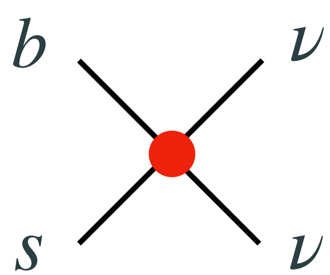
1. Charged-current semi-leptonic B-decays

$$R_D, R_{D^*}, R_{\Lambda_c}, R_{D^+}, \text{ etc}\,.$$ *(Talk by Markus + Patrick)*



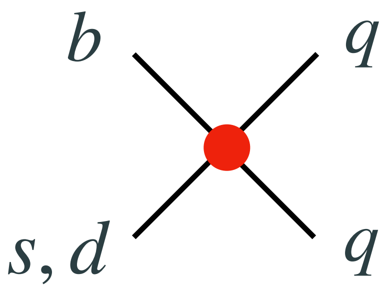
2. Neutral-current semi-leptonic B-decays

$$B \to K^{(*)}\nu\bar{\nu}$$ *(Talk by Caspar + Danny)*



3. Neutral-current non-leptonic B-decays

$$B_{s,d} \to K^{(*)}\bar{K}^{(*)}, \quad B_{s,d} \to K^{(*)}\phi$$



Neutral-current non-leptonic B-decays

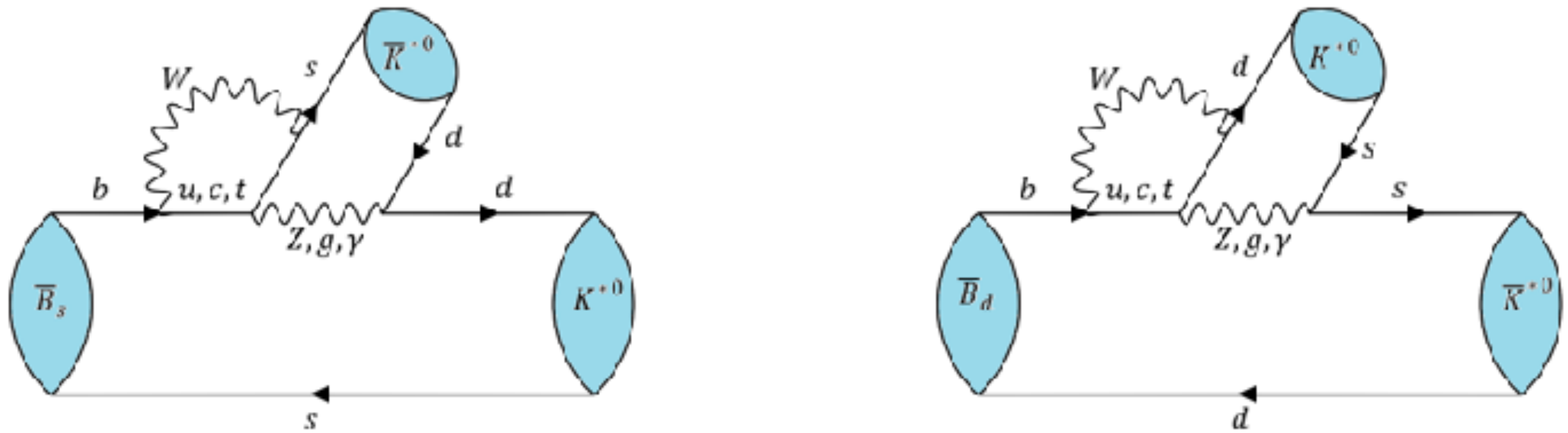
- Focus here on the $L_{K^{(*)}\bar{K}^{(*)}}$ observables:

$$L_{K^*\bar{K}^*} = \rho(m_{K^{*0}}, m_{K^{*0}}) \frac{\mathscr{B}(\bar{B}_s \to K^{*0}\bar{K}^{*0})}{\mathscr{B}(\bar{B}_d \to K^{*0}\bar{K}^{*0})} \frac{f_L^{B_s}}{f_L^{B_d}} = \frac{|A_0^s|^2 + |\bar{A}_0^s|^2}{|A_0^d|^2 + |\bar{A}_0^d|^2}$$

$$L_{K\bar{K}} = \rho(m_{K^0}, m_{K^0}) \frac{\mathscr{B}(\bar{B}_s \to K^0\bar{K}^0)}{\mathscr{B}(\bar{B}_d \to K^0\bar{K}^0)} = \frac{|A^s|^2 + |\bar{A}^s|^2}{|A^d|^2 + |\bar{A}^d|^2}$$

→ Longitudinal component of $B \to K^{(*)}\bar{K}^{(*)}$

**Disclaimer: Ratios reduce hadronic uncertainties but rescattering could be important in $b \to d$.*



[Algueró, Crivellin, Descotes-Genon, Matias, Novoa-Brunet, 2011.07867]
[Biswas, Descotes-Genon, Matias, Tetlalmatzi-Xolocotzi 2301.10542]

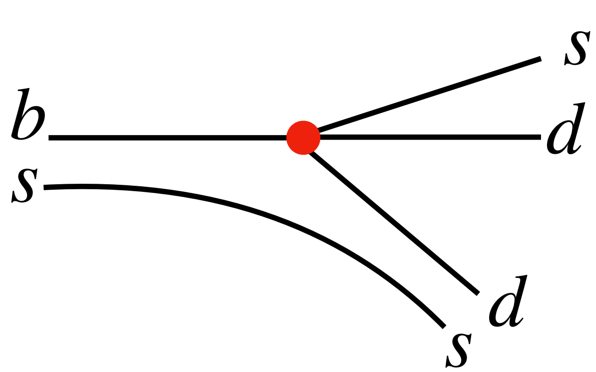
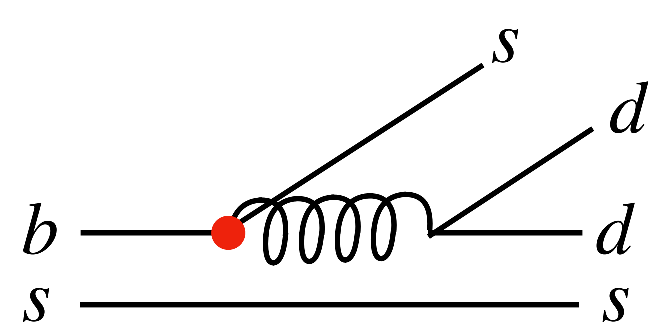
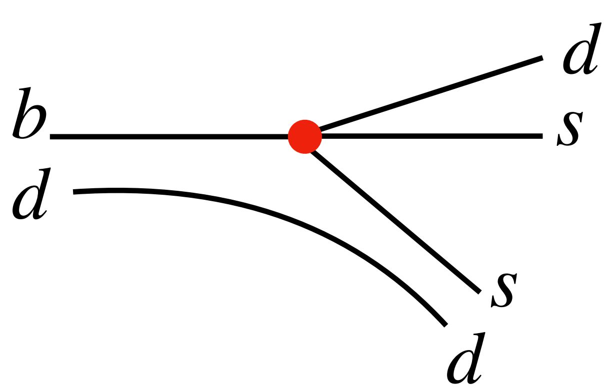
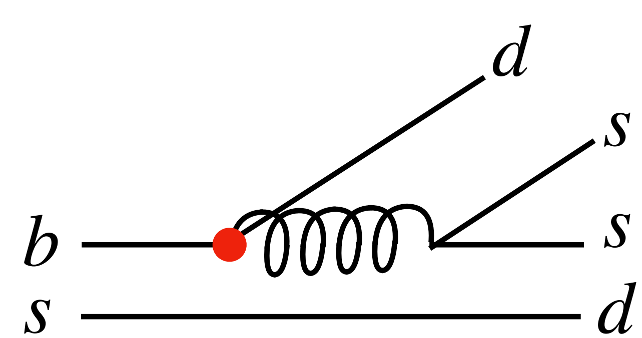
[Amhis, Grossman, Nir, 2212.03874]

Measurements and possible NP contributions

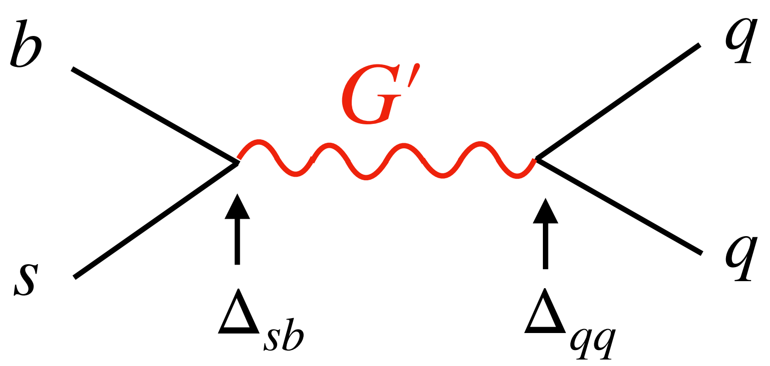
$$L^{\text{SM}}_{K^*\bar{K}^*} = 19.53^{+9.14}_{-6.64} \qquad L^{\text{exp}}_{K^*\bar{K}^*} = 4.43 \pm 0.92 \quad \longrightarrow \quad 2.6\sigma\,^*$$

$$L^{\text{SM}}_{K\bar{K}} = 26.00^{+3.88}_{-3.59} \qquad L^{\text{exp}}_{K\bar{K}} = 14.58 \pm 3.37 \quad \longrightarrow \quad 2.4\sigma\,^*$$

Discrepancies are not large + SM prediction is subject to theoretical challenges. Still, it is an interesting exercise to see if there are consistent short-distance NP explanations.

	4-quark op.	Gluon dipole
$\bar{B}_s \to K^{(*)}\bar{K}^{(*)}$ $(b \to s)$		
$\bar{B}_d \to K^{(*)}\bar{K}^{(*)}$ $(b \to d)$		

Going the 4-quark route seems difficult….



$$\mathcal{L} \supset \frac{2V_{tb}V_{ts}^*}{v_{\rm EW}^2} C_{4s} \, (\bar{s}_L^\alpha \gamma_\mu b_L^\beta)\,(\bar{q}^\beta \gamma_\mu q^\alpha)$$

$$\mathcal{L} \supset \Delta_{sb}^L (\bar{s}_L \gamma^\mu b_L)\, G'_\mu + \Delta_{sb}^R (\bar{s}_R \gamma^\mu b_R)\, G'_\mu + \sum_i \Delta_{qq} (\bar{q}_i \gamma^\mu q_i)\, G'_\mu$$

- $L_{K^{(*)}\bar{K}^{(*)}}$ observables: $\dfrac{\Delta_{sb}\Delta_{qq}}{m_{G'}^2} \sim \dfrac{1}{(5\,\mathrm{TeV})^2}$
- From di-jet searches: $\dfrac{\Delta_{qq}^2}{m_{G'}^2} \lesssim \dfrac{1}{(5\,\mathrm{TeV})^2}$

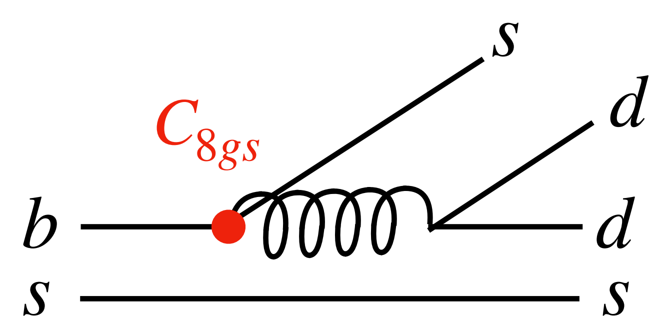
$$\frac{\Delta_{sb}}{m_{G'}} \sim \frac{1}{(5\,\mathrm{TeV})}$$

- B_s mixing: $\dfrac{\Delta_{sb}^2}{m_{G'}^2} \lesssim \dfrac{1}{(100\,\mathrm{TeV})^2}$

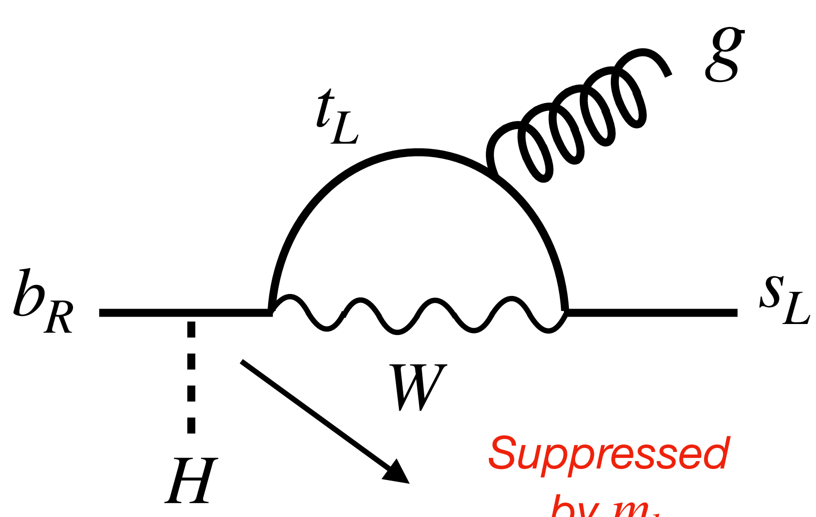
*Can fine tune Δ_{sb}^R: [Algueró, Crivellin, Descotes-Genon, Matias, Novoa-Brunet, 2011.07867]

Chromomagnetic (Gluon) Dipole

$$-\mathscr{L} \supset \frac{m_b V_{ts}^* V_{tb}}{4\pi^2\, v_{\rm EW}^2} C_{8gs} (\bar{s}_L \sigma_{\mu\nu} b_R) G^{\mu\nu}$$

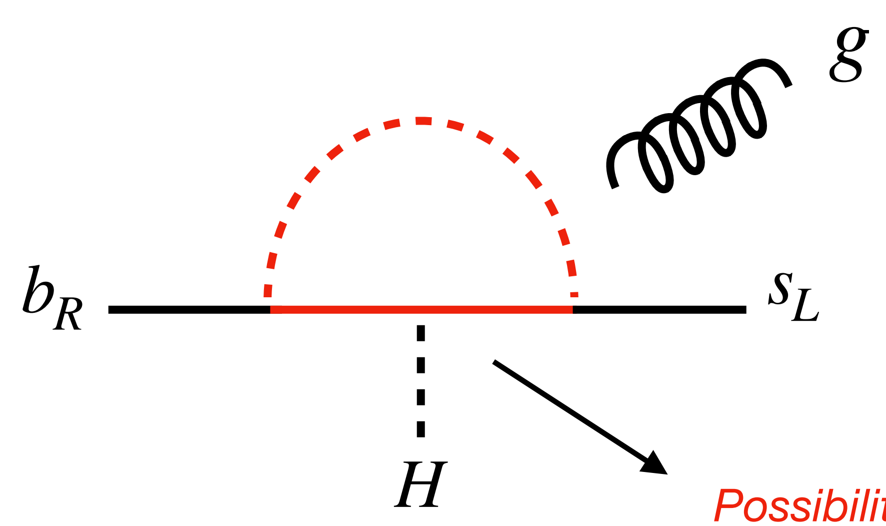


In the SM:



Suppressed by m_b

New physics:



Possibility to get a chiral enhancement!

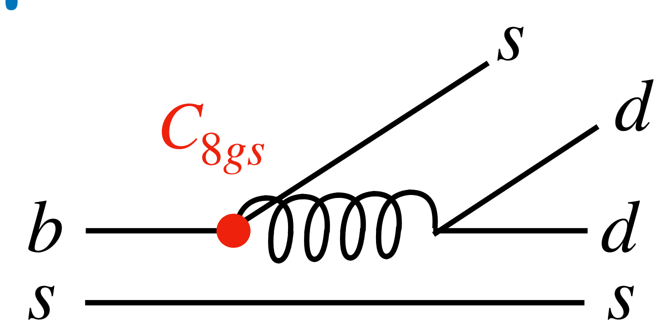
**We need a SM-sized effect, which points toward a low NP scale:*

We need
$C_{8gs}^{\rm NP} \sim C_{8gs}^{\rm SM} = -0.15$

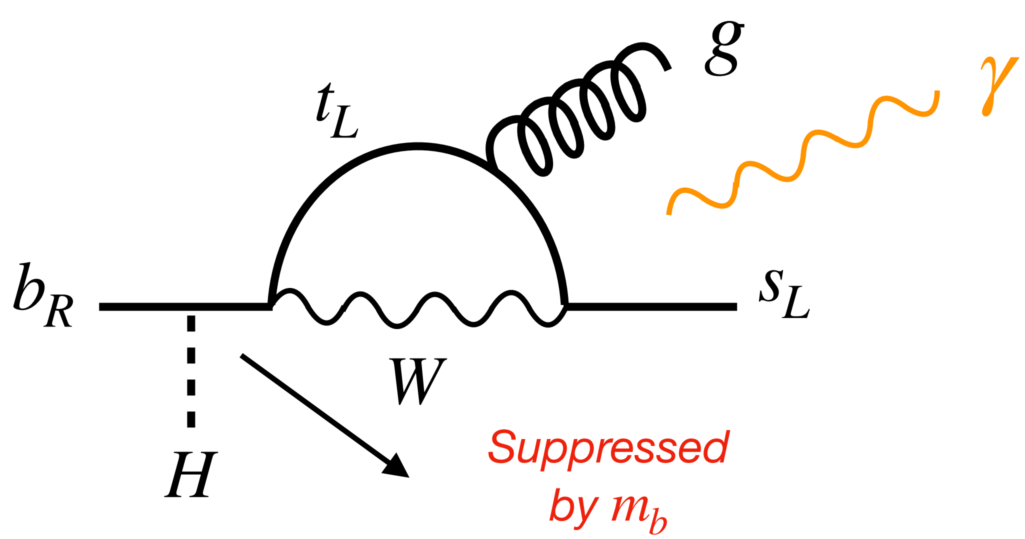
$$\Longrightarrow \quad \frac{m_b}{m_W^2} \approx \frac{m_t}{\Lambda_{\rm NP}^2} \quad \Longrightarrow \quad \Lambda_{\rm NP} \approx m_W \sqrt{\frac{m_t}{m_b}} \approx 500\ \text{GeV}$$

Comes with the electromagnetic dipole!

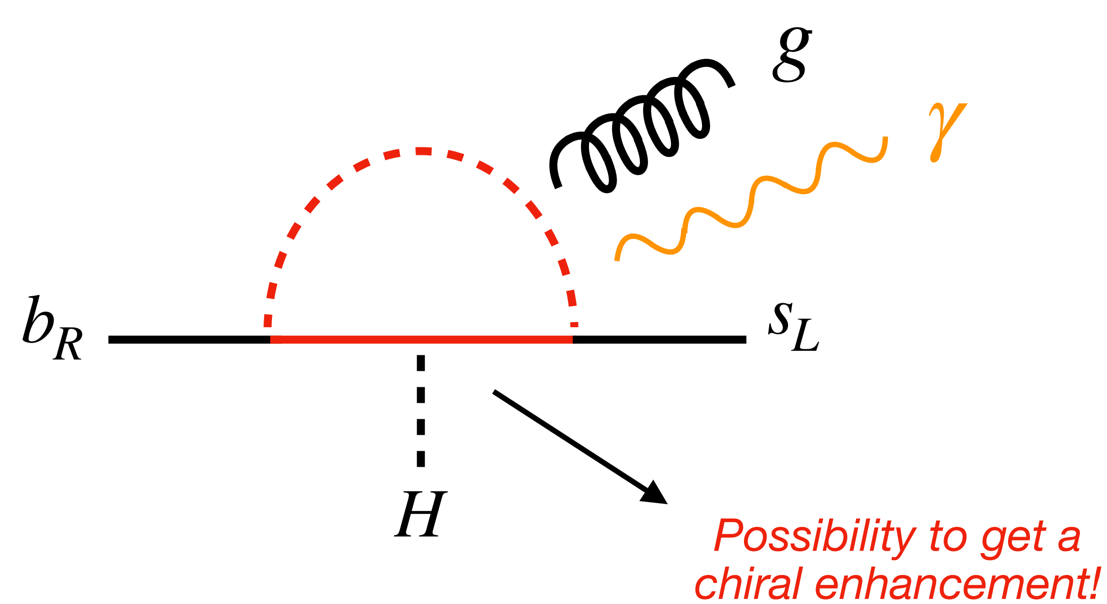
$$-\mathscr{L} \supset \frac{m_b V_{ts}^* V_{tb}}{4\pi^2\, v_{\mathrm{EW}}^2} C_{8gs}(\bar{s}_L \sigma_{\mu\nu} b_R) G^{\mu\nu}$$



In the SM:



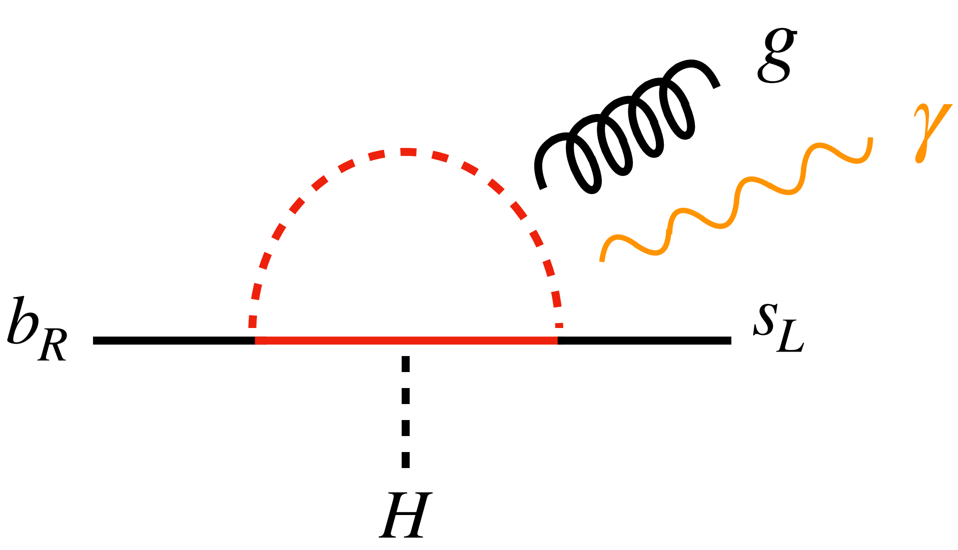
New physics:



- A challenge for any NP model generating the chromomagnetic dipole is then to explain how you pass the bounds from $B \to X_s \gamma$.

A closer look at the electromagnetic dipole

New physics:



- If color flow in the loop follows the flow of electric charge, then we have the tree-level prediction of:

$$C_{7\gamma s}/C_{8gs} = Q_{\text{loop}} = -1/3$$

$B \to X_s \gamma$ Th : $\mathcal{B}_{s\gamma} \times 10^4 = (3.39 \pm 0.17) - 2.10\ (3.93\, C_{7\gamma s} + C_{8gs})_{\mu_{\text{EW}}}$

Naively some partial accidental cancellation, could be a good model building starting point.

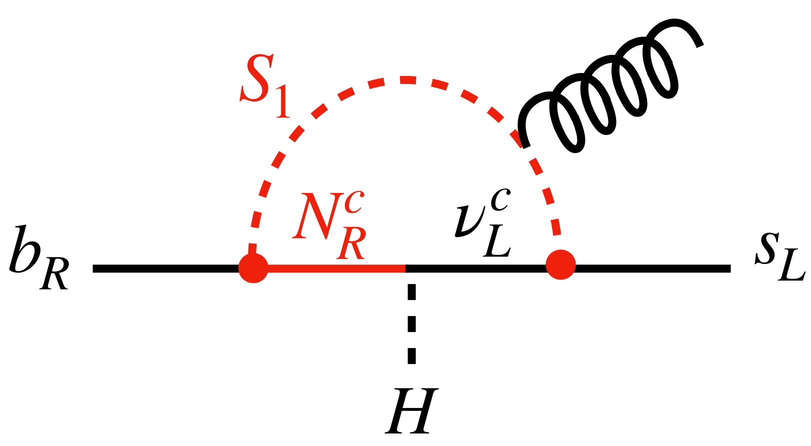
- Two options for the colored NP mediator: [Misiak, Rehman, Steinhauser, 2002.01548]

Color along the bosonic line — $S_1 \sim (\mathbf{3}, \mathbf{1}, -1/3)$

Color along the fermionic line — $Q \sim (\mathbf{3}, \mathbf{2}, 1/6)$

A scalar leptoquark model

- We go for the $S_1 \sim (\mathbf{3}, \mathbf{1}, -1/3)$ scalar leptoquark option, since it is one of three mediators that can explain the charged-current B anomalies (R_D, R_{D^*}, etc .)



$N_{L,R}$ → Psuedo-Dirac Singlet

Chiral Enhancement: TeV-scale N_R with an O(1) Yukawa

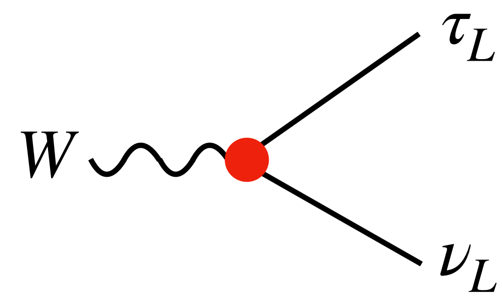
$$\mathscr{L} \supset -y_N \bar{\ell}_L^3 \tilde{H} N_R - M_R \bar{N}_L N_R - \frac{1}{2}\mu \bar{N}_L N_L^c$$

$$\nu_L^3 \to \cos(\theta_\tau)\,\nu_L^3 + \sin(\theta_\tau)\,N_L$$

Inverse seesaw

$$m_\nu \approx \frac{y_N^2 v^2}{2M_R^2}\mu \equiv \theta_\tau^2\,\mu$$

Correlated deviations in precision observables:



$\theta_\tau \lesssim 0.05$

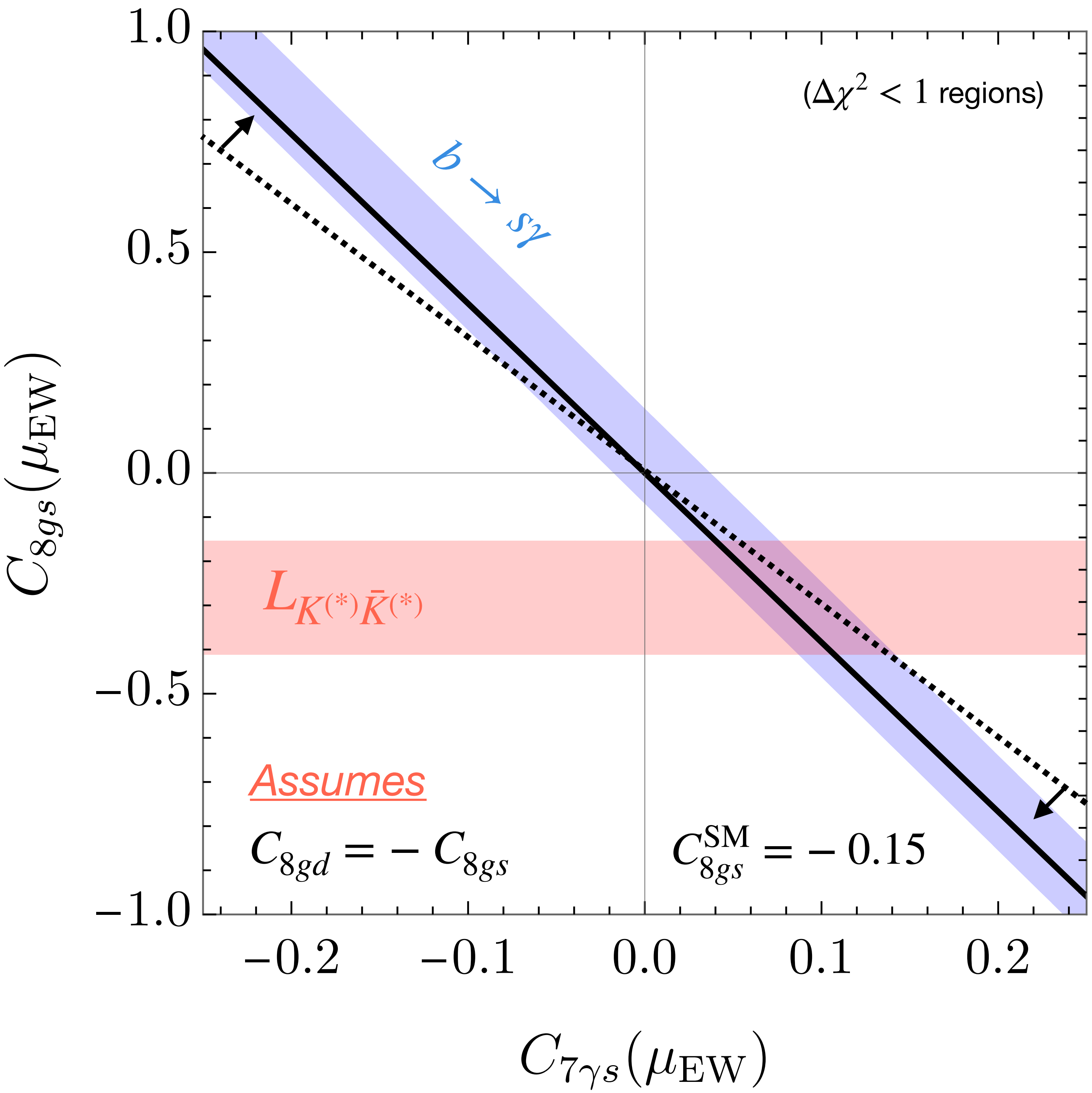
(EWPD + τ decays)

[Lizana, Matias, BAS, 2306.09178]

Evading the electromagnetic dipole

$S_1 \sim (\mathbf{3}, \mathbf{1})_{-1/3}$

$$\mathcal{B}_{s\gamma} \times 10^4 = (3.39 \pm 0.17) - 2.10\,(3.93\,C_{7\gamma s} + C_{8gs})_{\mu_{\rm EW}}$$



Dashed line: No RGE

$$\frac{C_{7\gamma s}}{C_{8gs}} \approx -\frac{1}{3}$$

Solid line: Includes RGE from $2\,\text{TeV}$ to $\mu_{\rm EW}$

$$\frac{C_{7\gamma s}}{C_{8gs}} \approx -\frac{1}{3.8}$$

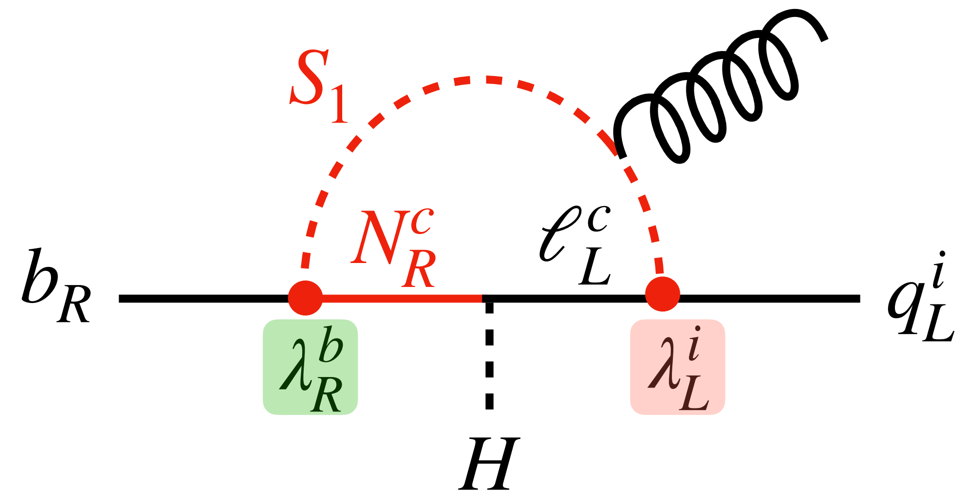
$$\begin{pmatrix} [C_{dG}]_{i3} \\ [C_{dB}]_{i3} \\ [C_{dW}]_{i3} \end{pmatrix}_{\mu_{\rm EW}} = \begin{pmatrix} 0.952 & 0.001 & -0.036 \\ 0.016 & 0.932 & -0.016 \\ -0.047 & -0.002 & 0.909 \end{pmatrix} \begin{pmatrix} [C_{dG}]_{i3} \\ [C_{dB}]_{i3} \\ [C_{dW}]_{i3} \end{pmatrix}_{M_1}$$

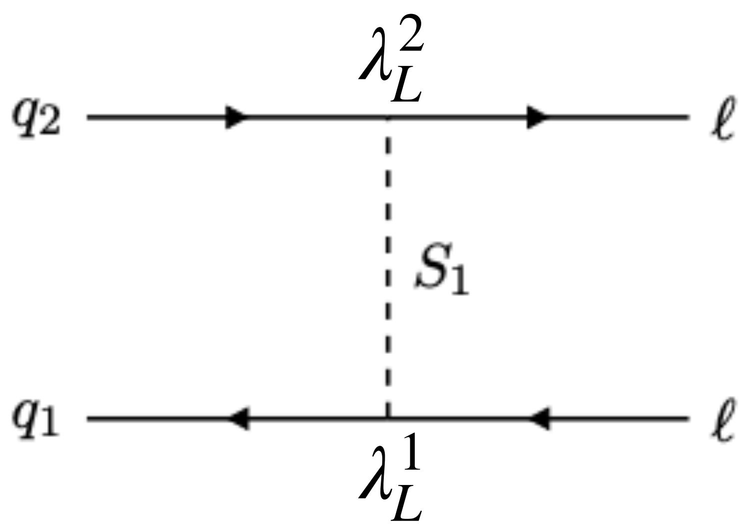
[Lizana, Matias, BAS, 2306.09178]

But, vanilla version has FCNC issues…

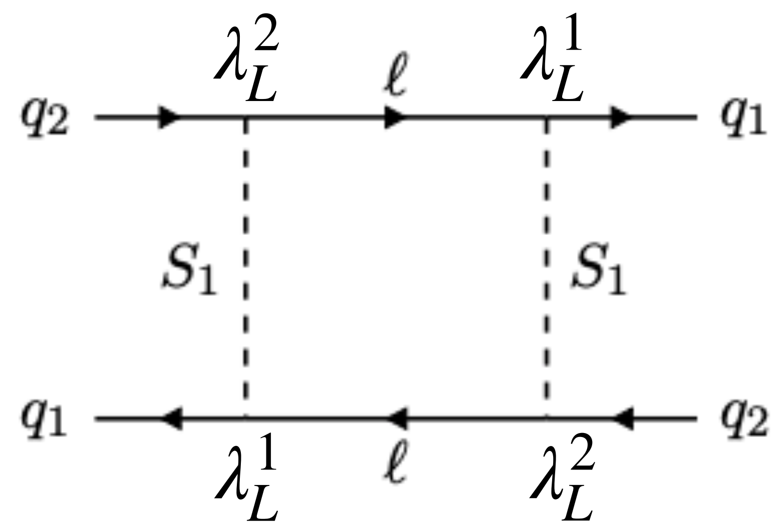
We need the following couplings:

$$\mathscr{L} \supset \lambda_L^i \bar{q}_L^{ci} \epsilon \ell_L^3 S_1 + \lambda_R^b \bar{b}_R^c N_R S_1$$





$K \to \pi\nu\bar{\nu}$



$K - \overline{K}$ / $D - \overline{D}$

- Fundamental issue here is that we need $\lambda_L^i \approx (-V_{td}/V_{ts}\,, 1) \times 0.3$, giving a larger-than-CKM breaking of $U(2)_q$.

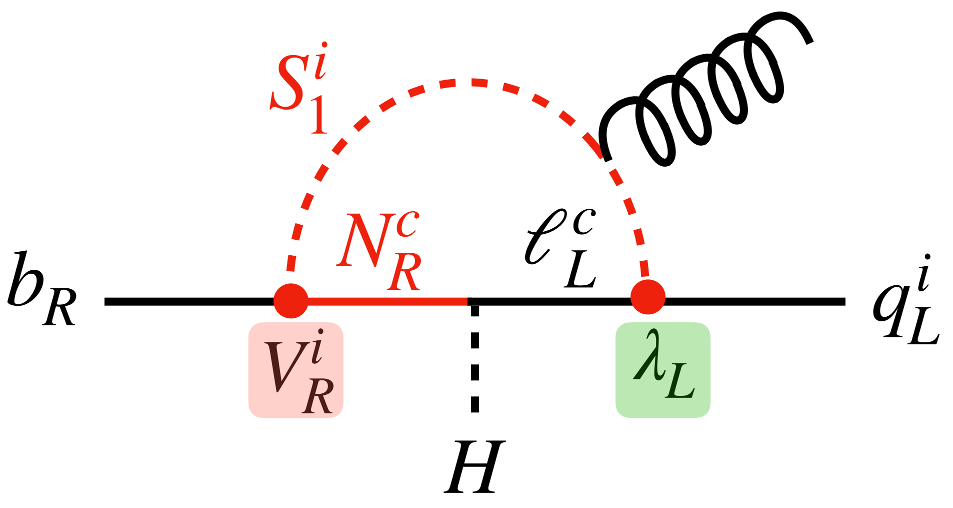
[Lizana, Matias, BAS, 2306.09178]

The way out: Add flavor to the LQ

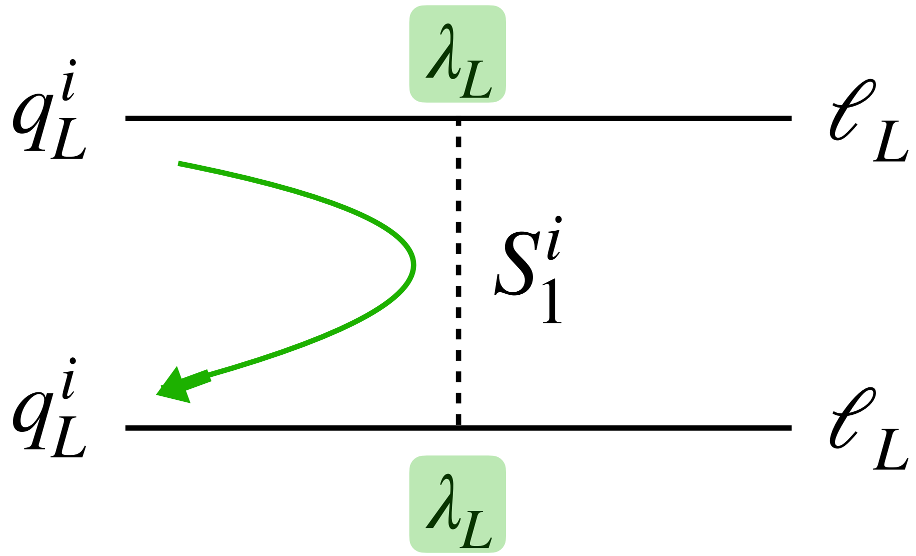
We promote:

$$S_1 \to S_1^i\,, \quad [\mathbf{2} \text{ of } U(2)_q]$$

$$\mathscr{L} \supset \lambda_L \bar{q}_L^{ci} \epsilon \ell_L^3 S_1^i + V_R^i \bar{b}_R^c N_R S_1^i$$



- FCNC's now protected since we have one LQ for each flavor (similar to squarks in SUSY). Shifts the breaking of $U(2)_q$ to the coupling V_R^i. At low energy, this coupling only enters via loops, like in the chromomagnetic dipole we need.

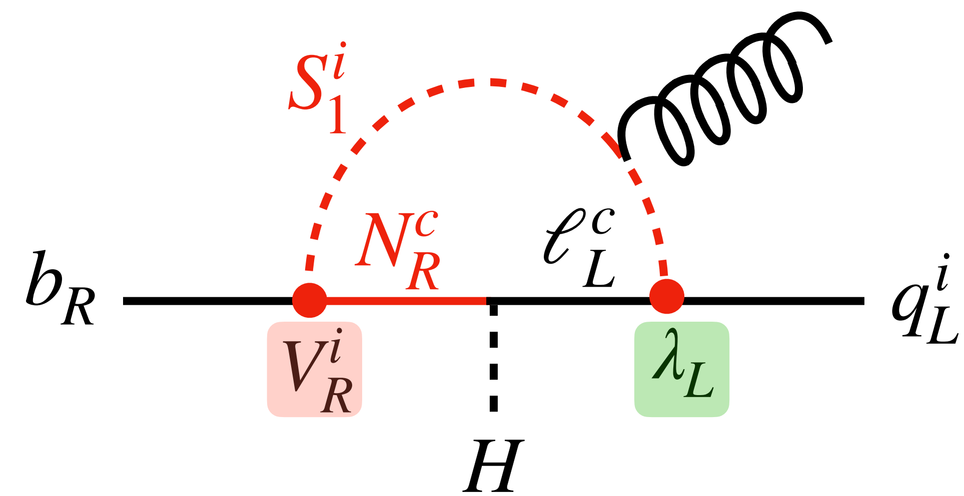


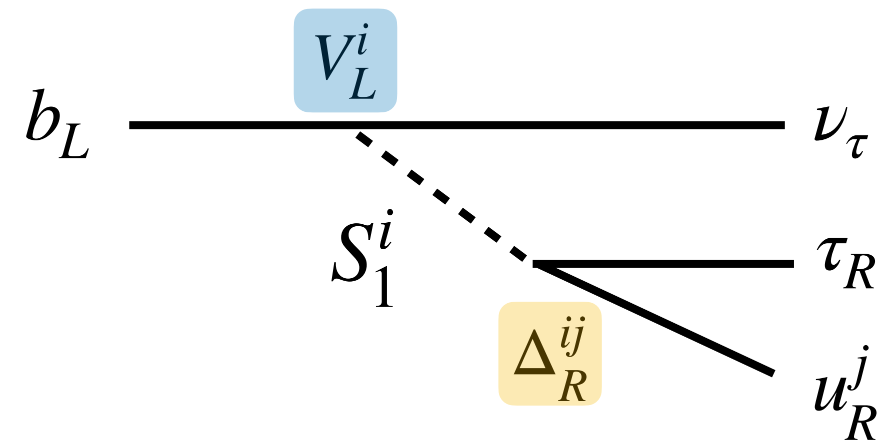
[Lizana, Matias, BAS, 2306.09178]

What about $b \to c\tau\nu$?

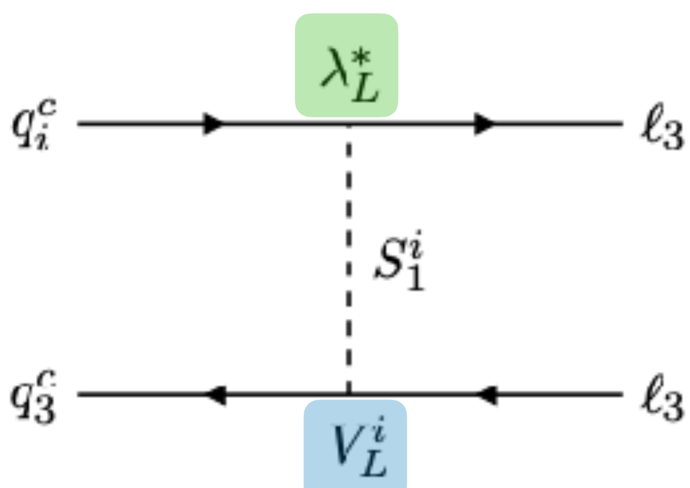
<u>The full model:</u>

$$\mathscr{L} \supset \lambda_L \bar{q}_L^{ci} \epsilon \ell_L^3 S_1^i + V_R^i \bar{b}_R^c N_R S_1^i + V_L^i \bar{q}_L^{c3} \epsilon \ell_L^3 S_1^i + \Delta_R^{ij} \bar{u}_R^{ci} \tau_R S_1^j$$

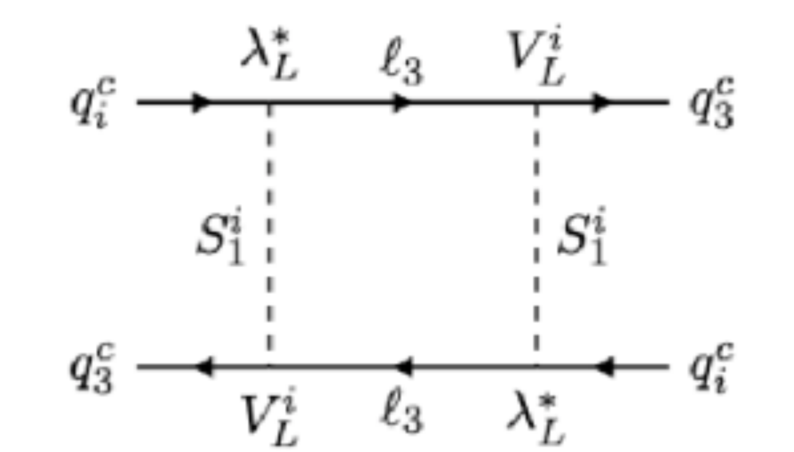




- New couplings needed for RD/RD*. Maybe not so nice at first glance, but these couplings are new U(2)-breaking sources. Generate new FCNC's:



$B \to K(\pi)\nu\bar{\nu}$



$B_{s,d} - \bar{B}_{s,d}$

$$V_L = (\epsilon_L, 1)\, \lambda_L^b$$

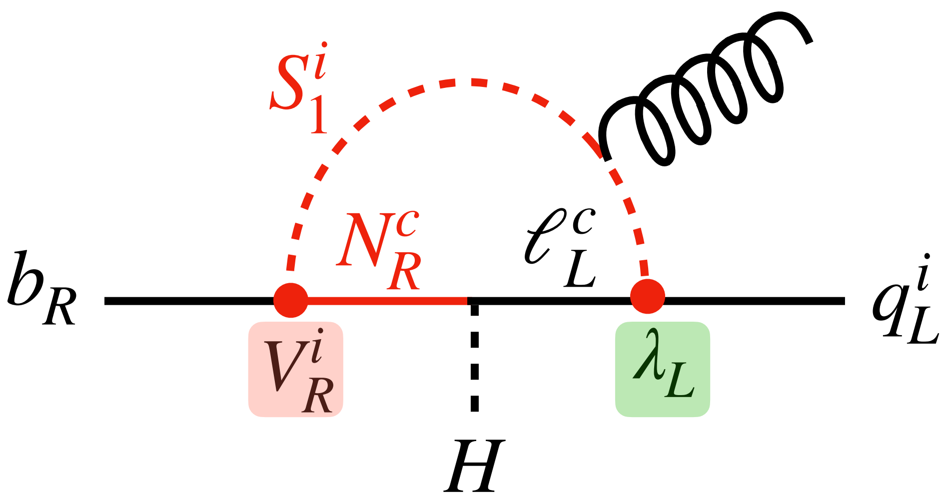
$$\Delta_R^{ij} = \lambda_R \frac{m_t}{m_c} y_u^{ij}$$

[Lizana, Matias, BAS, 2306.09178]

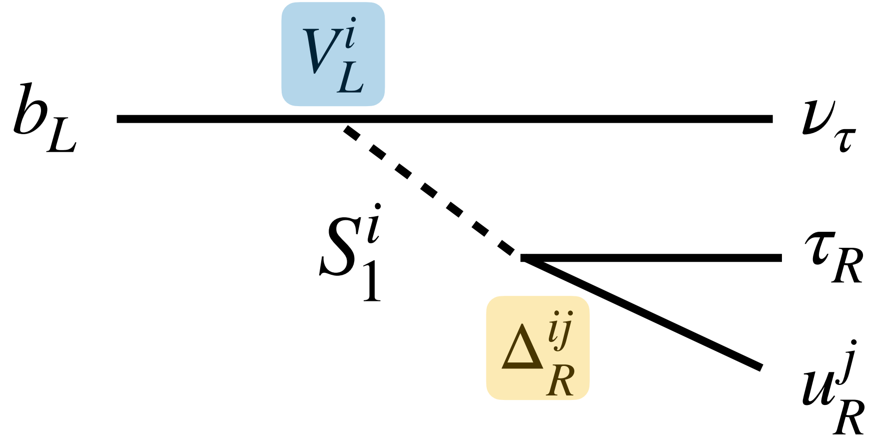
What about $b \to c\tau\nu$?

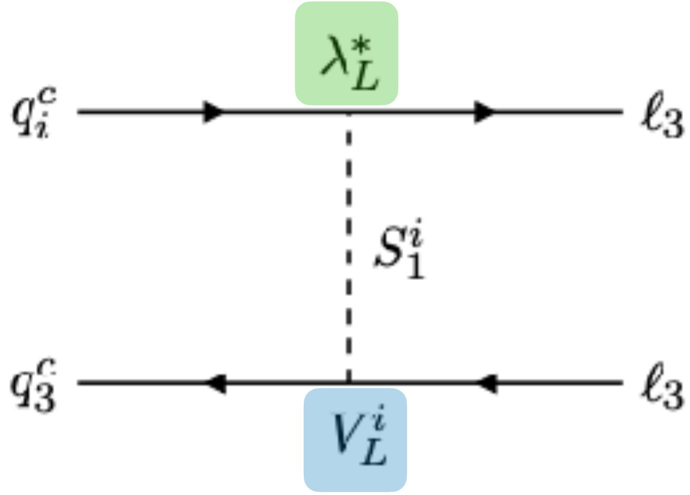
<u>The full model:</u>

$$\mathscr{L} \supset \lambda_L \bar{q}_L^{ci} \epsilon \ell_L^3 S_1^i + V_R^i \bar{b}_R^c N_R S_1^i + V_L^i \bar{q}_L^{c3} \epsilon \ell_L^3 S_1^i + \Delta_R^{ij} \bar{u}_R^{ci} \tau_R S_1^j$$



- New couplings needed for RD/RD*. Maybe not so nice at first glance, but these couplings are new U(2)-breaking sources. Generate new FCNC's:





$B \to K(\pi)\nu\bar{\nu}$

Combining the non-leptonic and charged-current anomalies predicts $B \to K\nu\bar{\nu} \propto \lambda_L V_L$. Same combination gives a sub-dominant vector contribution to RD/RD that improves the fit.*

[Lizana, Matias, BAS, 2306.09178]

Putting everything together

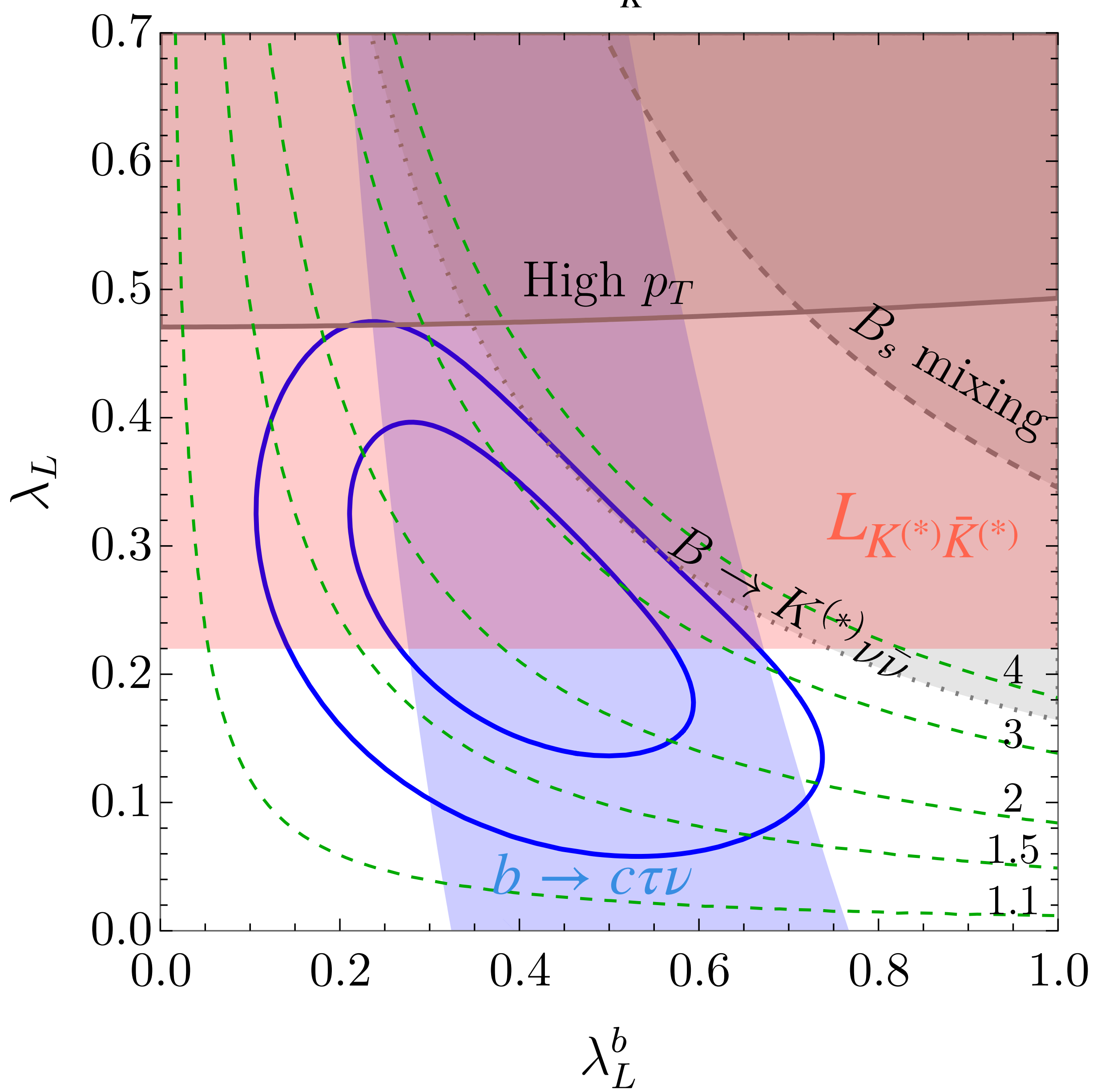
[Lizana, Matias, BAS, 2306.09178]

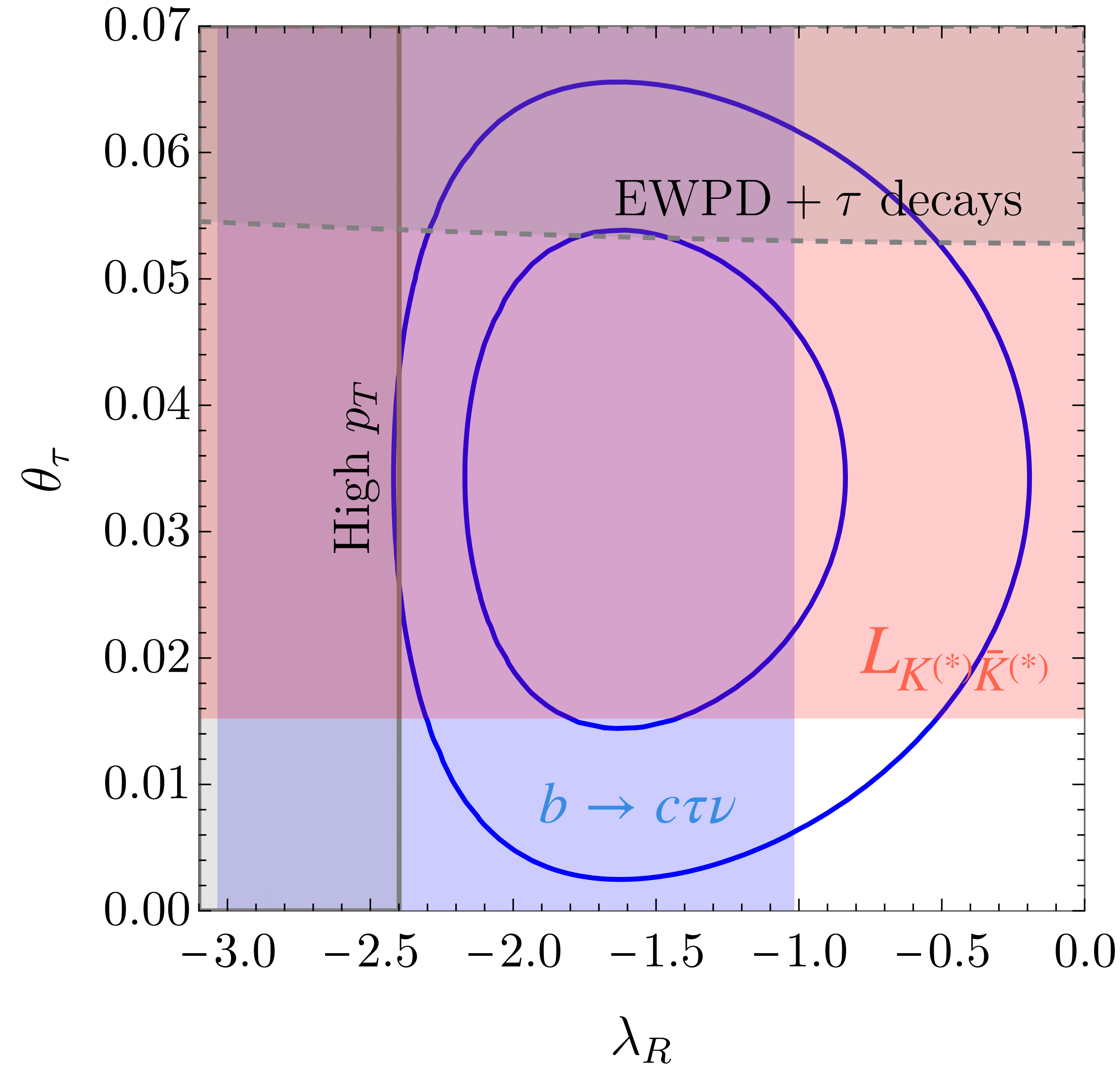
$B \to K\nu\bar{\nu}$ Average:

$$R_K^\nu = \frac{\mathscr{B}(B^+ \to K^+\nu\bar{\nu})_{\text{exp}}}{\mathscr{B}(B^+ \to K^+\nu\bar{\nu})_{\text{SM}}} = 2.8 \pm 0.8$$

$pp \to \tau \not{E}_T$ $pp \to \tau\tau$

- - - - R_K^ν contours





Conclusions

- Not so easy to have a consistent explanation of the $B_{s,d} \to K^{(*)}\bar{K}^{(*)}$ non-leptonic puzzle from heavy NP at short distances.

- The best option we found is going for NP in the gluon dipole, choosing a mediator with the right quantum numbers to pass the associated FCNC bound from $B \to X_s\gamma$.

- Interestingly, these criteria allow the S_1 LQ as a possible mediator, which is also 1 of only 3 mediators that can provide an explanation for hints of LFUV in $b \to c\tau\nu$ transitions.

- While the couplings needed are distinct for $B_{s,d} \to K^{(*)}\bar{K}^{(*)}$ and $b \to c\tau\nu$, their combined explanation necessarily leads to an enhancement in $\mathscr{B}(B \to K\nu\bar{\nu})$, as hinted by current data. It is intriguing that these three "curiosities" can be consistently connected via a single dynamical mediator.

Discussion points

- Interesting modes, plethora of observables, LHCb capabilities in reconstructing $B \to 4h$ with $h = \pi, K$ final states offer great precision on measurement related to these decays

- We are working towards update with full Run1+Run2 statistics + Run3 update of many different $B \to VV$ modes

    - $B_{(s)} \to K^{(*)}\bar{K}^{(*)}, B_s \to \phi K^{*,0}, B_d \to \rho K^{*,0}, B^+ \to \rho K^{*+}, B_{(s)} \to \omega K^{*,0} \ldots$

[Related talk by Aritra and Gilberto tomorrow!]

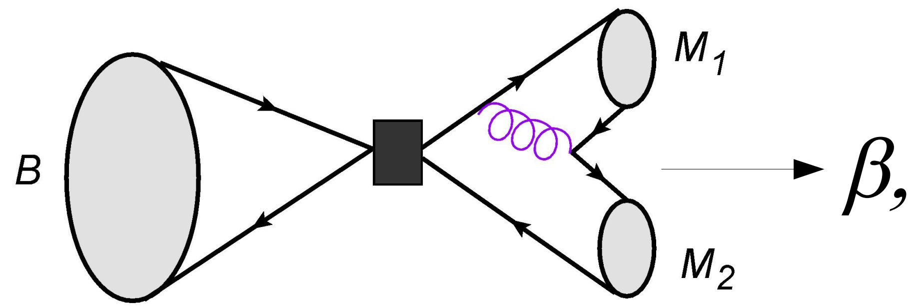
- Experimental precision on $L_{K^*\bar{K}^*}$ is $\sim 20\,\%$ while the theory prediction QCDf (naive SU(3)) is 14%-40% for PP and VV modes (18(?)% - 50%)

    - Experimental precision is expected to increase thanks to increase in statistics and usage of covariant formalism to reduce systematic uncertainties

    - Important to work towards reducing the SM prediction uncertainty

Discussion points

- Can data be used to constrain the contributions from annihilation topolgies?



  - Use the idea by T. Huber, G. Tetlalmatzi-Xolocotzi: constrain size QCD-factorisation amplitudes through SU(3) symmetry in $B \to V_1 V_2$ decays [EPJC 82 (2022) 3, 210]

- Two ways of representing the amplitudes $B \to M_1 M_2$ with $M_i = P, V$ mesons:

  - Topological decomposition, $SU(3)$ irreducible representation →Expand using QCD-factorisation and establish connections to implement constraint

  - Can we perform the same exercise as B->PP for VV to obtain a set of closed equations that allow to single out modes to constrain non factorisable contributions?

  - With a more global analysis we might get some more discriminating power against long dist contributions?

- And what about the form factors? Currently driving the $L_{M_1 M_2}$ theory uncertainty prediction especially in the VV case!

Decay amplitudes in QCDf

Nucl.Phys.B 675 (2003) 333-415

$$\begin{aligned}\sum_{p=u,c} A_{M_1M_2}\Big\{ & \boldsymbol{B M_1}\left(\alpha_1 \boldsymbol{U}_p + \alpha_4^p + \alpha_{4,\mathrm{EW}}^p \hat{\boldsymbol{Q}}\right)\boldsymbol{M_2 \Lambda}_p \\ & + \boldsymbol{B M_1 \Lambda}_p \cdot \mathrm{Tr}\left[\left(\alpha_2 \boldsymbol{U}_p + \alpha_3^p + \alpha_{3,\mathrm{EW}}^p \hat{\boldsymbol{Q}}\right)\boldsymbol{M_2}\right] \\ & + \boldsymbol{B}\left(\beta_2 \boldsymbol{U}_p + \beta_3^p + \beta_{3,\mathrm{EW}}^p \hat{\boldsymbol{Q}}\right)\boldsymbol{M_1 M_2 \Lambda}_p \\ & + \boldsymbol{B\Lambda}_p \cdot \mathrm{Tr}\left[\left(\beta_1 \boldsymbol{U}_p + \beta_4^p + b_{4,\mathrm{EW}}^p \hat{\boldsymbol{Q}}\right)\boldsymbol{M_1 M_2}\right] \\ & + \boldsymbol{B}\left(\beta_{S2} \boldsymbol{U}_p + \beta_{S3}^p + \beta_{S3,\mathrm{EW}}^p \hat{\boldsymbol{Q}}\right)\boldsymbol{M_1 \Lambda}_p \cdot \mathrm{Tr}\boldsymbol{M_2} \\ & + \boldsymbol{B\Lambda}_p \cdot \mathrm{Tr}\left[\left(\beta_{S1} \boldsymbol{U}_p + \beta_{S4}^p + b_{S4,\mathrm{EW}}^p \hat{\boldsymbol{Q}}\right)\boldsymbol{M_1}\right] \cdot \mathrm{Tr}\boldsymbol{M_2}\Big\},\end{aligned}$$

$$\boldsymbol{P} = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_q}{\sqrt{2}} + \frac{\eta'_q}{\sqrt{2}} & \pi^- & K^- \\ \pi^+ & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_q}{\sqrt{2}} + \frac{\eta'_q}{\sqrt{2}} & \bar{K}^0 \\ K^+ & K^0 & \eta_s + \eta'_s \end{pmatrix},$$

$$\boldsymbol{V} = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega_q}{\sqrt{2}} + \frac{\phi_q}{\sqrt{2}} & \rho^- & K^{*-} \\ \rho^+ & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega_q}{\sqrt{2}} + \frac{\phi_q}{\sqrt{2}} & \bar{K}^{*0} \\ K^{*+} & K^{*0} & \omega_s + \phi_s \end{pmatrix},$$

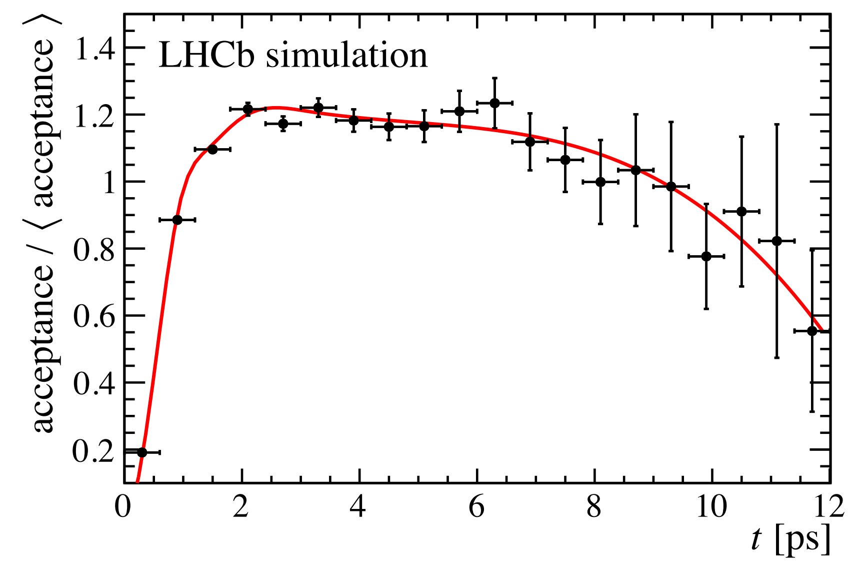
$$\boldsymbol{\Lambda}_p = \begin{pmatrix} 0 \\ \lambda_p^{(d)} \\ \lambda_p^{(s)} \end{pmatrix} \qquad \boldsymbol{B} = (B^-, \bar{B}^0, \bar{B}_s) \qquad \boldsymbol{U}_p = \begin{pmatrix} \delta_{pu} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad \hat{\boldsymbol{Q}} = \frac{3}{2}\boldsymbol{Q} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 \\ 0 & 0 & -\frac{1}{2} \end{pmatrix}.$$

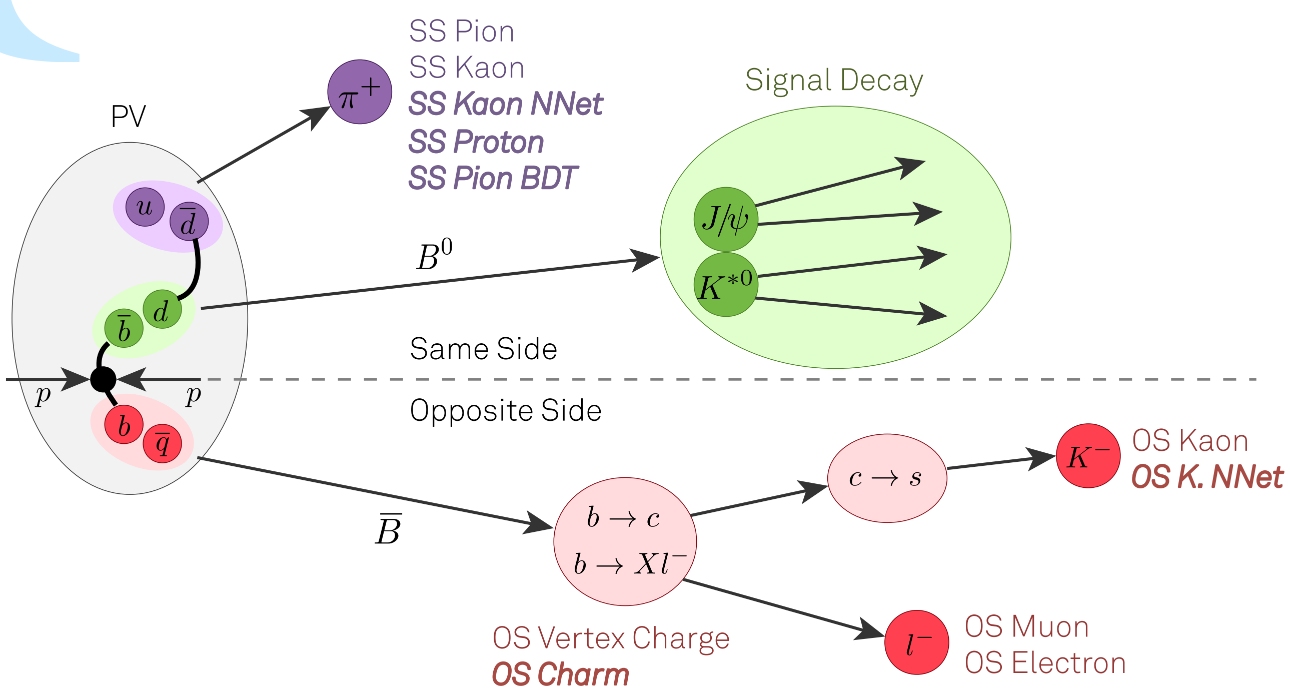
Backup

$B \to K^{(*)0} \bar{K}^{(*)0}$ measurements

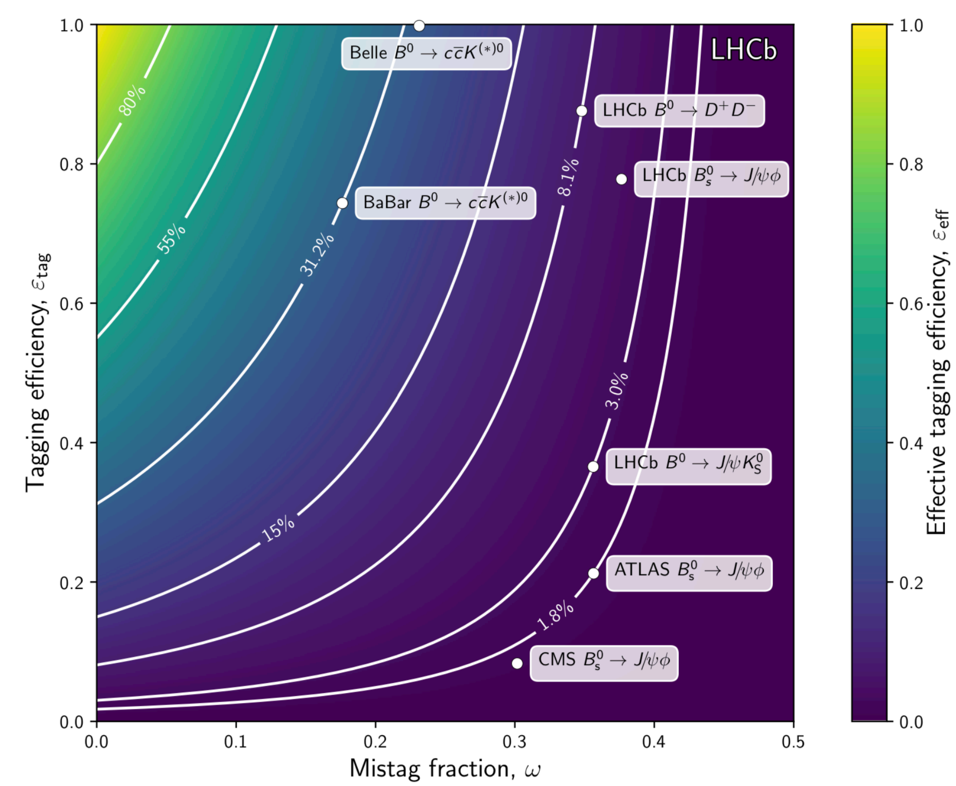
- Flavour tagged analyses allow to access to TD CP violation and measurement of the CKM angles β, β_s
    - Requires decay time acceptance modelling
    - Involves flavour tagging of the B at production

[LHCb: JHEP 03 (2018) 140]





[Credits to J. G. Pardiñas for the nice plot!]



[LHCb-FIGURE-2020-002]

Theory error budget on $L_{K^*\bar{K}^*}$

Form Factors

- LCSR from [Barucha, Straub, Zwicky]
- Main source of uncertainty
- Could be reduced using B_s and B_d correlations

Input	Relative Error $L_{K^*\bar{K}^*}$	Relative Error $\lvert P_s\rvert^2$	Relative Error $\lvert P_d\rvert^2$
f_{K^*}	$(-0.1\%, +0.1\%)$	$(-6.8\%, +7.1\%)$	$(-6.8\%, +7\%)$
$A_0^{B_d}$	$(-22\%, +32\%)$	–	$(-24\%, +28\%)$
$A_0^{B_s}$	$(-28\%, +33\%)$	$(-28\%, +33\%)$	–
λ_{B_d}	$(-0.6\%, +0.2\%)$	$(-4.6\%, +2.1\%)$	$(-4.1\%, +1.9\%)$
$\alpha_2^{K^*}$	$(-0.1\%, +0.1\%)$	$(-3.6\%, +3.7\%)$	$(-3.6\%, +3.6\%)$
X_H	$(-0.2\%, +0.2\%)$	$(-1.8\%, +1.8\%)$	$(-1.6\%, +1.6\%)$
X_A	$(-4.3\%, +4.4\%)$	$(-17\%, +19\%)$	$(-13\%, +14\%)$
κ	$(-1.4\%, +2.2\%)$	–	–
Others	$(-1.3\%, +1.1\%)$	$(-2.7\%, +2.5\%)$	$(-1.6\%, +1.6\%)$

[JHEP04(2021)066]

IR divergencies

- 100% uncertainty and free complex phase, influence substantially reduced in $L_{K^*\bar{K}^*}$
- U-spin correlation between B_s and B_d is parametrisation independent
- Even if X_A different for B_s and B_d still FF are dominating the error

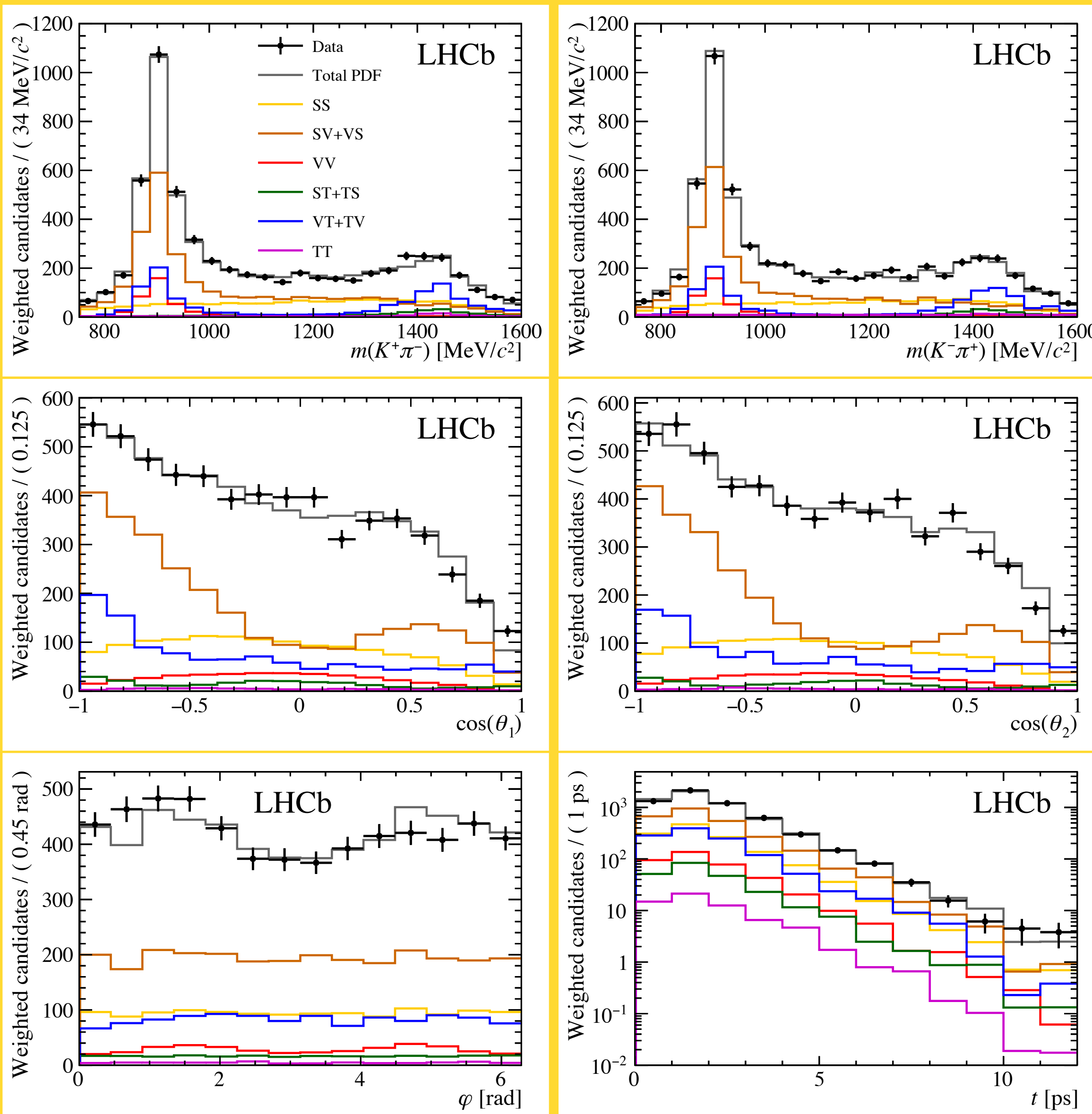
$$X_{A,H} = (1 + \rho_{A,H} e^{i\phi_{A,H}}) \ln\left(\frac{m_B}{\Lambda_h}\right)$$

$$\rho_{A,H} \in [0,1]\,,\ \phi_{A,H} \in [0, 2\pi]$$

[Beneke, Buchalla, Neubert, Sachrajda]

$B \to K^{*0}\bar{K}^{*0}$ at LHCb

Analysis performed in a large $m_{K\pi}$ window, involved spectroscopy work (19 polarisation amplitudes with scalar, vector and tensor components), six- dimensional fit



sation hierarchy $f_L \gg f_{\parallel,\perp}$ in

ng [Nucl.Phys.B774:64-101,2007] :

$.72^{+0.16}_{-0.21}$

TD CP asymmetries in $B_{(s)} \to K^{*0}\bar{K}^{0*}$ with 3fb^{-1} of data

- First measurement of the CP-violating phase

$$\phi_s^{s\bar{s}} = -0.10 \pm 0.13(\text{stat}) \pm 0.14(\text{syst})$$

- Low value of $f_L^{B_s}$ confirmed

$$f_L^{B_s} = 0.208 \pm 0.032(\text{stat}) \pm 0.046(\text{syst})$$

The crossed modes, $B_{(s)} \to K^{*0}\bar{K}^0$ and $B_{(s)} \to \bar{K}^{*0}K^0$

One can define L ratios using crossed PV and VP modes, depending on which spectator quark ends in up in a P or V meson:

$$\hat{L}_{K^*} = \rho(m_{K^0}, m_{\bar{K}^{*0}})\frac{\mathcal{B}(B_s \to K^{*0}\bar{K}^0)}{\mathcal{B}(B_d \to K^{*0}\bar{K}^0)} \quad \hat{L}_K = \rho(m_{K^0}, m_{\bar{K}^{*0}})\frac{\mathcal{B}(B_s \to K^0\bar{K}^{*0})}{\mathcal{B}(B_d \to K^0\bar{K}^{*0})}$$

Experimentally challenging as it requires flavour tagging for both the B^0 and B_s ,

$$L_{K^*} = 2\,\rho(m_{K^0}, m_{K^{*0}})\frac{\mathcal{B}(\bar{B}_s \to K^{*0}\bar{K}^0)}{\mathcal{B}(\bar{B}_d \to \bar{K}^{*0}K^0) + \mathcal{B}(\bar{B}_d \to \bar{K}^0K^{*0})}$$

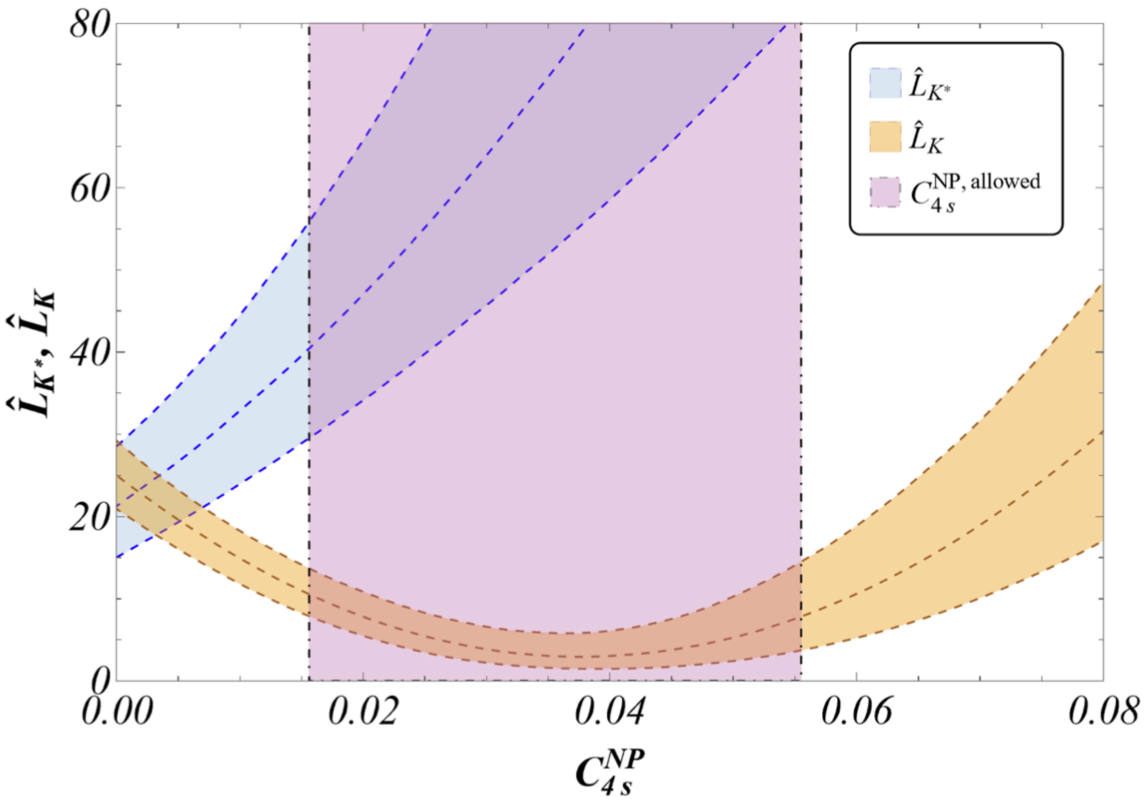
$$L_K = 2\,\rho(m_{K^0}, m_{K^{*0}})\frac{\mathcal{B}(\bar{B}_s \to K^0\bar{K}^{*0})}{\mathcal{B}(\bar{B}_d \to \bar{K}^{*0}K^0) + \mathcal{B}(\bar{B}_d \to \bar{K}^0K^{*0})}$$

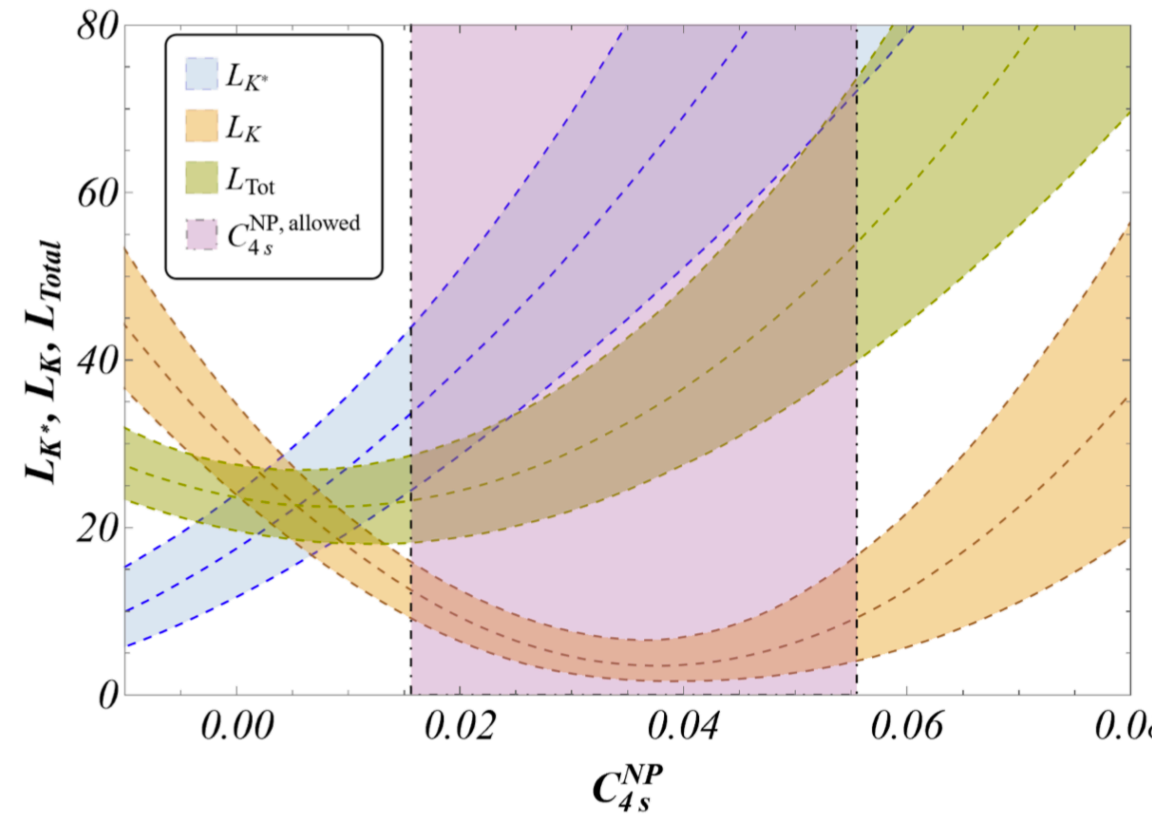
→ **FT required only on the B_s.**

$$L_{\text{total}} = \rho(m_{K^0}, m_{K^{*0}})\left(\frac{\mathcal{B}(\bar{B}_s \to K^{*0}\bar{K}^0) + \mathcal{B}(\bar{B}_s \to K^0\bar{K}^{*0})}{\mathcal{B}(\bar{B}_d \to \bar{K}^{*0}K^0) + \mathcal{B}(\bar{B}_d \to \bar{K}^0K^{*0})}\right)$$

→ **No FT required.**

The crossed modes, $B_{(s)} \to K^{*0}\bar{K}^0$ and $B_{(s)} \to \bar{K}^{*0}K^0$





- Measurements are only available for, Br($B_s \to K^0 + \bar{K}^{*0}$)+Br($B_s \to \bar{K}^0 + K^{*0}$) [JHEP 06 (2019) 114], No B_d results yet
- Current ongoing $B^0_{d,s} \to K^0_S K^*(892)$ to access L_{total}
- Plans to measure the tagged ones

Eur. Phys. J. A26, 125–134 (2005)

Covariant amplitude formalism

Amplitudes are expressed in terms of eigenstates of CP and angular momentum operators. Stated described by contracting polarisation tensors with corresponding orbital waves L

$$
\begin{aligned}
A_{VV}^{S}: &\quad S \propto \epsilon_\mu(V_1)\,\epsilon^\mu(V_2)\,,\\
A_{VV}^{P}: &\quad S \propto \varepsilon_{\mu\nu\alpha\beta}\,L^\alpha(V_1,V_2)\,\epsilon^\beta(V_1)\,\epsilon^\nu(V_2)\,p^\mu(B)\,,\\
A_{VV}^{D}: &\quad S \propto L_{\mu\nu}(V_1,V_2)\,\epsilon^\mu(V_1)\,\epsilon^\nu(V_2)\,,\\
A_{VS}^{+}: &\quad S \propto \epsilon_\mu(V_1)\,L^\mu(V_1,S_2) + \epsilon_\mu(V_2)\,L^\mu(S_1,V_2)\,,\\
A_{VS}^{-}: &\quad S \propto \epsilon_\mu(V_1)\,L^\mu(V_1,S_2) - \epsilon_\mu(V_2)\,L^\mu(S_1,V_2)\,,\\
A_{SS}: &\quad S \propto 1\,,
\end{aligned}
$$

$$V_{1,2} \equiv |K^\pm\pi^\mp\rangle_{J=1}$$

$$S_{1,2} \equiv |K^\pm\pi^\mp\rangle_{J=0}$$

Amplitude for each contribution

$$A_i(\Phi_4) = B_{L_B}(\Phi_4)\left[B_{L_{K^+\pi^-}}(\Phi_4)\,T_{K^+\pi^-}(\Phi_4)\right]\left[B_{L_{K^-\pi^+}}(\Phi_4)\,T_{K^-\pi^+}(\Phi_4)\right]S_i(\Phi_4)$$

Where:

- $B_{L_B}(B_{L_{K^\pm\pi^\mp}})$ are production barrier factors depending on the orbital momentum between $B(K^\pm\pi^\mp)$ decay products
- $T(s)$ are 2-body mass propagators
- S_i are spin densities from above

$$A(B^0_{(s)} \to (K^+\pi^-)(K^-\pi^+))(\Phi_4) = \sum_i a_i A_i(\Phi_4)$$

a_i complex coefficients, fit to data