$B \to D^{(*)}$ Form Factors in the Heavy Quark Expansion

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Beyond the Flavour Anomalies V
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b quarks on the Lattice

- On the lattice we want $am_q << 1$, so that discretisation effects are small.
- Work at fixed physical volume $V = (aL_x)^3$ large enough so that finite volume effects, $\sim e^{-M_\pi a L_x} << 1$, $\Rightarrow M_\pi a L_x >> 1$.
- This means that as we make a smaller, volume in lattice units L_x^3 must be increased.

For bottom quarks, small values of a such that $am_b << 1$ require large values of L.

$\Rightarrow$ **Fully relativistic b-quarks are very computationally expensive to simulate.**

Two options for computing $B \to D^{(*)}$ FFs:

- Use an effective action for the b quark, e.g. NRQCD, Fermilab heavy-quark action.
- Simulate relativistic heavy-quarks but with unphysically light mass $m_h \lesssim m_b$

Relativistic heavy-quarks on the Lattice

Use a heavy quark whose mass is below the physical b-quark mass. For some quantity, $F(m_b)$

- Compute $F^{\text{latt}}(m_h)$, using ensembles with different a, and different values of m_h.
- Fit those values to some function of the form

$$F^{\text{latt}}(m_h) = F^{\text{phys}}(m_h) + \delta F^{\text{disc}}(a\Lambda_{QCD}, am_h, ...)$$

  with a physical function determined from HQET, e.g.

$$F^{\text{phys}}(m_h) \sim \sum_n (\Lambda_{QCD}/m_h)^n$$

- Use the posterior distribution for the function to determine its physical value at the b-mass, $F^{\text{phys}}(m_b)$

This method allows for one or two results at or near the physical b-quark mass, while those results below am_b allow us to pin down the am_h discretisation effects.

This approach has a number of significant advantages over previous methods:

- Currents can be renormalised fully non-perturbatively, e.g. using RI-SMOM or PCVC/PCAC relations.
- Precision is **statistics limited**

Form Factor Definitions

There are 4 form factors needed to describe $B \to D^*$ decay within the Standard Model

$$\langle D^*|\bar{c}\gamma^\mu b|\overline{B}\rangle = i\sqrt{M_B M_{D^*}}\varepsilon^{\mu\nu\alpha\beta}\epsilon^*_\nu v'_\alpha v_\beta h_V,$$
$$\langle D^*|\bar{c}\gamma^\mu\gamma^5 b|\overline{B}\rangle = \sqrt{M_B M_{D^*}}\big[h_{A_1}(w+1)\epsilon^{*\mu} - h_{A_2}(\epsilon^*\cdot v)v^\mu - h_{A_3}(\epsilon^*\cdot v)v'^\mu\big].$$

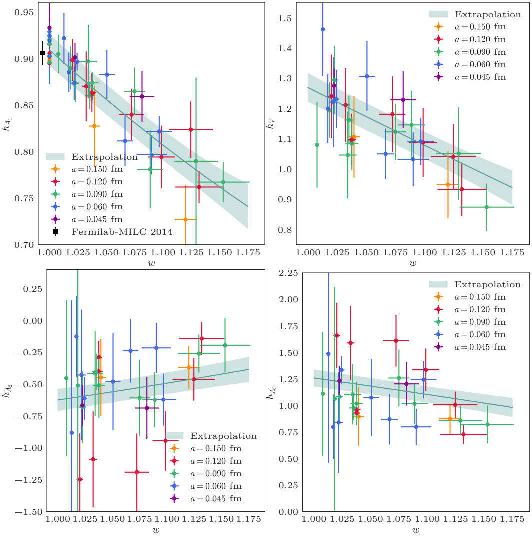
There are also 3 tensor form factors needed to include potential new physics:

$$\langle D^*|\bar{c}\sigma^{\mu\nu} b|\overline{B}\rangle = -\sqrt{M_B M_{D^*}}\varepsilon^{\mu\nu\alpha\beta}\big[h_{T_1}\epsilon^*_\alpha(v+v')_\beta + h_{T_2}\epsilon^*_\alpha(v-v')_\beta + h_{T_3}(\epsilon^*\cdot v)v_\alpha v'_\beta\big].$$

Three recent lattice calculations of the form factors by Fermilab-MILC, HPQCD and JLQCD.

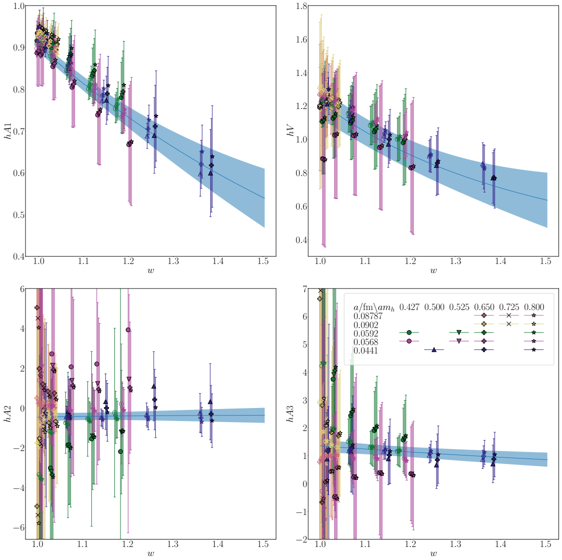
Fermilab-MILC 2105.14019

$B \to D^*$ FFs computed by Fermilab-MILC Collaboration using Fermilab heavy-quark action for c and b, on ensembles with 2+1 flavours of asqtad sea quarks



HPQCD 2304.03137

SM and Tensor FFs with HISQ for c and h quarks, up to $m_h \approx 0.9 m_b$, 2+1+1 flavours of HISQ sea quarks



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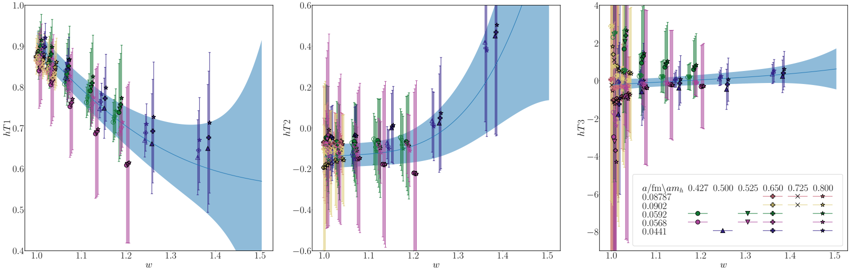
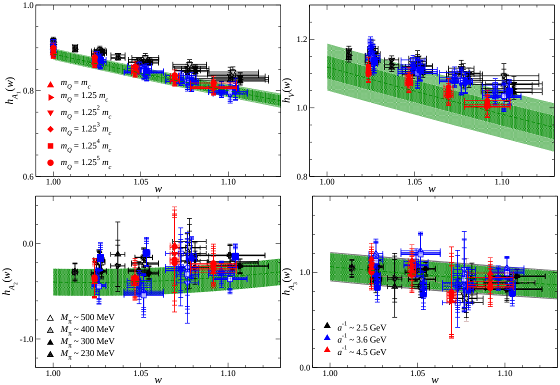


Also includes simultaneous analysis and updated results for $B_s \to D_s^*$.

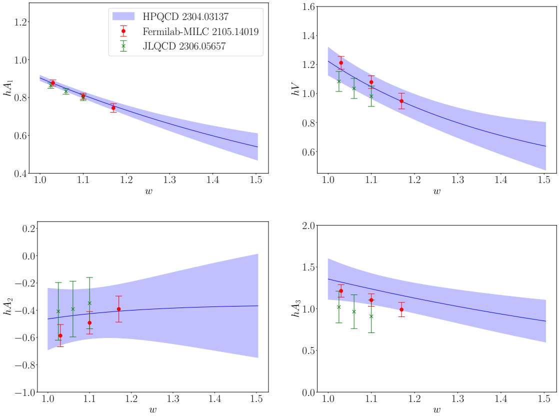
JLQCD

SM FFs with Móbius domain wall for c and h quarks, up to $m_h \approx 0.7 m_b$, 2+1 flavours of sea quarks



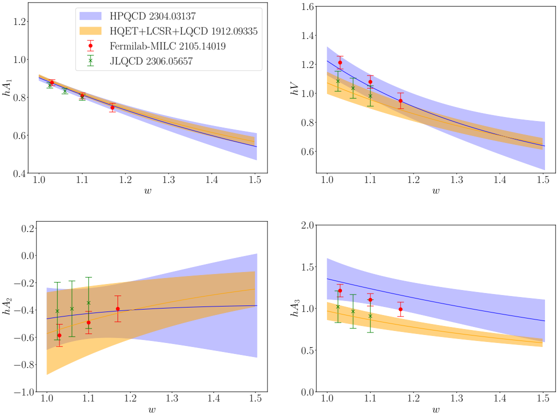
HPQCD and JLQCD calculations contain additional information about the $1/m_h$ dependence of the form factors.

Comparison of Lattice Results



Good agreement between lattice calculations.

Comparison of Lattice Results & HQE



$\mathcal{O}(\alpha_s, 1/m_c^2, 1/m_b)$ heavy quark expansion fit includes

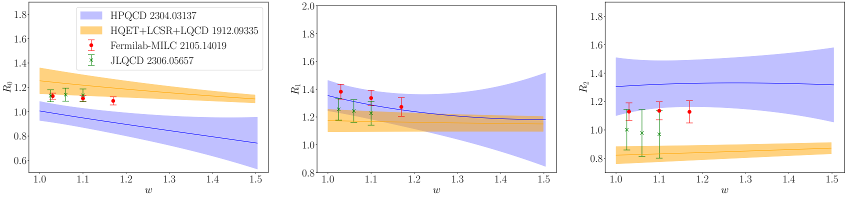
- Lattice QCD $B_{(s)} \to D_{(s)}$ and zero recoil $B_{(s)} \to D^*_{(s)}$
- LCSR & QCDSR

looks like reasonable agreement.

Comparison of Lattice Results & HQE

However, ratios do not seem to agree so well.

$$R_0 = \frac{1}{1+r}\left(w + 1 + w\frac{rh_{A_2} - h_{A_3}}{h_{A_1}} - \frac{h_{A_2} - rh_{A_3}}{h_{A_1}}\right), \quad R_1 = \frac{h_V}{h_{A_1}}, \quad R_2 = \frac{rh_{A_2} + h_{A_3}}{h_{A_1}}$$



Next steps:

- Check statistical compatibility with updated HQE fit to all available data
- Include $\mathcal{O}(\alpha_s, 1/m_c^2, 1/m_b)$ information in heavy-mass extrapolation of lattice data
- Include additional $1/m_h$ info from lattice in HQE fit

The Heavy Quark Expansion in a nutshell

The HQE exploits the fact that the b and c quarks are heavy

- Double expansion in $1/m_{b,c}$ and α_s
- The HQE symmetries relate $B^{(*)} \to D^{(*)}$ form factors
- At $1/m_{b,c}$ drastic reduction of independent degrees of freedom

With current precision we know we have to go beyond the $1/m_{b,c}$ order and we use the following form

$$F_i = \left(a_i + b_i \frac{\alpha_s}{\pi}\right) \xi + \frac{\Lambda_{\mathrm{QCD}}}{2m_b} \sum_j c_{ij} \xi^j_{\mathrm{SL}} + \frac{\Lambda_{\mathrm{QCD}}}{2m_c} \sum_j d_{ij} \xi^j_{\mathrm{SL}} + \left(\frac{\Lambda_{\mathrm{QCD}}}{2m_c}\right)^2 \sum_j g_{ij} \xi^j_{\mathrm{SSL}}$$

- total of 10 independent structures to be extracted from data

$$\xi^j_{\mathrm{SL}}(w) = \chi_2(w), \chi_3(w), \eta(w)$$
$$\xi^j_{\mathrm{SSL}}(w) = l_j(w)$$

Which datapoints we use?

Our "old" 2019 fits [Bordone, Gubernari, Jung, van Dyk, '19]

- Light-Cone Sum Rules: valid at $q^2 \lesssim 0$ [Gubernari, Kokulu, van Dyk, '18]
- FNAL/MILC and HPQCD $B \to D$ results [1503.07237, 1505.03925]
- FNAL/MILC anf HPQCD zero-recoil $B \to D^*$ results [1403.0635, 1711.11013]
- QCD Sum Rules to constrain sub-leading Isgur-Wise functions [Ligeti, Neubert, Nir '92,'93]
- Unitarity Bounds [Boyd, Grinstein, Lebel, '95, Caprini, Lellouch, Neubert, '97]

New players: Lattice QCD data away from zero-recoil

- Are they compatible with our previous fits?
- Can we learn something about the HQE from them?

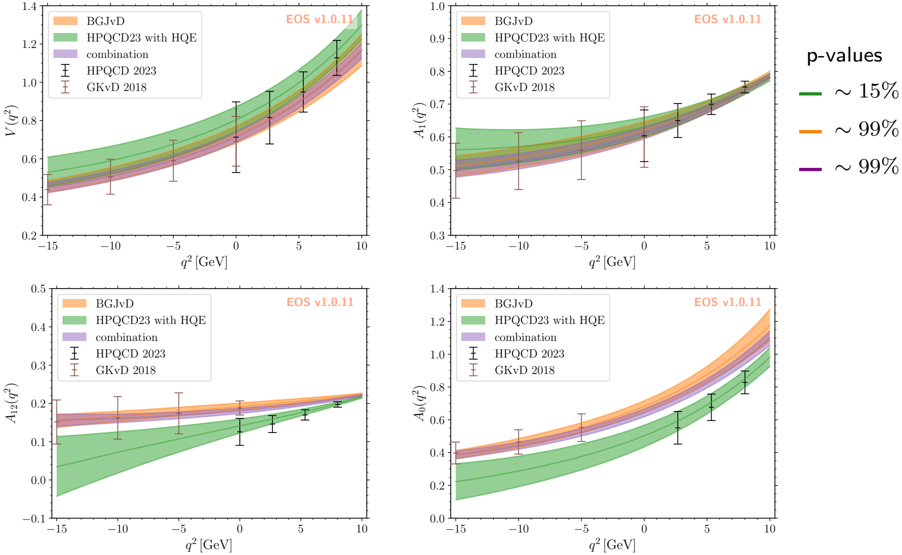
Fits and conventions

Default HQE fit from 2019

HQE up to $1/m_c^2$ fitted to the HPQCD'23 data w/ unitarity

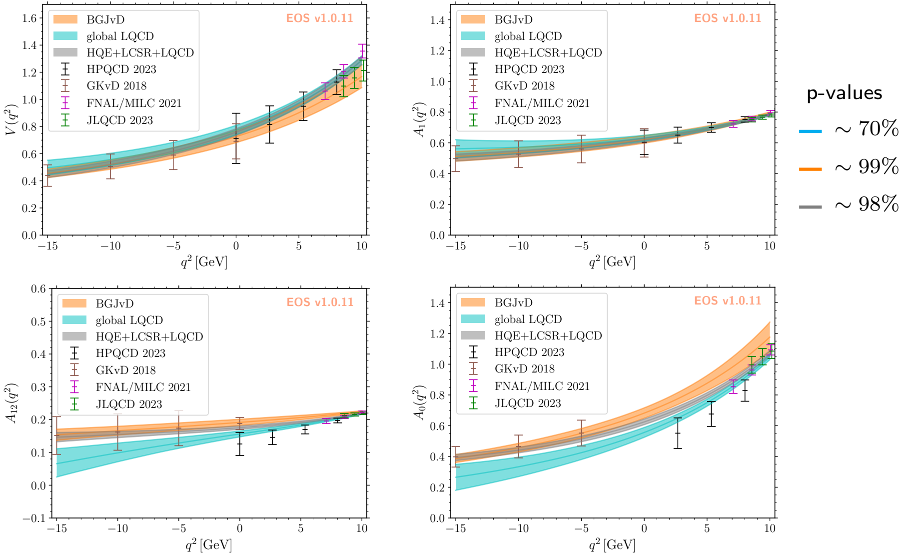
Combination of the previous two

Consider all coming results as very preliminary!!



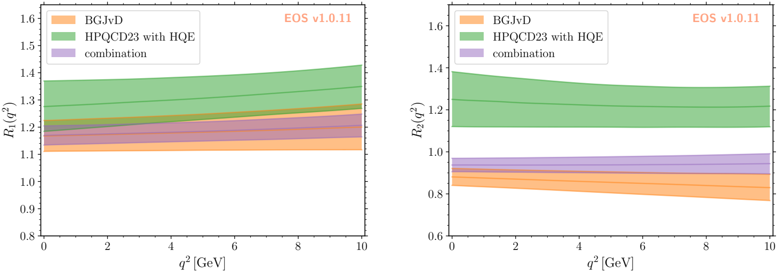
- If for V and A_1 everything aligns well, for A_{12} and A_0 there is an evident shift in slopes
- In the combination difference is milder

Combining with other LQCD results



- All LQCD $B \to D^*$ results can be described in a global fit to the HQE
- R_D is in tension with $B \to D$ LQCD w/o LCSRs

In the CLN basis



- Looking at ratios of form factors the tension is even larger
- In $R_2 \sim A_{12}$ there is a difference of the order of $\sim 10\%$

Fitting HPQCD lattice data at multiple m_h with HQE

Lattice data includes $m_h = 1.5 m_c$, $m_h = 0.9 m_b$

- need to add $1/m_h^2$ terms to continuum HQE parameterisation.
- Also add generic α_s and α_s^2 contributions

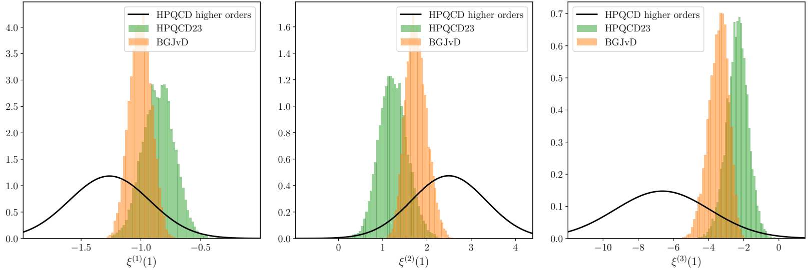
$$F_i = \left(a_i + b_i \frac{\alpha_s}{\pi} + k_i \frac{\alpha_s^2}{\pi^2} \right) \xi + \frac{\Lambda_{\rm QCD}}{2m_h} \sum_j c_{ij} \xi_{\rm SL}^j + \frac{\Lambda_{\rm QCD}}{2m_c} \sum_j d_{ij} \xi_{\rm SL}^j$$
$$+ \left(\frac{\Lambda_{\rm QCD}}{2m_c} \right)^2 \sum_j g_{ij} \xi_{\rm SSL}^j + \left(\frac{\Lambda_{\rm QCD}}{2m_h} \right)^2 \sum_j h_{ij} \xi_{\rm SSL}^j$$
$$+ \sum_{\substack{n,m=1,2 \\ q=h,c}} \left(\frac{\Lambda_{\rm QCD}}{2m_q} \right)^n \left(\frac{\alpha_s}{\pi} \right)^m \zeta_i^{n,m,q}$$

with $\zeta = \zeta(1) + \zeta'(1)(w-1)$

- Use the same uniform prior widths for ξ, $\xi_{\rm SL}^j$ and $\xi_{\rm SSL}^j$, use gaussian priors of 0 ± 10 for $\zeta_i^{n,m,q(\prime)}(1)$, h_{ij}, k_i
- discretisation and chiral effects added analogously to 2304.03137

What changes in terms of parameters?

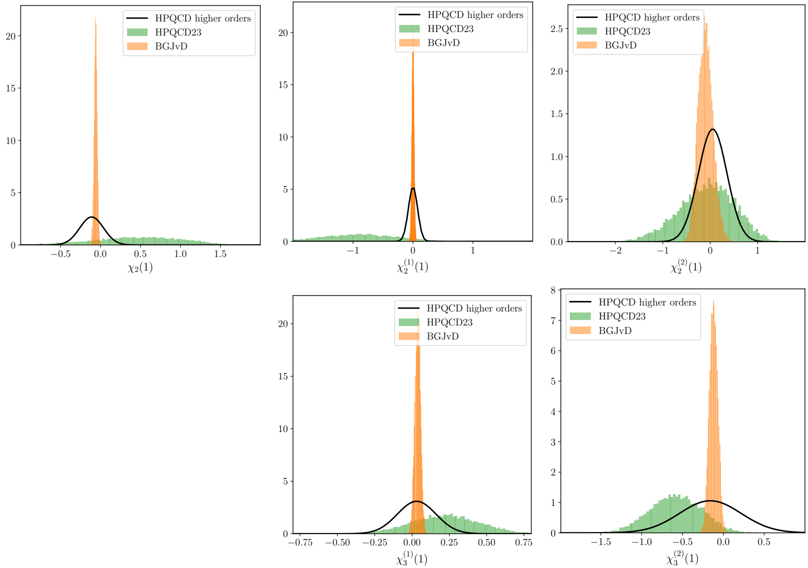
Isgur-Wise function: ξ



- Discrepancies arise in ξ at higher order in $(w-1)$, reflects difference seen in slope
- Similar situation comparing to combined fit posteriors

What changes in terms of parameters?

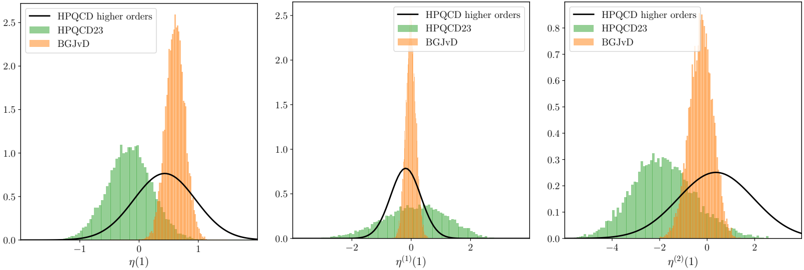
Sub-leading Isgur-Wise functions: χ_2, χ_3



- Information gained by including lattice $1/m_h$ dependence

What changes in terms of parameters?

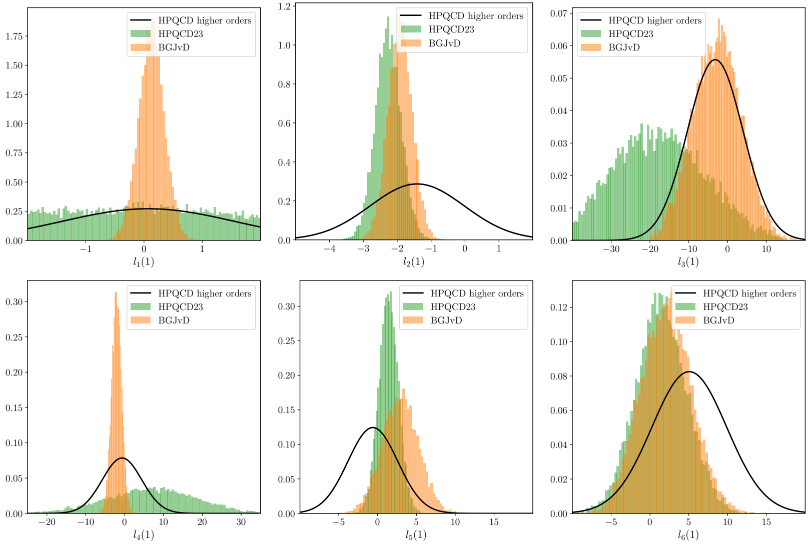
Sub-leading Isgur-Wise functions: η



- From QCDSR, $\eta(1)$ should be positive and different from zero

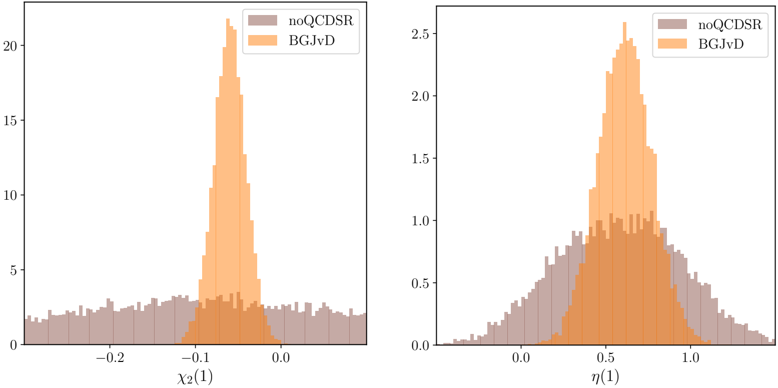
What changes in terms of parameters?

Sub-sub-leading Isgur-Wise functions: $l_i(1)$



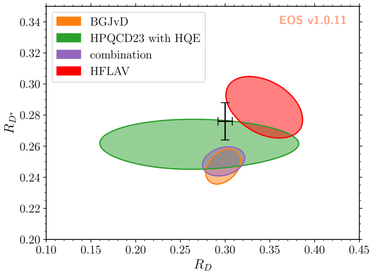
- Still some information to be gained by including lattice $1/m_h$ dependence for $l_i(1)$

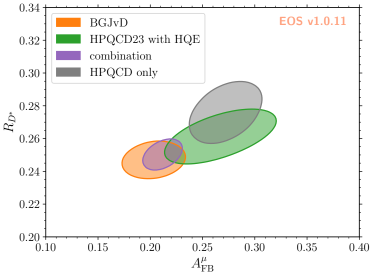
What happens if we remove QCDSRs?



- We lose sensitivity to some of the subleading IW functions
- the central value for $\eta(1)$ is very similar, but with larger uncertainties

Predictions

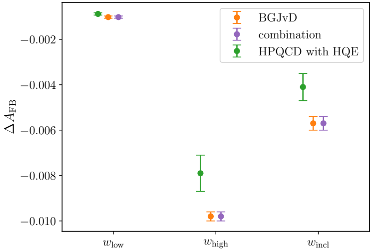


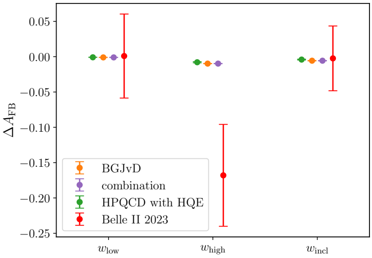


- Predictions for various observables are very different for the various fits
- For the HPQCD with HQE fit the sensitivity to R_D is reduced
- There is a non trivial correlation between A_{FB} and R_{D^*}
- If we fit all $B \to D$ and $B \to D^*$ LQCD results together to the HQE, we find

$$R_D = 0.301 \pm 0.004 \qquad R_{D^*} = 0.257 \pm 0.004$$

Predictions for ΔA_{FB}





- Various theory predictions disagree with each other
- In the comparison with experimental data, the main player are experimental uncertainties

Conclusions

- New Lattice QCD results can be accommodated in the HQE framework
- Studying the interplay between various datasets allows us to better understand correlations and sources of discrepancies
- The information on the subleading Isgur-Wise functions is crucial
- Relativistic lattice calculations done at multiple m_h can provide additional information about SL and SSL Isgur-Wise functions