$b \to s\ell\ell$ decays above the $D\bar{D}$ threshold

Beyond the Flavour Anomalies V

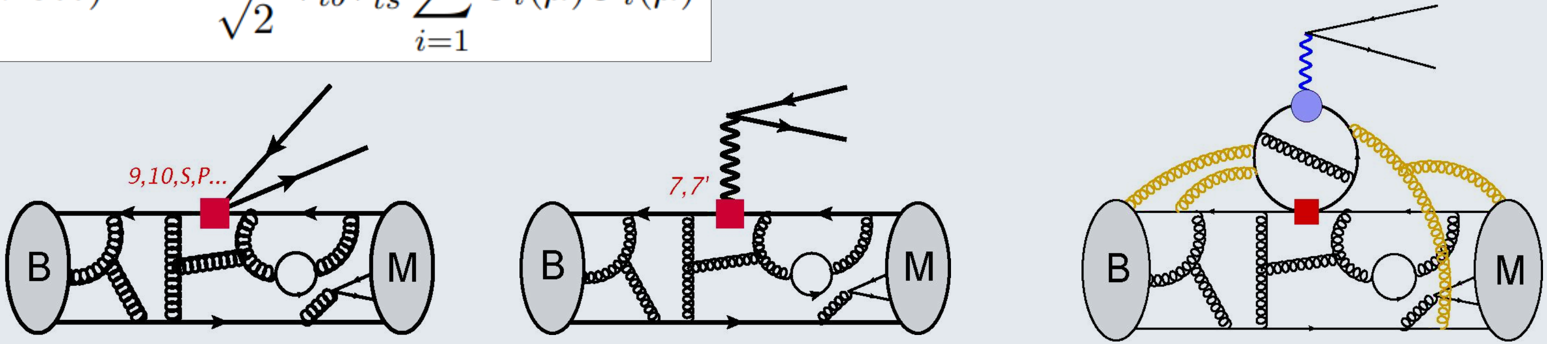
Siegen – 09/04/2024

Méril Reboud

Based on 2312.00619 [Hanhart, Kürten, MR, van Dyk]

Nonlocal Contributions

$$\mathcal{H}(b \to s\ell\ell) = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} C_i(\mu)\mathcal{O}_i(\mu)$$



$$\mathcal{A}_\lambda^{L,R}(B \to M_\lambda \ell\ell) = \mathcal{N}_\lambda \left\{ (C_9 \mp C_{10})\mathcal{F}_\lambda(q^2) + \frac{2m_b M_B}{q^2}\left[C_7 \mathcal{F}_\lambda^T(q^2) - 16\pi^2 \frac{M_B}{m_b}\mathcal{H}_\lambda(q^2)\right]\right\}$$

- B → $K^{(*)}$ μμ
- B_s → φ μμ, ...

Non-local form-factors:

$$\mathcal{H}_\lambda(k,q) = i\int d^4x\, e^{iq\cdot x}\mathcal{P}_\lambda^\mu \langle \bar{M}(k)|T\{Q_c[\bar{c}\gamma_\mu c](x), \mathcal{C}_i\mathcal{O}_i\}|\bar{B}(q+k)\rangle$$

Long story short

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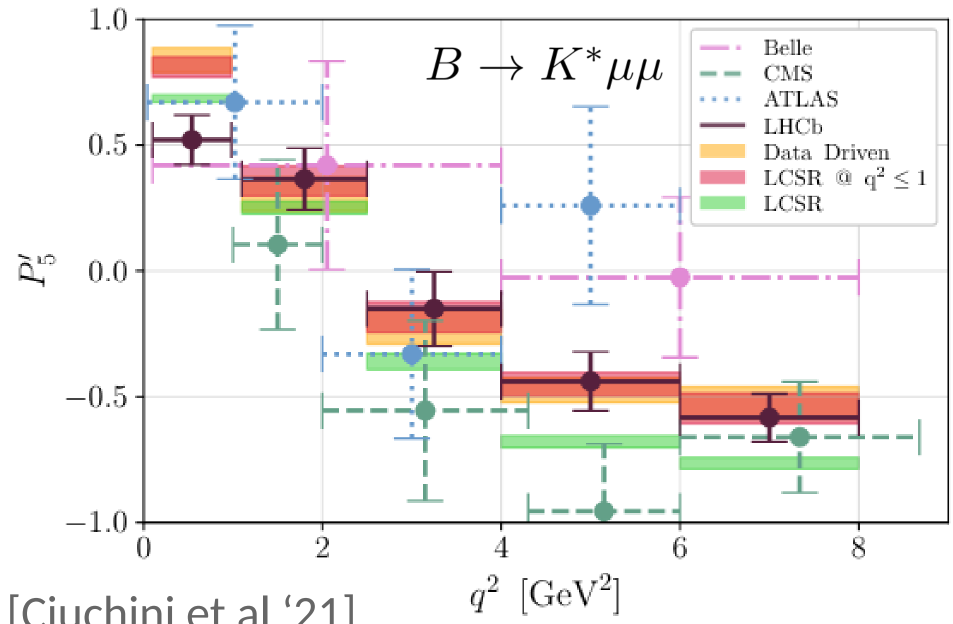

[Ciuchini et al '21]

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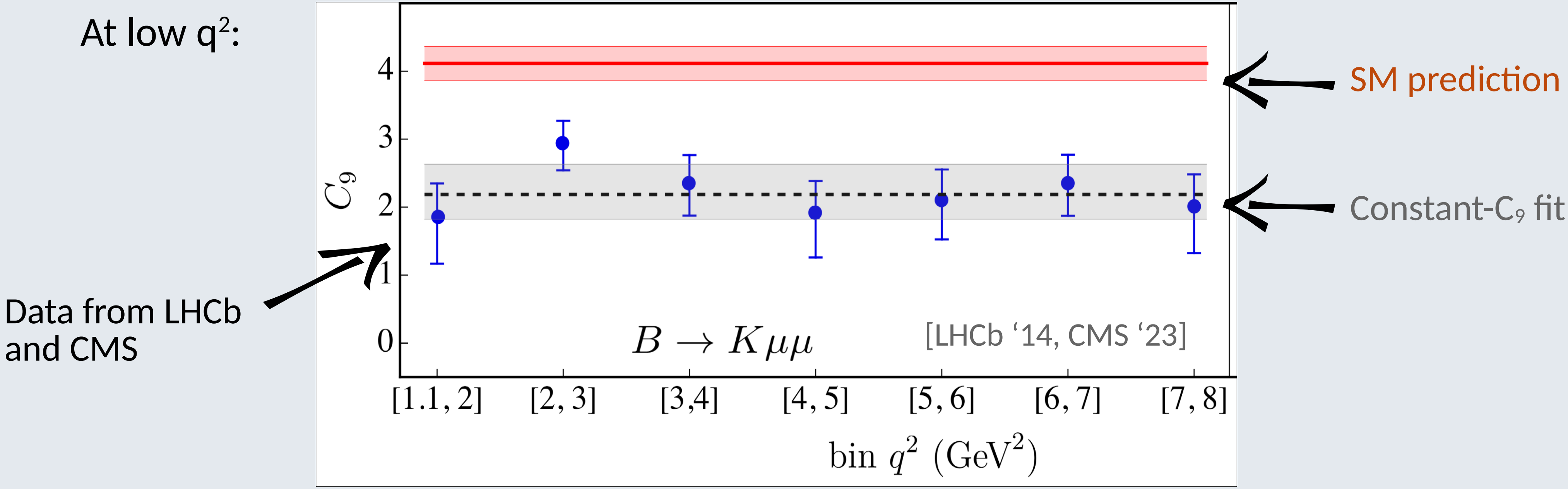
2) The contribution **mimics new physics** by shifting C_9
   - → *Pure* data-driven approaches *can't resolve SM and NP* [Ciuchini *et al* '21, '22]
   - → Data favors a constant shift in C_9 [Bordone, Isidori, Maechler, Tinari '24]

At low q^2:



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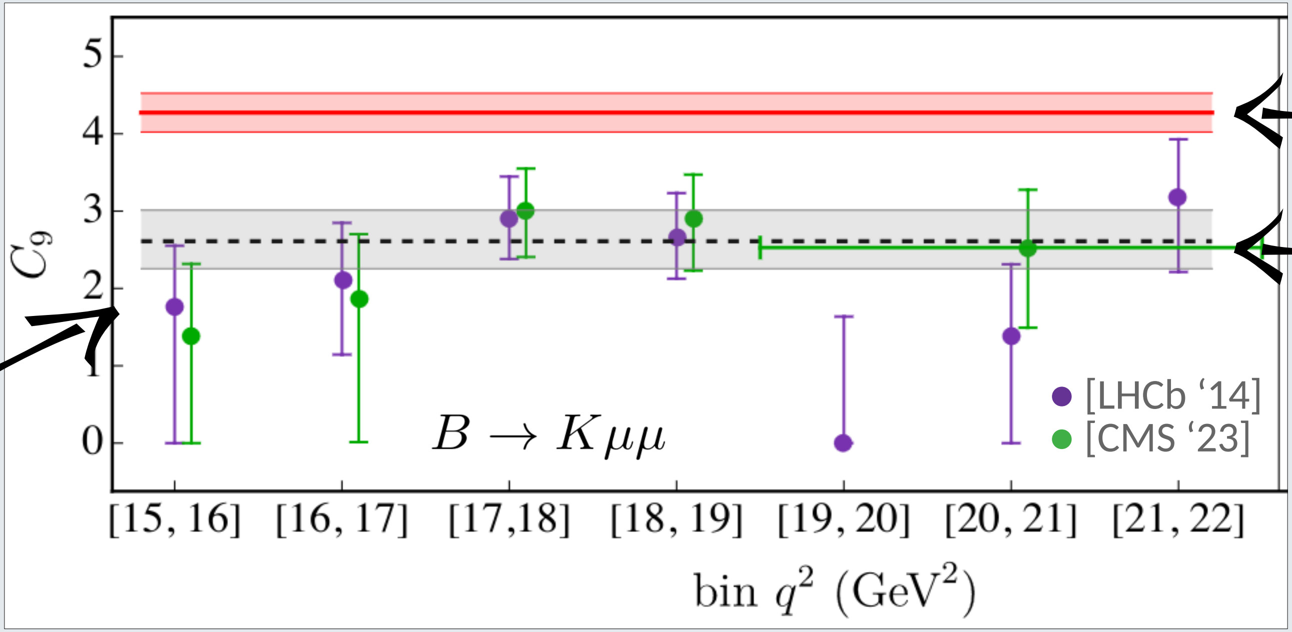
At high q^2:

Data from LHCb and CMS

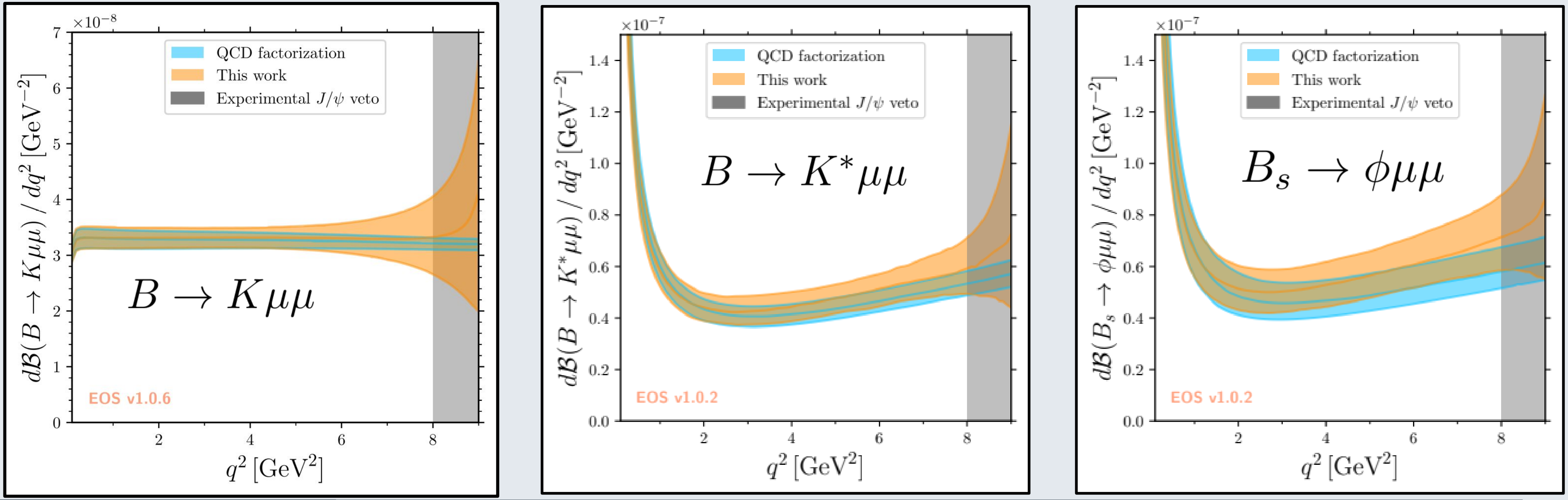


SM prediction

Constant-C_9 fit

Long story short

1) The contribution is **dominated by the charm loops** due to O_{1c} and O_{2c}
2) The contribution **mimics new physics** by shifting C_9
3) *Assuming that the analytic structure is well understood*, dispersive bounds and explicit calculation at negative q^2 allows to **control the charm-loop below the $D\bar{D}$ threshold** [Gubernari, MR, van Dyk, Virto '22]



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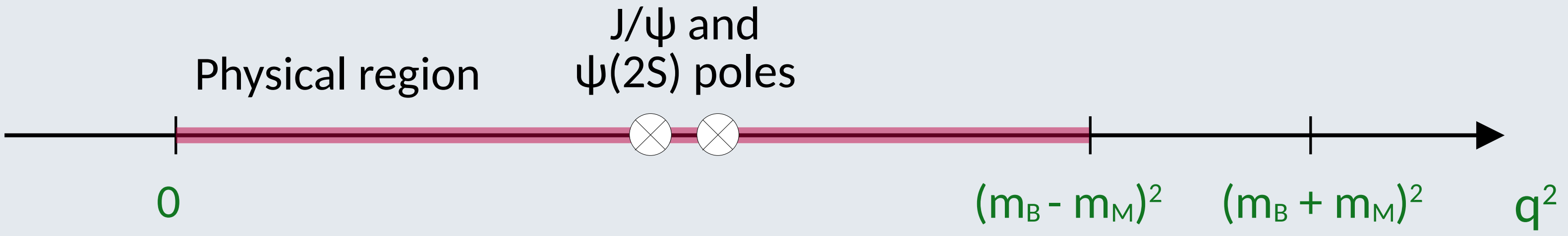
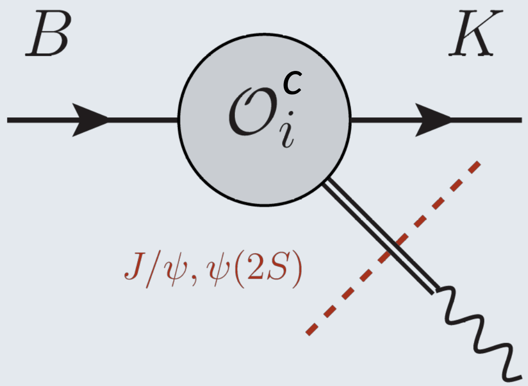
Can we say anything (just) above threshold?

Analyticity properties of H_μ



- Poles due to the narrow charmonium resonances



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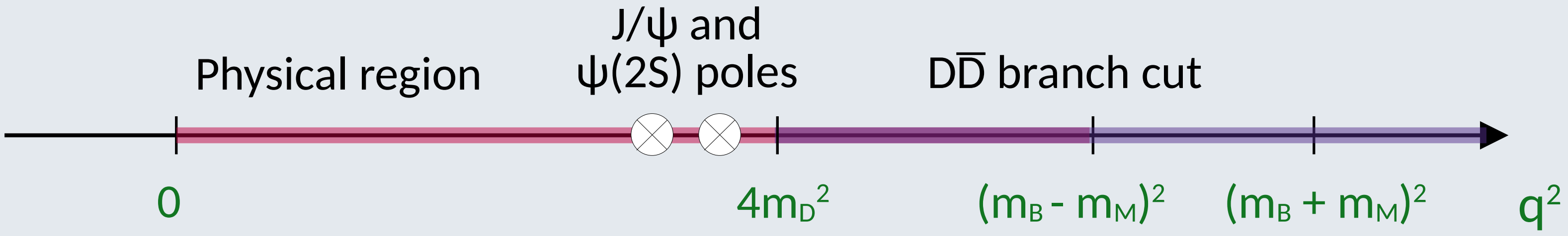
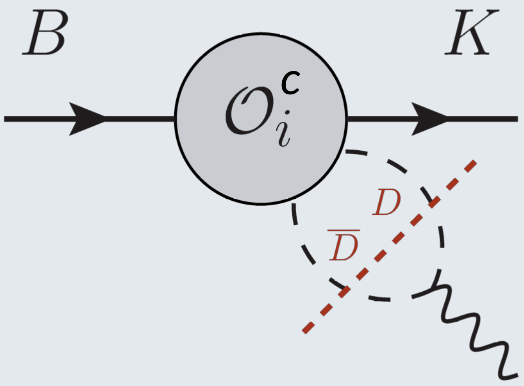


- Poles due to the narrow charmonium resonances
- Branch-cut starting at $4m_D^2$

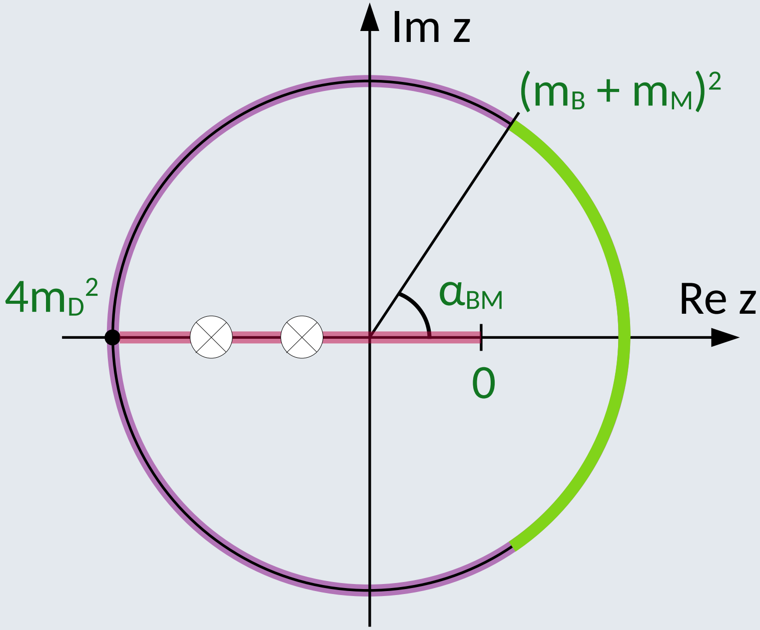


GRvDV parametrization

- Nonlocal form factors are expanded using **orthonormal polynomials** of the arc of the unit circle [Gubernari, MR, van Dyk, Virto '22]:

$$z(s) = \frac{\sqrt{4m_D^2 - s} - \sqrt{4m_D^2 - s_0}}{\sqrt{4m_D^2 - s} + \sqrt{4m_D^2 - s_0}}$$

$$\mathcal{H}_\lambda(z) = \frac{1}{\phi(z)\mathcal{P}(z)} \sum_{k=0}^{N} a_{\lambda,k}\, p_k(z)$$



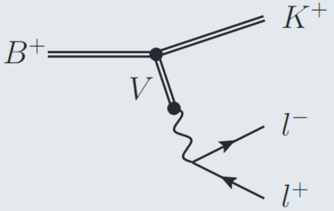
- The coefficients respect a **simple bound** [Gubernari, van Dyk, Virto '20]:

$$\sum_{n=0}^{\infty} \left\{ 2\left|a_{0,n}^{B\to K}\right|^2 + \sum_{\lambda=\perp,\parallel,0} \left[2\left|a_{\lambda,n}^{B\to K^*}\right|^2 + \left|a_{\lambda,n}^{B_s\to\phi}\right|^2 \right] \right\} < 1$$

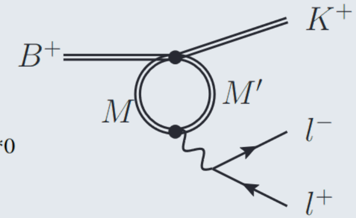
- The series converges on an arc of the unit circle but the convergence is slow and useless in practice

It is worth it! → see Andrea's talk

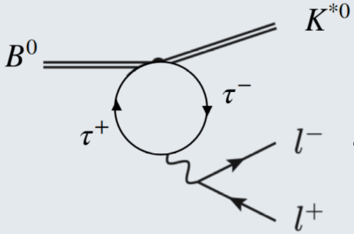
- Preliminary plot from Hadavizadeh's talk in Moriond
- Fitted with a dispersion relation that implements [Cornella *et al* '20]:



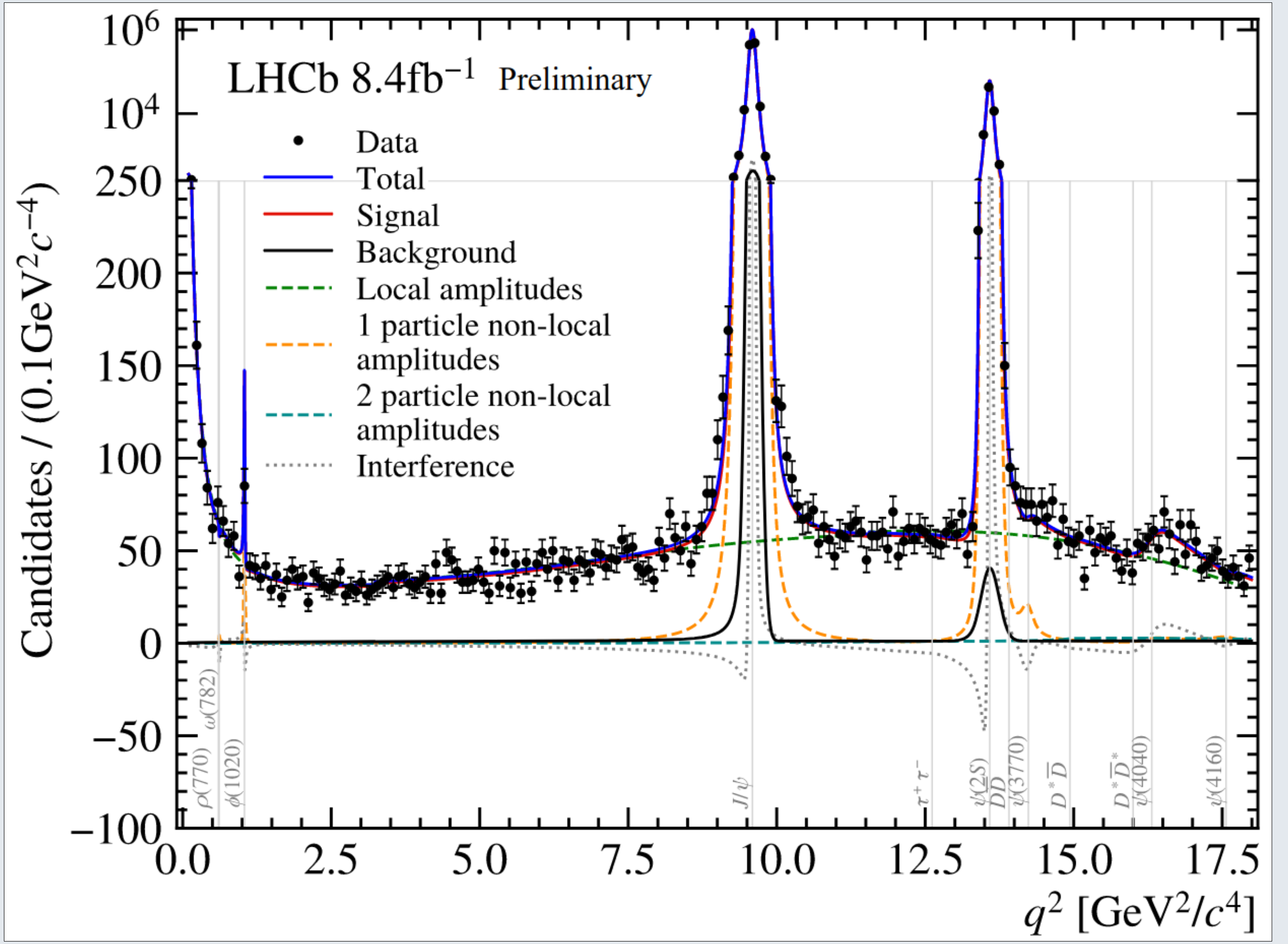
- 1pt contributions



- 2pt contributions

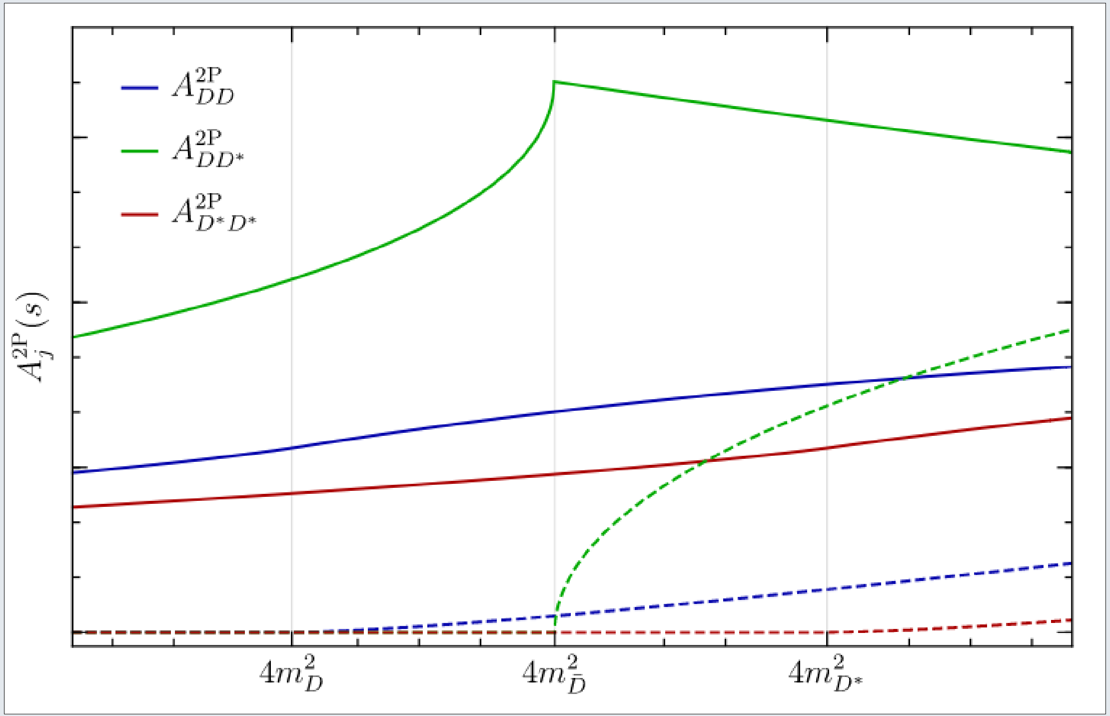


- tau contribution



The dispersive approach

- Implementing the contributions one by one in a dispersive approach has **several drawbacks**:
  - Unitarity is broken close to the resonances
  - Fuzzy distinction between resonant and non-resonant contributions
  - The model parameters need to be extracted from other observables and nothing ensures that they equally apply to the decay of interest



"Naive" Factorization

$$\mathcal{H}_\lambda(k,q) = i\int d^4x\, e^{iq\cdot x}\mathcal{P}^\mu_\lambda\,\langle \bar{M}(k)|T\{Q_c[\bar{c}\gamma_\mu c](x), \mathcal{C}_i\mathcal{O}_i\}|\bar{B}(q+k)\rangle$$

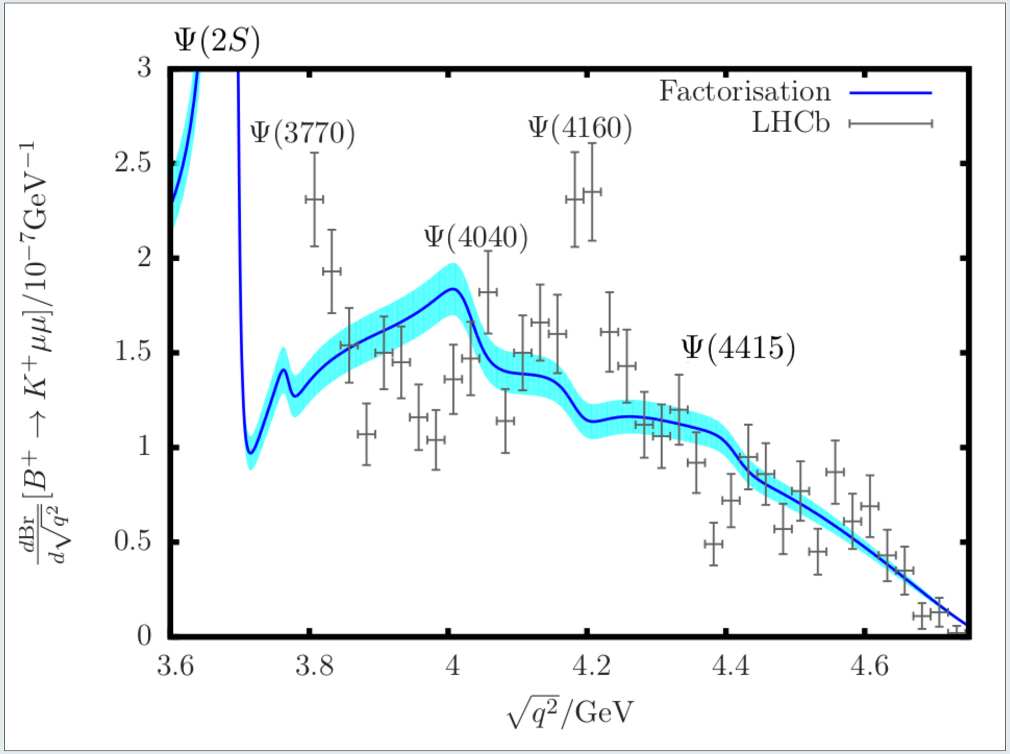
- Factorization approximation [Kruger & Sehgal `96; Lyon & Zwicky '14; Braß, Hiller *et al* '16]

$$\mathcal{H}^{\mathrm{KS}}_\lambda(q^2) = (C_F\mathcal{C}_1 + \mathcal{C}_2)\,\Pi(q^2)\,\mathcal{F}_\lambda(q^2)$$

- Needs a parametrization of the R-ratio

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu\mu)} \propto \mathrm{Im}\,\Pi(q^2)$$

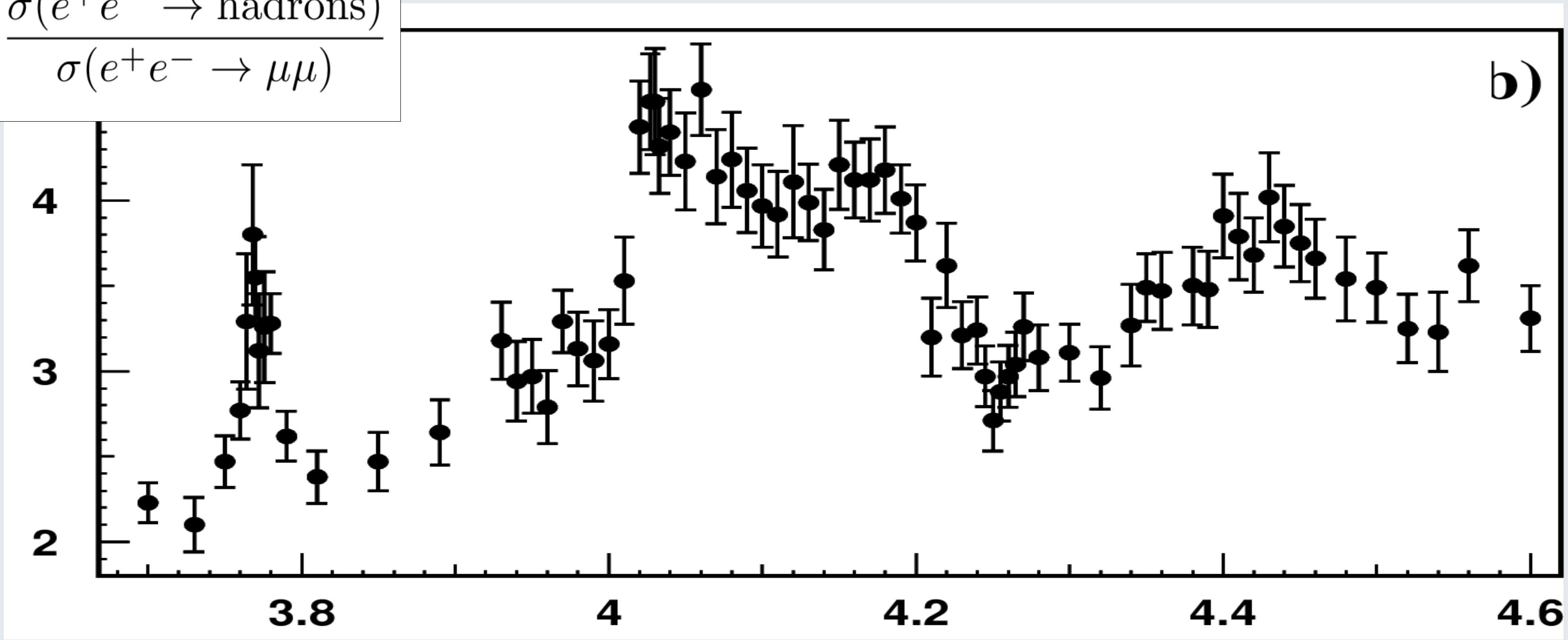
- Requires additional factors to fit the data → large non-factorizable effects?



The R ratio

[BES '01]

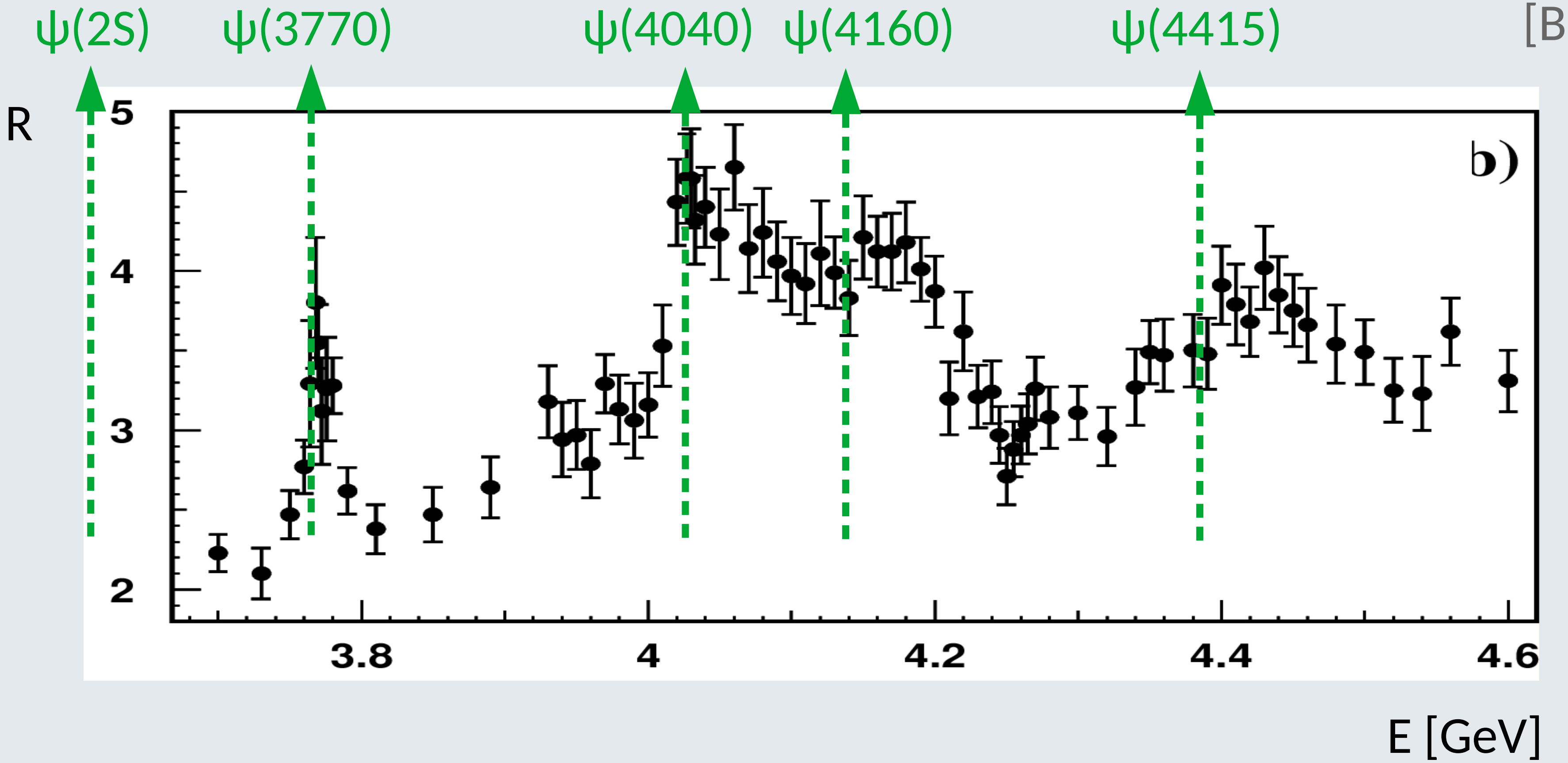
$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu\mu)}$$



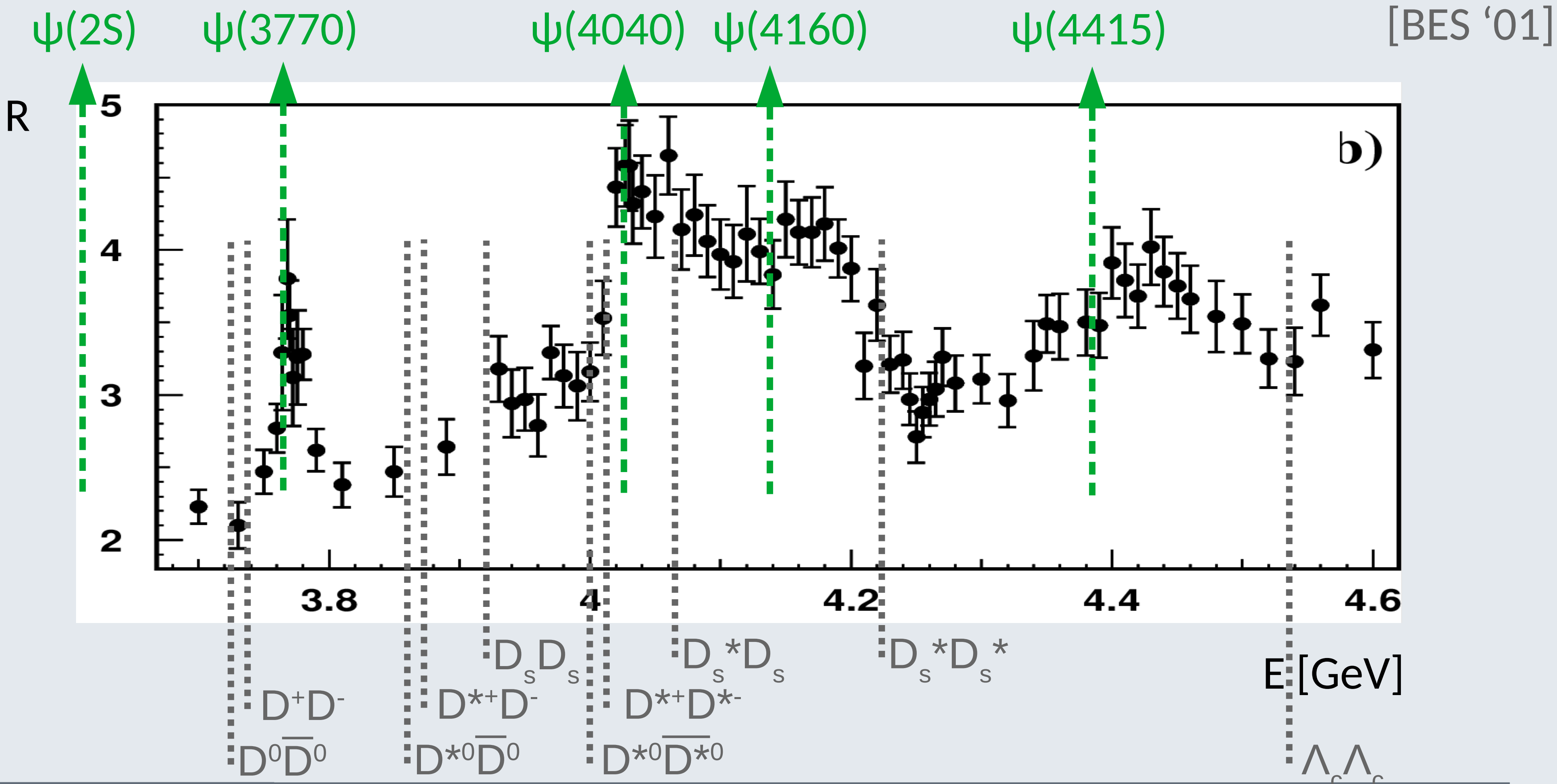
E [GeV]

The main $I^G(J^{PC}) = 0^-(1^{--})$ resonances

[BES '01]



Thresholds



ψ(2S)
ψ(3770)
ψ(4040)
ψ(4160)
ψ(4415)
[BES '01]
R
5
4
3
2
b)
3.8
4
4.2
4.4
4.6
E [GeV]
D^+D^-
$D^0\overline{D}^0$
$D^{*+}D^-$
$D^{*0}\overline{D}^0$
D_sD_s
$D^{*+}D^{*-}$
$D^{*0}\overline{D}^{*0}$
$D_s^*D_s$
$D_s^*D_s^*$
$\Lambda_c\Lambda_c$

K Matrix

- We have a **coupled multichannel problem:**

  $\psi \to e^+e^-$, $\psi \to D^{(*)}\bar{D}^{(*)}$, $(\psi \to BK^{(*)})$

- Resonances are **close to thresholds**
- **K-matrix** is the tool to use [Chung *et al.* '95, PDG's Resonances review]

Real valued couplings

$$\mathcal{M} = n\,[1 - \mathcal{K}\,\Sigma]^{-1}\,\mathcal{K}\,n$$

Analytic continuation of the phase space factors

$$\mathcal{K}_{ij}(s) = \sum_{r=1}^{N_R} \frac{g_i^r\, g_j^r}{m_r^2 - s} + c_{ij}$$

Kinematic factor:

$$n_k = (q_k/q_0)^{l_k} F_{l_k}(q_k/q_0)$$

e^+e^- and $D\bar{D}$ channels

$c\bar{c}$ resonances

Non-resonant contributions

Some details on the model

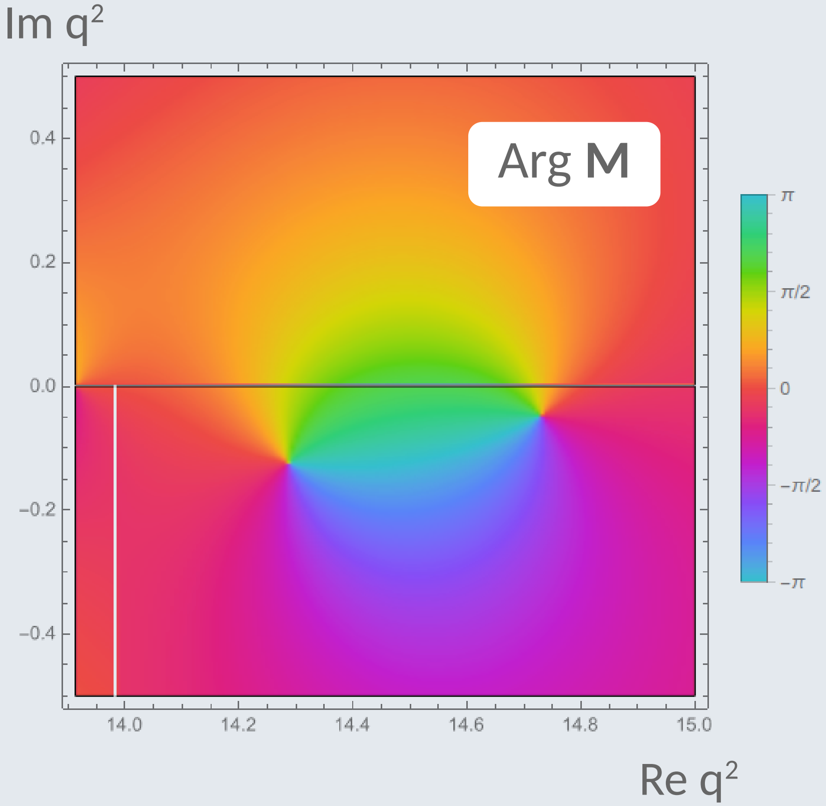
- Focus on the ψ(3770) region for a proof of concept

$$r \in \{\psi(2S), \psi(3770)\}$$

- Model the non-D$\bar{\text{D}}$ decays of the ψ(3770) with an effective 2-body P-wave channel

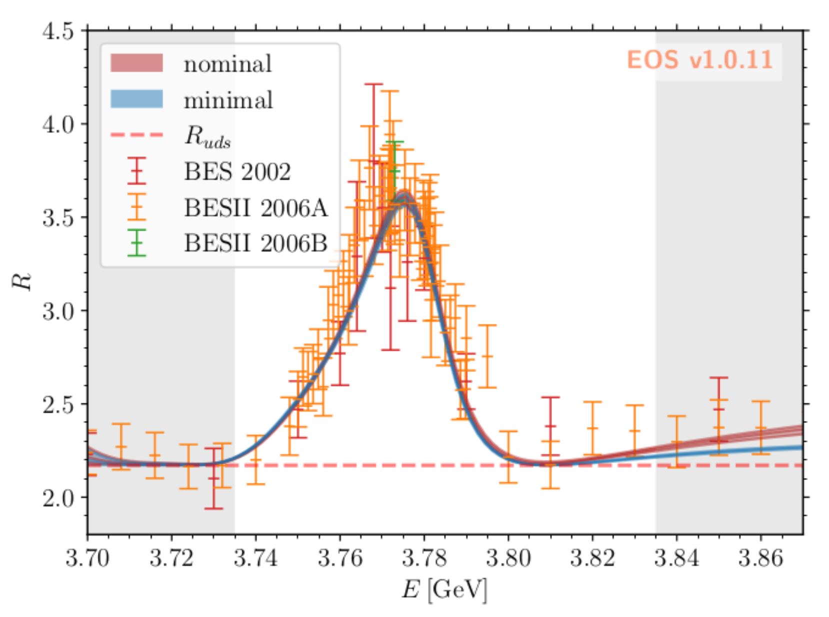
$$k \in \{e^+e^-, D^+D^-, D^0\bar{D}^0, \text{eff}_{\psi(2S)}, \text{eff}_{\psi(3770)}\}$$

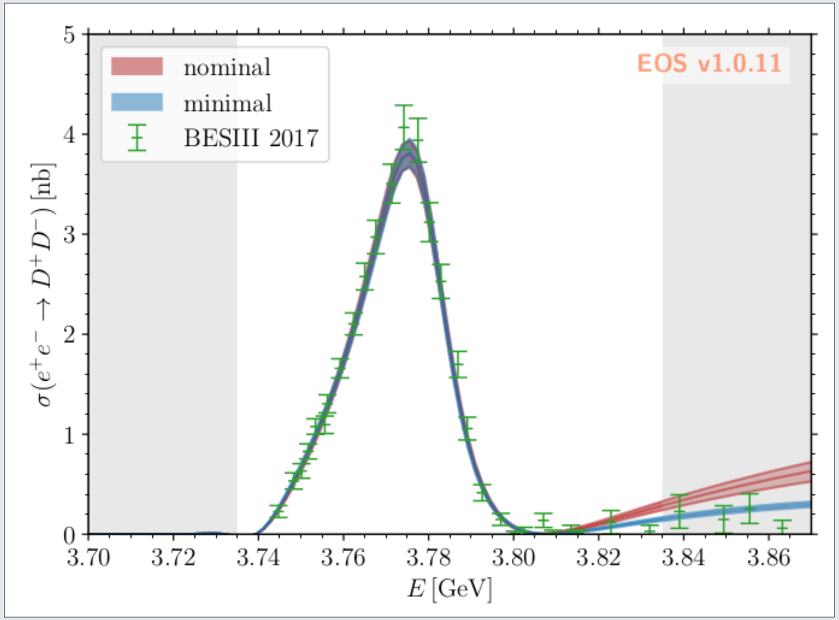
- The resonance pole and residues are extracted from the second Riemann sheet



Results for the ψ(3770) resonance

- Fit several models (with or without non-D$\bar{D}$ effective channel), excellent p-values
- Interference with the ψ(2S) crucial to reproduce the experimental shapes





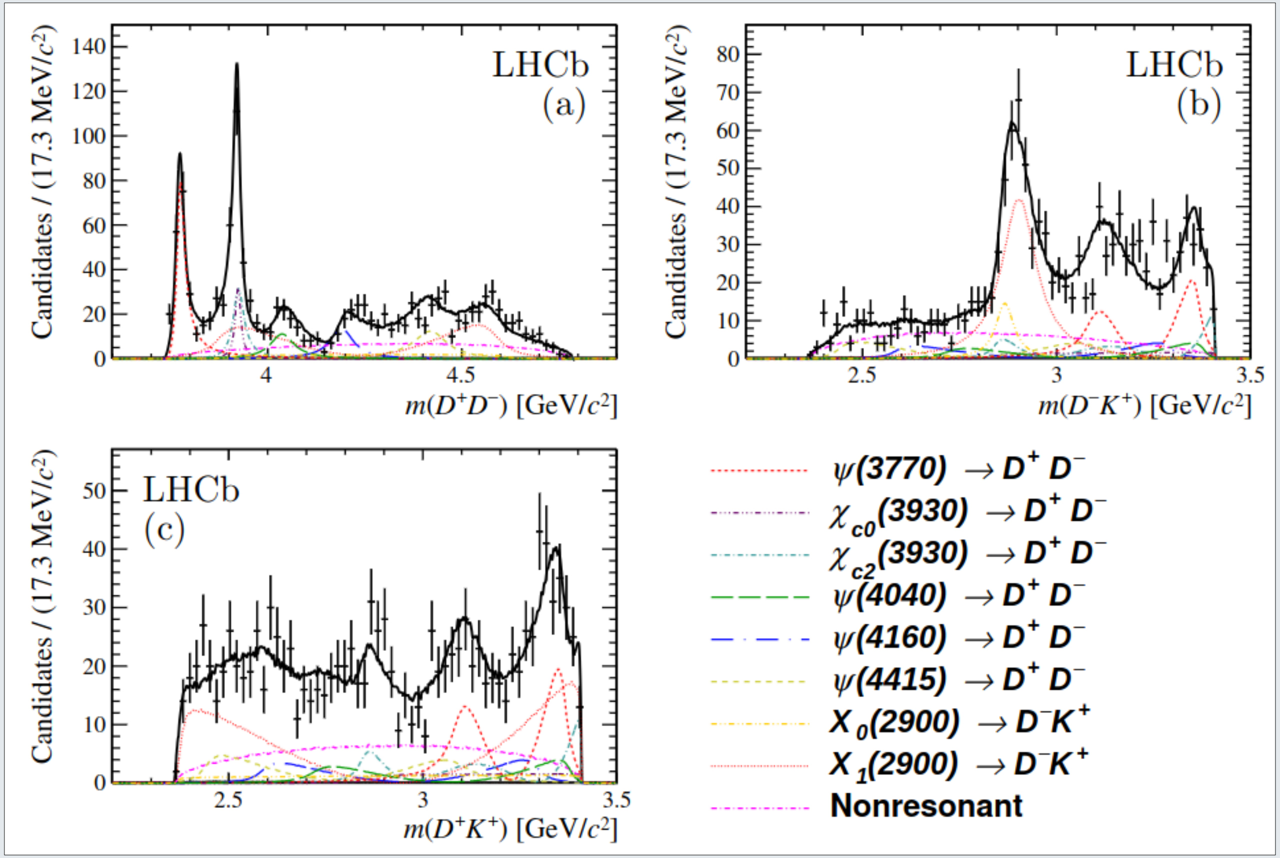
- Isospin symmetry is perfectly recovered

$$g^{\psi(3770)}_{D^0\bar{D}^0} / g^{\psi(3770)}_{D^+D^-} = 0.99 \pm 0.03$$

- ψ(3770) decays dominantly to D$\bar{D}$

$$\mathcal{B}(\psi(3770) \to \text{non-}D\bar{D}) < 6\% \text{ at } 90\% \text{ probability}$$

- Dalitz analysis of $B \to KD\bar{D}$ is available [LHCb '20]
- Problem: we need to single out the $D\bar{D}$ P-wave contribution
- Studied in a second LHCb paper [LHCb 2009.00025]
  - Expansion of the $D\bar{D}$ helicity angle in Legendre polynomials
  - LHCb provides moments of these distributions



Future hurdles

- Extending the ψ(3770) fit to larger q^2 will open the following issues:
  - **Analytic difficulties**:
    - Description of P-waves with different masses
    - Description of F-waves channels
    - Connection between the waves and the experimental helicities
  - **Numerical difficulties**:
    - Jump from 6 channels 2 resonances to 20 channels 5 resonances, i.e. from O(10) to O(100) parameters → assume isospin symmetry? U-spin?
    - Jump from 2 to 8 Riemann sheets
    - Huge number of experimental data points that need to be evaluated
- This work is **in progress** (it is fun, you can join if you feel unoccupied)
- Yesterday on the arXiv: K-matrix description including the ψ(4040) [Hüsken *et al* ‘24]

Conclusion & Outlook

- Nonlocal contributions to $b \to s\ell\ell$ decays are a **main source of theory uncertainties**.
- A systematic approach based on analyticity and unitarity allows for a description of these contributions **below the open-charm threshold**.
- We propose a new data-driven approach, based on a **K matrix description** of the $e^+e^- \to c\bar{c}$ and $b \to sc\bar{c}$ experimental observables to infer predict these contributions in the region of broad charmonium.

Back-up

Future work

	channel	type	related to
0	e^+e^-	PP (P wave)	-
1	eff(2S)	Effective	-
2	eff(3770)	Effective	-
3	eff(4040)	Effective	-
4	eff(4160)	Effective	-
5	eff(4415)	Effective	-
6	$D^0\ \bar{D}^0$	PP (P wave)	-
7	$D^+\ D^-$	PP (P wave)	6 (isospin)
8	$D^0\ \bar{D}^{*0}$	VP (P wave)	-
9	$D^{*0}\ \bar{D}^0$	VP (P wave)	8 (c.c.)
10	$D^+\ D^{*-}$	VP (P wave)	8 (isospin)
11	$D^{*+}\ D^-$	VP (P wave)	8 (c.c.)
12	$D_s^+\ D_s^-$	PP (P wave)	6 (u-spin)
13	$D^{*0}\ \bar{D}^{*0}$	VV (P wave, S=0)	-
14	$D^{*0}\ \bar{D}^{*0}$	VV (P wave, S=2)	-
15	$D^{*0}\ \bar{D}^{*0}$	VV (F wave, S=2)	-

	channel	type	related to
16	$D^{*+}\ D^{*-}$	VV (P wave, S=0)	13 (isospin)
17	$D^{*+}\ D^{*-}$	VV (P wave, S=2)	14 (isospin)
18	$D^{*+}\ D^{*-}$	VV (F wave, S=2)	15 (isospin)
19	$D_s^+\ D_s^{*-}$	VP (P wave)	8 (u-spin)
20	$D_s^{*+}\ D_s^-$	VP (P wave)	19 (c.c.)
21	$D_s^{*+}\ D_s^{*-}$	VV (P wave, S=0)	13 (u-spin)
22	$D_s^{*+}\ D_s^{*-}$	VV (P wave, S=2)	14 (u-spin)
23	$D_s^{*+}\ D_s^{*-}$	VV (F wave, S=2)	15 (u-spin)

Effective channels

Dilepton channel (assumes LFU)

$D_{(s)}\bar{D}_{(s)}$ channels

$D^*_{(s)}\bar{D}^*_{(s)}$ channels

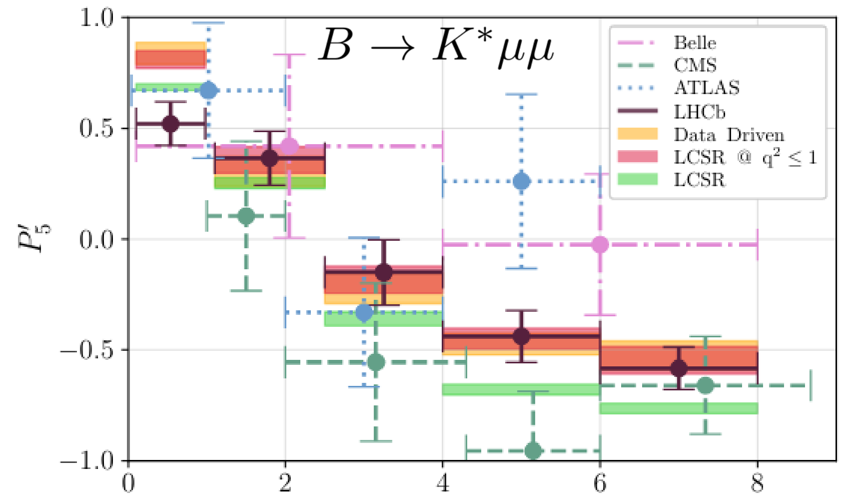
$D_{(s)}\bar{D}^*_{(s)}$ channels

q^2 parametrization

- **Simple q^2 expansion** [Jäger, Camalich '12; Ciuchini et al. '15]

$$\mathcal{H}_\lambda(q^2) = \mathcal{H}_\lambda^{\mathrm{QCDF}}(q^2) + h_\lambda(0) + \frac{q^2}{m_B^2} h'_\lambda(0) + \dots$$

Computed in [Beneke, Feldman, Seidel '01]



[Ciuchini et al '21]

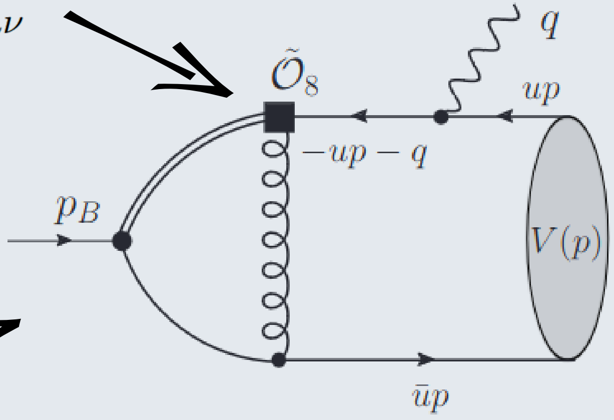
- The h_λ terms can be fitted or varied
- Fitting the h_λ terms on data gives a satisfactory fit but lacks predictive power
- This parametrization **cannot account** for the analyticity properties of $\mathcal{H}_\lambda$

Anatomy of H_μ in the SM

$C_1(\mu_b)$	$C_2(\mu_b)$	$C_3(\mu_b)$	$C_4(\mu_b)$	$C_5(\mu_b)$	$C_6(\mu_b)$	$C_7(\mu_b)$	$C_8(\mu_b)$	$C_9(\mu_b)$	$C_{10}(\mu_b)$
-0.2906	1.010	-0.0062	-0.0873	0.0004	0.0011	-0.3373	-0.1829	4.2734	-4.1661

- The contribution of O_8 is **negligible** [Khodjamirian, Mannel, Wang, '12; Dimou, Lyon, Zwicky '12]

$$\mathcal{O}_8 = \frac{g_s}{16\pi^2} m_b (\bar{s}_L \sigma^{\mu\nu} T^a b_R) G^a_{\mu\nu}$$



One of the non-factorizable contributions

Anatomy of H_μ in the SM

$C_1(\mu_b)$	$C_2(\mu_b)$	$C_3(\mu_b)$	$C_4(\mu_b)$	$C_5(\mu_b)$	$C_6(\mu_b)$	$C_7(\mu_b)$	$C_8(\mu_b)$	$C_9(\mu_b)$	$C_{10}(\mu_b)$
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- The contribution of O_8 is **negligible** [Khodjamirian, Mannel, Wang, '12]
- The contributions of $O_{3,4,5,6}$ are suppressed by **small Wilson coefficients**

$$\mathcal{O}_3 = (\bar{s}_L\gamma_\mu b_L)\sum_p(\bar{p}\gamma^\mu p)\,, \qquad \mathcal{O}_4 = (\bar{s}_L\gamma_\mu T^a b_L)\sum_p(\bar{p}\gamma^\mu T^a p)\,,$$

$$\mathcal{O}_5 = (\bar{s}_L\gamma_\mu\gamma_\nu\gamma_\rho b_L)\sum_p(\bar{p}\gamma^\mu\gamma^\nu\gamma^\rho p)\,, \qquad \mathcal{O}_6 = (\bar{s}_L\gamma_\mu\gamma_\nu\gamma_\rho T^a b_L)\sum_p(\bar{p}\gamma^\mu\gamma^\nu\gamma^\rho T^a p)\,,$$

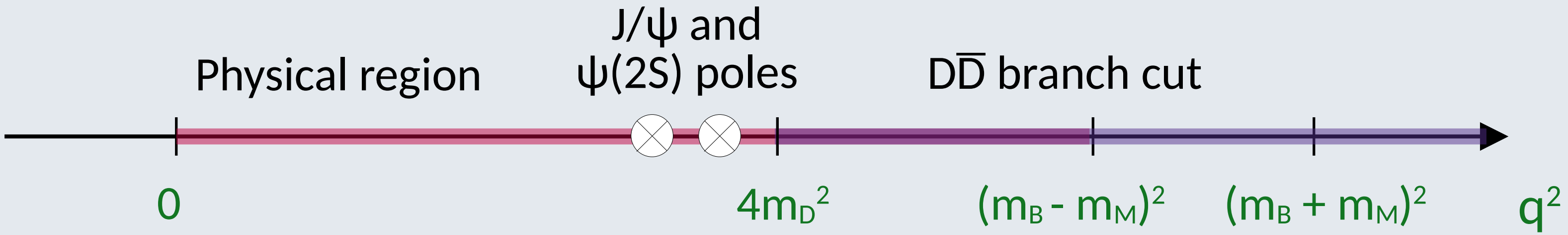
Anatomy of H_μ in the SM

$$\mathcal{O}_1^q = (\bar{s}_L \gamma_\mu T^a q_L)(\bar{q}_L \gamma^\mu T^a b_L)\,, \qquad \mathcal{O}_2^q = (\bar{s}_L \gamma_\mu q_L)(\bar{q}_L \gamma^\mu b_L)$$

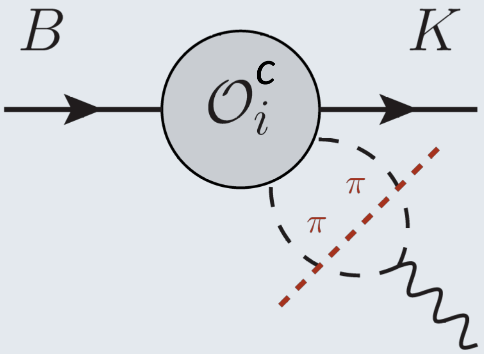
- Light-quark loops are CKM suppressed → **small contributions** even at the resonances [Khodjamirian, Mannel, Wang, '12]

Vector meson	ρ	ω	ϕ	J/ψ	$\psi(2S)$
f_V	221^{+1}_{-1}	195^{+3}_{-4}	228^{+2}_{-2}	416^{+5}_{-6}	297^{+3}_{-2}
$\lvert A_{\bar{B}^0 V \bar{K}^0}\rvert$	$1.3^{+0.1}_{-0.1}$	$1.4^{+0.1}_{-0.1}$	$1.8^{+0.1}_{-0.1}$	$33.9^{+0.7}_{-0.7}$	$44.4^{+2.2}_{-2.2}$
$\lvert A_{B^- V K^-}\rvert$	$1.2^{+0.1}_{-0.1}$	$1.5^{+0.1}_{-0.1}$	$1.8^{+0.1}_{-0.1}$	$35.6^{+0.6}_{-0.6}$	$42.0^{+1.2}_{-1.2}$

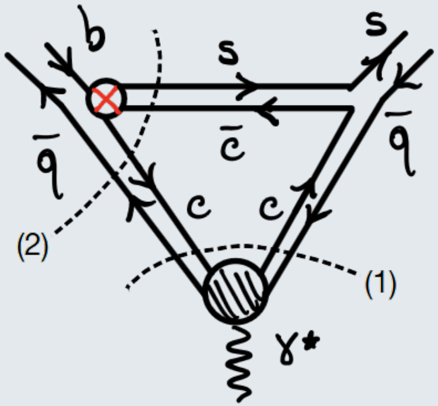
→ The main contribution comes from $\mathbf{O_1^c}$ and $\mathbf{O_2^c}$: **"charm loop"**

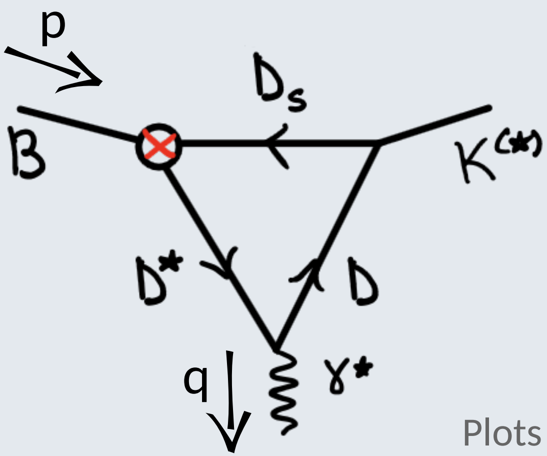


- Poles due to the narrow charmonium resonances
- Branch-cut starting at $4m_D^2$
- Branch-cut starting at $4m_\pi^2 \rightarrow$ negligible (OZI suppressed)



More involved analytic structure?





Plots from [Ciuchini et al. '22]

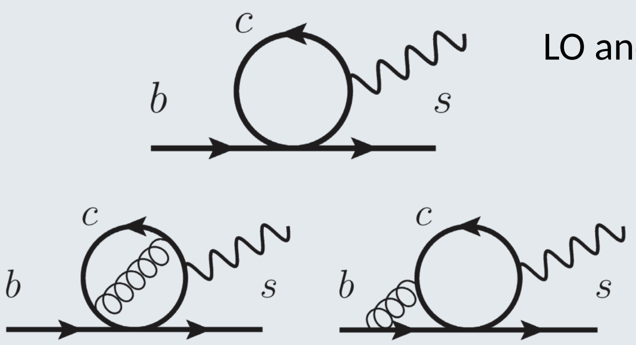
- $M_B > M_{D^*} + M_{D_s}$ → The function $H_\lambda(p^2,q^2)$ has a branch cut in p^2 and the physical decay takes place on this branch cut: **H_λ is complex-valued!**
- Triangle diagrams are known to create *anomalous* branch cuts in q^2 [e.g. Lucha, Melikhov, Simula '06] → Does this also apply here? We have no Lagrangian nor power counting!
- The presence and the impact of such a branch cut in our approach is under investigation

Theory inputs

$\mathcal{H}_\lambda$ can be calculated in **two kinematics regions**:

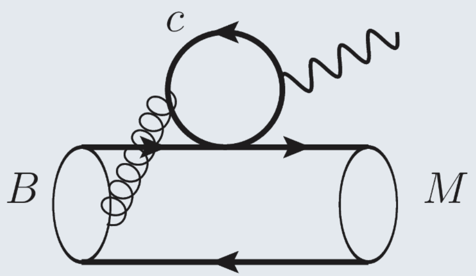
- **Local** OPE $|q|^2 \gtrsim m_b^2$ [Grinstein, Piryol '04; Beylich, Buchalla, Feldmann '11]

- **Light Cone** OPE $q^2 \ll 4m_c^2$ [Khodjamirian, Mannel, Pivovarov, Wang '10]



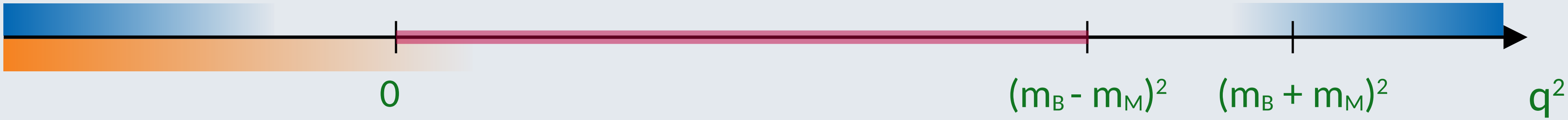
LO and α_s corrections

[Asatrian, Bieri, Greub, Walker '04; de Boer '17; **Asatrian, Greub, Virto '19**]



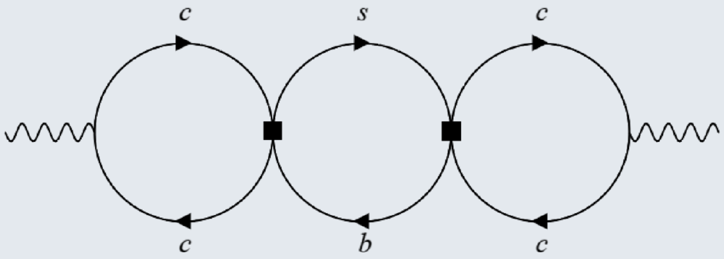
Non-perturbative soft gluon corrections

[Khodjamirian, Mannel, Pivovarov, Wang '10; **Gubernari, van Dyk, Virto '20**]



Dispersive bound

- **Main idea:** Compute the charm-loop induced, inclusive $e^+e^- \to \bar{b}s$ cross-section and relate it to $\mathcal{H}_\lambda$ [Gubernari, van Dyk, Virto '20]



+ other diagrams…

- The optical theorem gives a **shared bound** for **all the b → s processes**:

$$1 > 2 \int_{(m_B+m_K)^2}^{\infty} \left|\hat{\mathcal{H}}_0^{B\to K}(t)\right|^2 dt + \sum_\lambda \left[2 \int_{(m_B+m_{K^*})^2}^{\infty} \left|\hat{\mathcal{H}}_\lambda^{B\to K^*}(t)\right|^2 dt + \int_{(m_{B_s}+m_\phi)^2}^{\infty} \left|\hat{\mathcal{H}}_\lambda^{B_s\to \phi}(t)\right|^2 dt \right]$$

$$\text{known functions} \times \mathcal{H}_0^{B\to K}(t)$$

$$+ \Lambda_b \to \Lambda^{(*)} \ldots$$

Numerical analysis

- The parametrization is fitted to

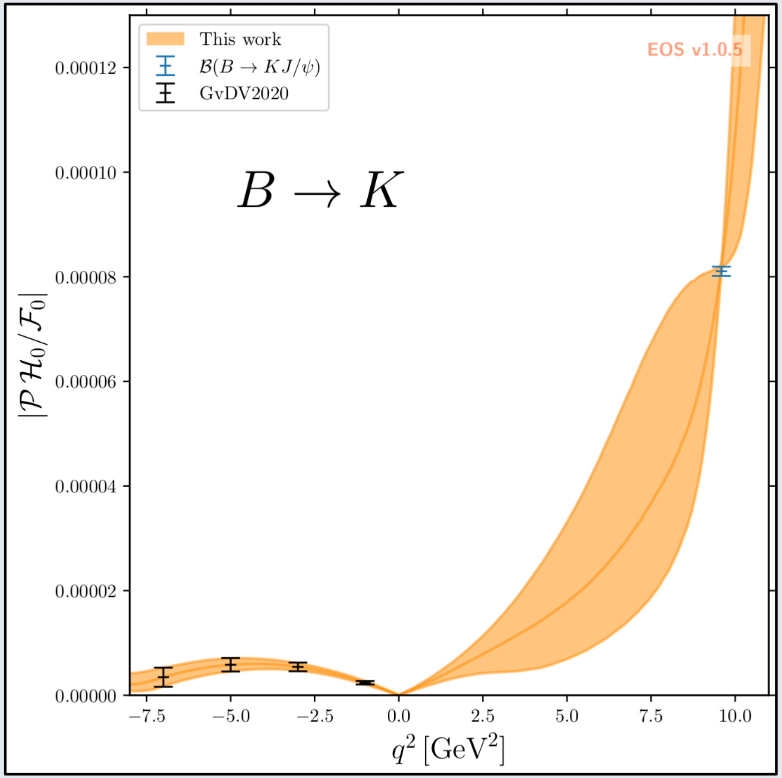
  $B \to K$, $B \to K^*$, $B_s \to \varphi$

  using:
  - 4 theory point at negative q^2 from the light cone OPE
  - Experimental results at the J/ψ
  - Use an **under-constrained fit** and allow for **saturation of the dispersive bound**

→ The uncertainties are **truncation order-independent**, i.e., increasing the expansion order does not change their size

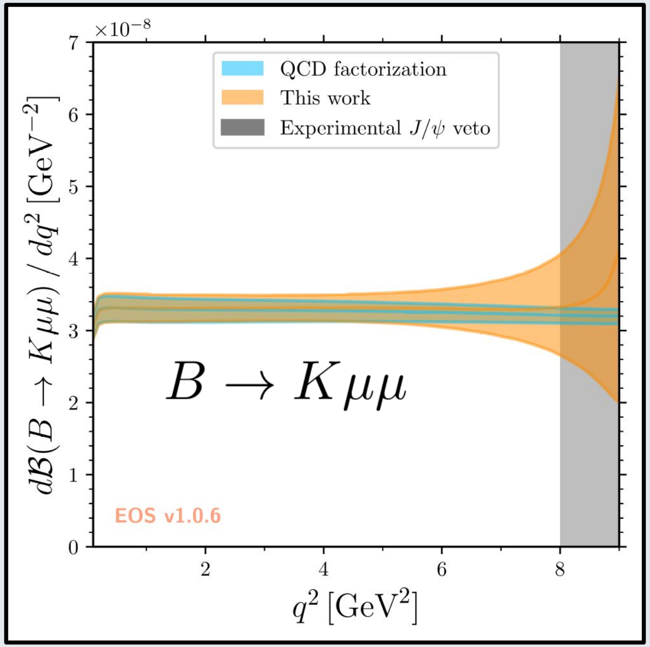
→ All p-values are larger than 11%

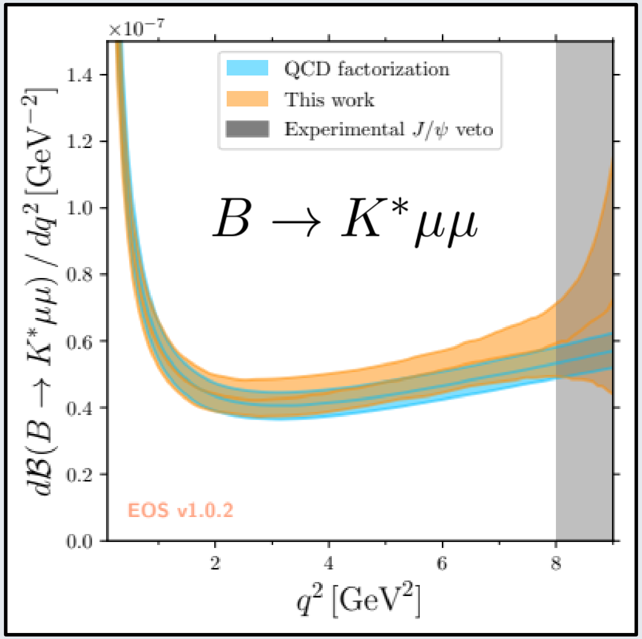
[Gubernari, MR, van Dyk, Virto ‘22]

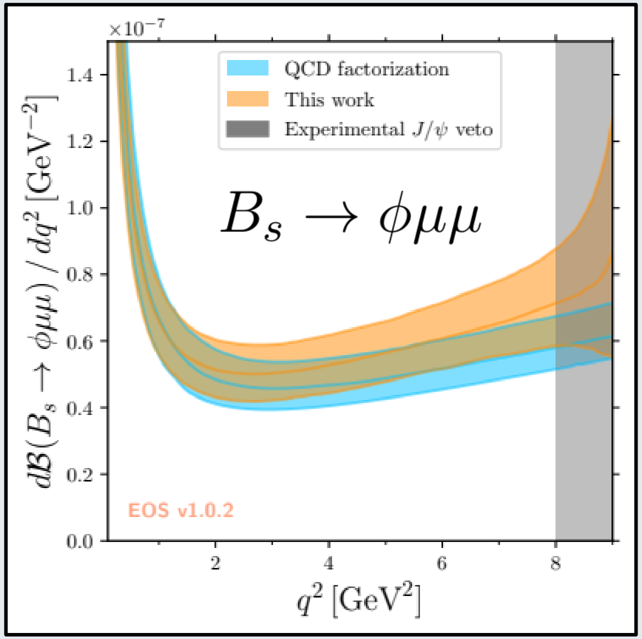


SM predictions

- **Good overall agreement** with previous theoretical approaches
  - Small deviation in the slope of $B_s \to \phi\mu\mu$
- **Larger** but **controlled** uncertainties especially near the J/ψ
  - The approach is **systematically improvable** (new channels, ψ(2S) data…)



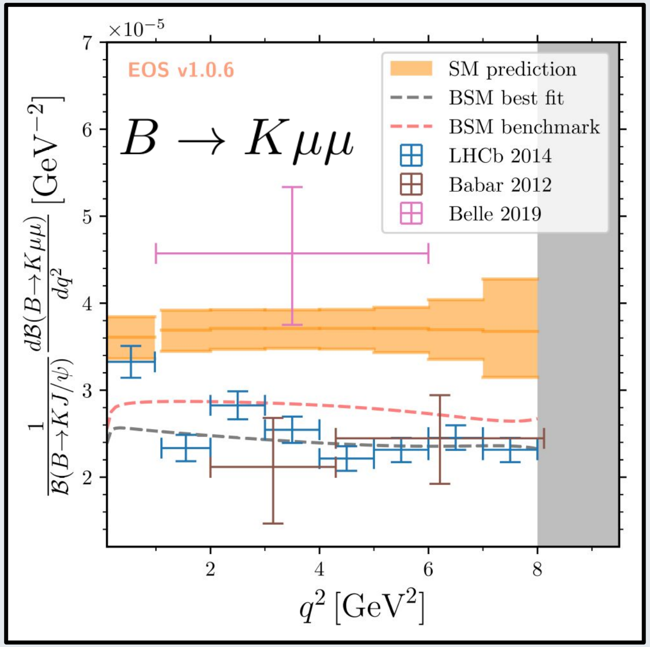


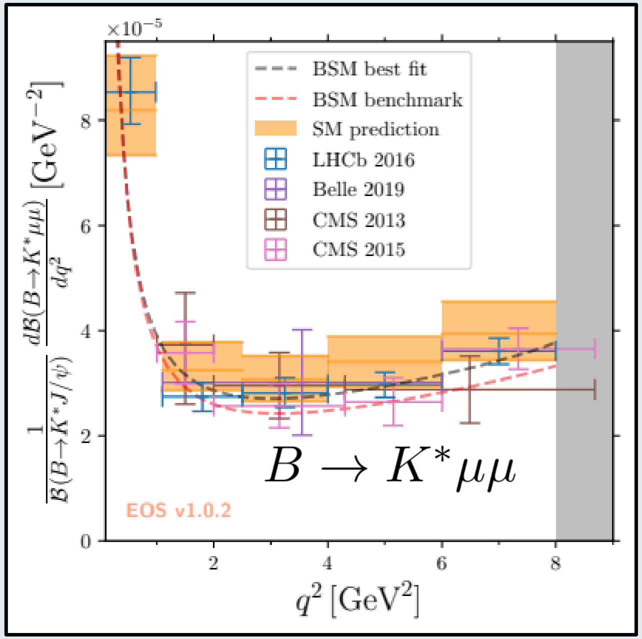


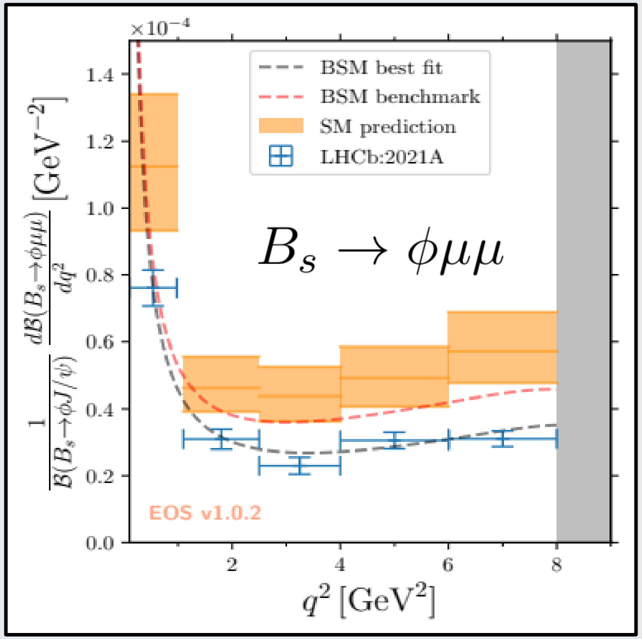
Confrontation with data

- This approach of the non-local form factors **does not solve the "B anomalies"**.
- In this approach, the greatest source of theoretical uncertainty now comes from **local form factors**.

Experimental results:
[Babar: 1204.3933; Belle: 1908.01848, 1904.02440; ATLAS: 1805.04000, CMS: 1308.3409, 1507.08126, 2010.13968, LHCb: 1403.8044, 2012.13241, 2003.04831, 1606.04731, 2107.13428]







Additional plots can be found in the paper: 2206.03797

Local form factors fit

- With this framework we perform a **combined fit** of $B \to K$, $B \to K^*$ and $B_s \to \varphi$ LCSR and lattice QCD inputs:
  - $B \to K$:
    - [HPQCD ’13 and ’22; FNAL/MILC ’17]
    - ([Khodjamiriam, Rusov ’17]) → large uncertainties, not used in the fit
  - $B \to K^*$:
    - [Horgan, Liu, Meinel, Wingate ’15]
    - [Gubernari, Kokulu, van Dyk ’18] (B-meson LCSRs)
  - $B_s \to \varphi$:
    - [Horgan, Liu, Meinel, Wingate ’15]
    - [Gubernari, van Dyk, Virto ’20] (B-meson LCSRs)
- Adding $\Lambda_b \to \Lambda^{(*)}$ form factors is possible and desirable

Details on the fit procedure

- The fit is performed in two steps…
  - Preliminary fits:
    - **Local** form factors:
      - BSZ parametrization (**8 + 19 + 19 parameters**)
      - Constrained on LCSR and LQCD calcultations
    - **Non-local** form factors:
      - order 5 GRvDV parametrization (**12 + 36 + 36 parameters**)
      - 4 points at negative q^2 + $B \to M\, J/\psi$ data

    → **130 nuisance parameters**
  - 'Proof of concept' fit to the WET's **Wilson coefficients**
- … using **EOS**: eos.github.io

BSM analysis

- A combined BSM analysis would be **very CPU expensive** (130 correlated, non-Gaussian, nuisance parameters!)
- Fit **separately** C_9 and C_{10} for the three channels:
  - $B \to K\mu^+\mu^- + B_s \to \mu^+\mu^-$
  - $B \to K^*\mu^+\mu^-$
  - $B_s \to \varphi\mu^+\mu^-$