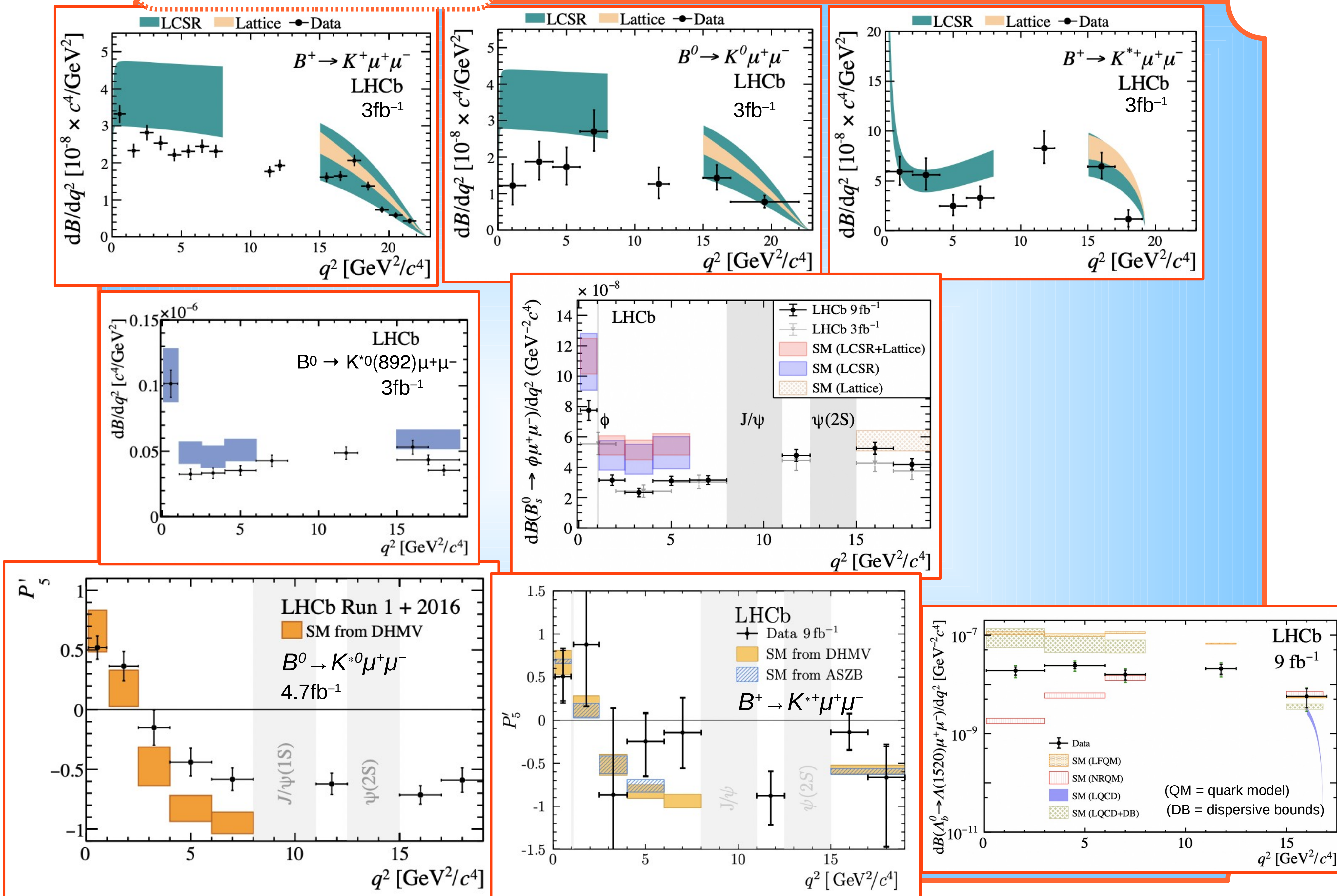
$B_{d,s} \to \mu^+ \mu^- \gamma$ phenomenology
– overview –

Diego Guadagnoli
CNRS, LAPTh Annecy

A novel, short-term way to cross-check
the existing tensions (“anomalies”) in $b \to s\ \mu\mu$ data

$b \to s$ data tensions



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- *The additional photon lifts chirality suppression*

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ee channel: enhancement is 5 orders of magnitude

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- *With Run 3 (⇨ hopefully comparable e and μ efficiencies), $B_s \to ee\,\gamma$ no more science fiction*

$B_s \rightarrow \mu\mu\ \gamma$ from $B_s \rightarrow \mu\mu$

$B_s \to \mu\mu\gamma$: "indirect" method

[Dettori, DG, Reboud, 2017]

Basic Idea *Extract $B_s \to \mu\mu\gamma$ from $B_s \to \mu\mu$ event sample, by enlarging $m_{\mu\mu}$ below B_s peak*

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- *Exploits rich and ever increasing $B_s \to \mu\mu$ dataset*
- *… to access $B_s \to \mu\mu\gamma$, that probes any $\mu\mu$ "anomaly"*
  - *more thoroughly (more EFT couplings)*
  - *in a different, not well tested, q^2 region*
  - *with a completely different exp approach*

Exp side

[thanks F. Dettori]

***Pros** (besides those already stated)*

- No need to reconstruct the γ (factor-of-20 loss in efficiency)

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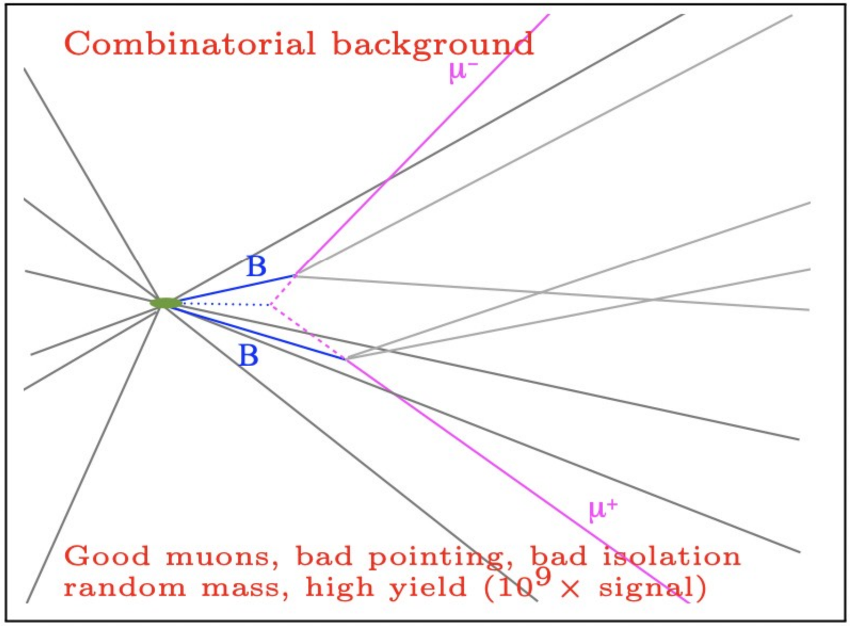
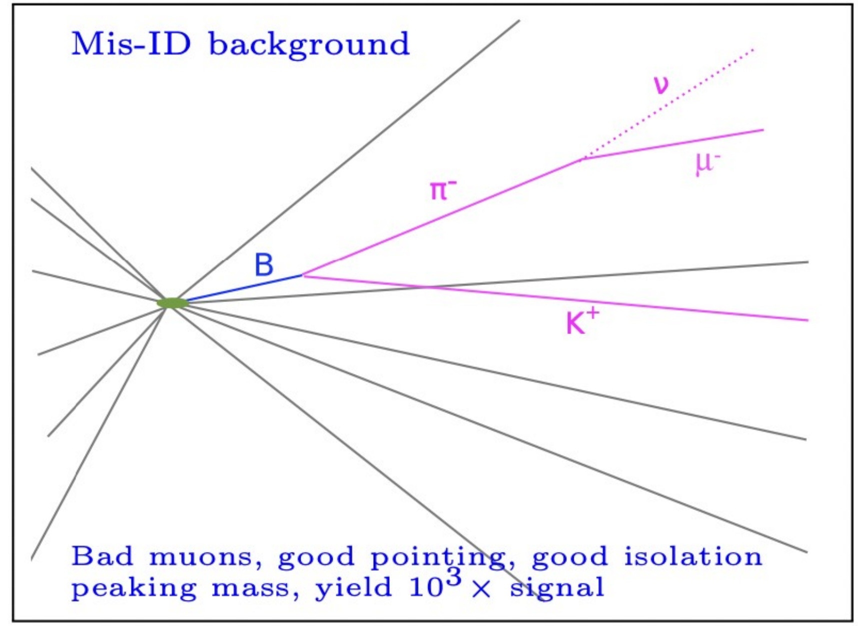
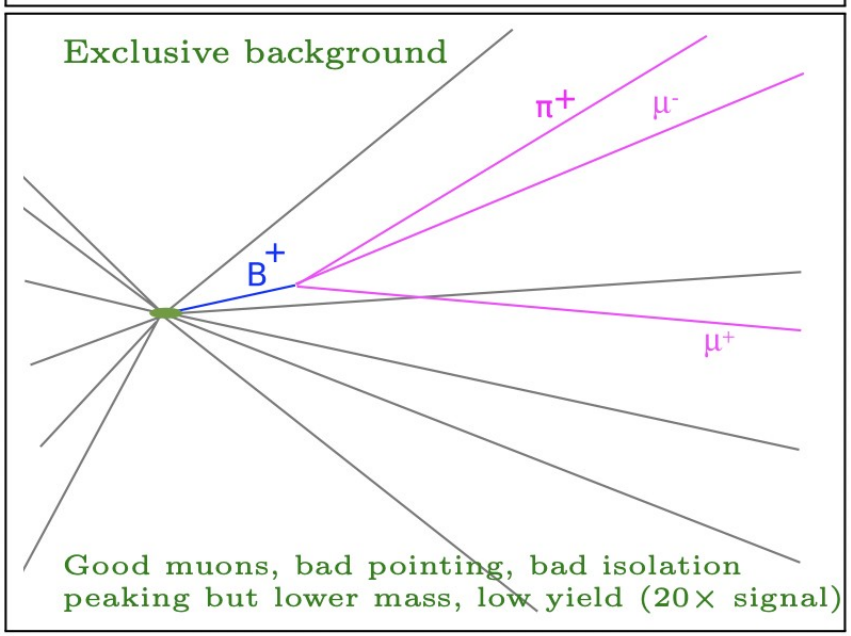
- Calibration not trivial – no “analogous” channel

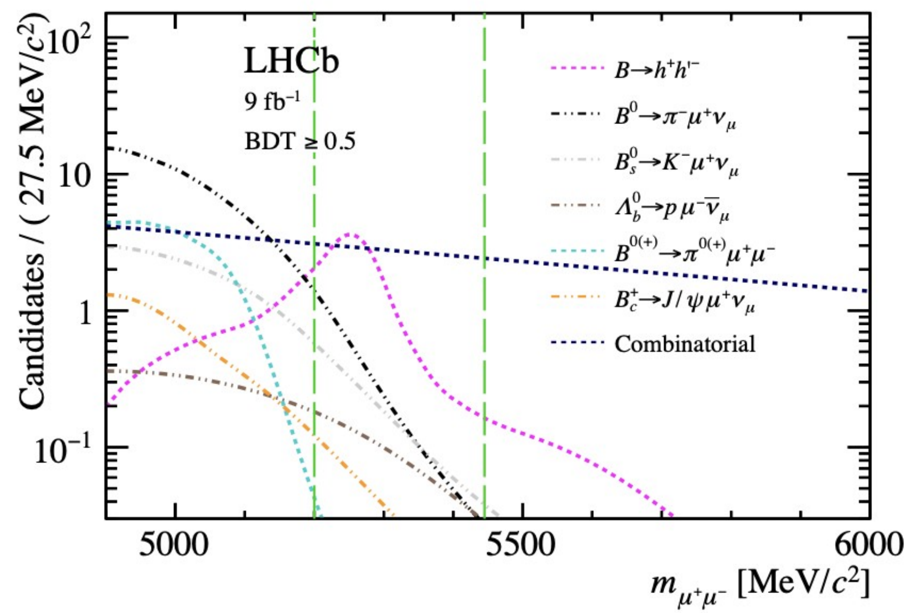
Backgrounds

[thanks F. Dettori]





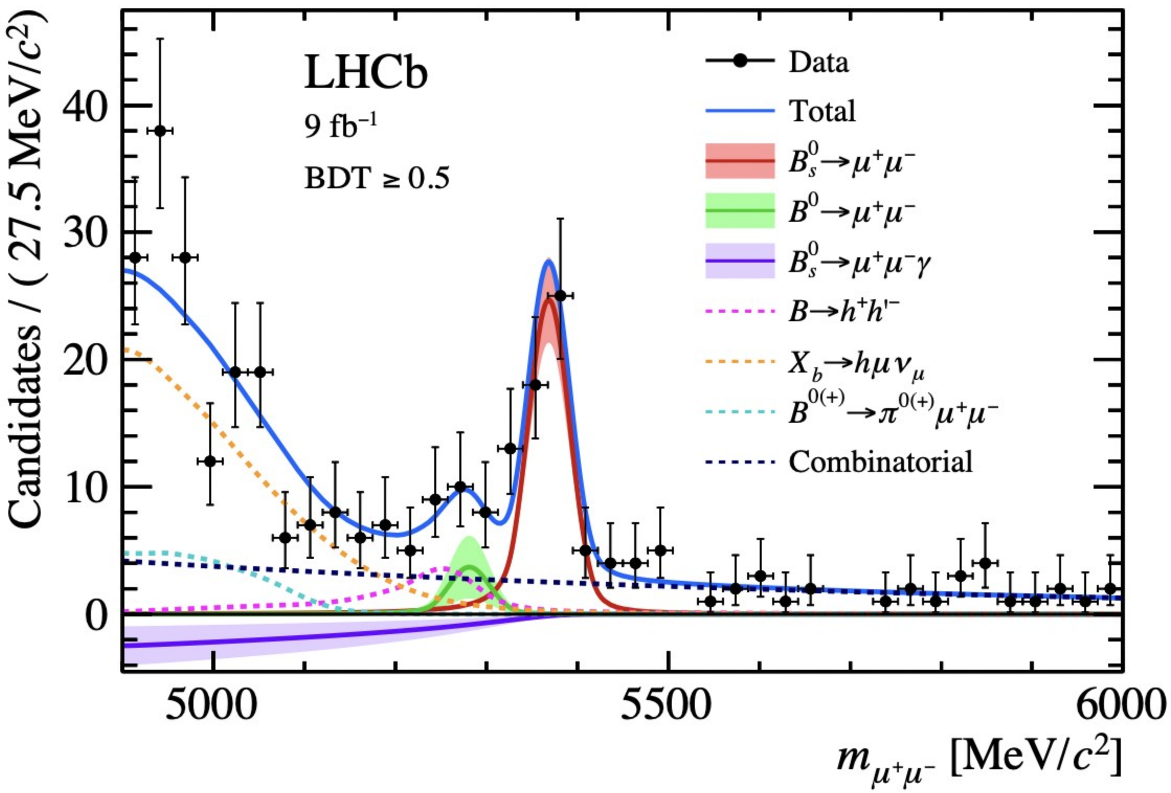




[LHCb-PAPER-2021-007] [LHCb-PAPER-2021-008]

Results

[thanks F. Dettori]



[LHCb-PAPER-2021-007] [LHCb-PAPER-2021-008]

$$\mathcal{B}(B_s^0 \to \mu^+\mu^-) = \left(3.09\,^{+\,0.46}_{-\,0.43}\,^{+\,0.15}_{-\,0.11}\right) \times 10^{-9}$$

$$\mathcal{B}(B^0 \to \mu^+\mu^-) = \left(1.2\,^{+\,0.8}_{-\,0.7} \pm 0.1\right) \times 10^{-10} < 2.6 \times 10^{-10}$$

$$\mathcal{B}(B_s^0 \to \mu^+\mu^-\gamma)_{m_{\mu\mu} > 4.9\,\mathrm{GeV}} = (-2.5 \pm 1.4 \pm 0.8) \times 10^{-9} < 2.0 \times 10^{-9}$$

No significant signal for $B^0 \to \mu^+\mu^-$ and $B_s^0 \to \mu^+\mu^-\gamma$, upper limits at 95%

First world limit on $B_s^0 \to \mu^+\mu^-\gamma$ decay

The elephant in the room (FFs)

Radiative leptonic FFs in LQCD

Novel ideas & applications, both at low q^2 (large E_γ) and high q^2 (small E_γ)

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$$\Gamma\left(E_\gamma^{\max}\right) = \Gamma_0 + \Gamma_1\left(E_\gamma^{\max}\right)$$

Total width w/ either 0 or 1 γ

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$\ell\ell'\gamma$ width, w/ $E_\gamma \leq E_\gamma^{max}$

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Requirement

E_γ^{max} small enough to justify scalar-QED approach in Γ_1

FFs at low q^2

within factorization

$B_s \to \mu\mu\gamma$ *with energetic* γ

[Beneke-Bobeth-Wang, *'20*]

- *For low $q^2 \leq 6$ GeV, $B_s \to \gamma^*$ f.f.'s can be calculated in a systematic expansion in $1/m_b$, $1/E_\gamma$*

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  resonance paramet'n

    - *actually dominant contribution by far*
    - *escapes first-principle description*

Bottom line

[Beneke-Bobeth-Wang, *'20*]

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- Prediction

$$\langle \mathcal{B} \rangle_{[4m_\mu^2,\,6.0]} = \left(12.51^{+3.83}_{-1.93}\right) \cdot 10^{-9}, \qquad \langle \mathcal{B} \rangle_{[2.0,\,6.0]} = \left(0.30^{+0.25}_{-0.14}\right) \cdot 10^{-9}$$

i.e. ϕ region gives 97.6% of the BR

FFs within LCSRs

[Janowski, Pullin, Zwicky, *'21*]
see also [Pullin, Zwicky, *'21;* Albrecht *et al.*, *19*]

- *FFs fitted to a z-expansion ansatz*

$$F_n^{\bar{B}\to\gamma}(q^2) = \frac{1}{1 - q^2/m_R^2}\left(\alpha_{n0} + \sum_{k=1}^{N} \alpha_{nk}(z(q^2) - z(0))^k\right)$$

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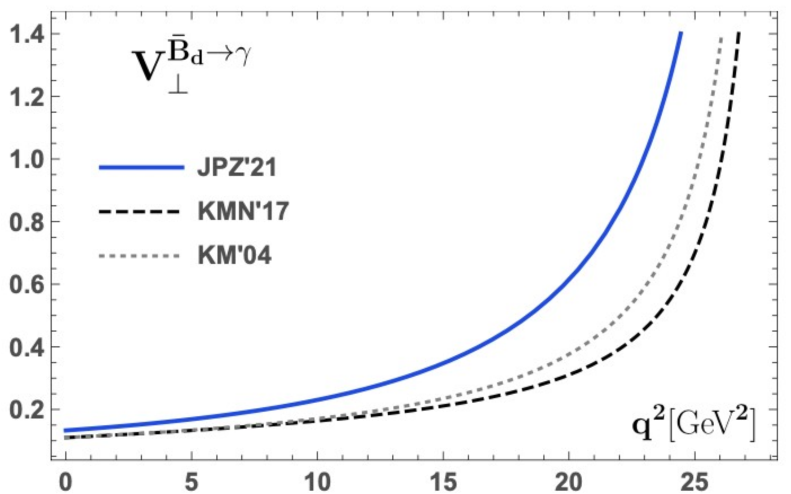
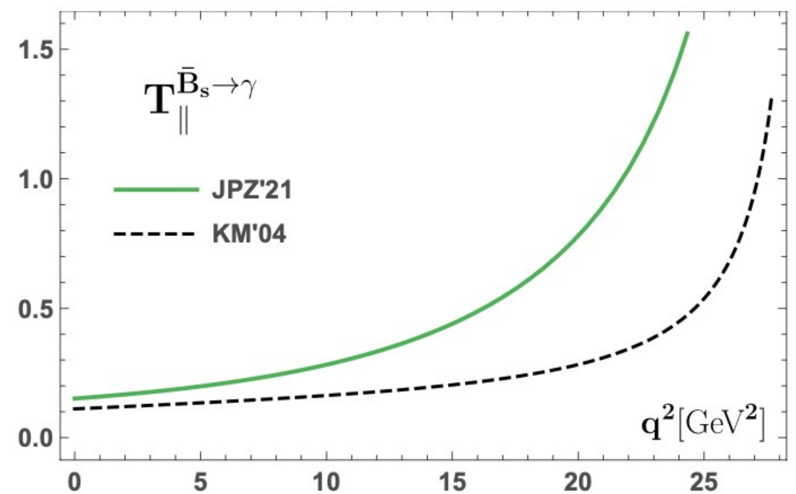
- *Comparison with the quark-model FF parameterizations in*

[Melikhov, Nikitin, '04; Kozachuk, Melikhov, Nitikin, '17]





FFs at high q^2

A phenomenological approach
using LQCD and heavy-quark symmetry

Our approach.zip

[DG, Normand, Simula, Vittorio, *'23*]

① *Use available $D_s \to \gamma$ LQCD data*

(directly computed in very range of interest)

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[• *Validate as much as possible*]

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Our region of interest is high $q^2 \in [4.2, 5.0]^2$ GeV^2

In precisely this region, LQCD has directly computed $D_s \to \gamma$ *FFs*

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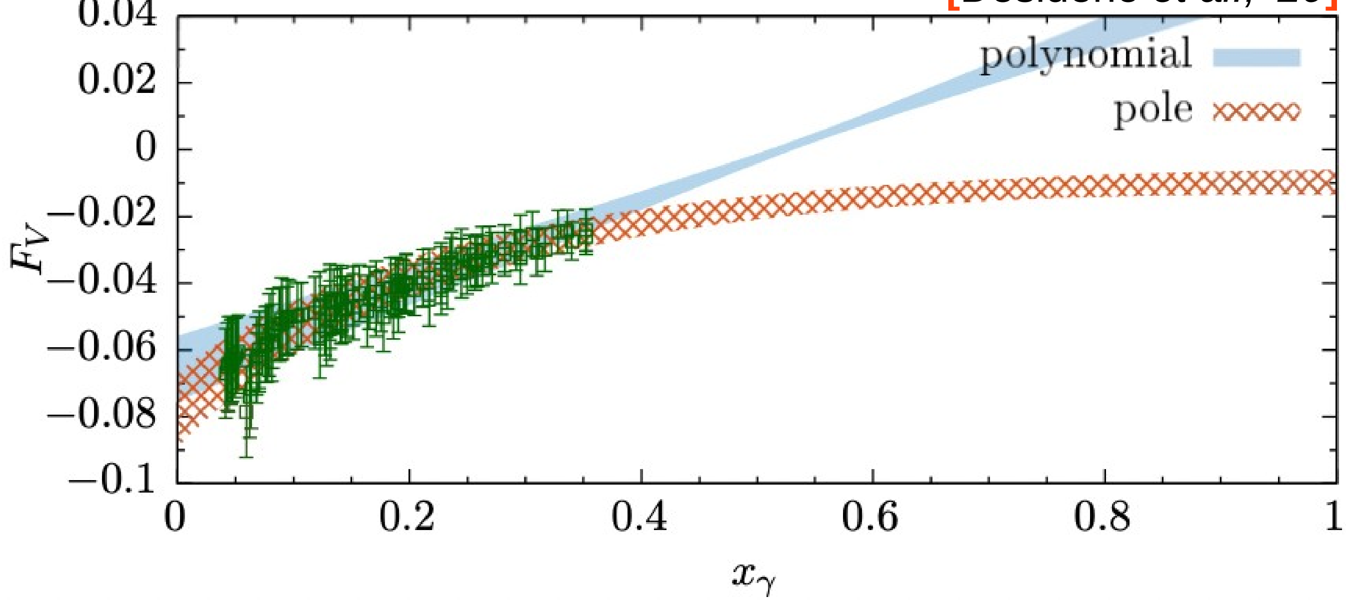
- High q^2 means low $x_\gamma \equiv 1 - q^2 / m^2_{Ds}$

$q^2 \in [4.2, 5.0]^2$ GeV^2 ⟺ $x_\gamma \in [0.39, 0.13]$

[Desiderio *et al., '20*]



② *Frame LQCD data within Vector Meson Dominance*

High q^2 means small E_γ

⇨ The nearest vector- (or axial-)meson dominates

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$$\langle \gamma | \bar{s} \gamma_\mu b | \bar{B}_s \rangle \simeq \sum_\lambda \frac{\langle 0 | \bar{s} \gamma_\mu b | B_s^*(\varepsilon_\lambda) \rangle \langle B_s^*(\varepsilon_\lambda) | B_s \gamma \rangle}{q^2 - m_{B_s^*}^2}$$

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$$\propto \ V_\perp(q^2) = \frac{1}{\pi} \int_0^\infty dt \frac{\mathrm{Im}[V_\perp(t)]}{t - q^2} = \frac{r_\perp}{1 - q^2/m_{B_s^*}^2} + \ldots$$

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$$\langle \gamma | \bar{s} \gamma_\mu b | \bar{B}_s \rangle \simeq \sum_\lambda \frac{\langle 0 | \bar{s} \gamma_\mu b | B_s^*(\varepsilon_\lambda) \rangle \langle B_s^*(\varepsilon_\lambda) | B_s \gamma \rangle}{q^2 - m_{B_s^*}^2}$$

$\propto \; m_{B_s^*} f_{B_s^*}$

$\propto$ "tri-coupling" $g_{B_s^* B_s \gamma}$

$$\propto \; V_\perp(q^2) = \frac{1}{\pi} \int_0^\infty dt \frac{\mathrm{Im}[V_\perp(t)]}{t - q^2} = \frac{r_\perp}{1 - q^2/m_{B_s^*}^2} + \dots$$

② *Frame LQCD data within Vector Meson Dominance*

High q^2 means small E_γ

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One can thus relate the (fitted) residue to the (otherwise unknown) tri-coupling

$$r_\perp = \frac{m_{B_s} f_{B_s^*}}{m_{B_s^*}} g_{B_s^* B_s \gamma}$$

② *VMD: fit ansaetze*

FFs are described as a sum of poles + cuts

Description useful if one or two terms dominate

Try minimal fit ansaetze. See if coherent picture emerges.

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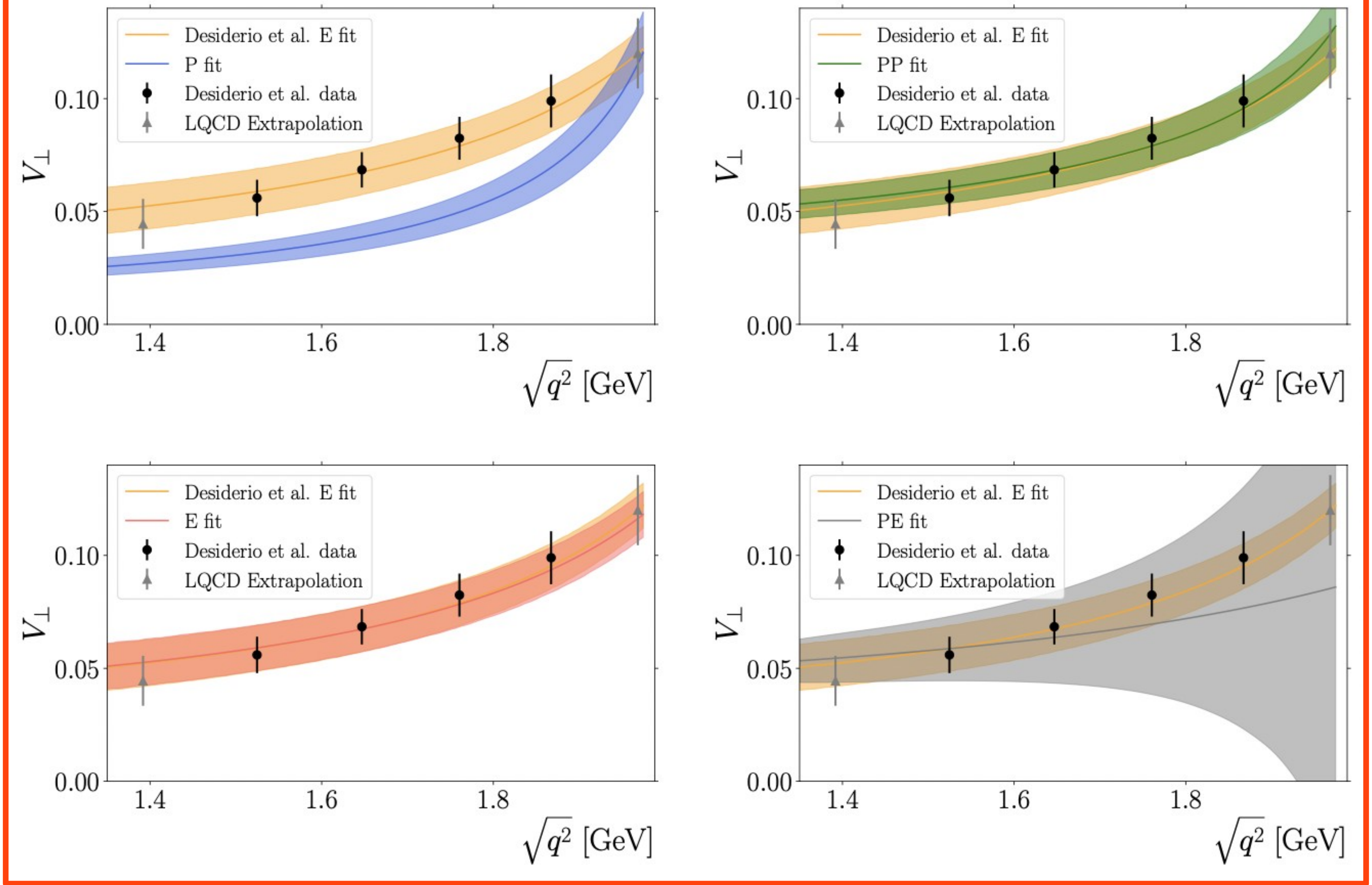
E fit	One effective pole	⇨	Fit for residue & pole mass
PE fit	One phys & one eff pole		

...

② VMD: the vector-FF example



③ *From the D_s to the B_s*

Basic idea:

$$\text{Tri-coupling} = \sum_{i\,=\,\text{valence quarks}} (\pm\ \text{e.m. charge})_i \times \underbrace{(\text{magn. moment})_i}_{\propto\ 1\,/\,m_i}$$

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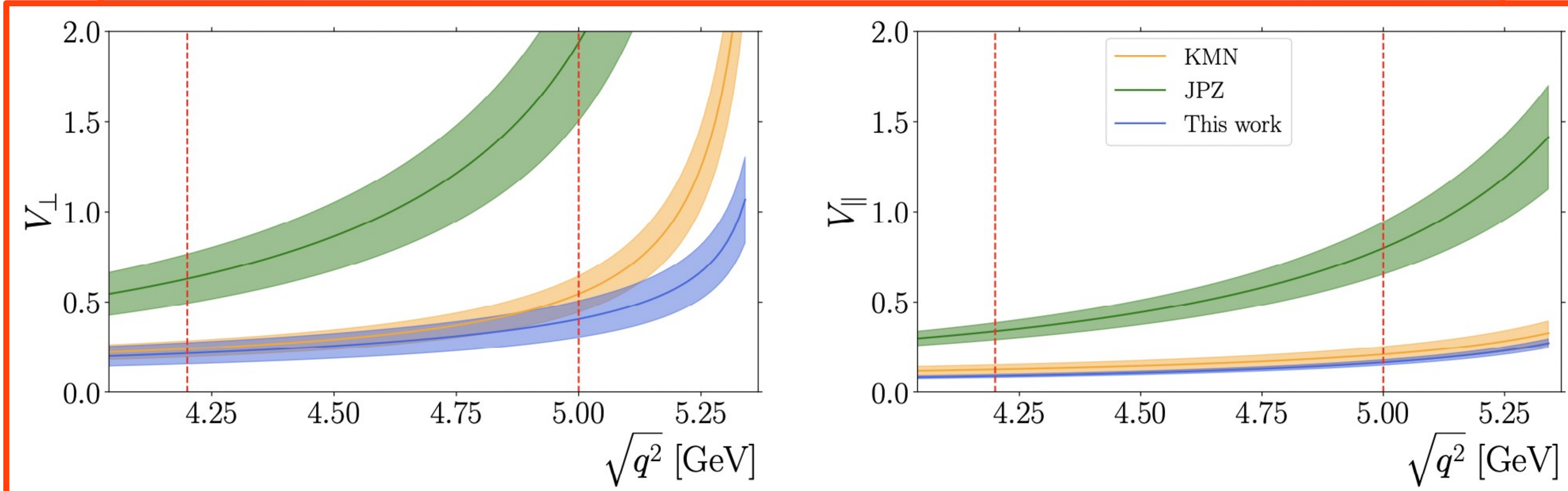
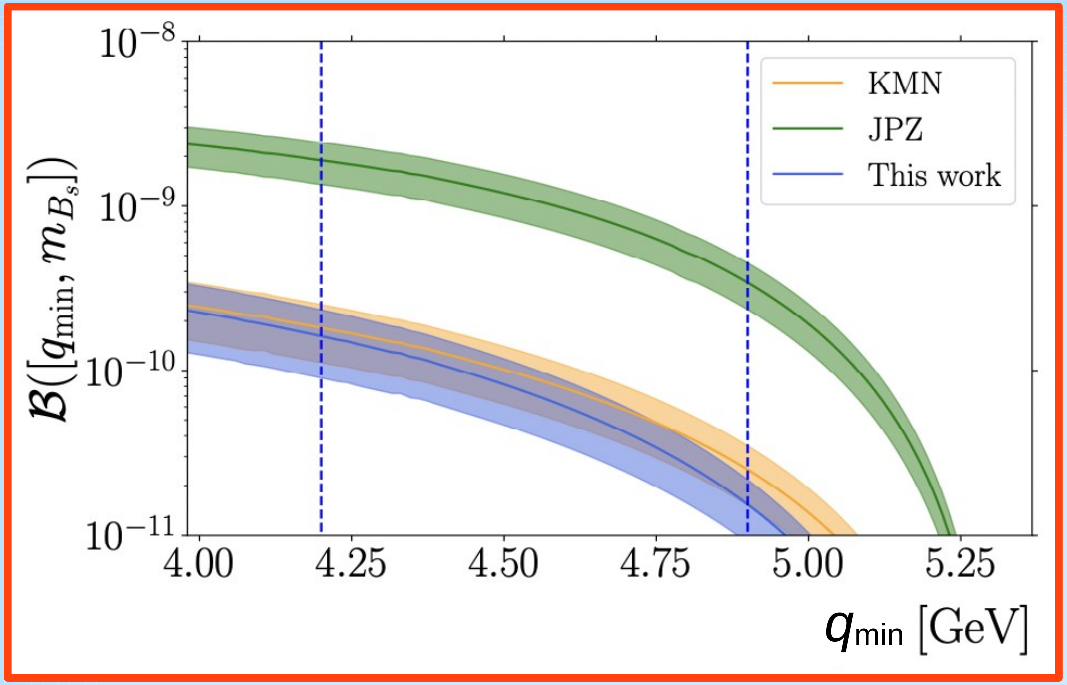


BR($B_s \to \mu^+ \mu^- \gamma$) prediction



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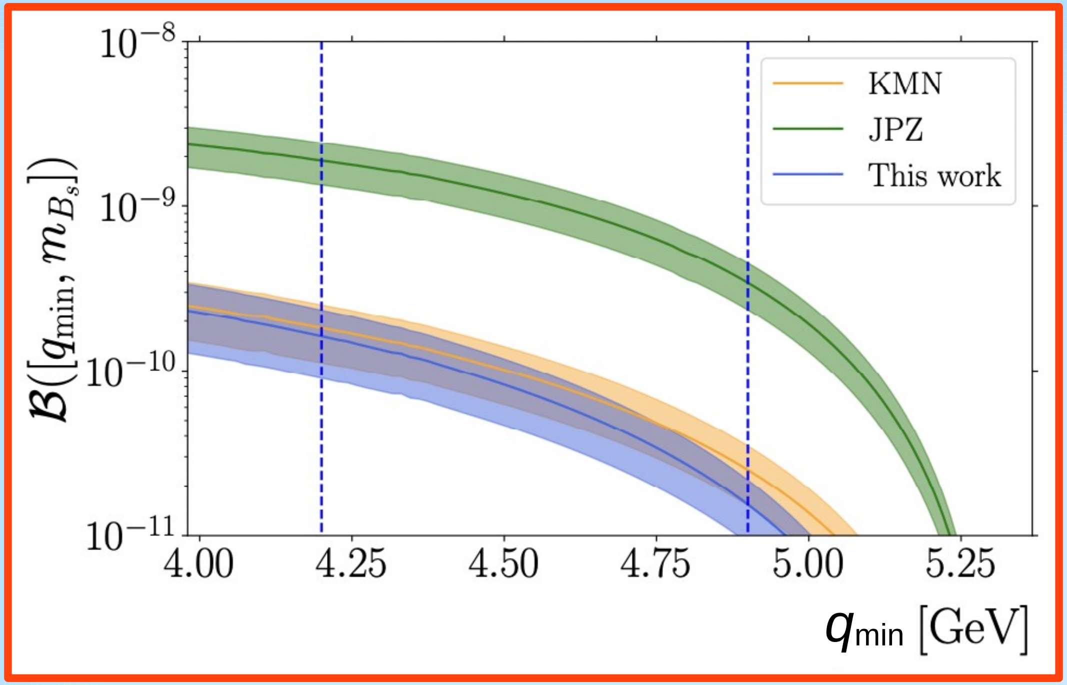
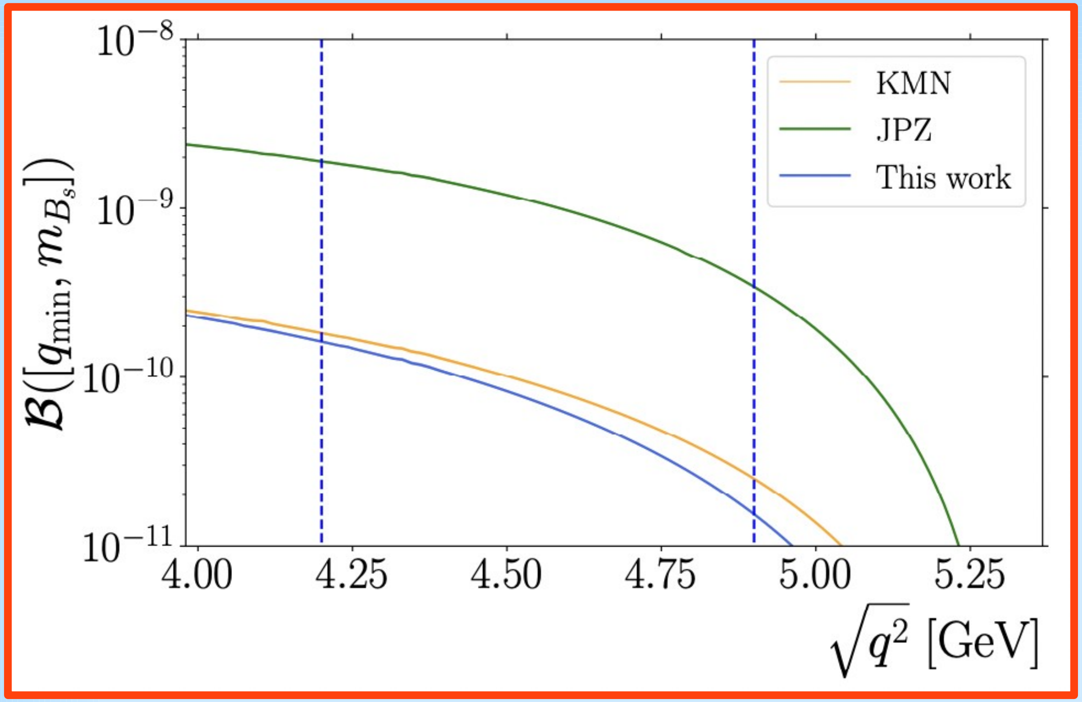


Below ~ 4.4 GeV there is broad-$c\bar{c}$ pollution

These contributions are incalculable from first principles

How large is their share of the total error?

BR($B_s \to \mu^+ \mu^- \gamma$) prediction



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Tiny!

- Low impact of broad $c\bar{c}$ encouraging, given that this systematics inherently escapes a rigorous description

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- f.f. uncertainty, even if still large, in principle “reducible”
- Maybe worthwhile to look for more observables with such properties

Example: the $B_s \to \mu\mu\gamma$ effective lifetime

[Carvunis et al., '21]

- *Natural exp observable: untagged rate*

$$\langle\Gamma(B_s(t) \to f)\rangle \;\equiv\; \Gamma(B_s^0(t) \to f) + \Gamma(\bar{B}_s^0(t) \to f)$$

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Recalling the time dependence of the |amplitudes|^2

$$\begin{aligned} |\overset{(-)}{\mathcal{A}}_f(t)|^2 &= \frac{e^{-\Gamma_s t}}{2}\Big[\Big(|\mathcal{A}_f|^2 + |q/p|^2|\bar{\mathcal{A}}_f|^2\Big)\cosh(\Delta\Gamma_s\, t/2) \pm \Big(|\mathcal{A}_f|^2 - |q/p|^2|\bar{\mathcal{A}}_f|^2\Big)\cos(\Delta M_s\, t) \\ &- 2\,\mathrm{Re}\left(q/p\,\bar{\mathcal{A}}_f\mathcal{A}_f^*\right)\sinh(\Delta\Gamma_s\, t/2) \mp 2\,\mathrm{Im}\left(q/p\,\bar{\mathcal{A}}_f\mathcal{A}_f^*\right)\sin(\Delta M_s\, t)\end{aligned}$$

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yields the following quantity sensitive to new CPV

$$A^f_{\Delta\Gamma_s} = \frac{-2 \int_{\mathrm{PS}} \mathrm{Re} \left(q/p \, \bar{\mathcal{A}}_f \mathcal{A}_f^* \right)}{\int_{\mathrm{PS}} \left(|\mathcal{A}_f|^2 + |q/p|^2 |\bar{\mathcal{A}}_f|^2 \right)}$$

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- *$A_{\Delta\Gamma}$ can be extracted from (an accurate measurement of) the effective lifetime*

Conclusions

$B_s \to \mu\mu\gamma$ is interesting in many respects

- *It's new – never measured*

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- *It's now measurable from* $B_s \to \mu\mu$ *– for high* q^2
- *High* q^2 *offers several TH advantages*
  - *Probes in complementary kin. region the tensions reported in semi-lep BRs*

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  - Probes in complementary kin. region the tensions reported in semi-lep BRs
  - Test is strong, given the very different underlying exp method
  - Preferred region for lattice QCD

Spares

Impact of broad $c\bar{c}$

[*Carvunis et al., '21*]

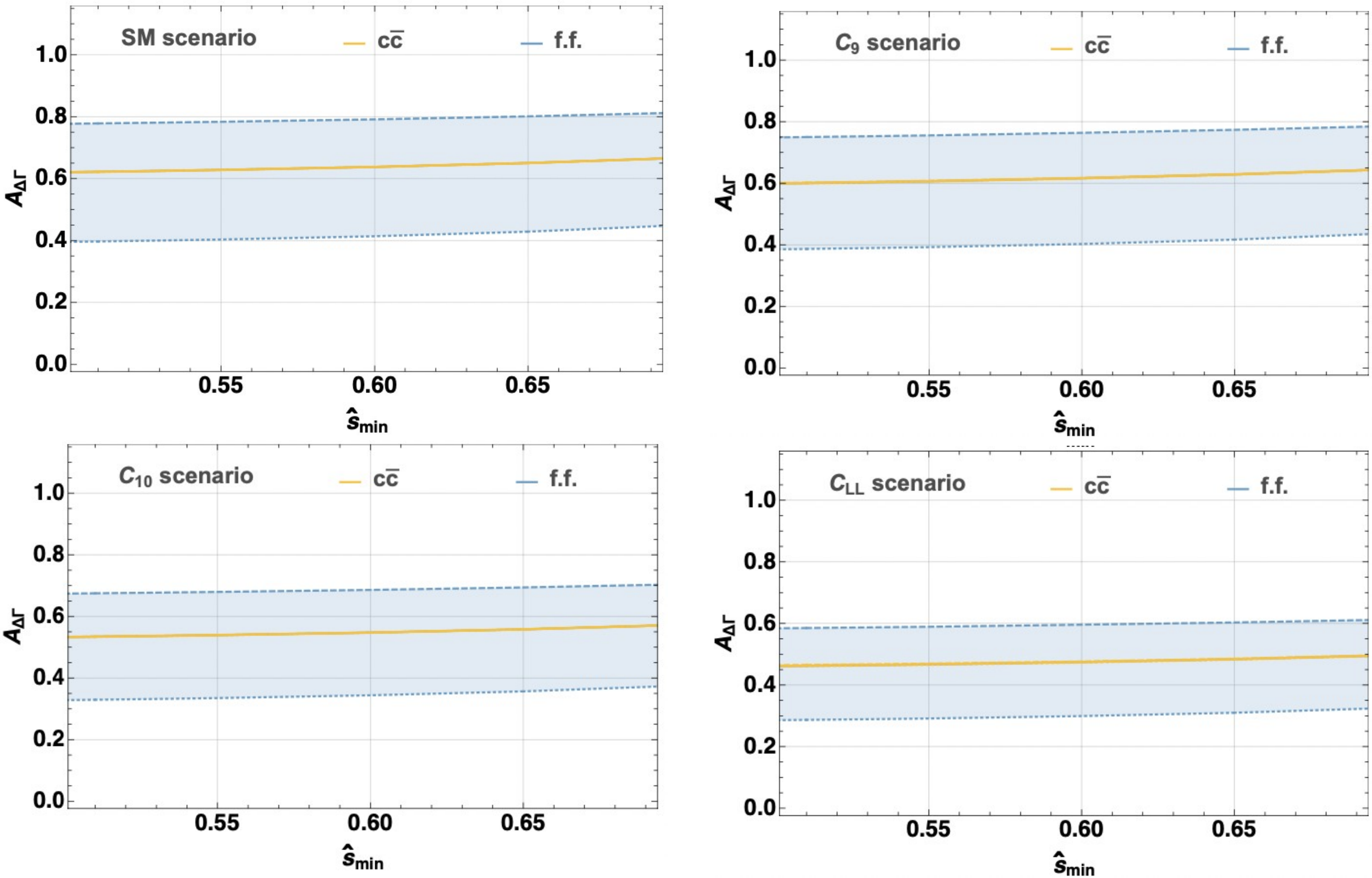
- Parameterize the effect most generally (e.g. discussion in [Lyon, Zwicky, *'14*])

$$C_9 \to C_9 - \frac{9\pi}{\alpha^2} \bar{C} \sum_V |\eta_V| e^{i\delta_V} \frac{\hat{m}_V \, \mathcal{B}(V \to \mu^+\mu^-) \, \hat{\Gamma}^V_{\rm tot}}{\hat{q}^2 - \hat{m}_V^2 + i \hat{m}_V \hat{\Gamma}^V_{\rm tot}}$$

  - $|\eta_V| \in [1, 3]$ & $\delta_V \in [0, 2\pi)$ (uniformly and independently for the 5 resonances)
  - for $s_{min} \in [0.5, 0.7]\ m_{Bs}^2$

    $s_{\psi(2S),\ \psi(3770),\ \psi(4040),\ \psi(4160),\ \psi(4415)} = \{0.47, 0.49, 0.57, 0.61, 0.68\}$
  - for all TH scenarios

Impact of broad $c\bar{c}$



Impact of broad $c\bar{c}$

[*Carvunis et al., '21*]

- Bottom line: broad $c\bar{c}$ has surprisingly small impact on $A_{\Delta\Gamma}$

  But broad-$c\bar{c}$ shift to C_9 typically O(5%) – and with random phase

  ⮡ Far from obvious why such a small impact on $A_{\Delta\Gamma}$

- Closer look (App. D for an analytic understanding)

  Cancellation is a conspiracy between

  - Complete dominance of contributions quadratic in C_9 and C_{10}
  - Multiplying f.f.'s $F_V, F_A \in \mathbb{R}$
  - Broad $c\bar{c}$ can be treated as small modif. of (numerically large) C_9

  ⮡ Ease cancellations between num & den in $A_{\Delta\Gamma}$

Radiative leptonic FFs in LQCD

Large E_γ

- *The required correlator (weak & e.m. current insertion between a B and the vac) has always the desired large-Euclidean-t behavior*

  [Kane, Lehner, Meinel, Soni, *'19*]

  Note that this is non-trivial – e.g. it doesn't seem to hold if there are hadronic final states

- *However, the low-q^2 spectrum is dominated by resonant contributions (~98% of the BR), that LQCD is unable to capture*

Amplitude structure

[Beneke-Bobeth-Wang, '20]

- *Take the weak operators as $O_i \equiv J_i^{(l)} \cdot J_i^{(q)}$*

  and i = 9,10 for definiteness (and simplicity)

$$\overline{A} \propto \epsilon^*_\mu \left\{ \sum_i C_i \left[T_i^{\mu\nu} \langle \ell\bar{\ell} | \, J^{(l)}_{i\,\nu}(0) \, |0\rangle \right.\right.$$
$$\left.\left. + \underbrace{S_\nu^{(i)} \, \mathrm{FT}_x \langle \ell\bar{\ell} | \, T\{J^\mu_{\rm em}(x), J_i^{(l)\,\nu}(0)\}|0\rangle}_{} \right] \right\}$$

FSR: only $S_\nu^{(10)} \neq 0$ ($\propto m_\ell$) tiny

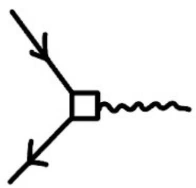
Main object to calculate

$$T_i^{\mu\nu} \propto \mathrm{FT}_x \langle 0 | \, T\{J^\mu_{\rm em}(x), J_i^{(q)\,\nu}(0)\}|B\rangle$$

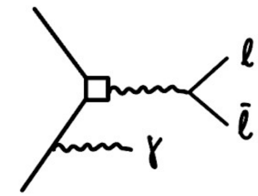
Notes on structure

[Beneke-Bobeth-Wang, *'20*]

- O_7 :

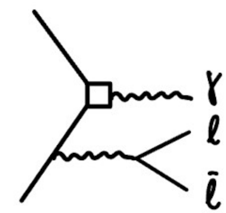


$T^{\mu\nu}_{7A}$:



but also

$T^{\mu\nu}_{7B}$:



- $$T_i^{\mu\nu} = T_i^{\mu\nu}(k,q) \propto (g^{\mu\nu} k \cdot q - q^\mu k^\nu) \overbrace{(F_L^{(i)} - F_R^{(i)})}^{= F_A^{(i)}} + i\varepsilon^{\mu\nu q k} \underbrace{(F_L^{(i)} + F_R^{(i)})}_{= F_V^{(i)}}$$

- For $E_\gamma \gg \Lambda_{\mathrm{QCD}}$ $\quad F_R^{(i)} \sim \frac{\Lambda_{\mathrm{QCD}}}{E_\gamma} F_L^{(i)} \Rightarrow F_A^{(i)} \approx F_V^{(i)}$

Two-step matching onto SCET

[Beneke-Bobeth-Wang, '20]

- *Decoupling of h modes $O(m_b^2)$ in QCD $\rightarrow$ SCET$_I$ matching*

$$\sum_i^9 \eta_i C_i \, T_i^{\mu\nu} \; = \; \sum_i^9 C_i H_i(q^2) \cdot$$

$$\cdot \, \mathrm{FT}_x \langle 0 | \, T\{ J^\mu_{\mathrm{em,SCET_I}} \underbrace{(x), \, [\bar{q}_{\mathrm{hc}} \gamma_L^{\nu_\perp} h_v](0)}_{} \} | B \rangle$$

separation $x \sim 1/\sqrt{E_\gamma \Lambda_{\mathrm{QCD}}}$

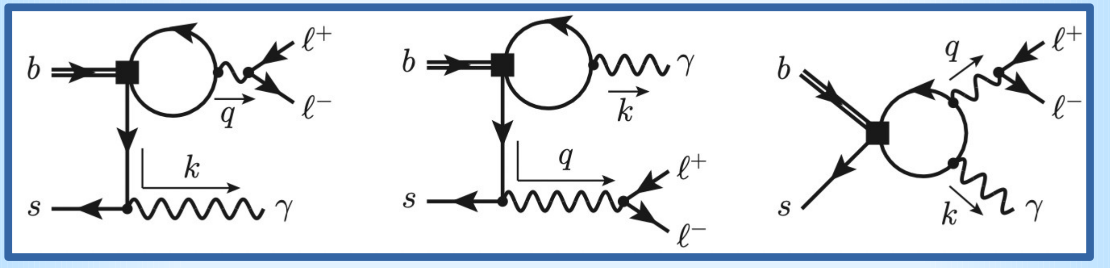
i.e. intermediate propagator is hc

- *Decoupling of hc modes $O(E_\gamma \, \Lambda_{QCD} \, ; \, m_b \, \Lambda_{QCD})$ in SCET$_I$ $\rightarrow$ SCET$_{II}$*

NLP

[Beneke-Bobeth-Wang, '20]

- *Three sources*
  - *coupling of γ to b quark*
  - *power corr's to SCET$_I$ correlator at tree level*
  - *annihilation-type insertions of 4q operators* ⇨ *local*



- *Two soft FFs*
  - $\xi(E_\gamma)$: *computable as in $B_u \to \ell\, \nu\, \gamma$* [Beneke-Rohrwild, '11]
  - *For B-type contributions:* $\tilde{\xi}(E_\gamma)$
    Its Im develops resonances, thus escaping a factorization description

Resonances

[Beneke-Bobeth-Wang, *'20*]

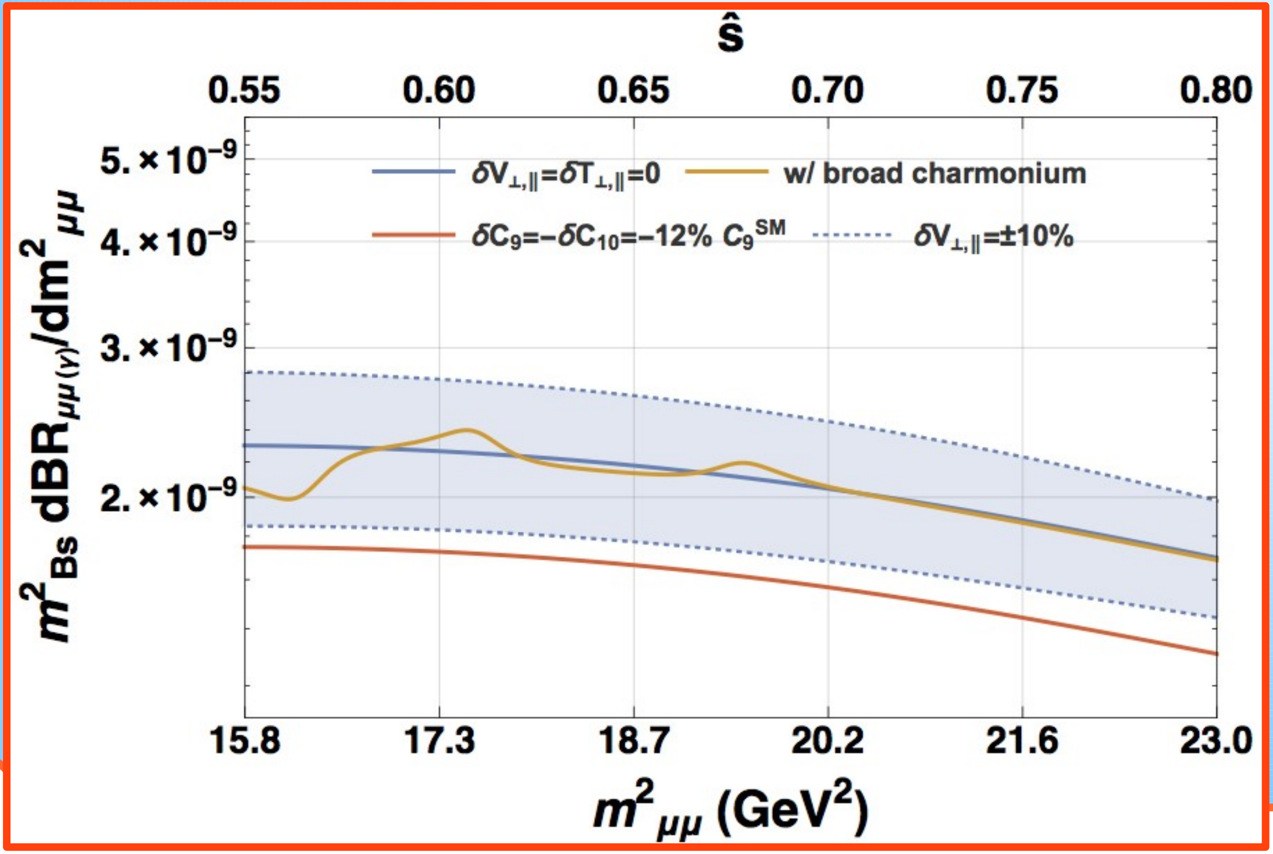
- $T_{7B}^{\mu\nu}$ *leads to* $\overline{A}_{res}$
  - *standard spectral repr. (à la BW)*
  - *formally power-suppressed*

    hence inclusion won't lead to double counting of some short-distance contributions

$B_s \to \mu\mu\gamma$ spectrum

- *In [DG, Reboud, Zwicky, '17] resonant ansatz used to rewrite low-q^2 BR in terms of the measured BR($B_s \to \phi\gamma$)*

- *Then main focus on large-q^2 region, above narrow charmonium. Broad-charmonium pollution estimated with similar resonant ansatz*



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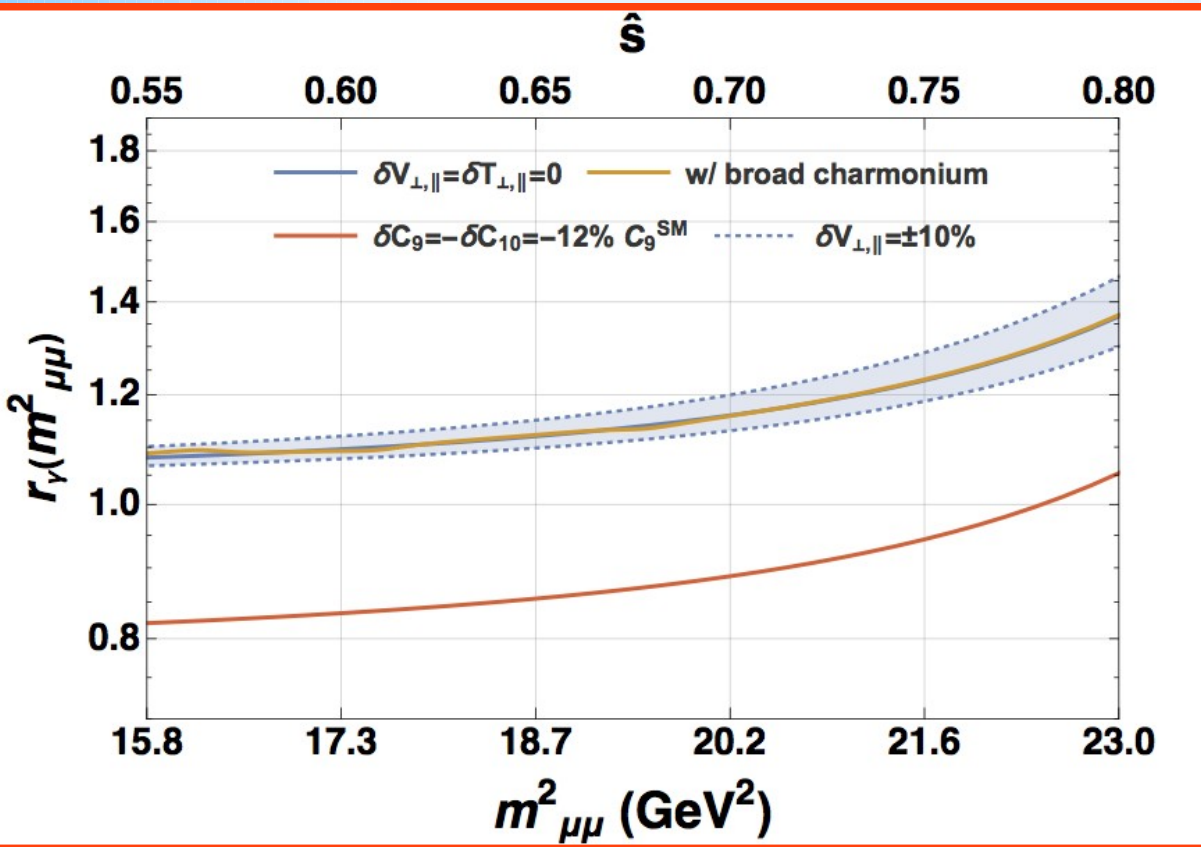

$$r_\gamma \equiv \frac{d\,BR(B_s \to \mu\mu\gamma)/dq^2}{d\,BR(B_s \to e e \gamma)/dq^2}$$