Beyond the Flavour Anomalies V - Siegen, 9-11 April 2024

Recent theory developments in $b \to c\ell\nu$ transitions

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based on arXiv:2305.15457 in collaboration with:

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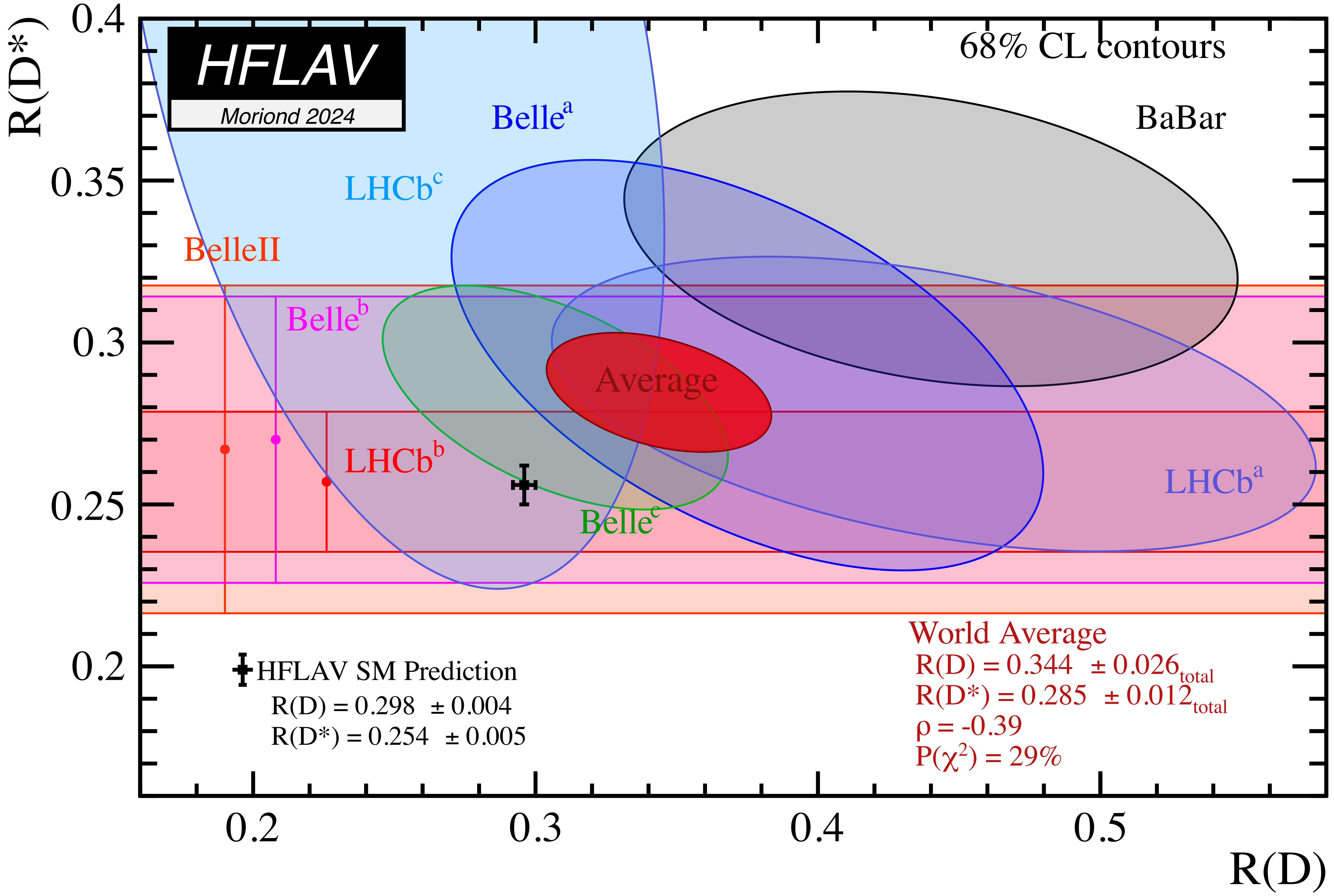
Introduction to $b \to c$ anomalies

- Tree level, theoretically clean processes with large Br (~ few %)
- Sensitive to NP via LFUV tests

$$R(D^{(*)}) = \frac{\mathcal{B}(\overline{B} \to D^{(*)}\tau\overline{\nu}_\tau)}{\mathcal{B}(\overline{B} \to D^{(*)}\ell\overline{\nu}_\ell)} \qquad \ell = e, \mu$$



Experimental average (HFLAV):

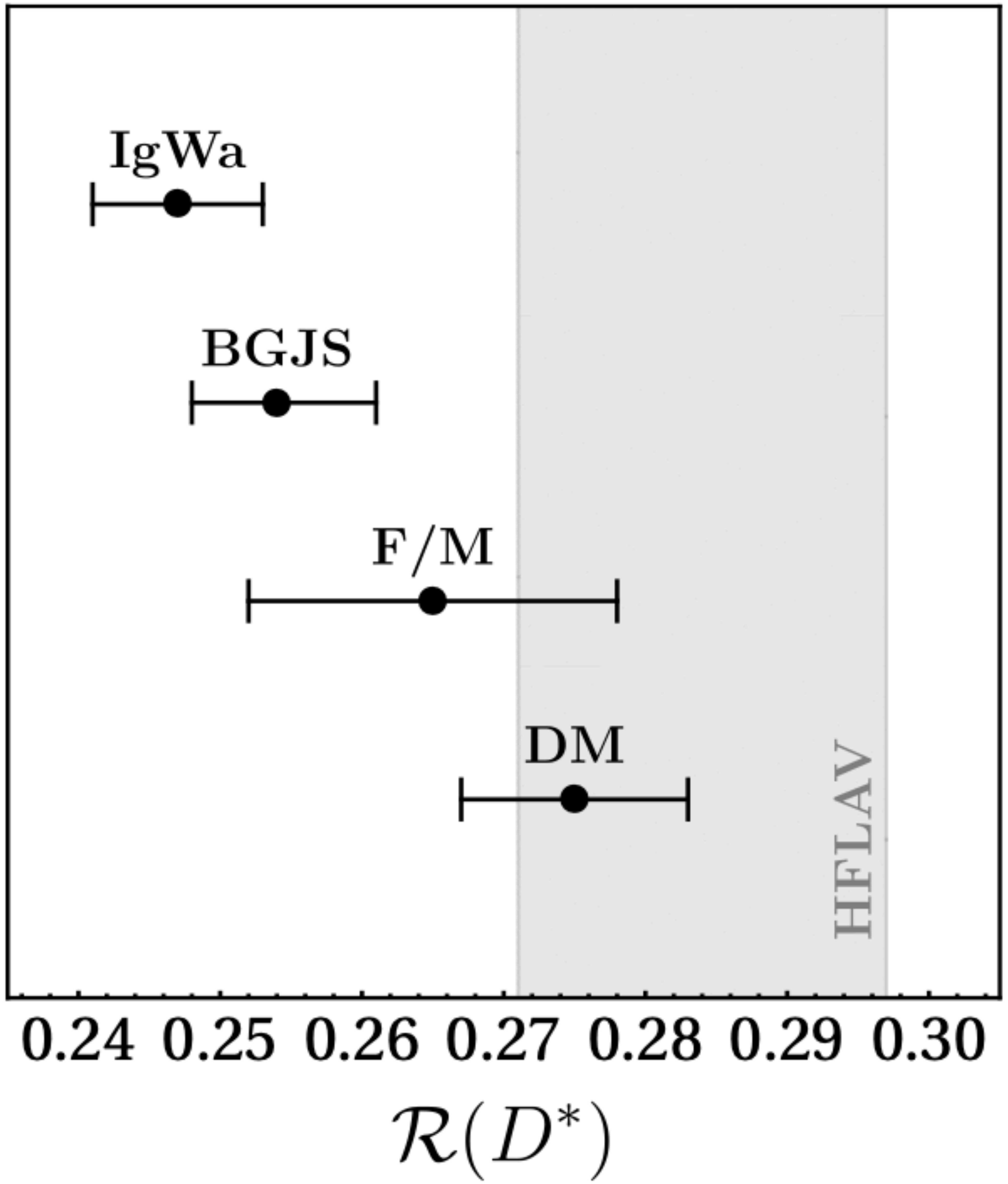
$$R(D) = 0.344 \pm 0.026$$
$$R(D^*) = 0.285 \pm 0.012$$

SM predictions:

$$R(D) = 0.298 \pm 0.004$$
$$R(D^*) = 0.254 \pm 0.005$$

Comb. discrepancy at ~3.2σ level hinting at τ over-abundance

What if it's a FF issue?



The SM prediction for $R(D^*)$ might not be as stable as originally thought!

Different Form Factors approaches have different predictions, with noticeable increase on the prediction for the latest determinations (and strongly correlated to $|V_{cb}^{excl}|$ determination)

Could the discrepancy actually arise from issues on the FF estimates?

The IgWa approach

Developed by Bordone, Jung, van Dyk to go beyond original HQET formulation:

expand the FF $h_X(w) = \xi(w)\hat{h}_X(w)$, with $\xi(w)$ the leading Isgur-Wise function, in α_s and $1/m_{b,c}$

$$\hat{h}_X = \hat{h}_{X,0} + \frac{\alpha_s}{\pi}\delta\hat{h}_{X,\alpha_s} + \frac{\bar{\Lambda}}{2m_b}\delta\hat{h}_{X,m_b} + \frac{\bar{\Lambda}}{2m_c}\delta\hat{h}_{X,m_c} + \left(\frac{\bar{\Lambda}}{2m_c}\right)^2 \delta\hat{h}_{X,m_c^2}$$

$\quad$ $\propto m_i$ $\quad$ $\propto$ sub-lead. I-W functs. $\xi_3(w), \chi_{2,3}(w)$ $\quad$ $\propto$ sub-lead. I-W functs. $\ell_{1-6}(w)$

Expand each of the 10 I-W functs. as a power of z, and fit to theory (LCSR and QCDSR) and experiment data up to a different order for each of the functions, selected by goodness-of-fit

$$f(w) = f^{(0)} + 8f^{(1)}z + 16\left(f^{(1)} + 2f^{(2)}\right)z^2 + \frac{8}{3}\left(9f^{(1)} + 48f^{(2)} + 32f^{(3)}\right)z^3 + \mathcal{O}(z^4)$$

1908.09398 Bordone, Jung, van Dyk

2004.10208 Iguro, Watanabe

The BGJS approach

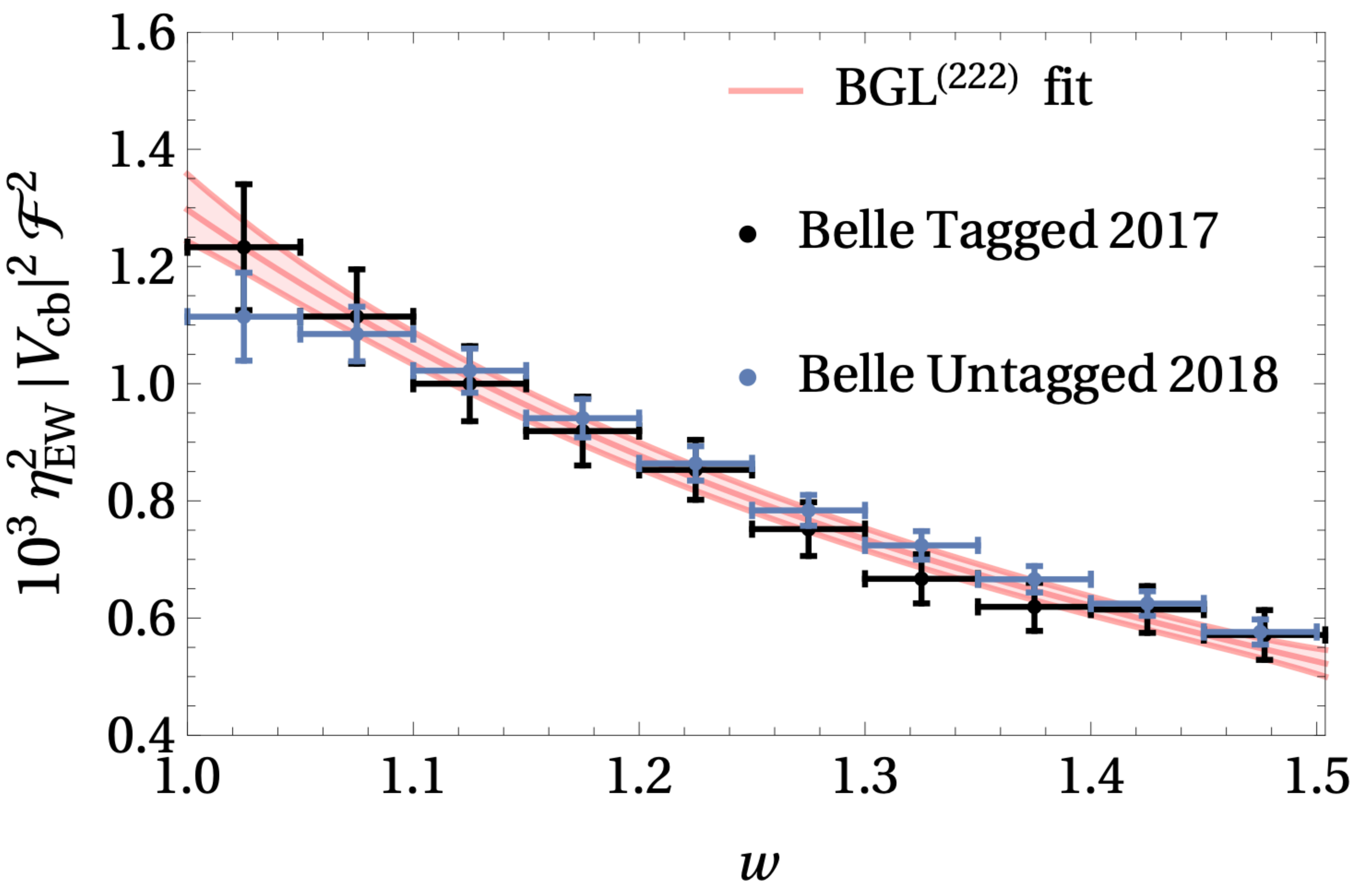
Expand the FF as a series in $z = (\sqrt{w+1} - \sqrt{2})/(\sqrt{w+1} + \sqrt{2})$, where $w = (m_B^2 + m_{D*}^2 - q^2)/(2m_B m_{D*})$

$$f_i(z) = \frac{1}{P_i(z)\phi_i(z)} \sum_{k=0}^{n_i} a_k^i z^k$$

Different expansion order for each FF (selected by goodness-of-fit)

Weak unitarity constraints imposed on series coefficients to ensure a rapid convergence of the series in the physical region, $0 < z < 0.056$

$$\sum_{k=0}^{n_g} (a_k^g)^2 < 1, \quad \sum_{i=0}^{n_f} (a_k^f)^2 + \sum_{k=0}^{n_{F_1}} (a_k^{\mathcal{F}_1})^2 < 1$$

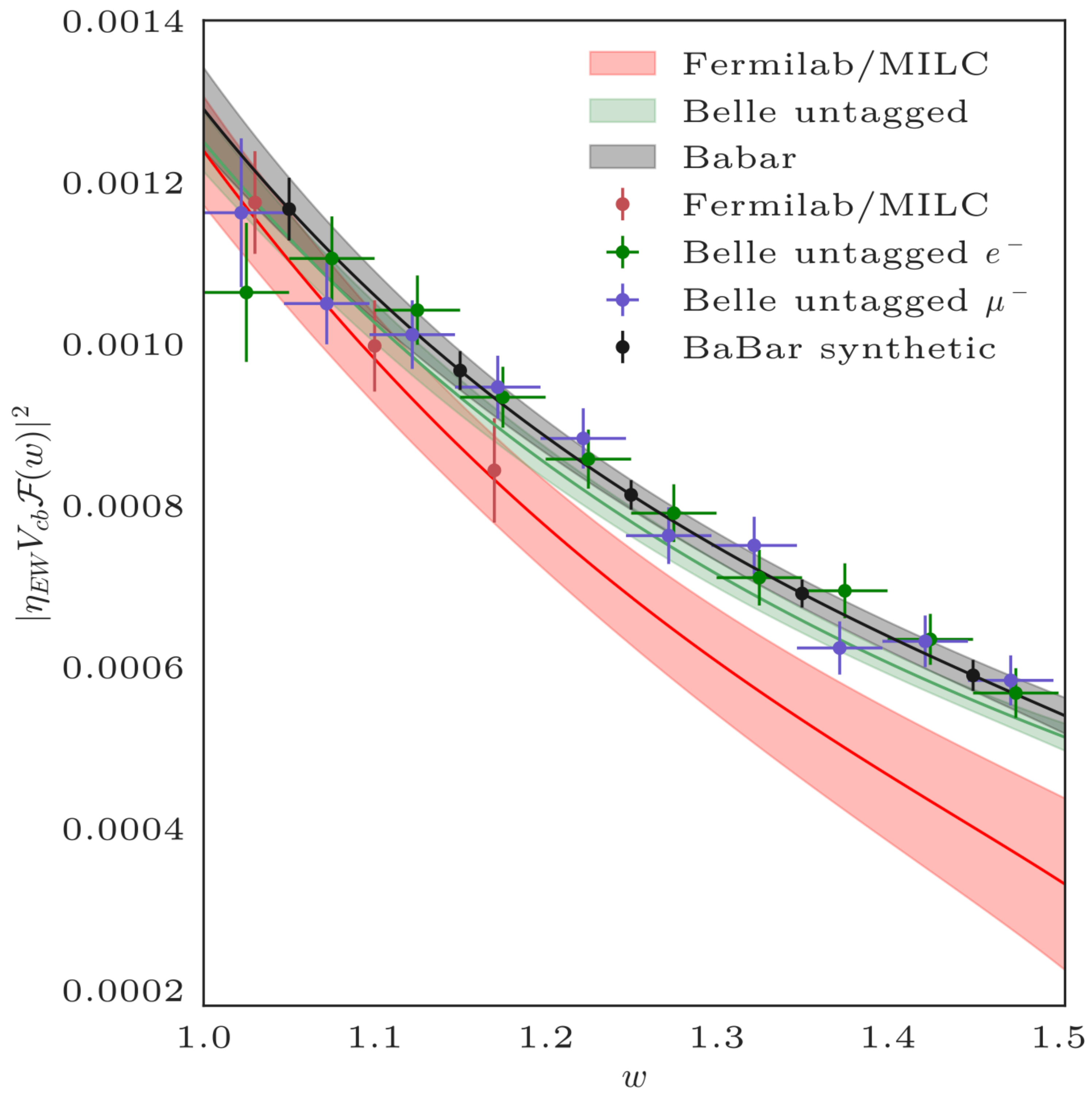


Additional input coming from HQET required for pseudoscalar FF

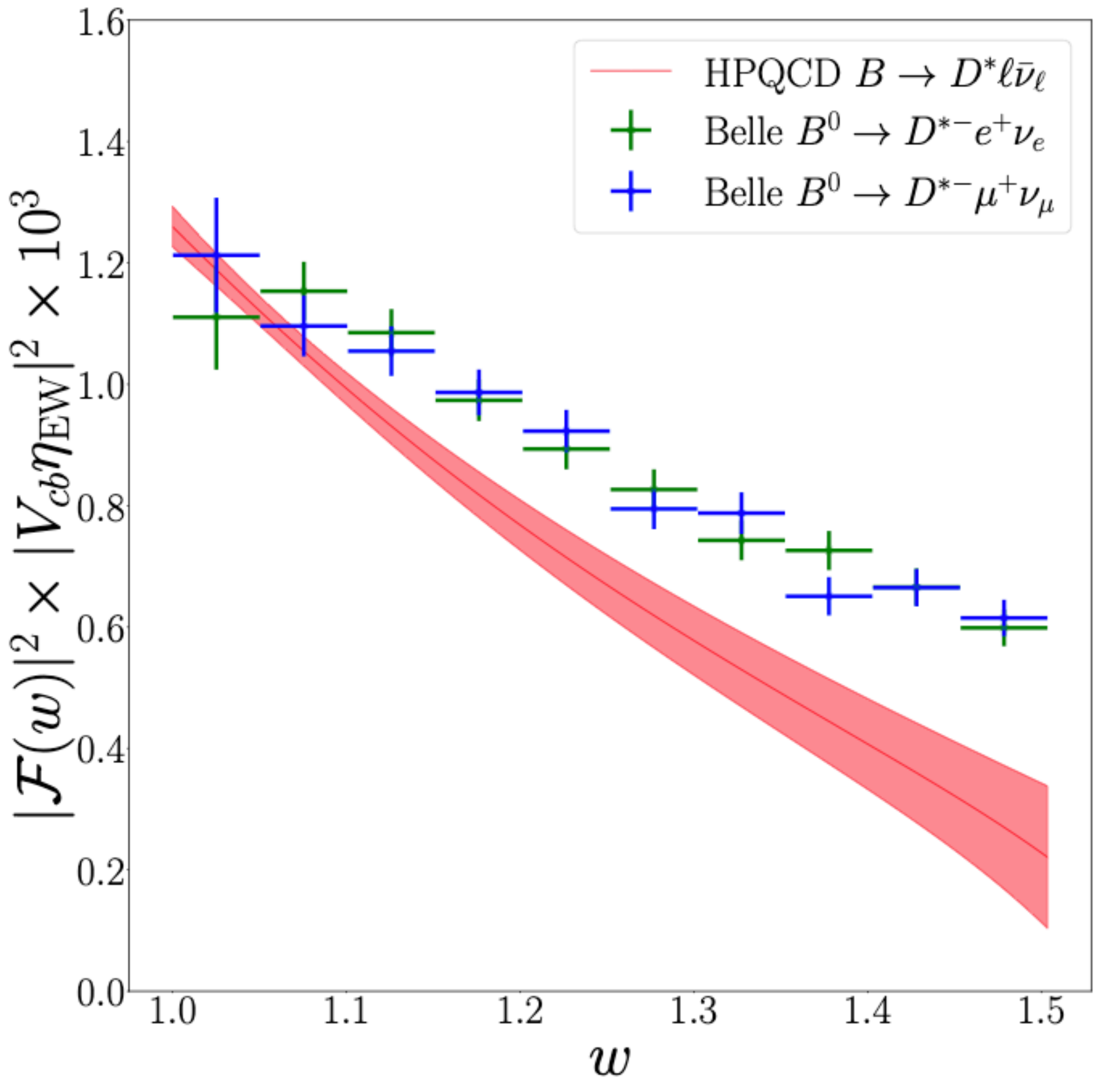
1905.08209
Gambino, Jung, Schacht

The Lattice approach

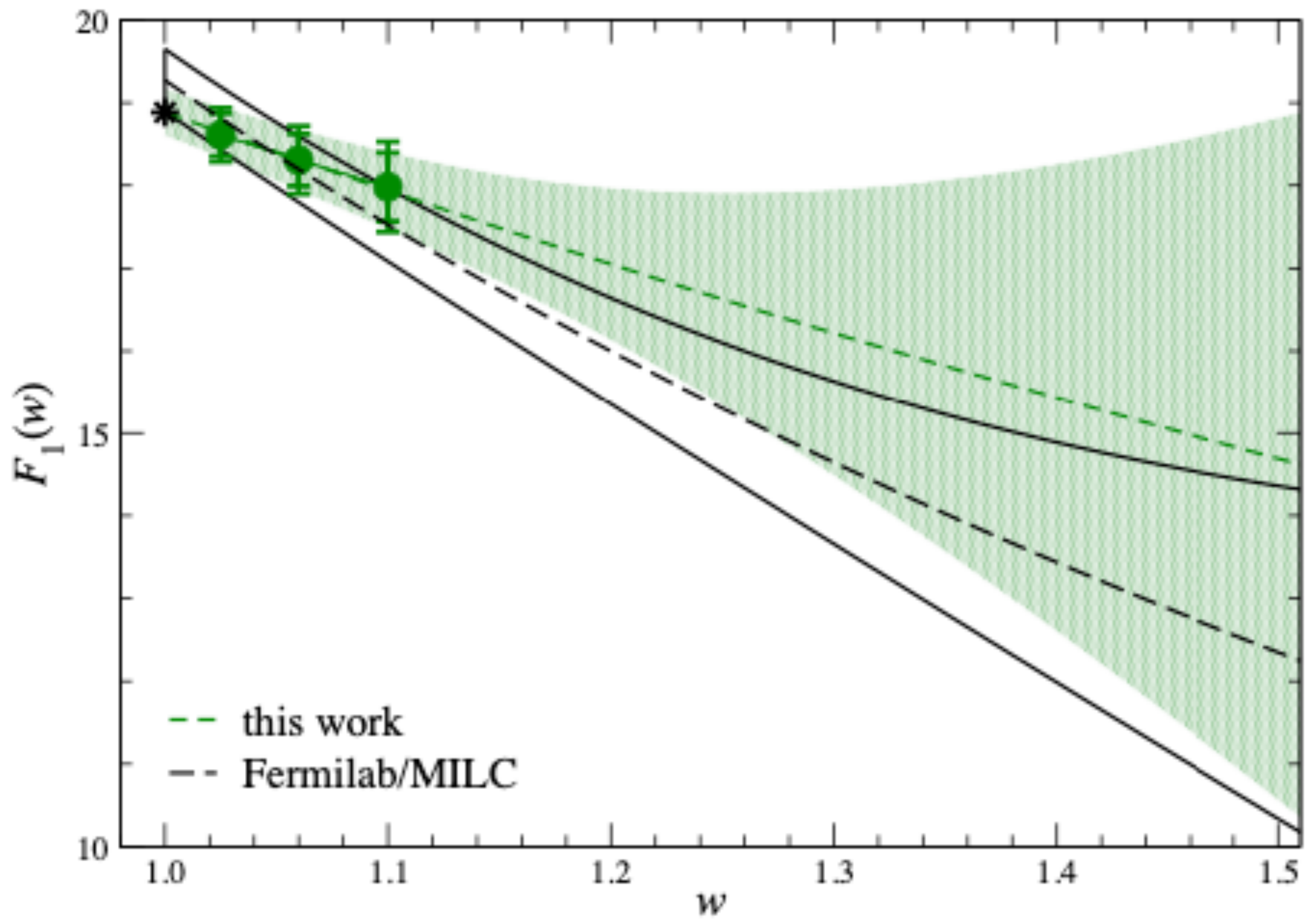
Same parameterization as the BGL approach, results beyond non-zero recoil recently obtained



2105.14019
FNAL/MILC



2304.03137
HPQCD



2306.05657
JLQCD

Results of F/M and HPQCD are mostly in agreement, however not well reproducing with data.
JLQCD seems more compatible with data, but larger errors. Do we have a problem with the slope?

Disclaimer: I am NOT a DM developer, just an ambassador! Be kind :)

The Dispersive Matrix approach

Goal: determine FFs f(t) starting from known theoretical values of $f(t_i)$ (e.g. Lattice), that can be therefore used to extract V_{cb} from all the independent differential measurements

The starting point is the introduction of 2 ingredients: inner product and auxiliary function:

$$\langle g|h\rangle = \frac{1}{2\pi i}\oint_{|z|=1}\frac{dz}{z}\bar{g}(z)h(z)$$

$$g_t(z) \equiv \frac{1}{1-\bar{z}(t)z}$$

$$\Rightarrow \quad \mathbf{M} \equiv \begin{pmatrix} \langle \phi f|\phi f\rangle & \langle \phi f|g_t\rangle & \langle \phi f|g_{t_1}\rangle & \cdots & \langle \phi f|g_{t_N}\rangle \\ \langle g_t|\phi f\rangle & \langle g_t|g_t\rangle & \langle g_t|g_{t_1}\rangle & \cdots & \langle g_t|g_{t_N}\rangle \\ \langle g_{t_1}|\phi f\rangle & \langle g_{t_1}|g_t\rangle & \langle g_{t_1}|g_{t_1}\rangle & \cdots & \langle g_{t_1}|g_{t_N}\rangle \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \langle g_{t_N}|\phi f\rangle & \langle g_{t_N}|g_t\rangle & \langle g_{t_N}|g_{t_1}\rangle & \cdots & \langle g_{t_N}|g_{t_N}\rangle \end{pmatrix}$$

Matrix built out of inner products, hence its determinant is by construction positive semidefinite

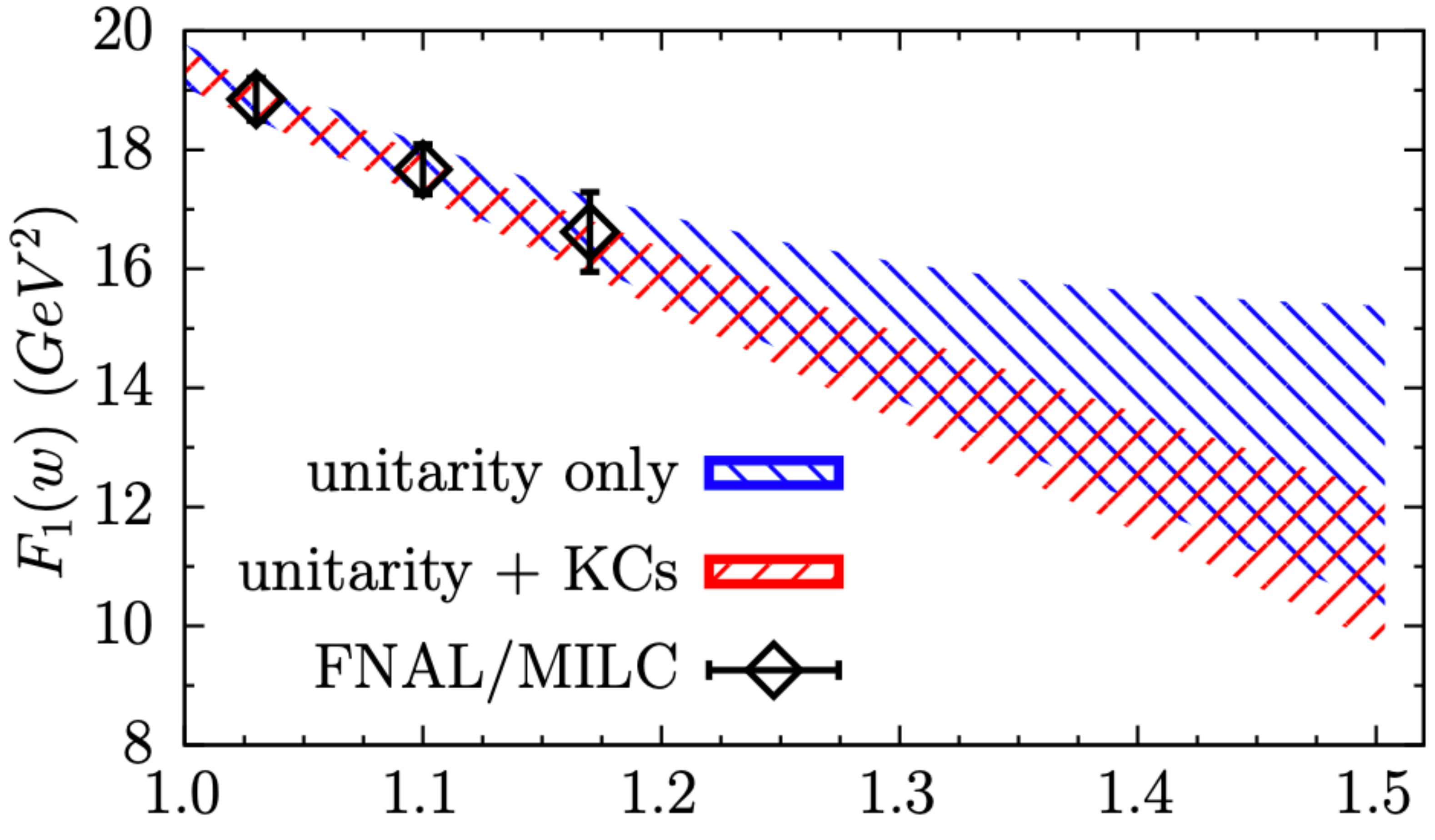
2105.07851, 2105.08674, 2109.15248
Martinelli, Simula, Vittorio

The Dispersive Matrix approach

M_{11} obeys to the dispersion relation $\frac{1}{2\pi i}\oint_{|z|=1}\frac{dz}{z}|\phi(z)f(z)|^2 \leq \chi \longrightarrow 0 \leq \langle \phi f | \phi f \rangle \leq \chi$

The Cauchy theorem allows to compute the remaining terms, and the semidefinite positiveness is not spoiled by replacing M_{11} by its upper limit

$$\Rightarrow \quad \mathbf{M}_\chi = \begin{pmatrix} \chi & \phi f & \phi_1 f_1 & \ldots & \phi_N f_N \\ \phi f & \frac{1}{1-z^2} & \frac{1}{1-z z_1} & \ldots & \frac{1}{1-z z_N} \\ \phi_1 f_1 & \frac{1}{1-z_1 z} & \frac{1}{1-z_1^2} & \ldots & \frac{1}{1-z_1 z_N} \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ \phi_N f_N & \frac{1}{1-z_N z} & \frac{1}{1-z_N z_1} & \ldots & \frac{1}{1-z_N^2} \end{pmatrix}$$



Requiring the positiveness of the determinant allows to obtain a band for the FF, representing the envelope of the results of all possible (un)truncated z-expansions, like BGL ones

$$\beta(z) - \sqrt{\gamma(z)} \leq f(z) \leq \beta(z) + \sqrt{\gamma(z)}$$

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The Dispersive Matrix approach

$$\beta(z) - \sqrt{\gamma(z)} \le f(z) \le \beta(z) + \sqrt{\gamma(z)}$$

$$\beta(z) \equiv \frac{1}{\phi(z)d(z)} \sum_{j=1}^{N} \phi_j f_j d_j \frac{1-z_j^2}{z-z_j} ,$$

$$\gamma(z) \equiv \frac{1}{1-z^2} \frac{1}{\phi^2(z)d^2(z)} (\chi - \chi_{\rm DM}) ,$$

$$\chi_{\rm DM} \equiv \sum_{i,j=1}^{N} \phi_i f_i \phi_j f_j d_i d_j \frac{(1-z_i^2)(1-z_j^2)}{1-z_i z_j} ,$$

$$d(z) \equiv \prod_{m=1}^{N} \frac{1-z z_m}{z-z_m} ,$$

$$d_j \equiv \prod_{m\neq j=1}^{N} \frac{1-z_j z_m}{z_j-z_m} .$$

$\Rightarrow$

Unitarity requires $\gamma(z) \geq 0$, which implies $\chi \geq \chi_{\rm DM}$. Therefore, the FF at any given z is given by the convolution of $\gamma(z)$ and $\beta(z)$ with the distribution of input data with $\chi > \chi_{\rm DM}$: input data is therefore *filtered* by unitarity!

The *filter* can be interpreted, in terms of a z-expansion, as the existence of (at least) one (un)truncated BGL fit that satisfies unitarity and *exactly* reproduces input data (i.e. Lattice)

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Not all that glitters is gold…

DM results shown here are based on F/M (only Lattice result published at time of the study), implications of HPQCD and JLQCD will be shown in a few slides

The DM FF approach is capable to address tension in $R(D^*)$ (and $|V_{cb}|$ incl. vs excl. discrepancy), but however in tension with new F_L^ℓ and $A_{\rm FB}^\ell$ data!

2305.15457
MF, Blanke, Crivellin, Iguro, Nierste, Simula, Vittorio

Where is this coming from?

In order to understand the origin of this pattern, it's instrumental to take a look at the helicity amp.

$$H_0(w) = \frac{\mathcal{F}_1(w)}{\sqrt{m_B^2 + m_{D^*}^2 - 2m_B m_{D^*} w}} \qquad H_\pm(w) = f(w) \mp m_B m_{D^*} \sqrt{w^2-1}\, g(w)$$

which are used to build

$$\frac{1}{|V_{cb}|^2}\frac{d\Gamma^\ell}{dw} \propto |H_0(w)|^2 + |H_+(w)|^2 + |H_-(w)|^2$$

$$F_L^\ell(w) = \frac{|H_0(w)|^2}{|H_0(w)|^2 + |H_+(w)|^2 + |H_-(w)|^2}$$

$$A_{\rm FB}^\ell(w) = \frac{|H_-(w)|^2 - |H_+(w)|^2}{|H_0(w)|^2 + |H_+(w)|^2 + |H_-(w)|^2}$$

$\Rightarrow$

$\mathcal{F}_1(w)$	$\frac{1}{\lvert V_{cb}\rvert^2}\frac{d\Gamma^\ell}{dw}$	$\lvert V_{cb}\rvert$	$\mathcal{R}(D^*)$	$A_{\rm FB}^\ell$	F_L^ℓ
↗	↗	↘	↘	↘	↗
↘	↘	↗	↗	↗	↘

A change in the shape of $F_1(w)$ has a direct proportional impact on $R(D^*)$, $|V_{cb}|$, A_{FB}^ℓ and F_L^ℓ

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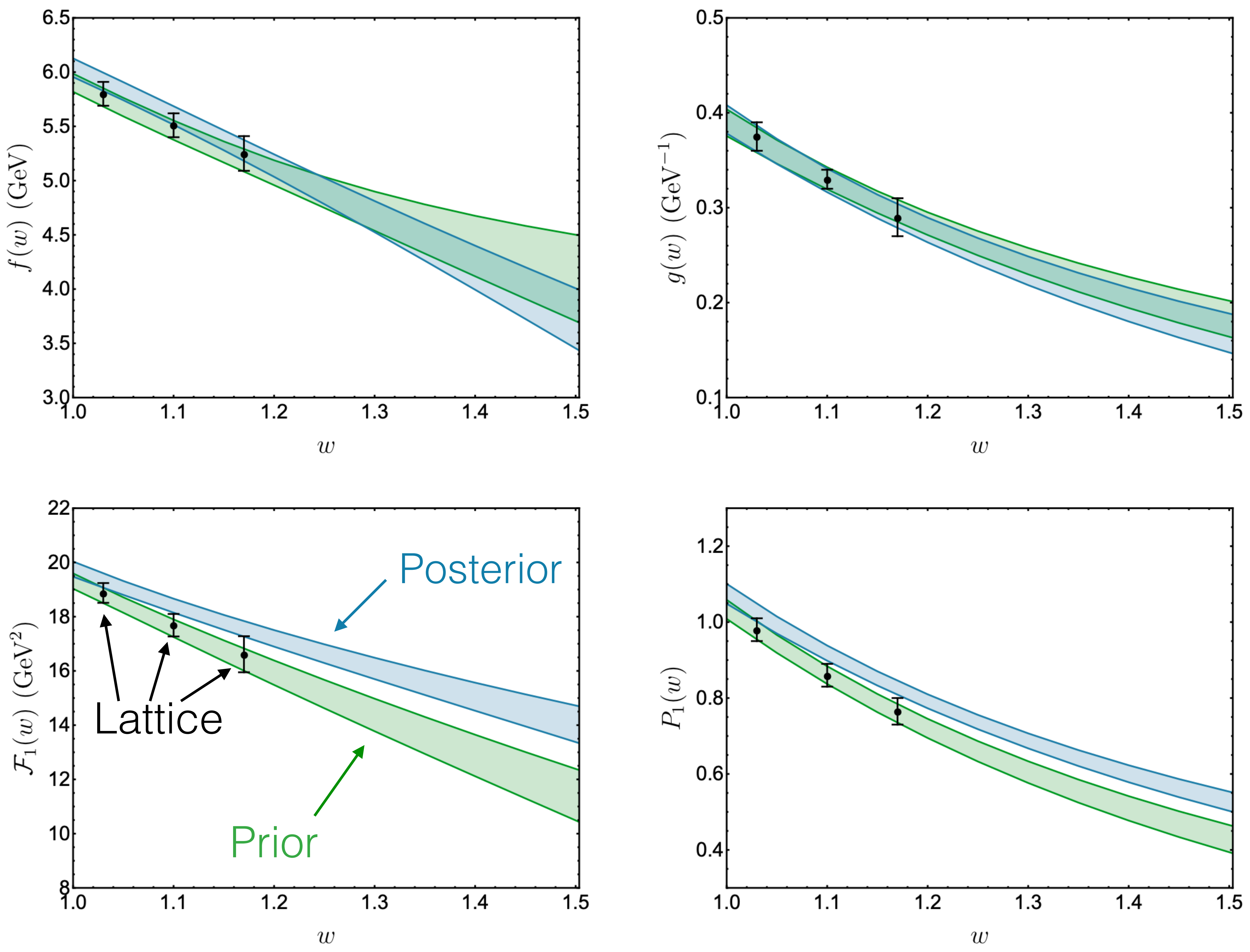
What if we try to perform a fit to this data?

Goal: perform a fit to A_{FB}^{ℓ} and F_L^{ℓ} using DM results for the FF as priors

$$\mathcal{R}(D^*)_{\rm fit} = 0.265 \pm 0.005$$
$$F^{\ell}_{L,\,\rm fit} = 0.515 \pm 0.005$$
$$A^{e}_{\rm FB,\,fit} = 0.227 \pm 0.007$$
$$A^{\mu}_{\rm FB,\,fit} = 0.222 \pm 0.007$$

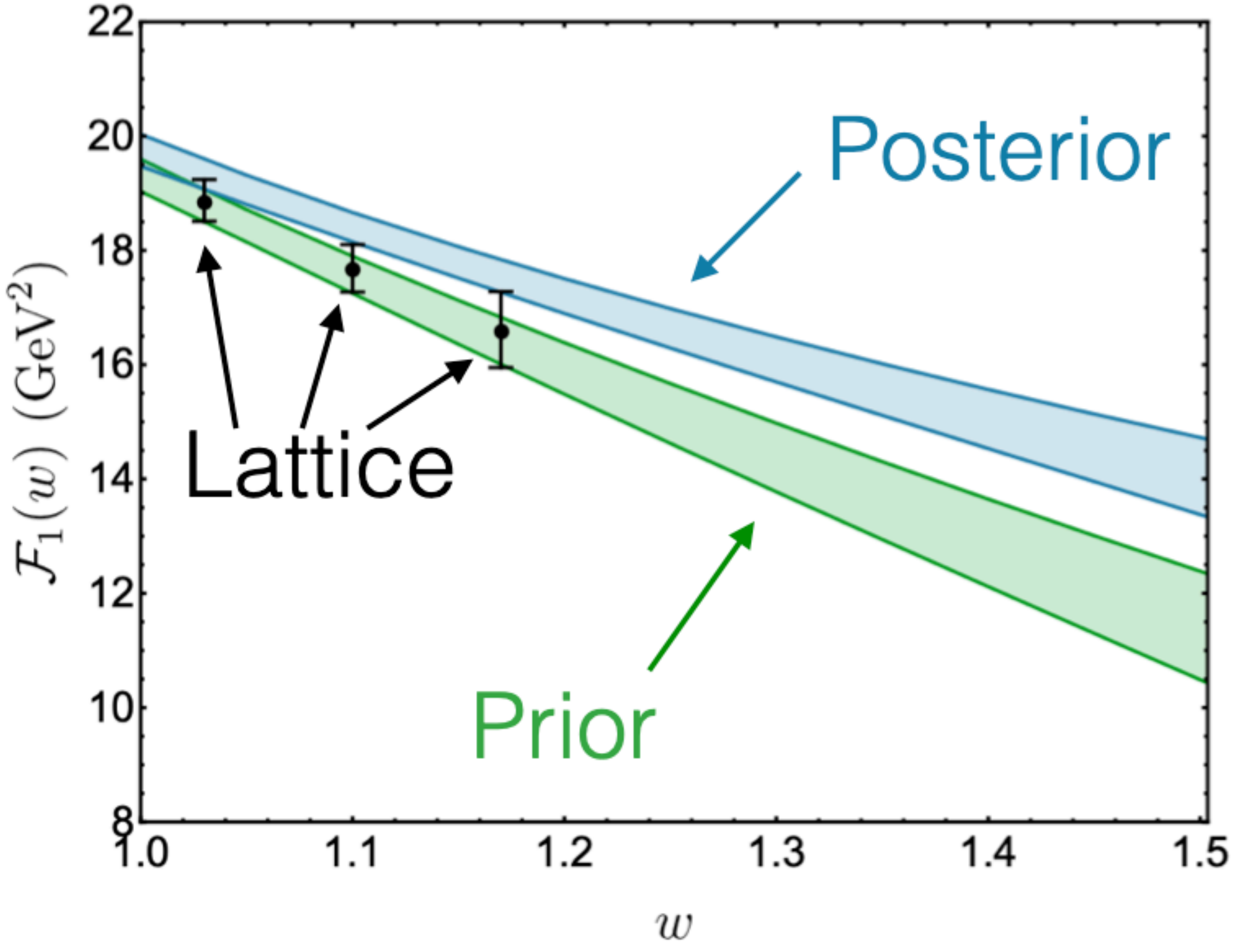
$F_1(w)$ shape is changed to accommodate for F_L^{ℓ} and $A_{\rm FB}^{\ell}$, reintroducing the $R(D^*)$ anomaly

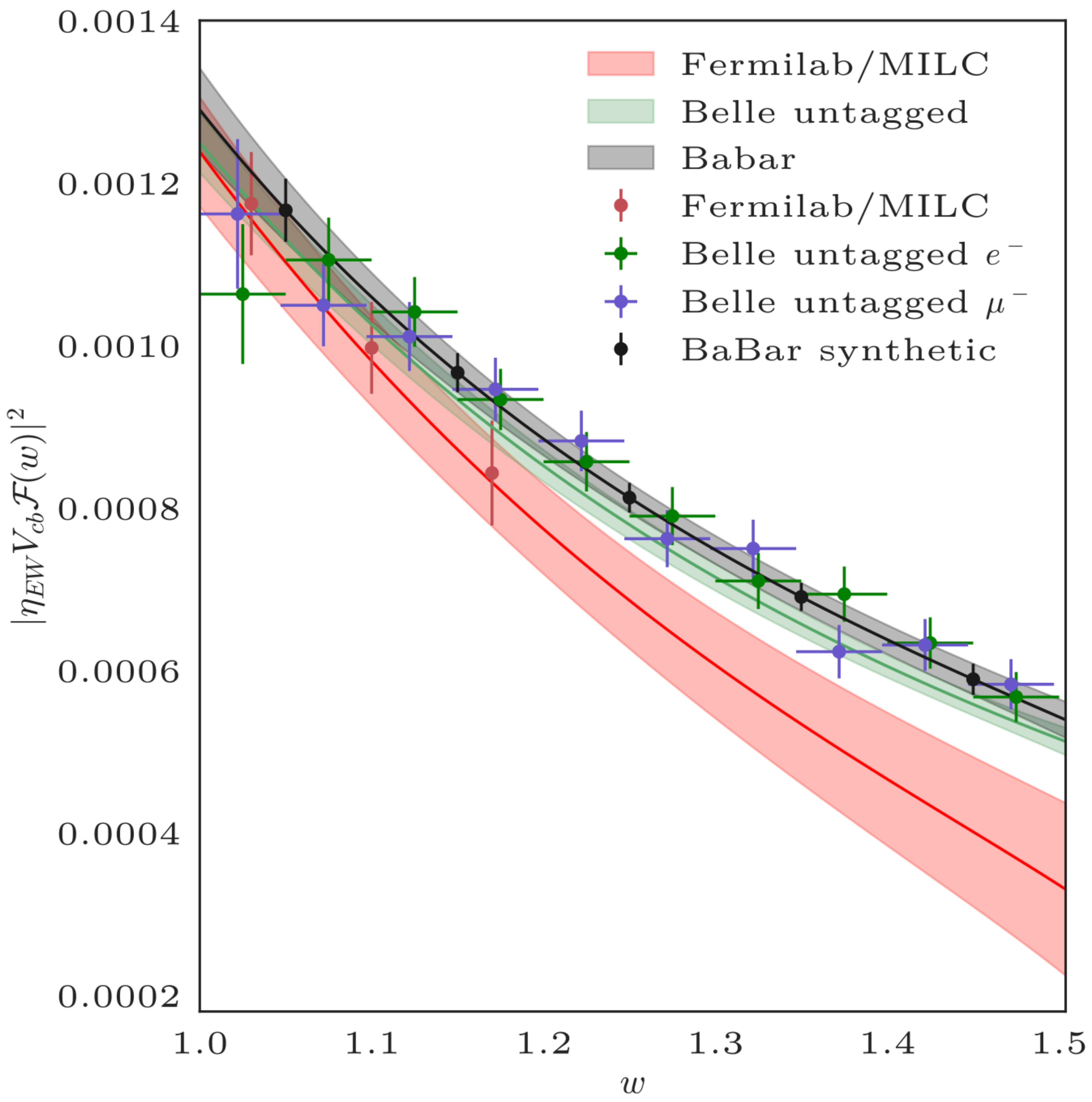


Strong discrepancy between prior and posterior values, lattice results not even reproduced anymore!

[2305.15457](2305.15457)
MF, Blanke, Crivellin, Iguro, Nierste, Simula, Vittorio

But this is an healthy cross-check





What data is telling us is that A_{FB}^{ℓ} and F_L^{ℓ} measurements are in agreement with differential distributions: a further hint that there's something fishy with FF…

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Can we reproduce everything introducing NP in light leptons?

The DM FF offer the unique possibility to employ NP in light leptons to address anomalies (forbidden in other scenarios due to CKM limits)

Could this fix the issue?

Only evidence found for g_{V_L}; however F_L^ℓ and $A_{\rm FB}^\ell$ are ratios, hence insensitive to it!

The absence of an hint for scalar/tensor WCs is due to more precise measurements in light lepton channel, together with m_ℓ suppression in interference terms with SM

$$g_{V_L} = -0.054 \pm 0.015$$
$$g_{V_R} \in [-0.04, 0.01]$$
$$g_{S_L} \in [-0.07, 0.02]$$
$$g_{S_R} \in [-0.05, 0.03]$$
$$g_T \in [-0.01, 0.02]$$

$\Rightarrow$ If the FF prediction for F_L^ℓ and $A_{\rm FB}^\ell$ does not reproduce data, this **cannot be fixed by introducing NP effects** in light leptons as could be done for $R(D^*)$!

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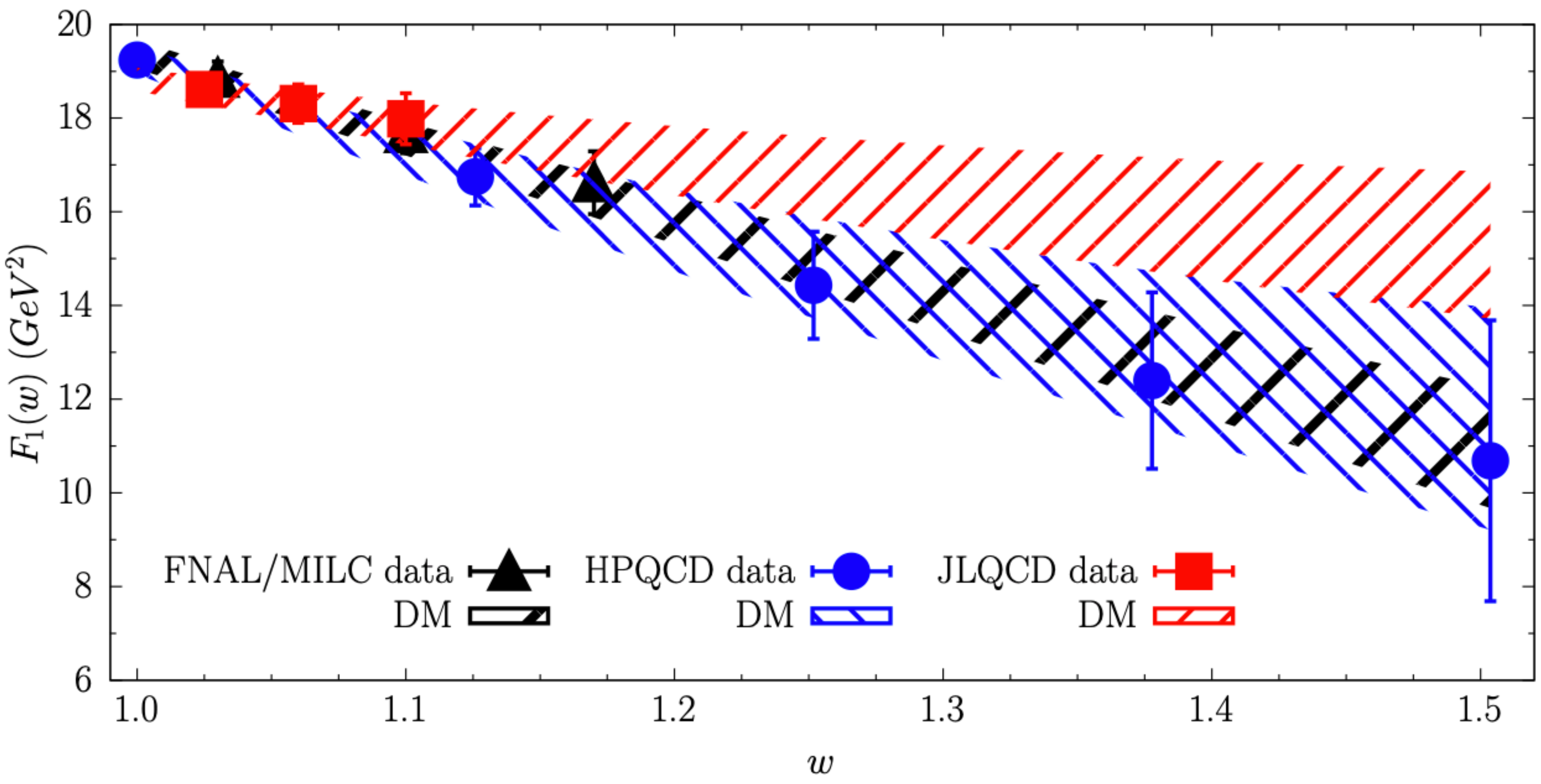
DM confronts with all lattice estimates…



As expected, FNAL/MILC and HPQCD give similar results (with different precision), while JLQCD is somewhat different (notice increase of precision thanks to DM filter)

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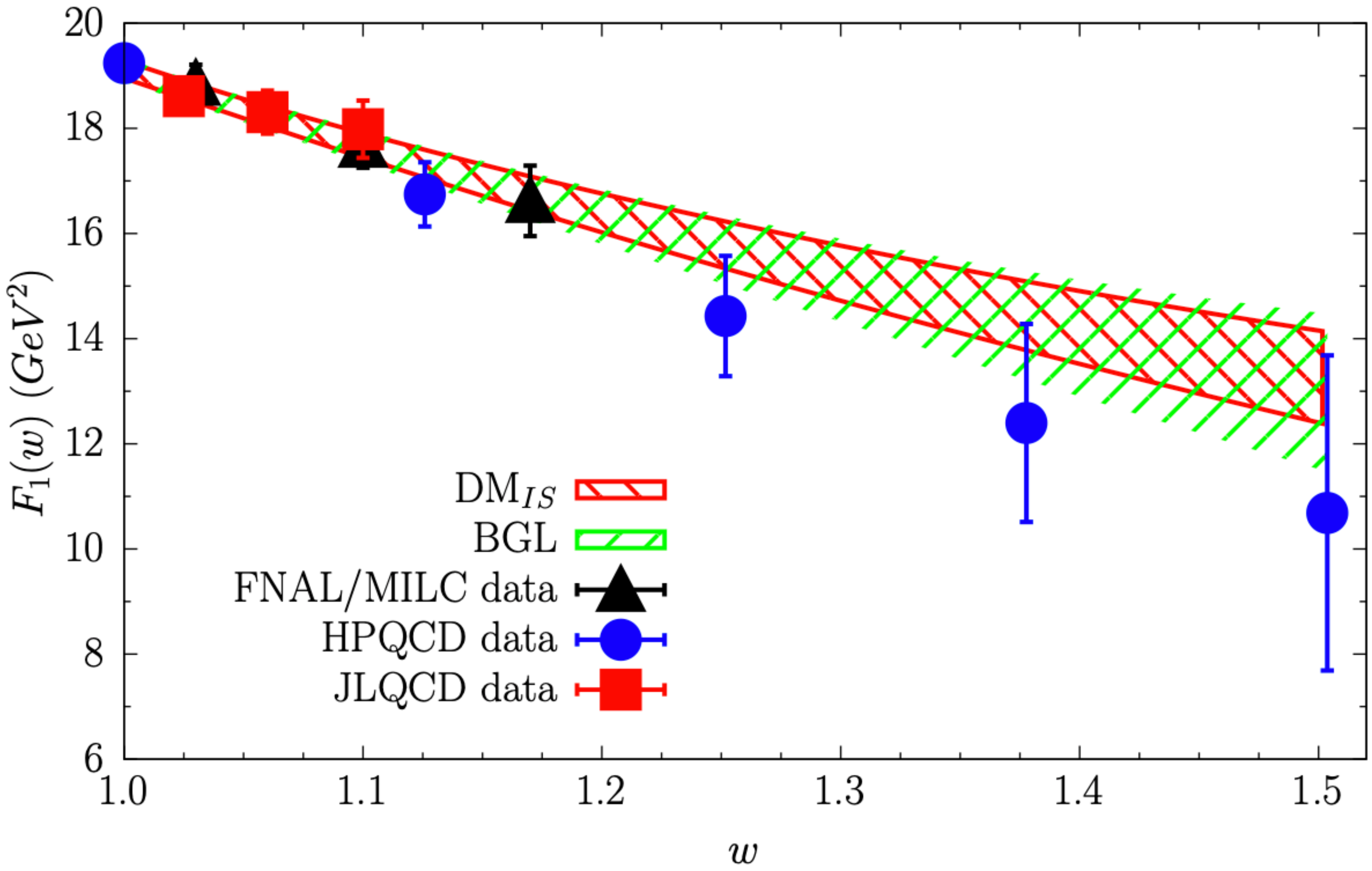
… and can combine them, too



Finally, a combined extraction to all datasets à la BGL or by means of Importance Sampling (IS) is done, with results mainly driven by FNAL/MILC & HPQCD

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Implications on the prediction to F_L^ℓ and $A_{\rm FB}^\ell$

Lattice FFs	$R(D^*)$	$P_\tau(D^*)$	$F_{L,\tau}$	$F_{L,\ell}$	$A_{FB,\ell}$
FNAL/MILC [15]	0.275(8)	-0.529(7)	0.418(9)	0.450(19)	0.261(14)
HPQCD [16]	0.266(12)	-0.543(18)	0.399(23)	0.435(42)	0.265(30)
JLQCD [17]	0.247(8)	-0.509(11)	0.448(16)	0.516(29)	0.220(21)
Average [15]-[17] (PDG scale factor)	0.262(9) (1.8)	-0.525(7) (1.3)	0.422(10) (1.4)	0.465(22) (1.5)	0.251(13) (1.2)
Combined [15]-[17]	0.259(5)	-0.521(6)	0.425(7)	0.473(14)	0.252(10)
Experimental value	0.284(12) [36]	$-0.38 \pm 0.51^{+0.21}_{-0.16}$ [38]	0.49(8) [39, 40]	0.520(6) [13, 14]	0.232(10) [13, 14]

We have an analogous pattern: either we reproduce $R(D^*)$ but observe a tension with new F_L^ℓ and $A_{\rm FB}^\ell$ data (FNAL/MILC & HPQCD) or viceversa (JLQCD)!

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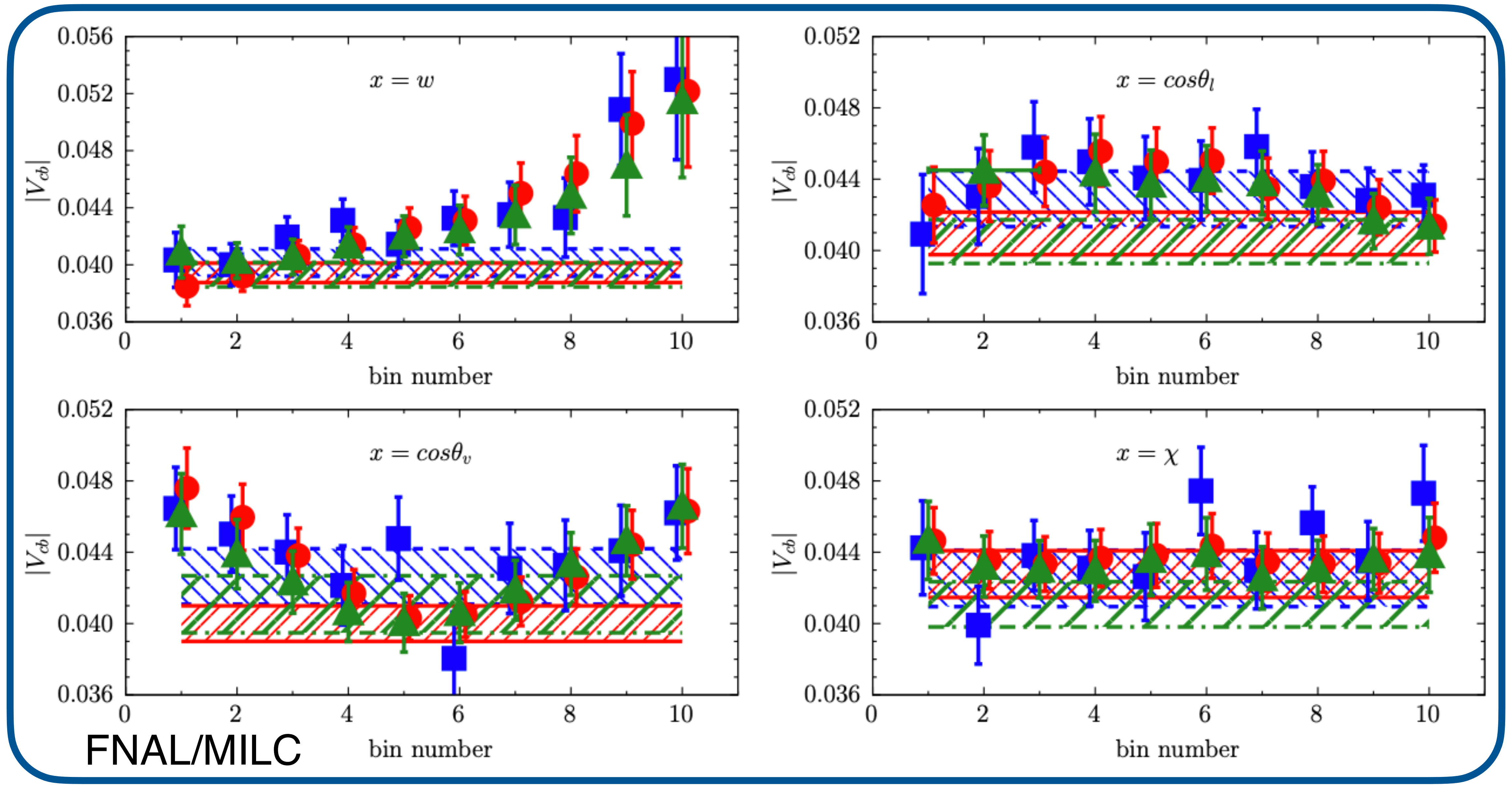
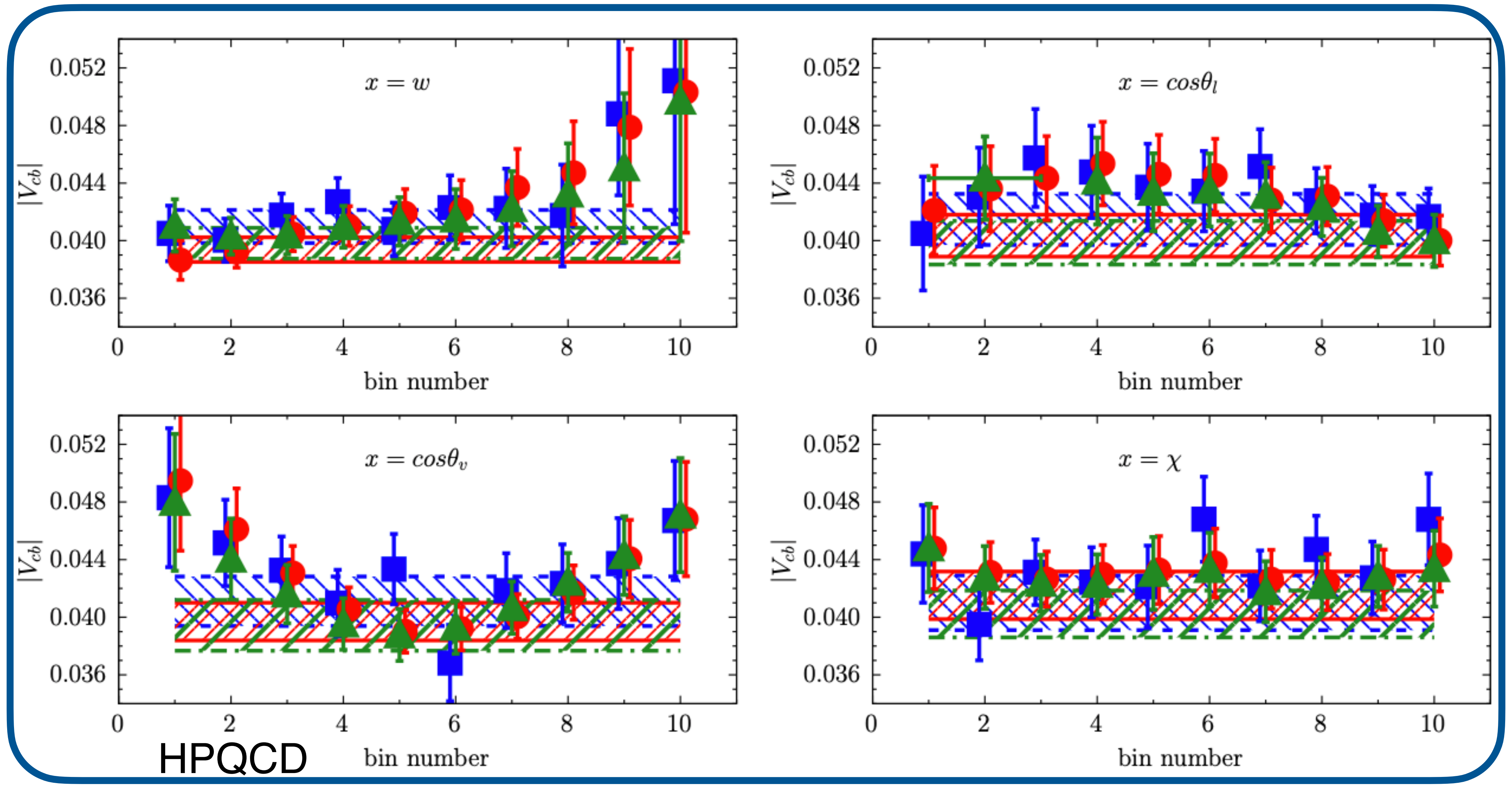
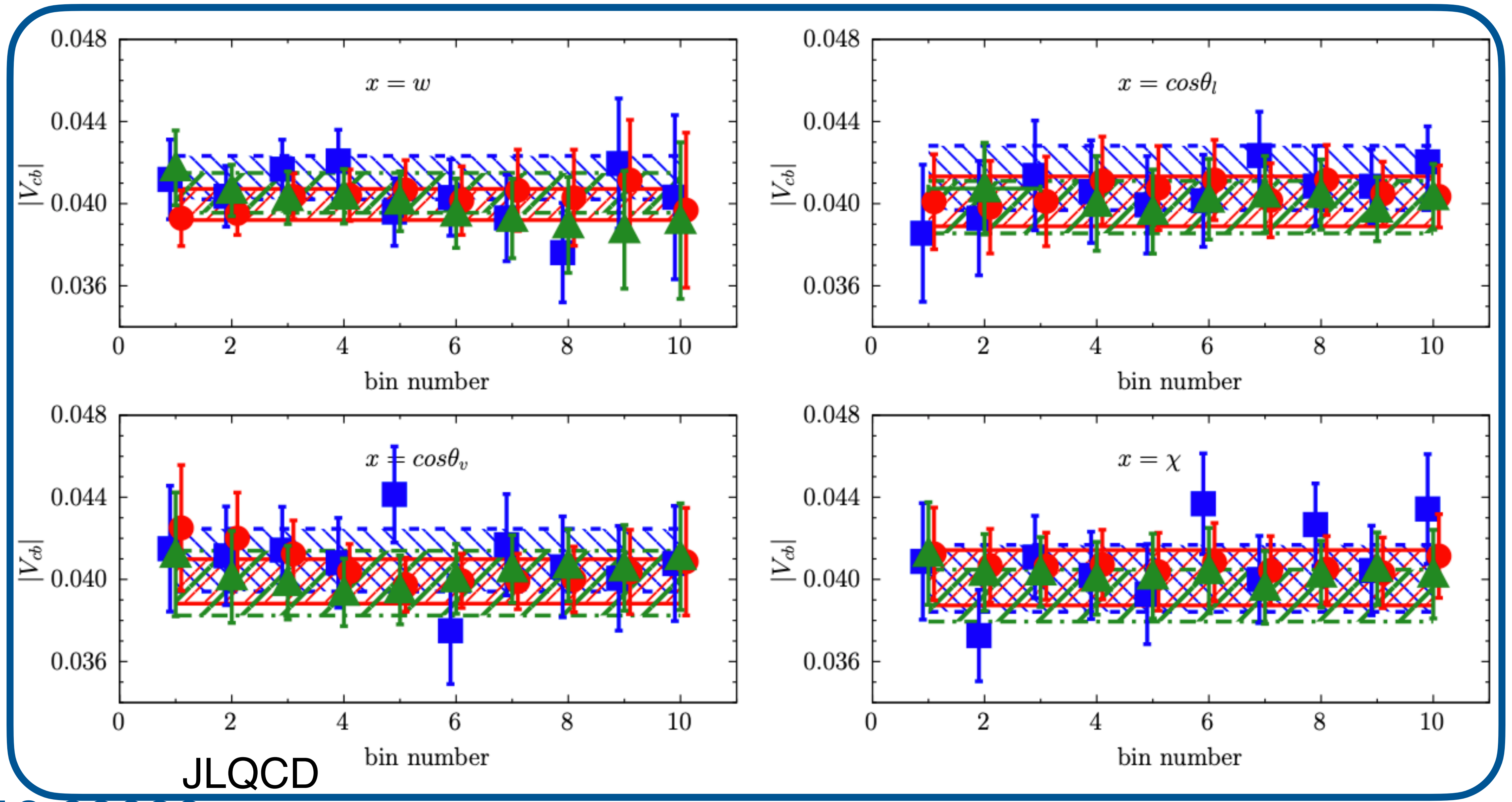
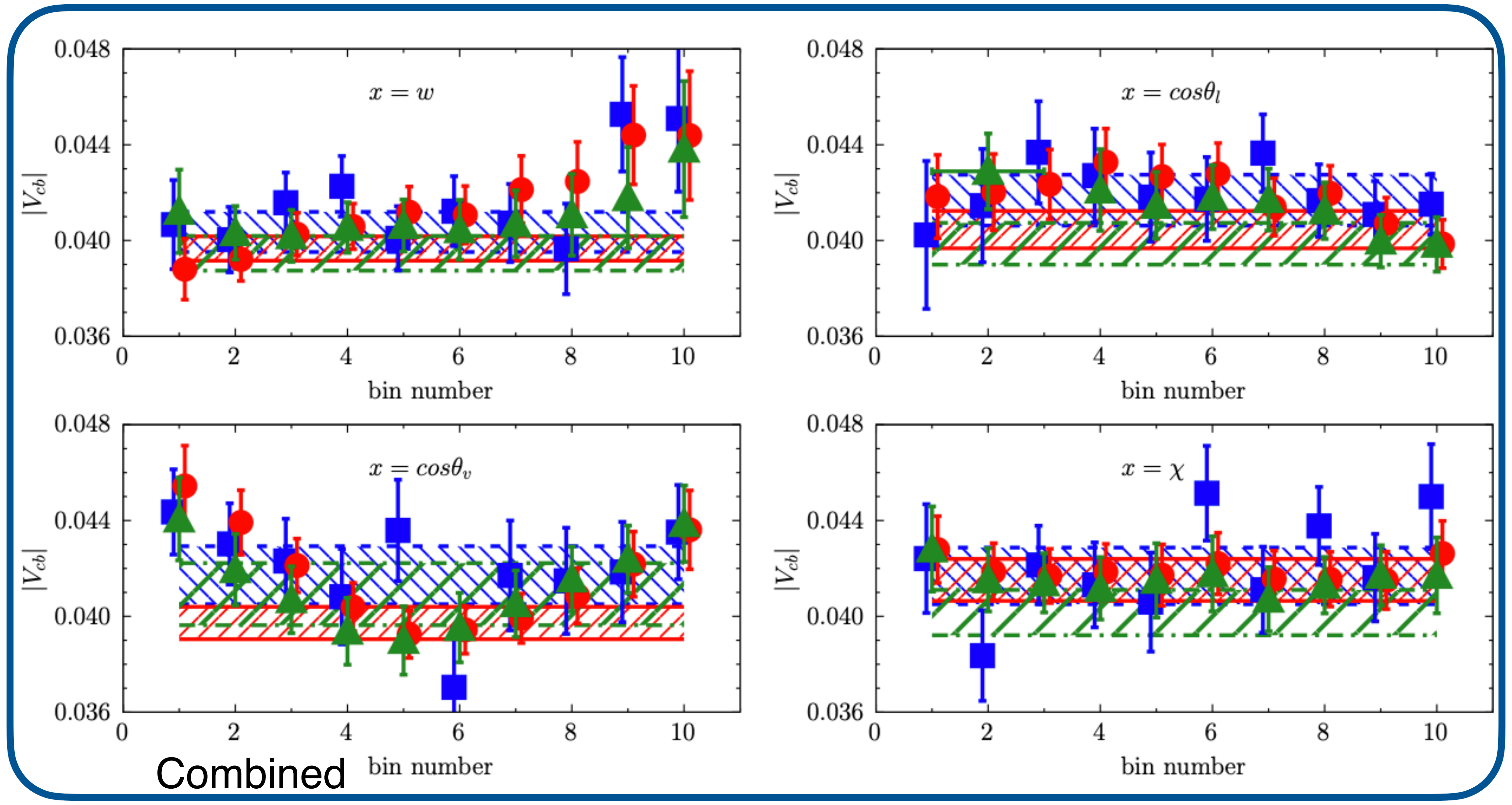
Implications to V_{cb} determinations









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Take Home Messages

- For the experimental community: please, give us as much differential data as possible (also for angular obs!), it will help us better understand the shape of the Form Factors!

- For the lattice community: it is great that we have various groups computing the same things, keep up on your hard work!

- For everyone computing FFs: I hope I convinced you that while a discrepancy in $d\Gamma/dw$ might be due to NP, this is not the case for F_L^ℓ and A_{FB}^ℓ. Please, take a look at 'em!

Conclusions

- Recent determination of A_{FB}^{ℓ} and F_L^{ℓ} have become available from Belle and Belle II, already with great precision!

- Theory prediction of A_{FB}^{ℓ} and F_L^{ℓ} strongly correlated to the one of $R(D*)$; while the latter can be modified by NP effects, the former are strongly NP-insensitive…

- Theory determinations of FF should therefore take in great attention their implications of the predictions for A_{FB}^{ℓ} and F_L^{ℓ}, and the consequent impact on the extraction of $|V_{cb}^{excl}|$!