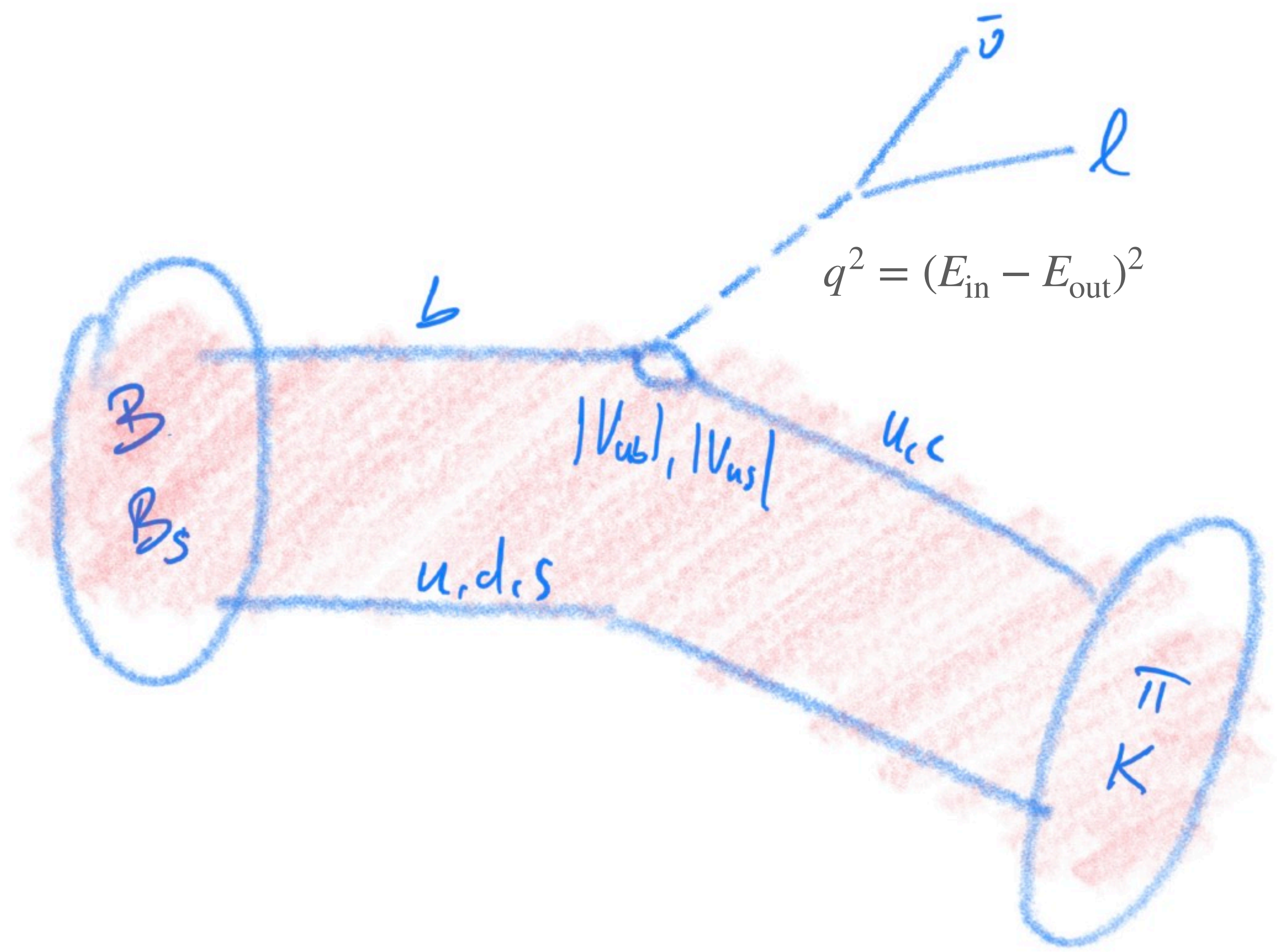
$B_s \to K\ell\nu$ decays — theory

9-11.04.2024
Kulturhaus Lÿz, Siegen

Andreas Jüttner

CERN
University of Southampton

Exclusive semileptonic meson decay

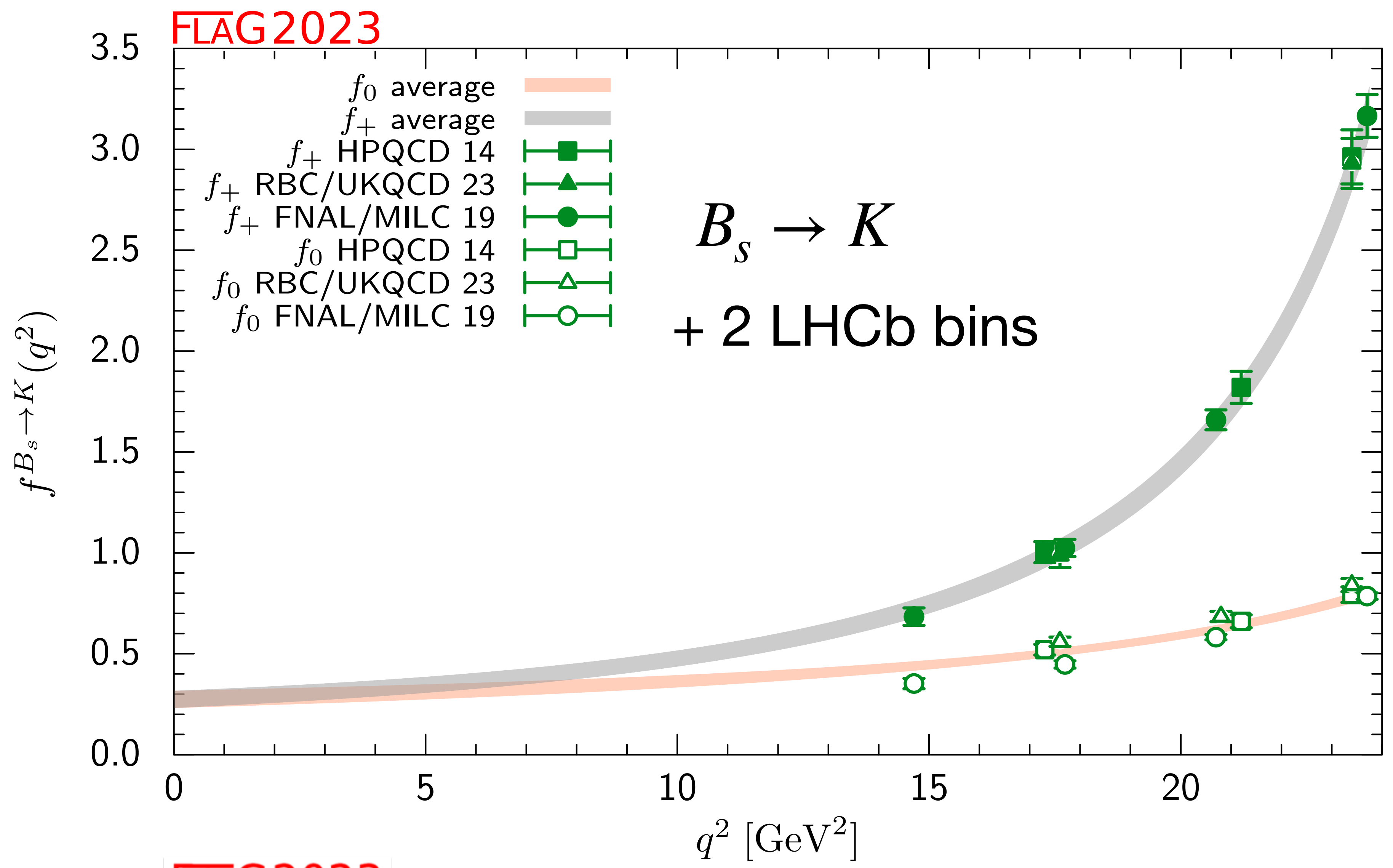


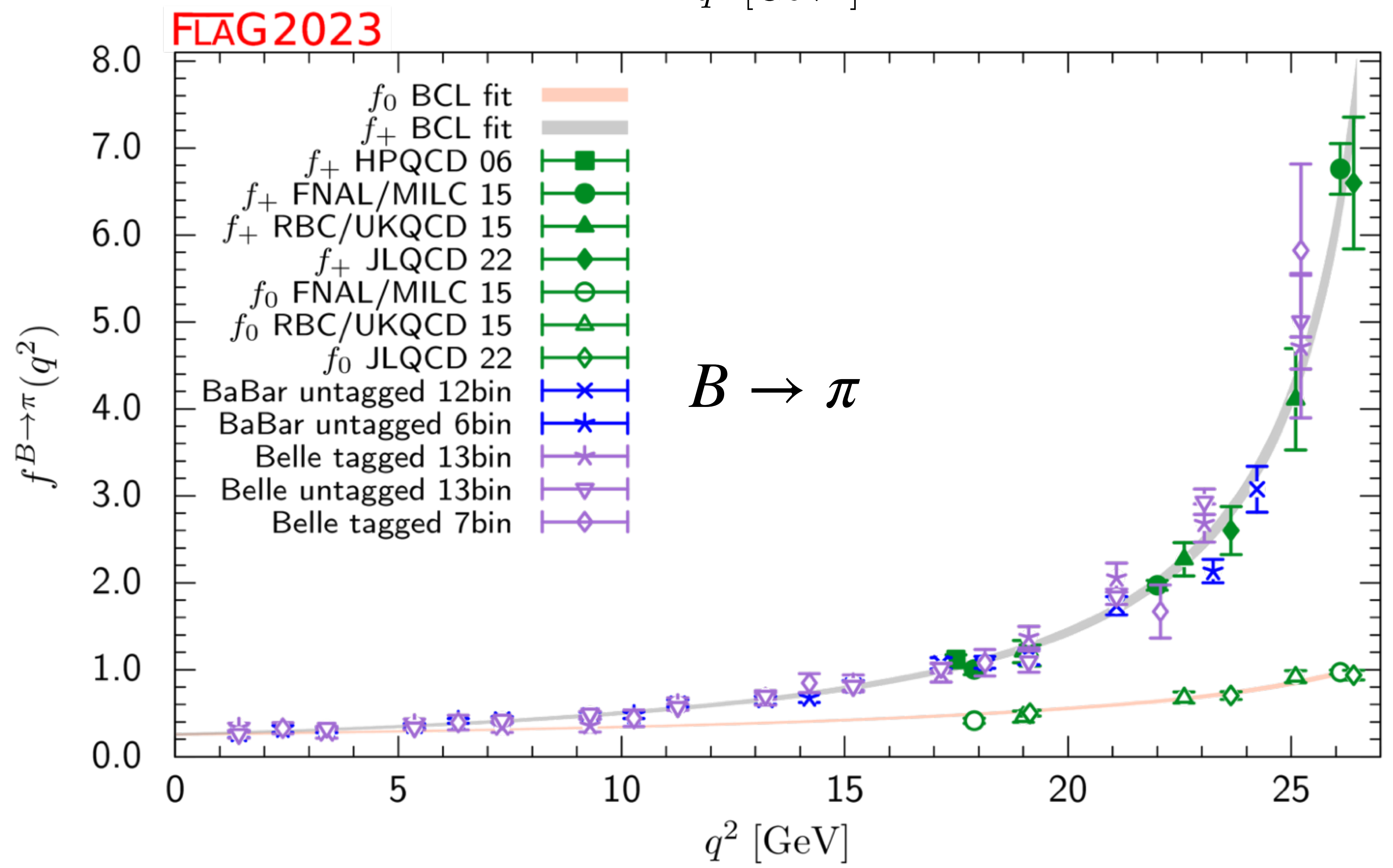
$$\frac{d\Gamma(B_s \to P\ell\nu)}{dq^2} = \eta_{\text{EW}} \frac{G_F^2 |V_{xb}|^2}{24\pi^3} \frac{(q^2 - m_\ell^2)^2 |\vec{k}|}{(q^2)^2} \left[\left(1 + \frac{m_\ell^2}{2q^2}\right) \vec{k}^2 |f_+(q^2)|^2 + \frac{3m_\ell^2}{8q^2} \frac{(M_{B_s}^2 - M_P^2)^2}{M_{B_s}^2} |f_0(q^2)|^2 \right]$$

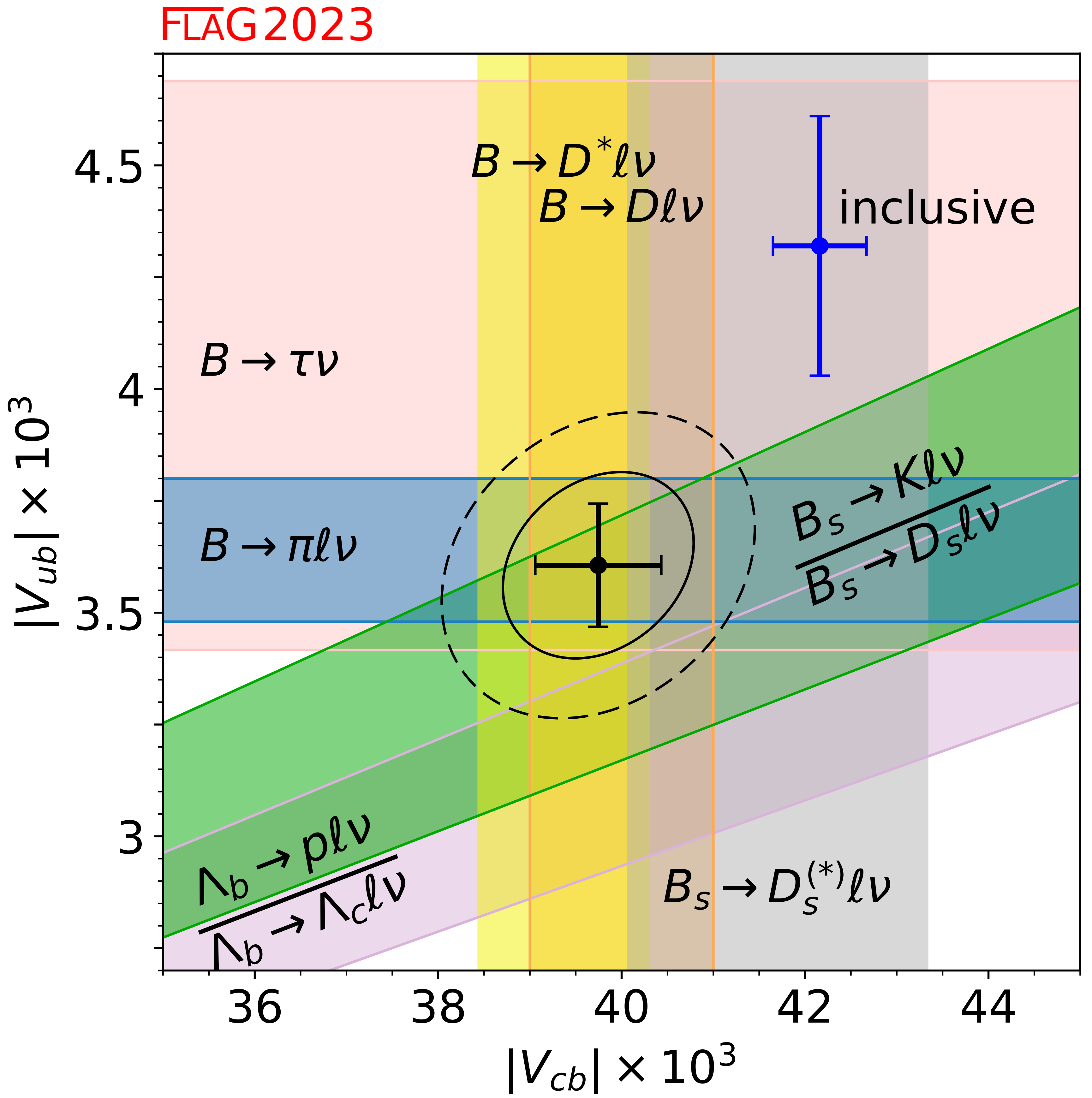
$\langle \text{had}' | H_W | \text{had} \rangle_{QCD}$

- form factors computed on lattice QCD (see FLAG21 and updates on the FLAG website)
- considered a *relatively* standard computation for tree-level
- heavily used for CKMology but also for lepton-flavour-universality tests

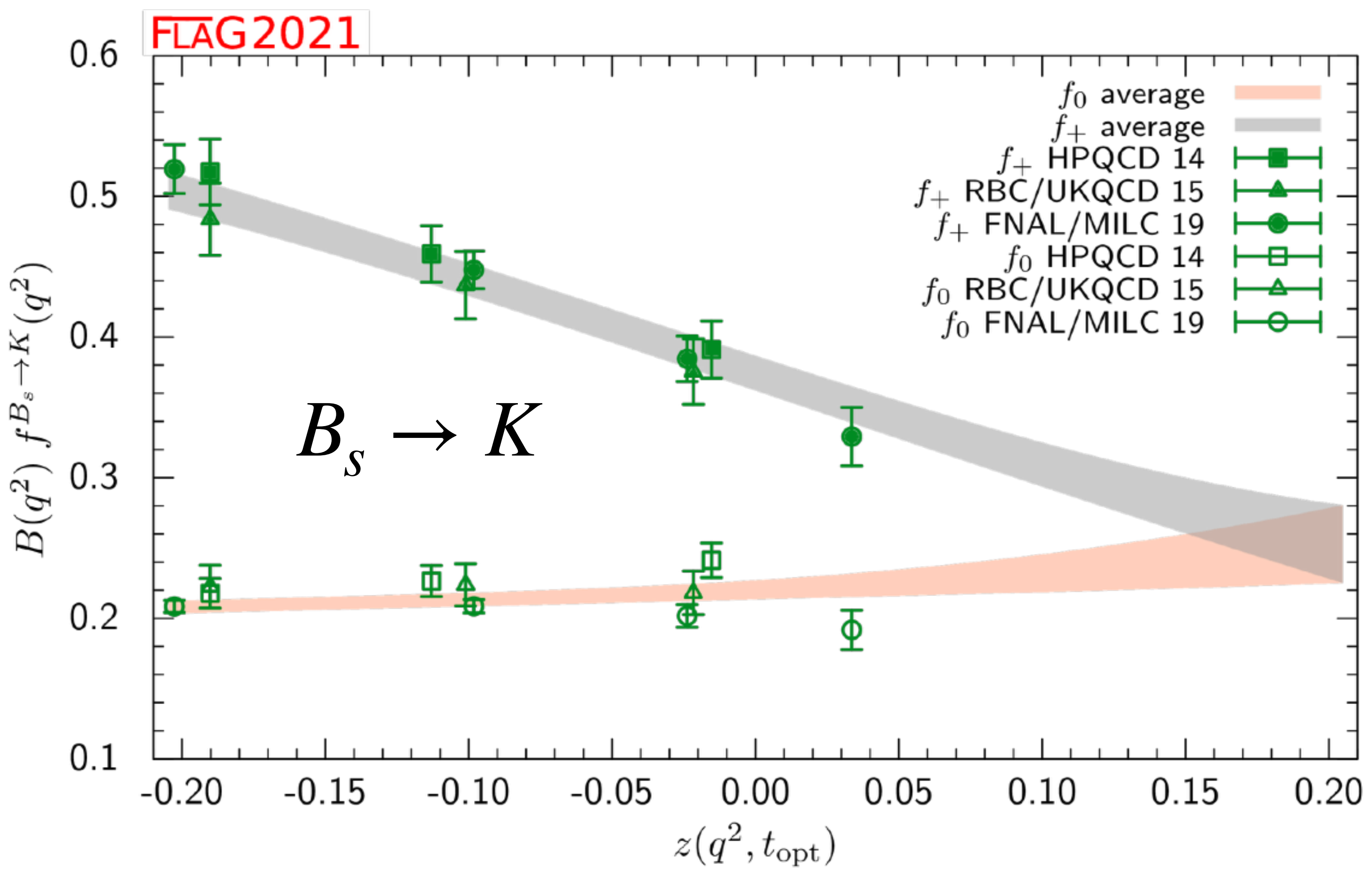
$b \to u$ exclusive: $|V_{ub}|$

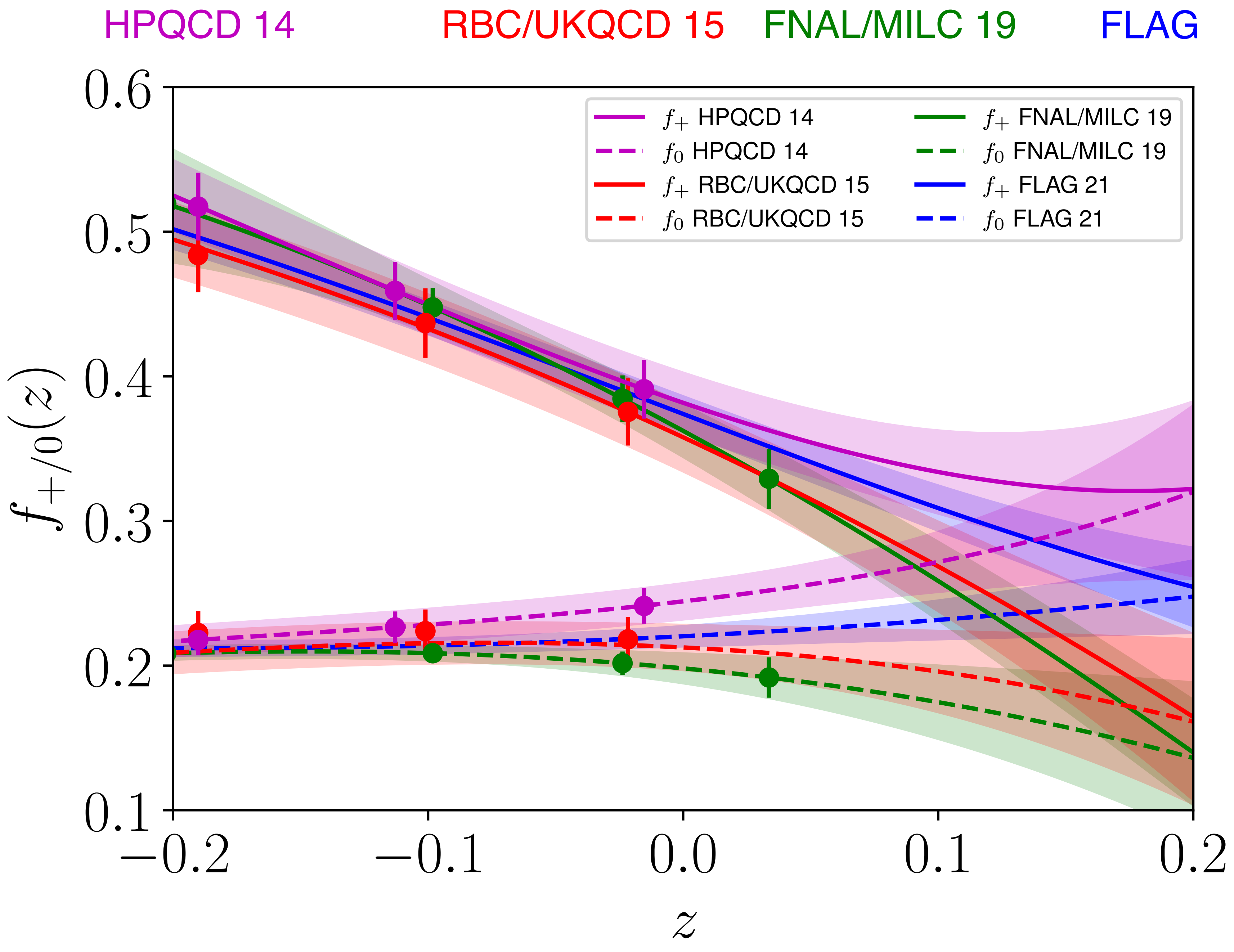




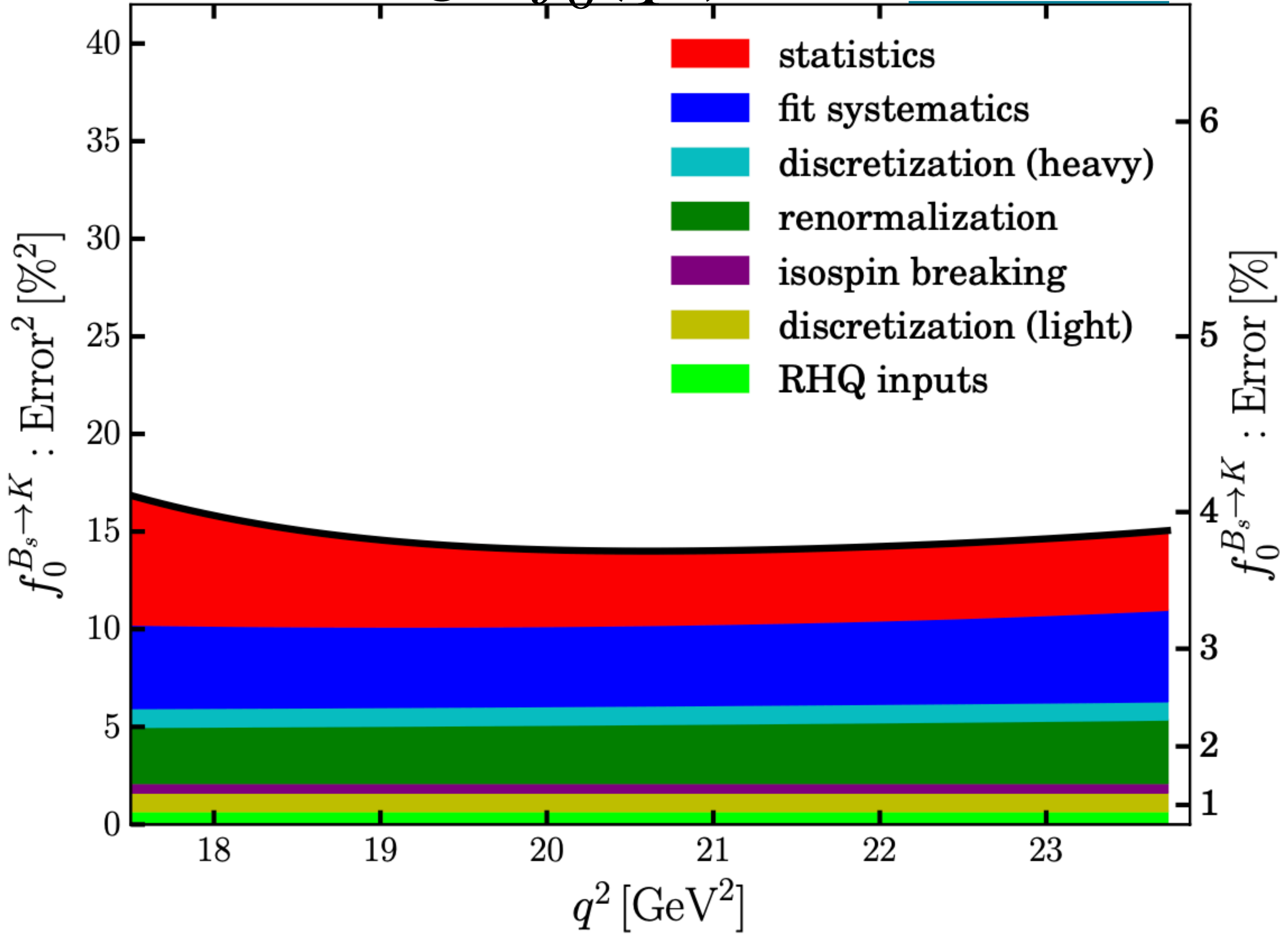


Work required on the lattice (I)





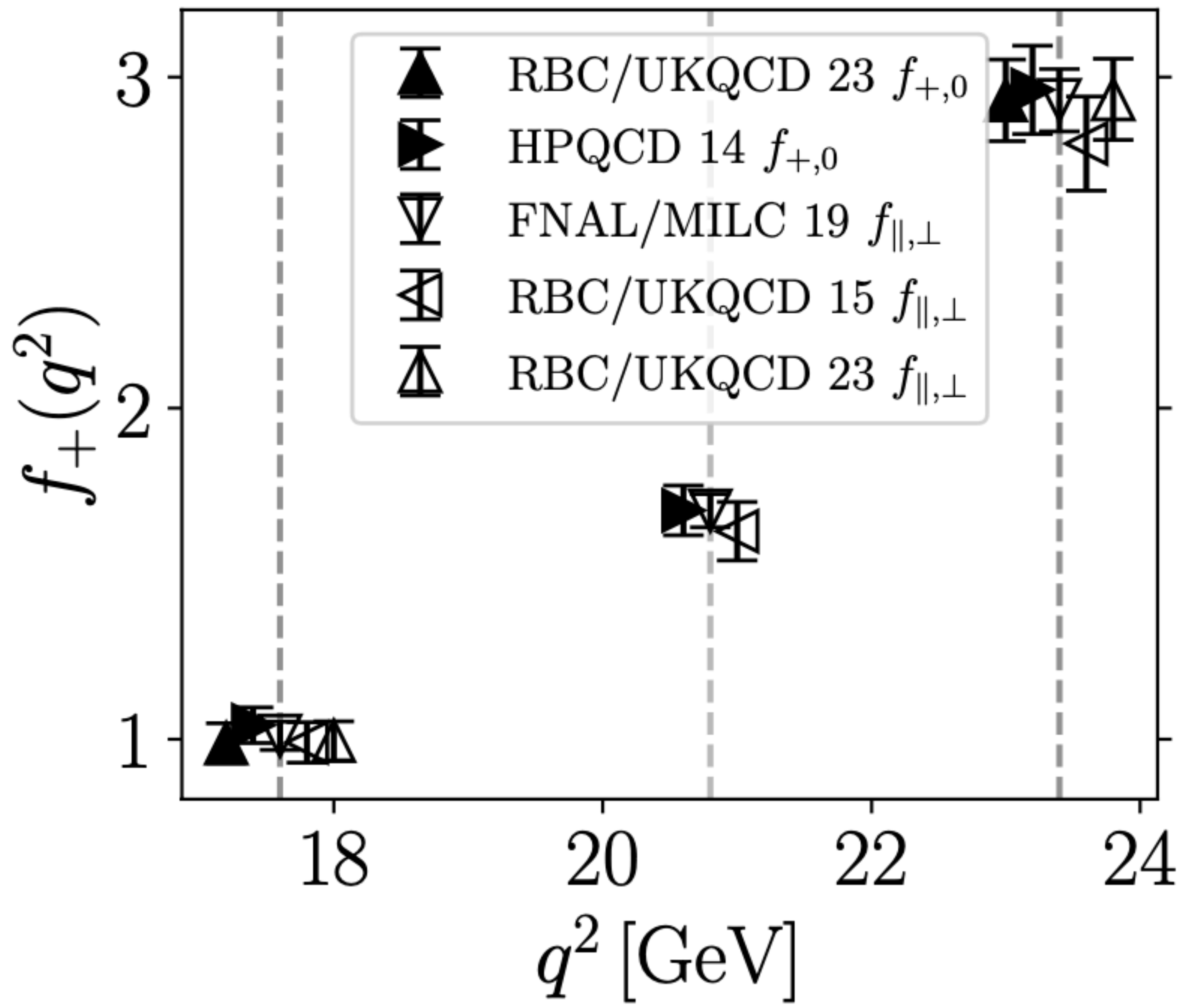
error budget $f_0(q^2)$ **RBC/UKQCD 23 arXiv:2303.11280**

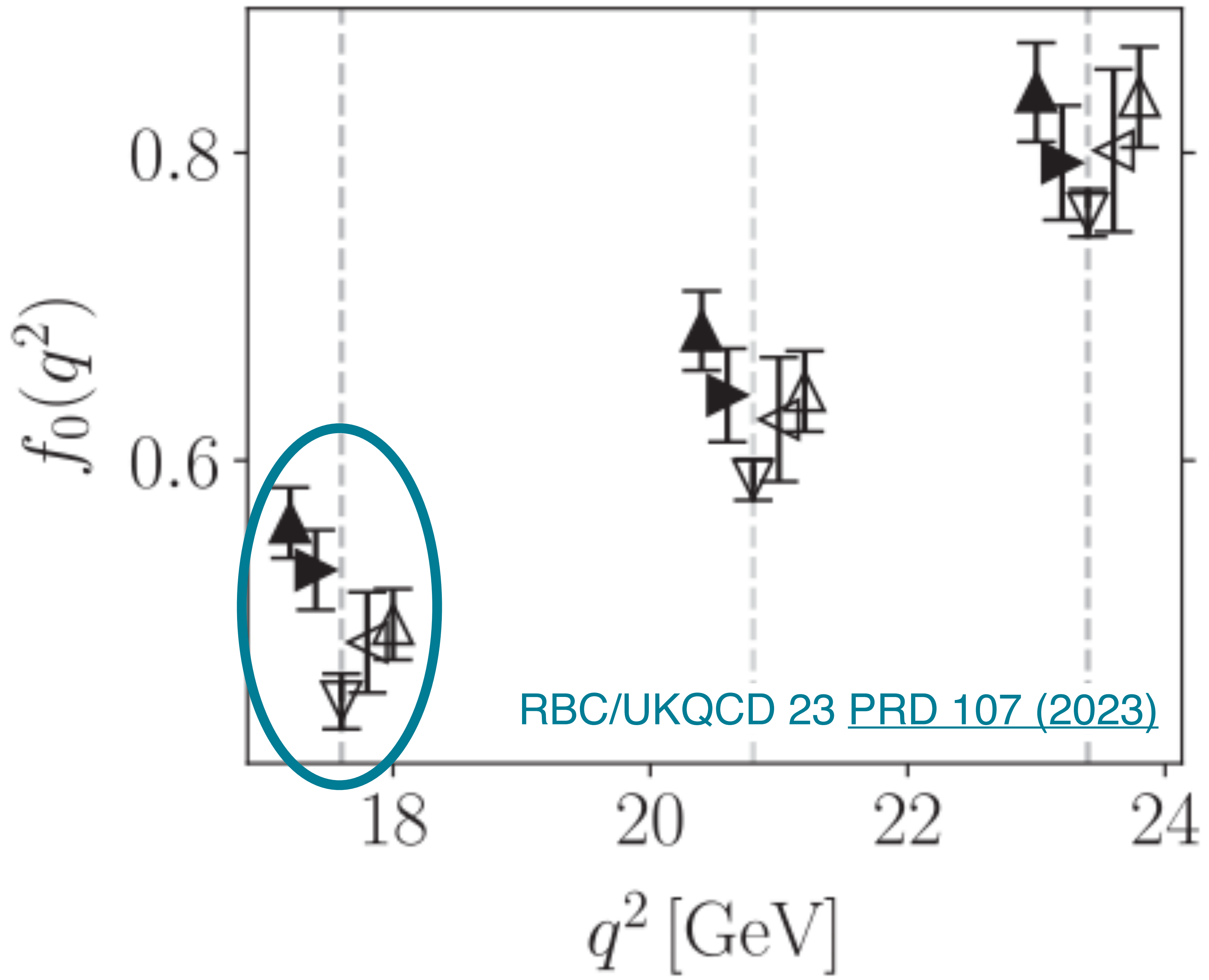


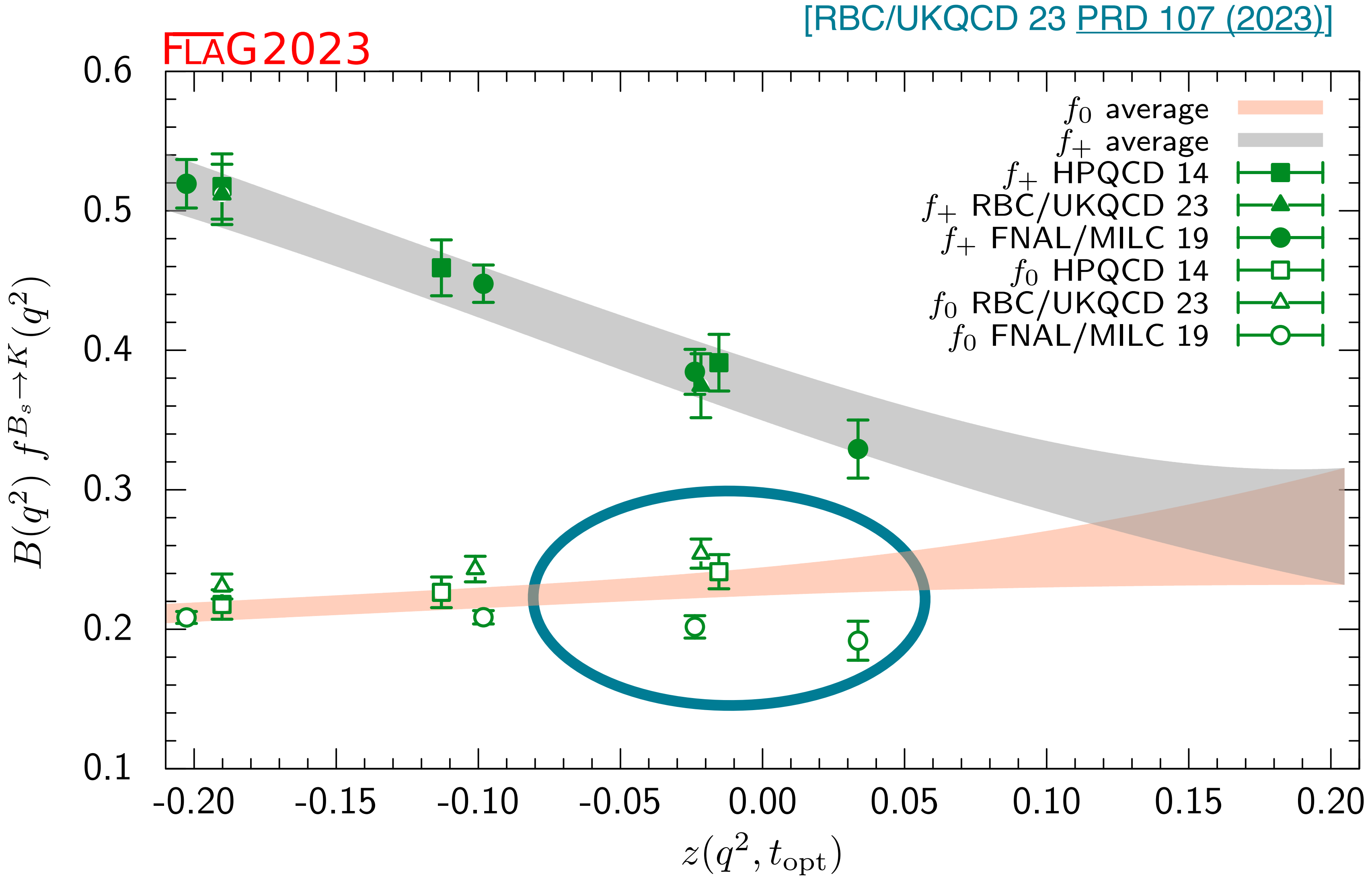
- Lattice data sets show tension $B \to \pi$, $B_s \to K$ (combination requires PDG-inflation *factor)*
- by definition they should agree
- reasons yet to be understood

Work required on the lattice (I)

[HPQCD 14 PRD 90 (2014)]
[FNAL/MILC 19 PRD 100 (2019)]
[RBC/UKQCD 23 PRD 91 (2015)]
[RBC/UKQCD 23 PRD 107 (2023)]







Results for $f_0(q^2)$ incompatible; origin of discrepancy understood in RBC/UKQCD 23
[RBC/UKQCD PRD 107 (2023)]

note that kinematic constraint $f_0(0) = f_+(0)$ is imposed on data!

→ here: use only HPQCD 14 and RBC/UKQCD 23

Work required on the lattice (II)

$$\langle O_{B_s}(t) O^\dagger_{B_s}(0)\rangle = \sum_{n=1}^{N_n^{\max}} \frac{|\langle 0|O_{B_s}(0)|n\rangle|^2}{2E^{(n)}_{B_s}} e^{-E^{(n)}_{B_s} t}$$

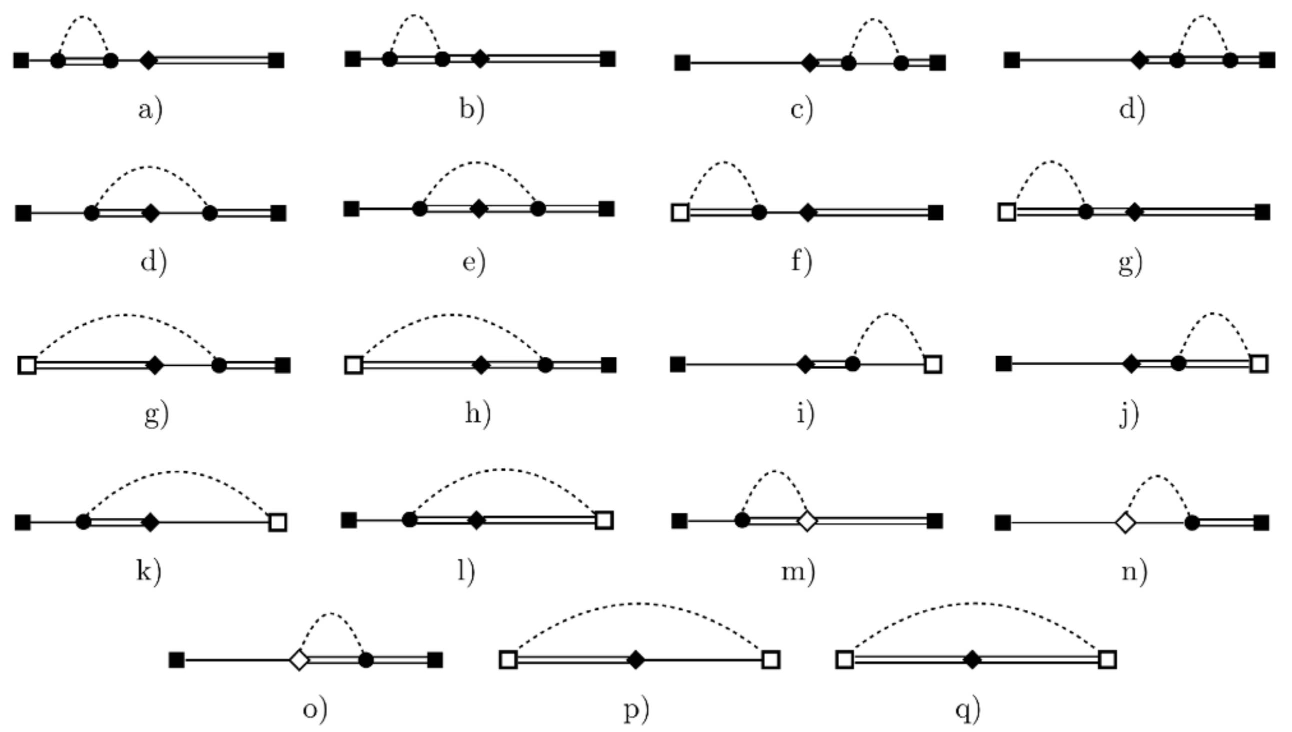
from Euclidean 2pt function

$$\langle O_K(t_{\rm snk}) V_\mu(t) O^\dagger_{B_s}(0)\rangle = \sum_{m,n=1}^{N_n^{\max},N_m^{\max}} \frac{1}{4E^{(n)}_{B_s} E^{(m)}_K} \langle 0|O_K(t_{\rm snk})|m\rangle \langle m|V_\mu(t)|n\rangle \langle n|O_{B_s}(0)|n\rangle e^{-E^{(m)}_{B_s}(t_{\rm snk}-t)} e^{-E^{(n)}_K t}$$

from Euclidean 3pt function

Extraction of $\langle K|V_\mu|B_s\rangle$ is essentially a data-analysis problem:

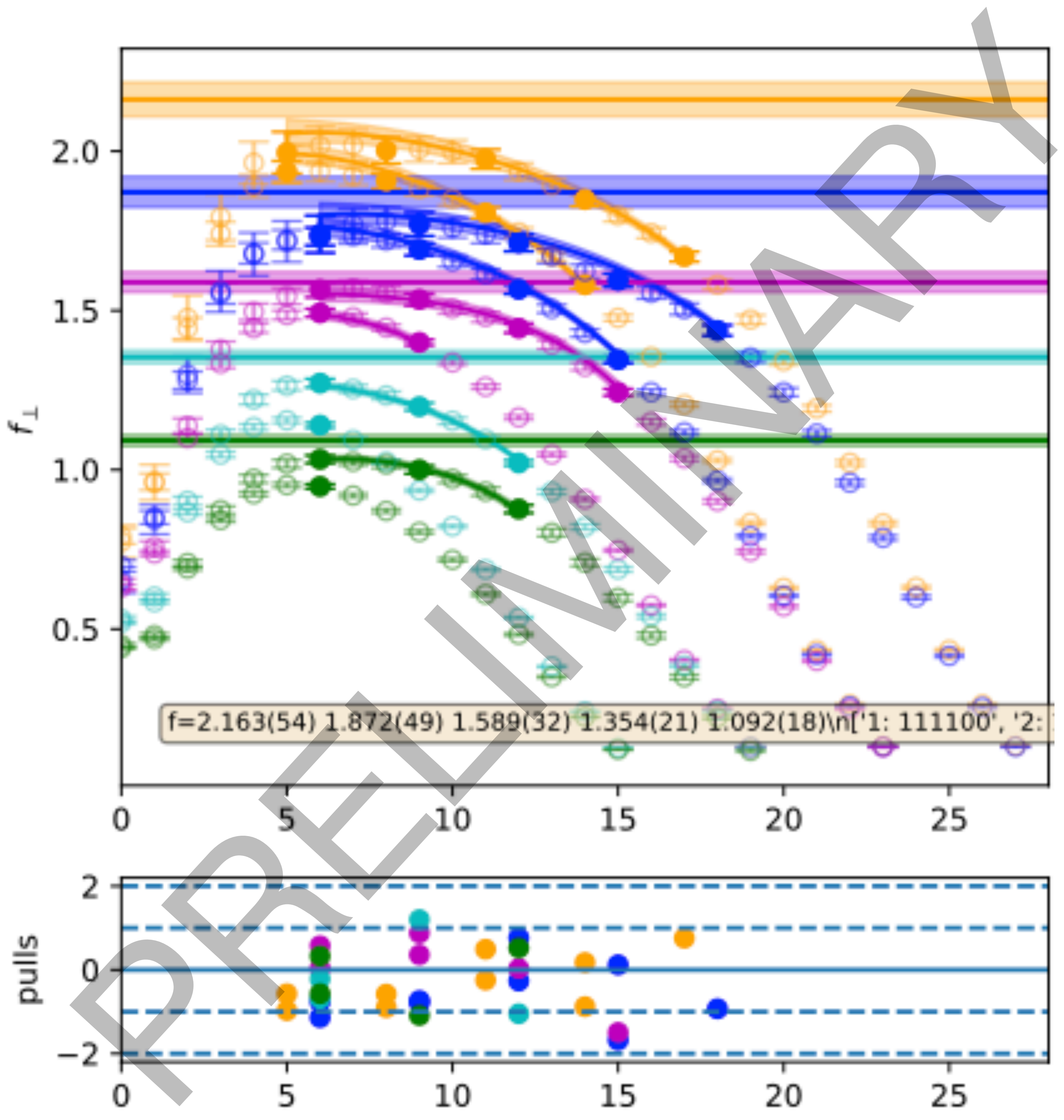
- $N^{\max}_{m,n}$ limited by statistical precision of lattice results for 2pt/3pt function
- couplings to particular excited states enhanced (e.g. $B^*\pi$) in 3pt compared to 2pt function
- potential of mis-identification/representation of excited-states contribution, and hence, ground state



[Bär, Broll, Sommer, arXiv:2306.02703]

A so-far under-estimated/ignored systematic effect?

Work required on the lattice (II)



Preliminary data for $B_s \to K$

- b, s, l at physical quark masses
- fit enforcing ES in 2pt and 3pt functions to be the same is **not working**
  NOTE: the ES spectrum in QCD is the same — the corresponding amplitudes may just be suppressed/enhanced and resolution in data not sufficient to correctly identify!
- fit leaving ES independent between 2pt and 3pt functions **is working**

What to do?

- Here: identify nature of ES
- in general:
  - improve statistics
  - do GEVP
  - be careful!

Even more work required on the lattice…

finite-volume errors
continuum limit
QED and iso-spin-breaking effects
…

In any case, lattice QCD predicts form factors at reference-q^2 values; they need to be combined with experiment in some way

Fitting strategies

A

- fit parameterisation to lattice data
- compute theory prediction for $d\Gamma/dq^2/|V_{ub}|^2$ bin-by-bin by integration
- combine with experimental data for bin-by-bin prediction for $|V_{ub}|$
- final $|V_{ub}|$ from weighted average over bins

Clean separation of SM and exp. measurement

B

- fit parameterisation simultaneously to lattice form factors and results for experimental data for diff. decay rate (use shape-information from both experiment and lattice)
- determine $|V_{ub}|$ directly from such a global fit

Unitarity constraint and fit-ansatz imposed on experimental data (which may contain BSM)

SM correct — A and B should result in compatible predictions

Form-factor parameterisation

$$f_X(q_i^2) = \frac{1}{B_X(q_i^2)\phi_X(q_i^2, t_0)} \sum_{n=0}^{K_X-1} a_{X,n} z(q_i^2)^n$$

unitarity constraint: $|\mathbf{a}_X|^2 \leq 1$

Boyd, Grinstein, Lebed, PRL 74 (1995)

Determine all $a_{X,n}$ from finite set of theory data

Frequentist fit:
- $N_{\mathrm{dof}} = N_{\mathrm{data}} - K_X \geq 1$
  $\rightarrow$ in practice truncation K at low order
- induced systematic difficult to estimate
- meaning of Frequentist with unitarity constraint?

Bayesian fit:
- fit including higher order z expansion meaningful
- unitarity regulates and controls higher-order coefficients [Flynn, AJ, Tsang JHEP 12 (2023) 175]
- well-defined meaning of unitarity constraint

Recommendation: Combined Frequentist + Bayesian perspective

Strategy A

$B_s \to K\ell\nu$ — $|V_{ub}|$

Experiment

Lattice

two bins from LHCb [LHCb PRD 108 (2023)]

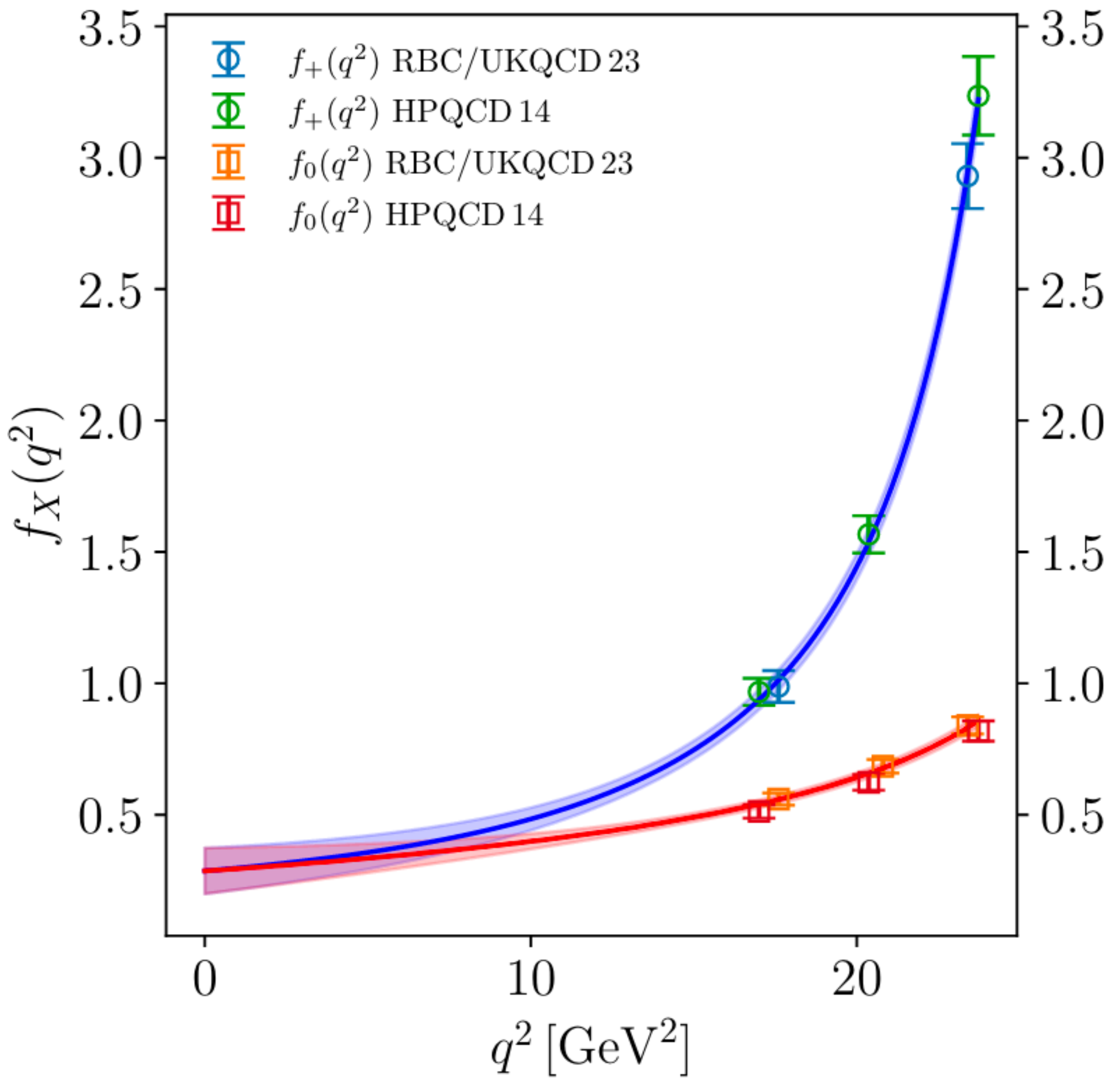
$$R_{\rm BF} = \frac{\mathcal{B}(B_s^0 \to K^-\mu^+\nu_\mu)}{\mathcal{B}(B_s^0 \to D_s^-\mu^+\nu_\mu)}$$

$$R_{BF}^{\rm low} = 1.66(08)(09) \times 10^{-3} \quad (q^2 \leq 7\,{\rm GeV}^2)$$

$$R_{BF}^{\rm high} = 3.25(21)(^{+18}_{-19}) \times 10^{-3} \quad (q^2 \geq 7\,{\rm GeV}^2)$$

$$R_{BF}^{\rm total} = 4.89(21)(^{+24}_{-25}) \times 10^{-3}$$

$$\mathcal{B}(B_s^0 \to D_s^-\mu^+\nu_\mu) = 2.49(12)(21) \times 10^{-2},$$ [LHCb PRD 101 (2020)]



$$|V_{ub}| = \sqrt{\frac{R_{BF}^{\rm bin}\mathcal{B}(B_s^0 \to D_s^-\mu^+\nu_\mu)}{\tau_{B_s^0}\Gamma_0^{\rm bin}(B_s \to K\ell\nu)}},$$

$$\frac{d\Gamma(B_s \to K\ell\nu)}{dq^2} = \eta_{\rm EW}\frac{G_F^2|V_{ub}|^2}{24\pi^3}\frac{(q^2-m_\ell^2)^2|\vec{k}|}{(q^2)^2} \times \left[\left(1+\frac{m_\ell^2}{2q^2}\right)\vec{k}^2|f_+(q^2)|^2 + \frac{3m_\ell^2}{8q^2}\frac{(M_{B_s}^2-M_P^2)^2}{M_{B_s}^2}|f_0(q^2)|^2\right]$$

$B_s \to K\ell\nu - |V_{ub}|$

Let's fit just HPQCD 14 and RBC/UKQCD 23 (FNAL/MILC 19 excluded):

Frequentist works:

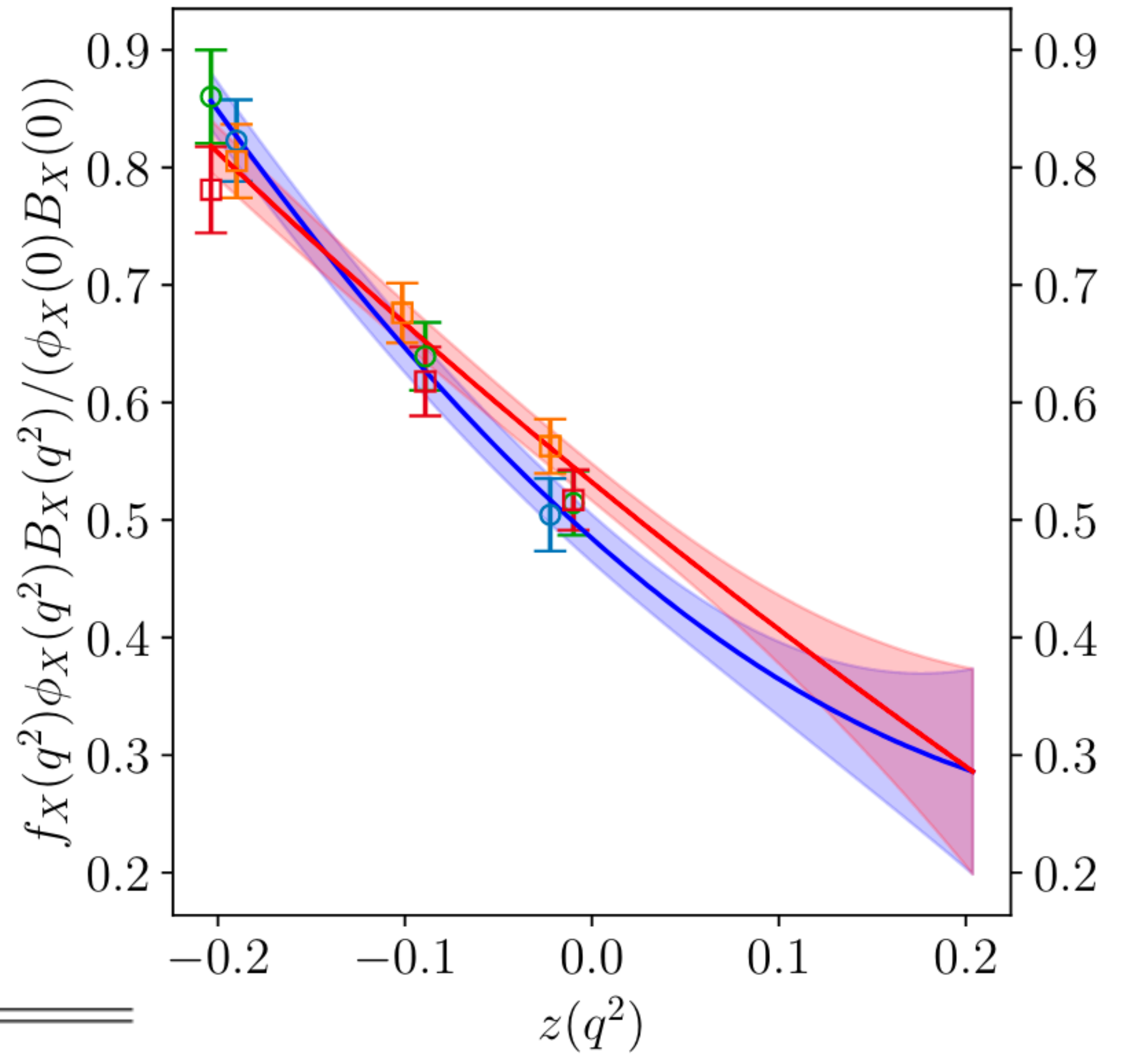
K_+	K_0	$a_{+,0}$	$a_{+,1}$	$a_{+,2}$	$a_{+,3}$	$a_{+,4}$	p	$\chi^2/N_{\rm dof}$	$N_{\rm dof}$
3	3	0.01095(48)	-0.0202(43)	0.079(26)	-	-	0.14	1.61	6
5	5	0.0129(22)	0.11(13)	2.5(2.5)	16.1(20.3)	34.9(64.6)	0.25	1.38	2

K_+	K_0	$a_{0,0}$	$a_{0,1}$	$a_{0,2}$	$a_{0,3}$	$a_{0,4}$	p	$\chi^2/N_{\rm dof}$	$N_{\rm dof}$
3	3	0.0752(24)	-0.250(24)	0.61(11)	-	-	0.14	1.61	6
5	5	0.0830(85)	0.29(55)	11.7(11.9)	92.9(115.0)	270.2(401.8)	0.25	1.38	2

Bayesian fit works:

K_+	K_0	$f(q^2=0)$	$R^{\rm impr}_{B_s\to K}$	$R_{B_s\to K}$	$\frac{\Gamma^\tau}{\lvert V_{ub}\rvert^2}\,[\frac{1}{\rm ps}]$	$\frac{\Gamma^\mu}{\lvert V_{ub}\rvert^2}\,[\frac{1}{\rm ps}]$	$V^{\rm low}_{\rm CKM}$	$V^{\rm high}_{\rm CKM}$	$V^{\rm full}_{\rm CKM}$
3	3	0.341(45)	1.650(34)	0.635(40)	5.11(47)	8.1(1.2)	0.00277(38)	0.00346(33)	0.00316(33)
5	5	0.349(50)	1.689(41)	0.642(47)	5.02(46)	7.9(1.2)	0.00279(41)	0.00354(35)	0.00322(35)
10	10	0.327(44)	1.687(40)	0.661(42)	4.87(43)	7.4(1.0)	0.00294(40)	0.00361(35)	0.00332(35)

K_+	K_0	$I[\mathcal{A}^\tau_{\rm FB}]\,[\frac{1}{\rm ps}]$	$I[\mathcal{A}^\mu_{\rm FB}]\,[\frac{1}{\rm ps}]$	$\not{\mathcal{A}}^\tau_{\rm FB}$	$\not{\mathcal{A}}^\mu_{\rm FB}$	$I[\mathcal{A}^\tau_{\rm pol}]\,[\frac{1}{\rm ps}]$	$I[\mathcal{A}^\mu_{\rm pol}]\,[\frac{1}{\rm ps}]$	$\not{\mathcal{A}}^\tau_{\rm pol}$	$\not{\mathcal{A}}^\mu_{\rm pol}$
3	3	1.47(15)	0.058(13)	0.2875(36)	0.00703(65)	0.34(12)	8.0(1.1)	0.066(22)	0.9805(17)
5	5	1.45(14)	0.059(14)	0.2891(37)	0.00744(79)	0.22(13)	7.7(1.2)	0.042(26)	0.9792(21)
10	10	1.40(13)	0.053(12)	0.2876(34)	0.00711(70)	0.22(13)	7.3(1.0)	0.045(25)	0.9801(19)

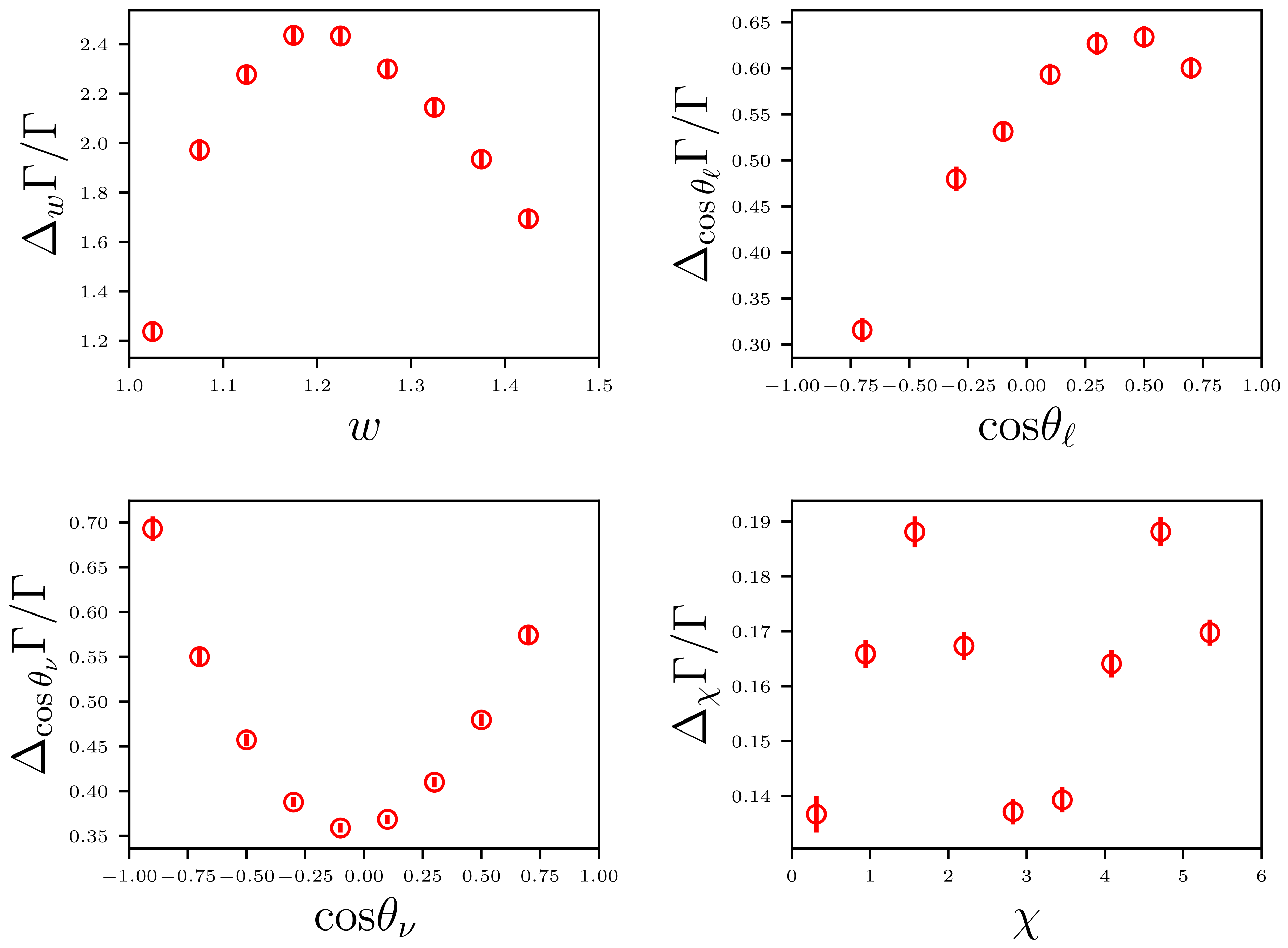


[Flynn, AJ, Tsang JHEP 12 (2023) 175]

Strategy A — 10 $B \to D^{*}\ell\nu$ bins

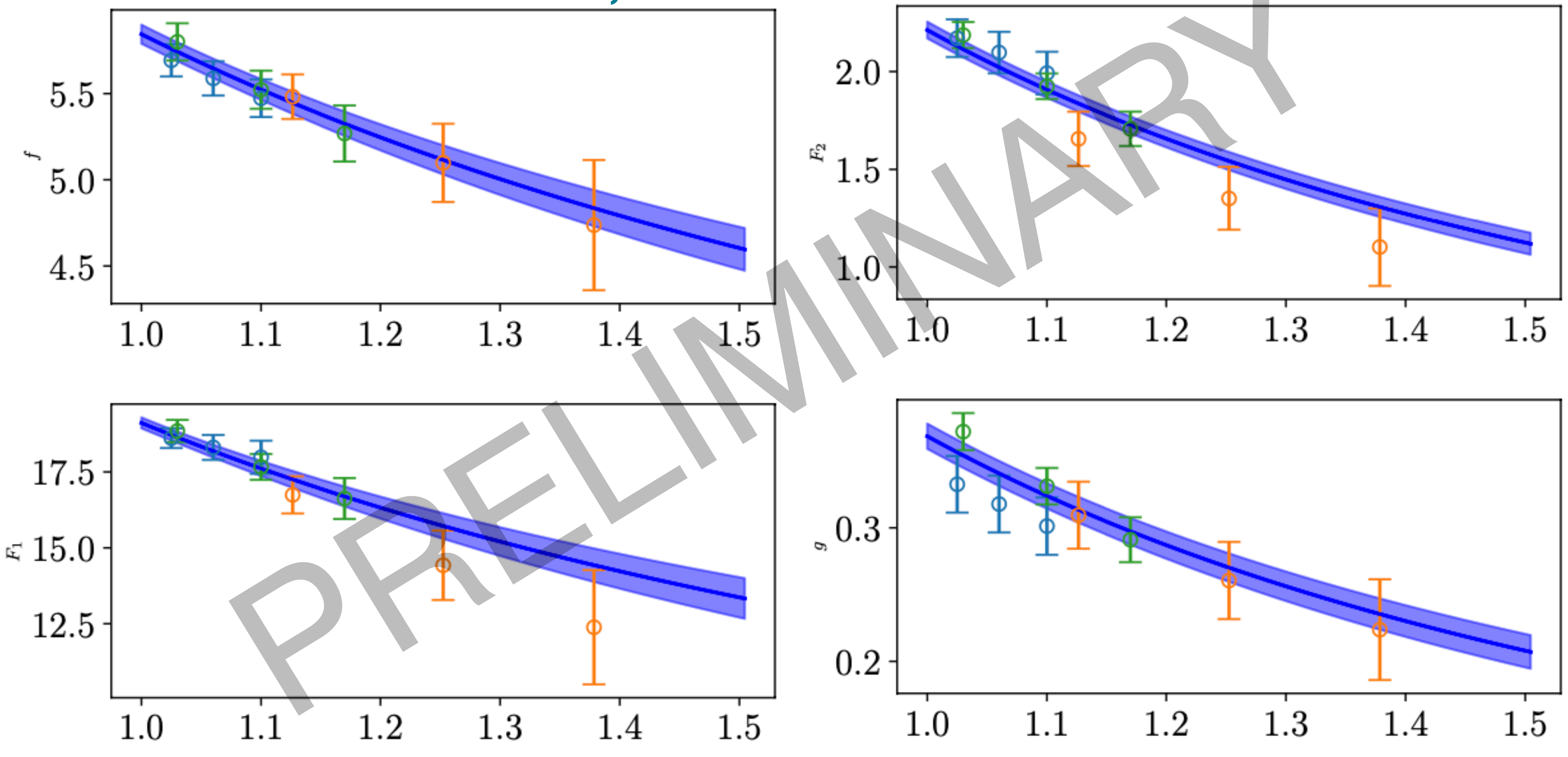
Experiment

BelleII [Phys.Rev.D 108 (2023)]



Lattice

BGL fit to HPQCD, FNAL/MILC and JLQCD data sets

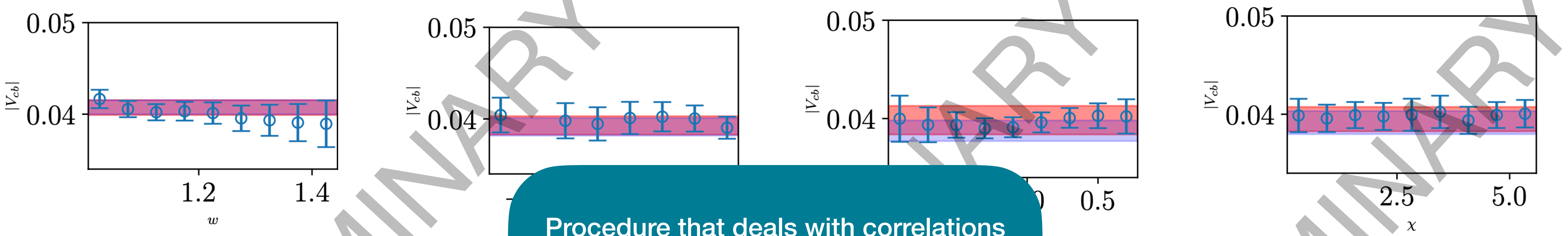


[Bordone and AJ, in preparation]

Strategy A

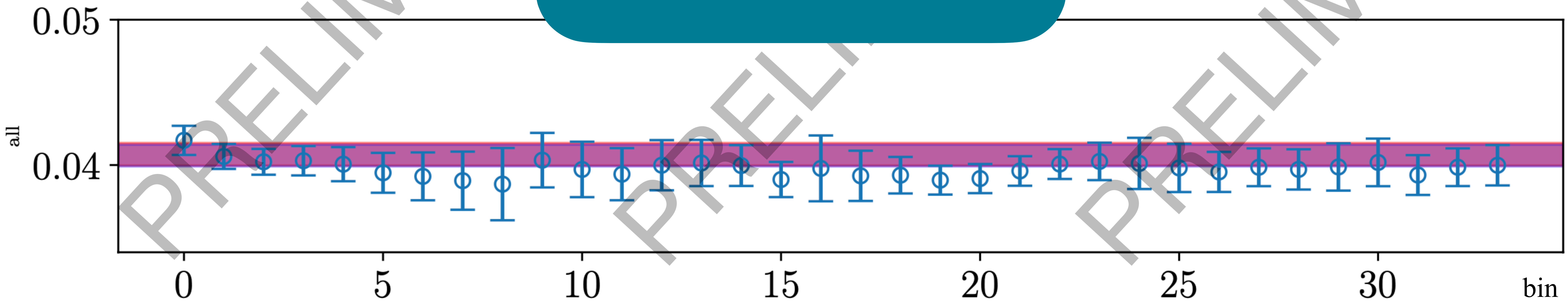
bin-by-bin results for $|V_{cb}|$ based on JLQCD 23 (lattice) and BelleII (experiment)

fits to four different differential decay rates



combined fit to all differential decay rates

Procedure that deals with correlations needs to be defined carefully



[Bordone and AJ, in preparation]

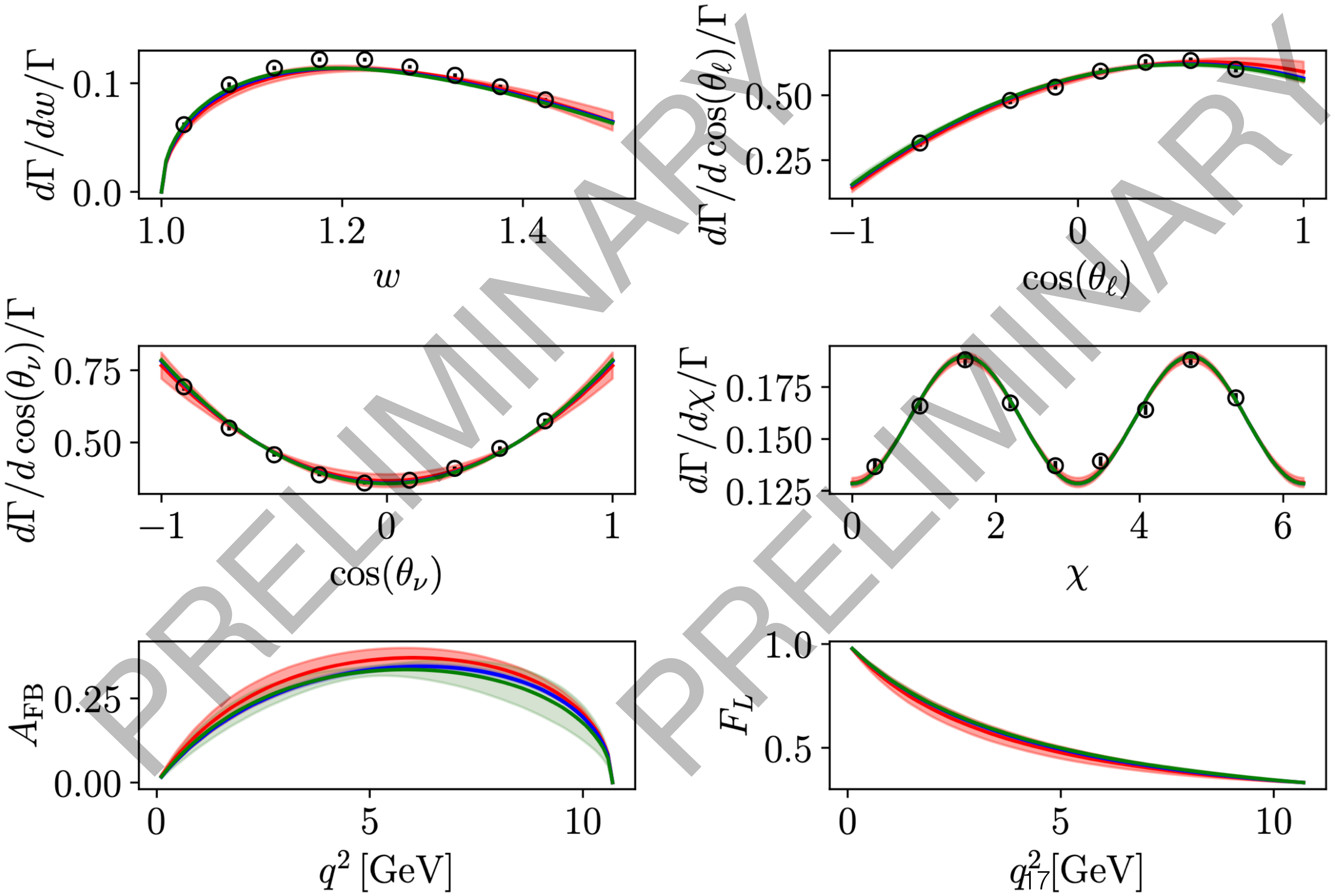
See also [Martinelli et al. arXiv:2404.00334]

Strategy B

not much I can say at this tage for $B_s \to K$ (only 2 bins),
so let's look at $B \to D^*$

Strategy B

BelleII and JLQCD 23



BGL fits to:

- **experiment** BelleII
- **lattice** JLQCD 2023
- **experiment + lattice** BelleII+JLQCD 2023

- fits all of acceptable quality
- theory and exp agree on shape
- compare results for variety of observables from variations of fitting strategy

[Bordone and AJ, in preparation]

See also: Fedele et al. PRD 108, 055037 (2023)

Yet another fit variation: combined fit of $B \to \pi l \nu$ and $B_s \to K l \nu$

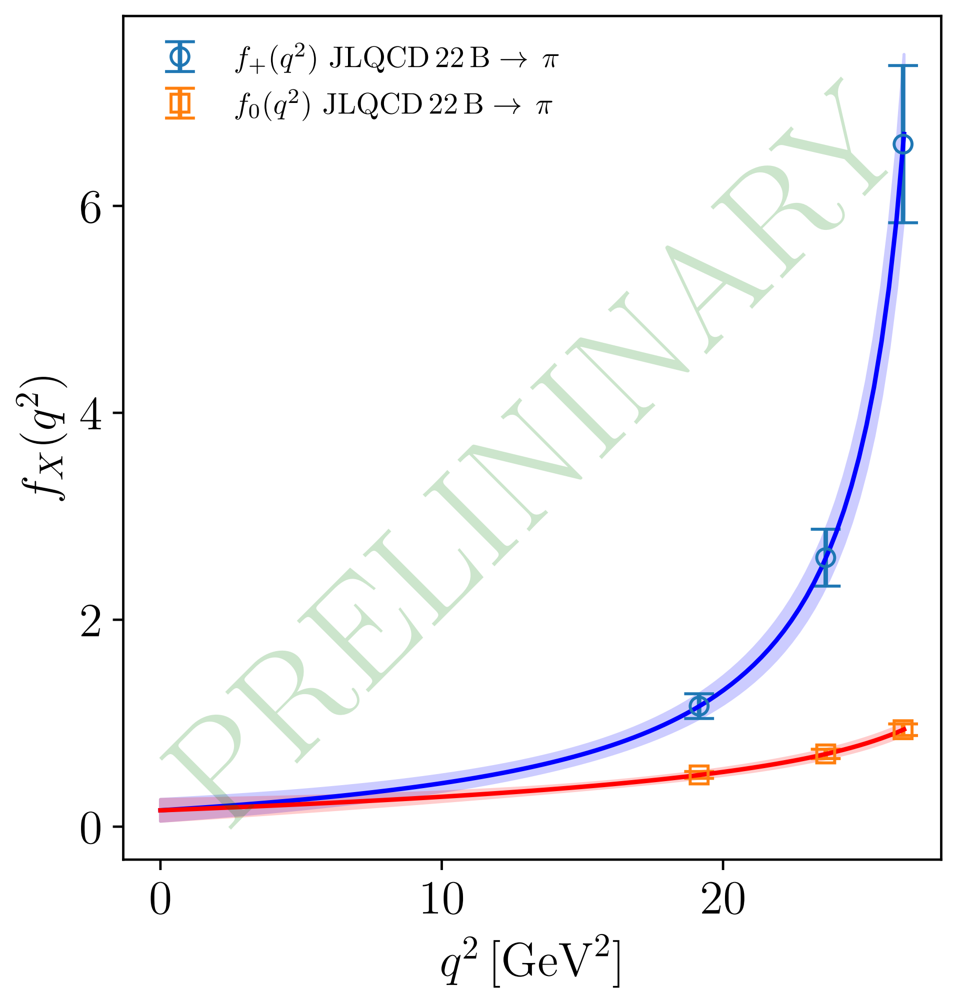
Idea: simultaneous Bayesian fit over both channels subject to combined unitarity constraint

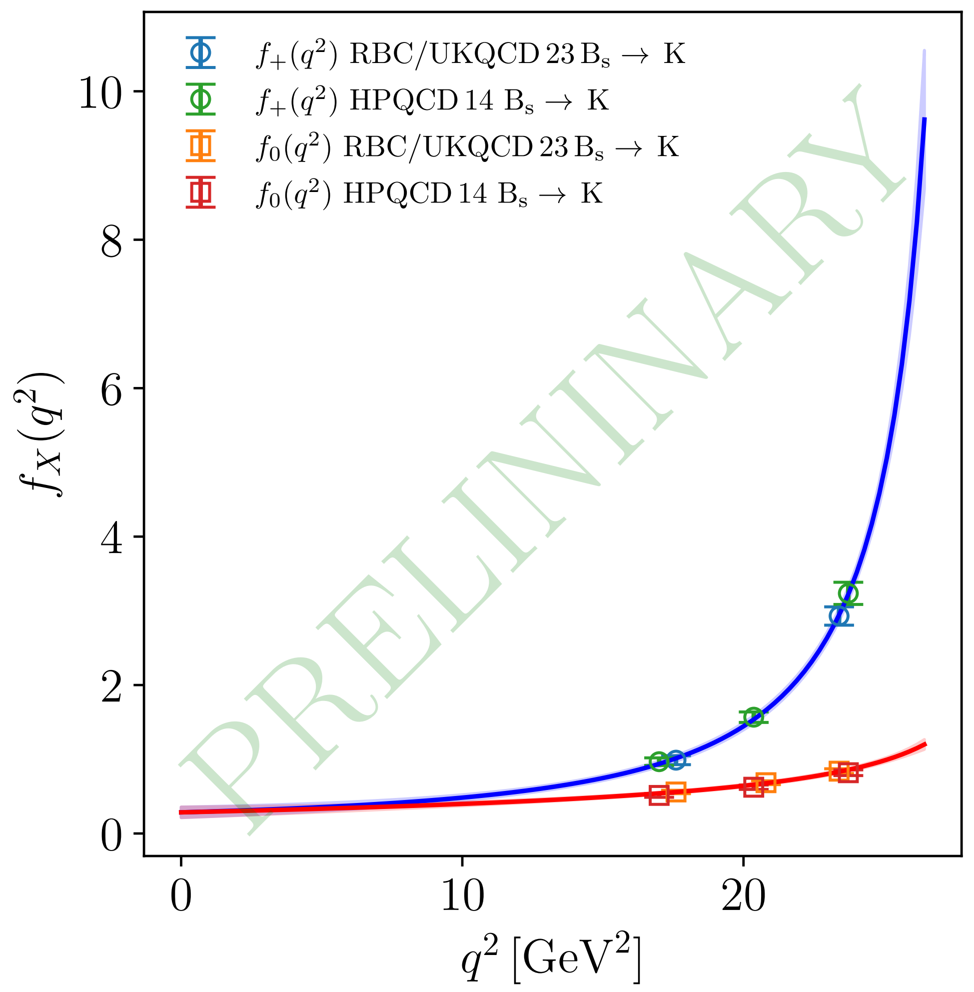
dispersion relation

$$\chi_X \geq \int_{t_{B\pi}}^{\infty} dt \frac{W_{B\pi}(t)\,|f_X^{B\to\pi}(t)|^2}{(t-q^2)^{n_X}} + \int_{t_{B_sK}}^{\infty} dt \frac{W_{B_sK}(t)\,|f_X^{B_s\to K}(t)|^2}{(t-q^2)^{n_X}} \rightarrow \boxed{1 \geq |\vec{a}_X^{B\to\pi}|^2 + |\vec{a}_X^{B_s\to K}|^2} \qquad (X = +, 0)$$

Lattice data: $B_s \to K$ [HPQCD 14 PRD 90 (2014), RBC/UKQCD 23 PRD 107 (2023)]

$B \to \pi$ [JLQCD 22 PRD 106 (2022)]





- we find simultaneous unitarity constraint stronger than individual
- effect will depend on channel
- in case at hand coefficients have noticeably smaller error
- work in progress

Conclusions

- bread-and-butter may actually not be as bread-and-butter — issues in existing and future lattice computations need to be addressed and resolved

- expect updates for lattice results

- there are many different ways in which we can analyse the experimental and lattice data and we should explore them all

- it's exciting to get semileptonic data of such high quality and precision — we are ready for analysing new bins for $B_s \to K\ell\nu$ (and other channels)
  - please provide detailed list of all input
  - please provide stat. and syst. correlations/covariances for **all** results
  - please provide as much detail as possible

- exciting interplay between experiment and theory over coming years!