Analysis of $B \to K^*\mu^+\mu^-$ decays: experiment & theory

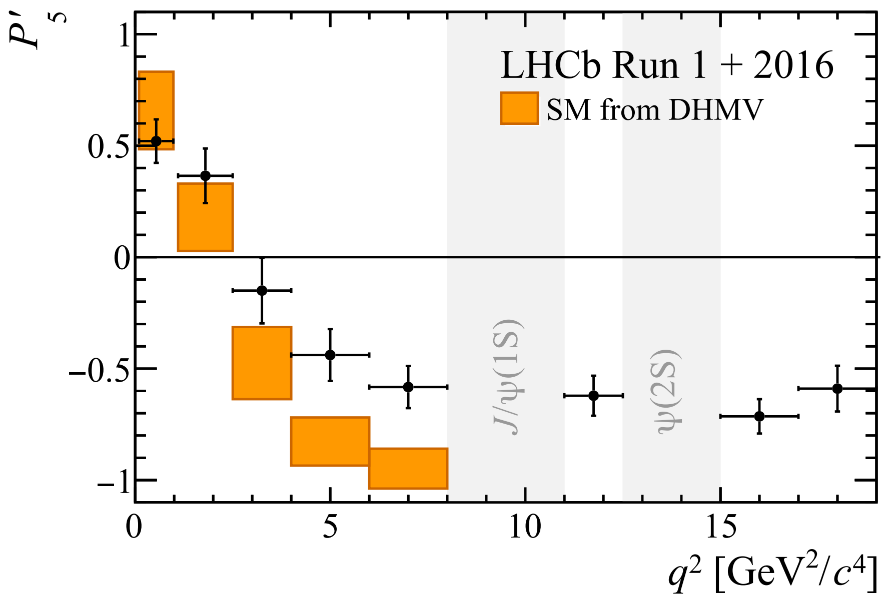
Beyond the flavour anomalies, Siegen

Mark Smith, Andrea Mauri, Arianna Tirani

9 April 2024

$$
\begin{aligned}
\frac{d^4\Gamma_P}{dq^2 d\Omega} = \frac{9}{32\pi} \big[& J_{1s}\sin^2\theta_K + J_{1c}\cos^2\theta_K \\
& + (J_{2s}\sin^2\theta_K + J_{2c}\cos^2\theta_K)\cos 2\theta_\ell \\
& + J_3\sin^2\theta_K\sin^2\theta_\ell\cos 2\phi + J_4\sin 2\theta_K\sin 2\theta_\ell\cos\phi \\
& + J_5\sin 2\theta_K\sin\theta_\ell\cos\phi + J_{6s}\sin^2\theta_K\cos\theta_l \\
& + J_7\sin 2\theta_K\sin\theta_\ell\sin\phi + J_8\sin 2\theta_K\sin 2\theta_\ell\sin\phi \\
& + J_9\sin^2\theta_K\sin^2\theta_\ell\sin 2\phi \big]
\end{aligned}
$$

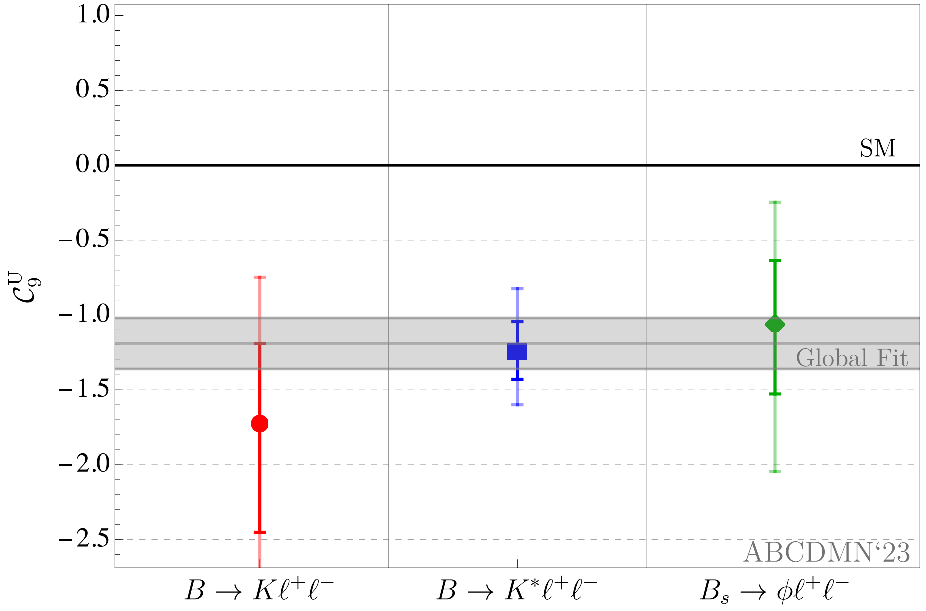
[PRL 125 (2020) 011802]



Why we do it:

- A measurement of *observables*
  - Unambiguous(ish), average, re-interpret
  - What is your $m_{K\pi}$ window?

[EPJC 83 (2023) 648]



Why we do it:

- A measurement of *observables*
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- (Almost) model independent
  - Scalars, tensors, *CP*-asymmetries

[PRD 87 (2013) 3]

$$\frac{4}{3}J_{1s} = \frac{(2+\beta_\ell^2)}{4}\left[|A_\perp^L|^2 + |A_\parallel^L|^2 + (L\to R)\right] + \frac{4m_\ell^2}{q^2}\mathrm{Re}\left(A_\perp^L A_\perp^{R*} + A_\parallel^L A_\parallel^{R*}\right)$$
$$+ 4\,\beta_\ell^2\left(|A_{0\perp}|^2 + |A_{0\parallel}|^2\right) + 4\left(4-3\beta_\ell^2\right)\left(|A_{t\perp}|^2 + |A_{t\parallel}|^2\right)$$
$$+ 8\sqrt{2}\frac{m_\ell}{\sqrt{q^2}}\mathrm{Re}\left[(A_\parallel^L + A_\parallel^R)A_{t\parallel}^* + (A_\perp^L + A_\perp^R)A_{t\perp}^*\right],$$

$$\frac{4}{3}J_{1c} = |A_0^L|^2 + |A_0^R|^2 + \frac{4m_\ell^2}{q^2}\left[|A_t|^2 + 2\,\mathrm{Re}(A_0^L A_0^{R*})\right] + \beta_\ell^2|A_S|^2$$
$$+ 8\,(2-\beta_\ell^2)|A_{t0}|^2 + 8\,\beta_\ell^2|A_{\parallel\perp}|^2 + 16\,\frac{m_\ell}{\sqrt{q^2}}\mathrm{Re}\left[(A_0^L + A_0^R)A_{t0}^*\right],$$

$$\frac{4}{3}J_{2s} = \frac{\beta_\ell^2}{4}\left[|A_\perp^L|^2 + |A_\parallel^L|^2 + (L\to R) - 16\left(|A_{t\perp}|^2 + |A_{t\parallel}|^2 + |A_{0\perp}|^2 + |A_{0\parallel}|^2\right)\right.$$

$$\frac{4}{3}J_{2c} = -\beta_\ell^2\left[|A_0^L|^2 + |A_0^R|^2 - 8\left(|A_{t0}|^2 + |A_{\parallel\perp}|^2\right)\right],$$

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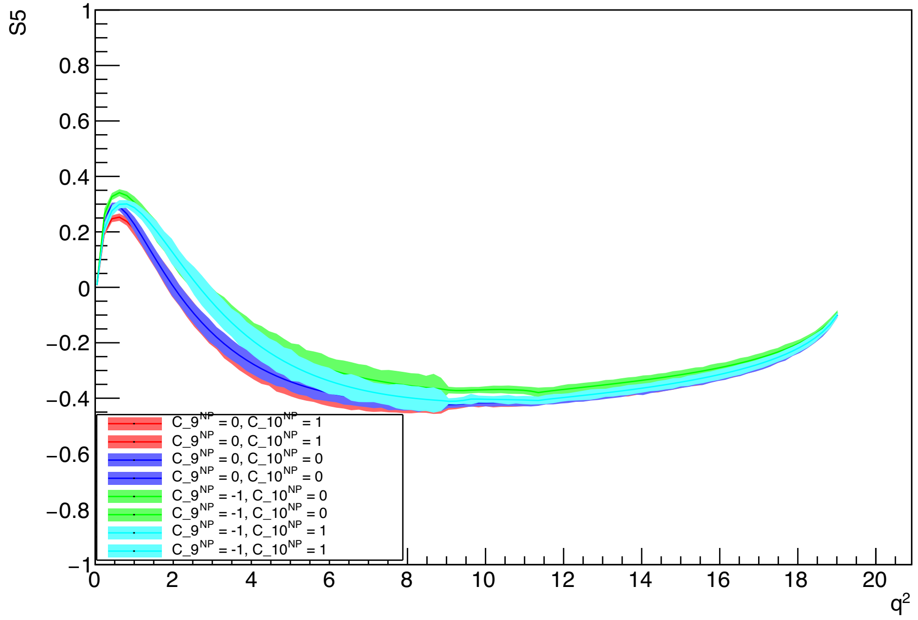
What they lack:

- Full value for the data
  - Binning inherently loses information

Flavio / Michael McCann:



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What they lack:

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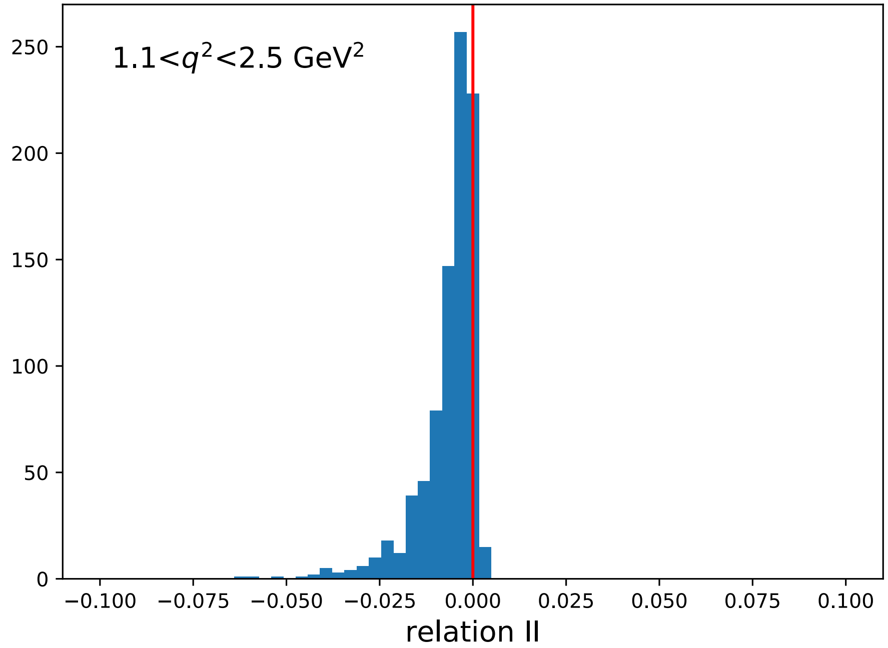
- Too many observables

[JHEP 12 (2021) 085]

I. From $i = 0$ in Eq.(3.12) one finds $|n_0|^2 = a_0(n_0^\intercal n_\parallel) + b_0(n_0^\intercal n_\perp)$ yielding the first relation:

$$0 = +J_{2c}(16J_{2s}^2 - 4J_3^2 - \beta^2 J_{6s}^2 - 4J_9^2) + 2(J_3(4J_4^2 + \beta^2(-J_5^2 + J_7^2) - 4J_8^2) \\ +2J_{2s}(4J_4^2 + \beta^2(J_5^2 + J_7^2) + 4J_8^2) - 2(\beta^2(J_4J_5J_{6s} + J_{6s}J_7J_8 + J_5J_7J_9) - 4J_4J_8J_9))\,. \qquad (3.15)$$



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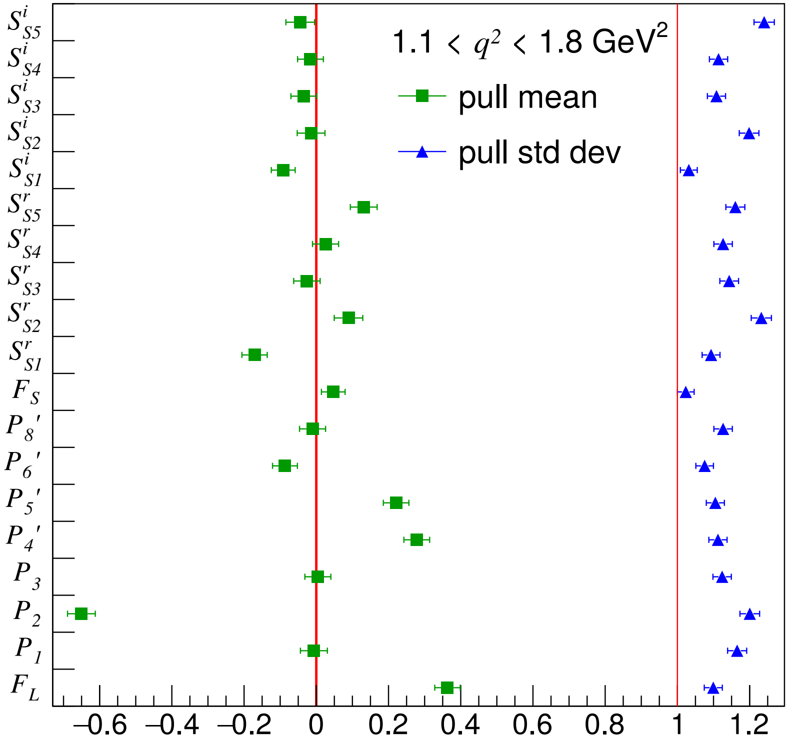
What they lack:

- Full value for the data
  - Binning inherently loses information
- Too many observables
- **Nice statistical behaviour**
  - Care for theory fits

[JHEP 12 (2021) 085]

$1.1 < q^2 < 1.8\ \text{GeV}^2$



Expected LHCb Run 2 yield

- We quote a central value $\pm\sigma$ and correlation matrix
  - Implicitly assuming (symmetric) parabolic $-2NLL$ up to $\infty\,\sigma$ around central value
  - What if we quote $y.yy^{+\sigma^+}_{-\sigma^-}$? Do the WC fits really use a bifurcated Gaussian?
- What if we know the central value is biased?
  - Estimate the size of the bias? It depends on the unknown true value
  - What does this mean for the correlation matrix?
- Coverage correction with Feldman-Cousins method?
  - How would a WC coefficient fit use a confidence interval?
  - What if the fitted central value is outside the quoted interval?
  - What about the correlation matrix?
  - Practically can only use Feldman-Cousins method for ≤ 2 observables at once
  - Physical boundaries may be defined by the combination of several observables

- Model independent $\frac{d\mathcal{B}}{dq^2}$
- Complete CP-asymmetries
- Complete S-wave & interference
- Symmetry check
- and more...

from q^2 bins to q^2-*unbinned* analyses

Different type of analysis

Modelling of the decay amplitudes

Generalised model
Model *independent*

Specific model
Model *dependent*

Binned

Unbinned

Binned fit → **Binned angular observables**

Amplitude ansatz → $A^{L,R}(q^2)$

coming soon...

Z-expansion → $C^{(\prime)}_{9,10}$

+ non-local polynomial

PRL 132 (2024) 131801
PRD 109 (2024) 052009

Dispersion relation → $C^{(\prime)}_{9,10}$ C^{τ}_{9}

+ Non-local phases & magnitudes

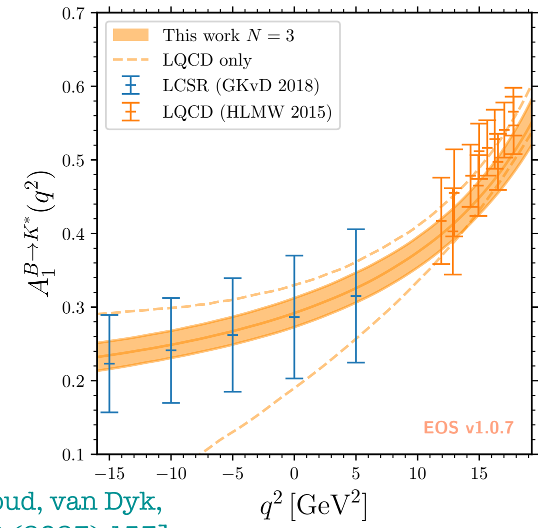
LHCb-PAPER-2024-011
in preparation

z-expansion vs dispersion relation

Commonalities

- Usual experimental treatment of B → K*mumu (acceptance, combinatorial, etc.)
- *Local form factors* (FFs)

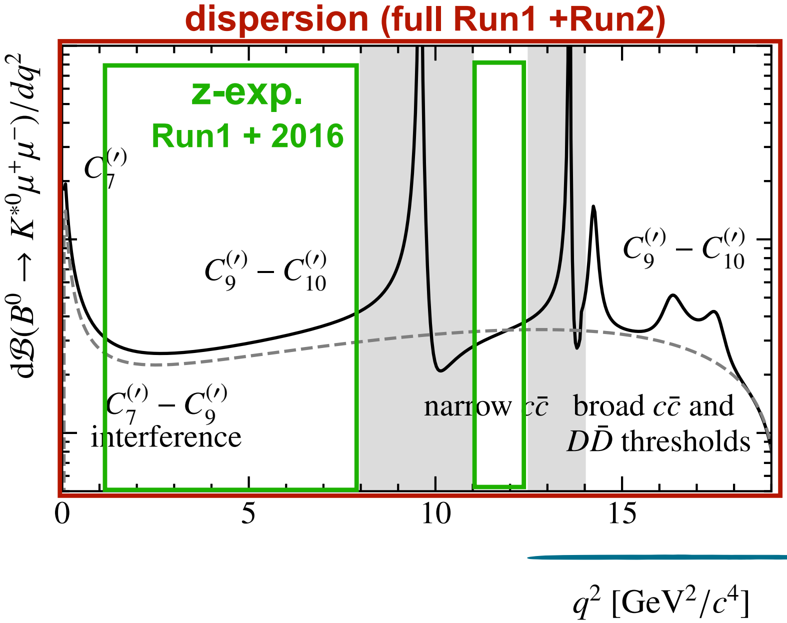
  Constrained to:
  - light-cone sum rules
  - lattice QCD



[Gubernari, Reboud, van Dyk, Virto; JHEP 12 (2023) 153]

Differences

- Modelling of the *non-local* contribution
- q^2 range
- dataset



z-expansion

based on: Bobeth, Chrzaszcz, van Dyk, Virto; EPJC 78 (2018) 451
Gubernari, van Dyk, Virto; JHEP 02 (2021) 088
Gubernari, Reboud, van Dyk, Virto; JHEP 09 (2022) 133

Decay amplitudes:

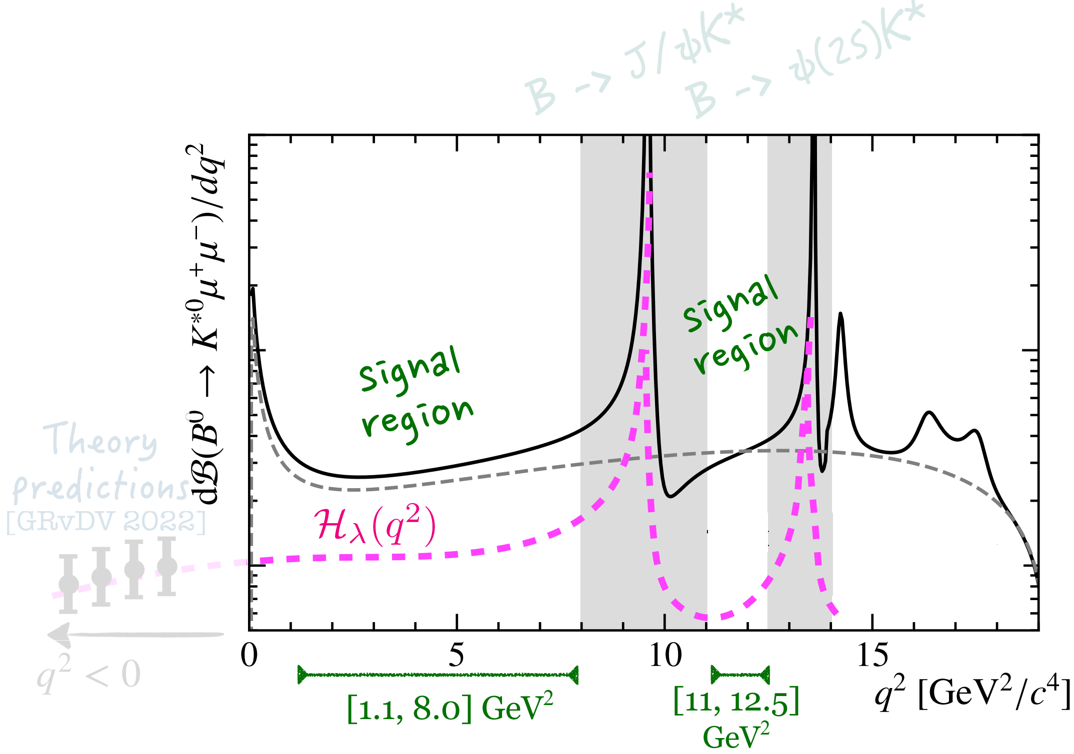
non-local contributions

$$\mathcal{A}_\lambda^{L,R} = \mathcal{N}_\lambda \left\{ \left[(\mathcal{C}_9 \pm \mathcal{C}_9') \mp (\mathcal{C}_{10} \pm \mathcal{C}_{10}') \right] \mathcal{F}_\lambda(q^2, k^2) + \frac{2 m_b M_B}{q^2} \left[(\mathcal{C}_7 \pm \mathcal{C}_7') \mathcal{F}_\lambda^T(q^2, k^2) - 16\pi^2 \frac{M_B}{m_b} \mathcal{H}_\lambda(q^2) \right] \right\}$$

$\lambda = \perp, \parallel, 0$

polynomial expansion

$$\mathcal{H}_\lambda(z) = \frac{1 - z z_{J/\psi}^*}{z - z_{J/\psi}} \frac{1 - z z_{\psi(2S)}^*}{z - z_{\psi(2S)}} \times \ldots \times \sum_n \alpha_{\lambda,n} z^n$$



- Add information to constrain non-local parameters
  1. experimental measurements on $B^0 \to \psi_n K^{*0}$ decays
  2. theory predictions at $q^2 < 0$
     - reliable for $q^2 \ll 4m_c^2$

BR, pol. frac, phase diff. from B-factories
[PRD 76 (2007) 031102]
[PRD 88 (2013) 074026]
[PRD 90 (2014) 112009]
[PRD 88 (2013) 052002]
[EPJC 72 (2012) 2118]

Two fit configurations:
- $q^2 < 0$ *constraints*
- $q^2 > 0$ *only*

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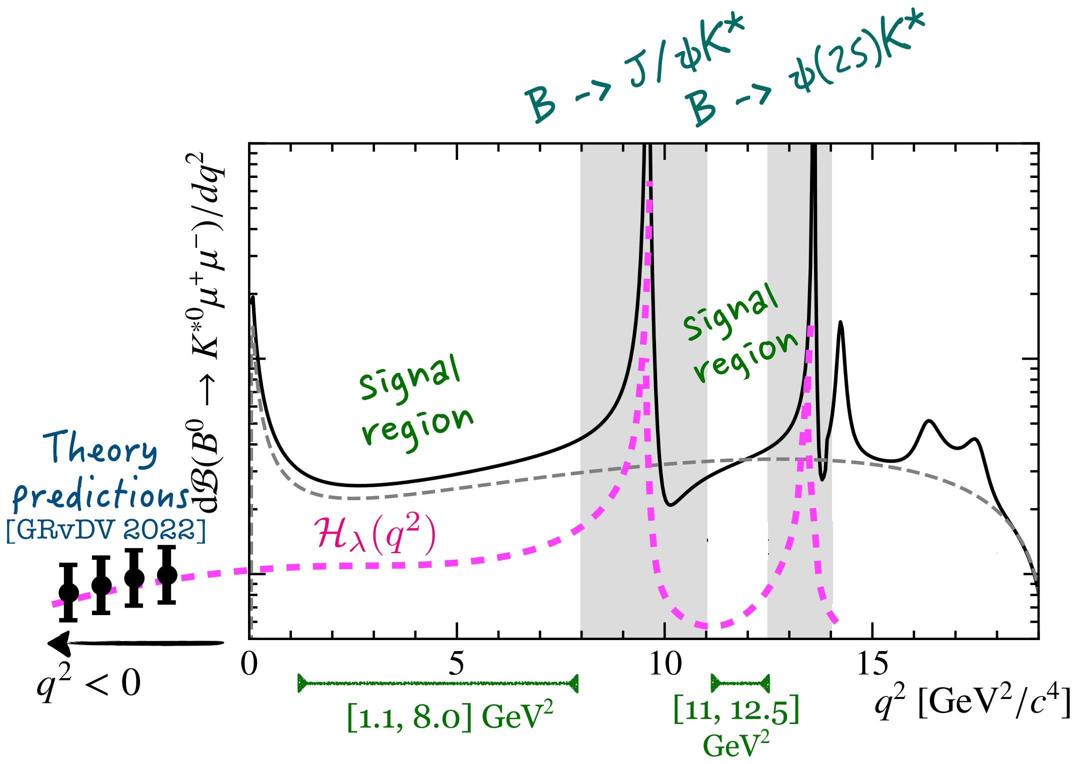

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Dispersion relation

Dispersion Relation

Local — Non-local contributions

$$C_9^{\text{eff},\lambda}(q^2) = C_9^{\mu} + Y_{c\bar{c}}^{(0),\lambda} + Y_{c\bar{c}}^{1P,\lambda}(q^2) + Y_{\text{light}}^{1P,\lambda}(q^2) + Y_{c\bar{c}}^{2P,\lambda}(q^2) + Y_{\tau\bar{\tau}}(q^2)$$

$$C_7^{\text{eff},\lambda}(q^2) = C_7^{\mu} + \epsilon^{\lambda} e^{i\omega^0}$$

ΔC_7^{λ}

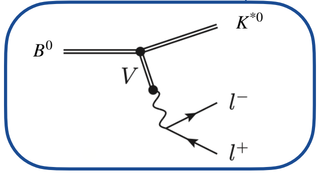
Polarisation dependent shift to C_7

This is determined theoretically at negative q^2 values

Subtraction term

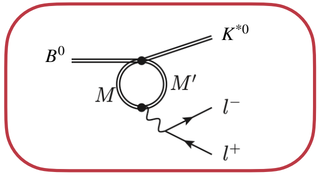
Asatrian, Greub, Virto [JHEP 04 (2020) 012]

Negligible impact from light quarks



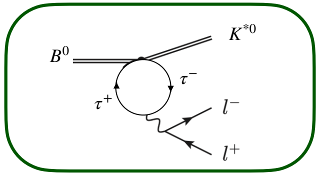
1-particle contributions

Includes:
$\omega(782)$, $\rho(770)$, $\phi(1020)$, J/ψ, $\psi(2S)$, $\psi(3770)$, $\psi(4040)$, $\psi(4160)$



2-particle contributions

Includes:
$D\overline{D}$,
$D^*\overline{D}$,
$D^*\overline{D}^*$



Tau loop contribution

Sensitive to C_9^{τ}

C. Cornella, G. Isidori, M. König, S. Liechti, P. Owen, N. Serra [Eur.Phys.J.C 80 (2020) 12, 1095]

Signal parameters

z-expansion

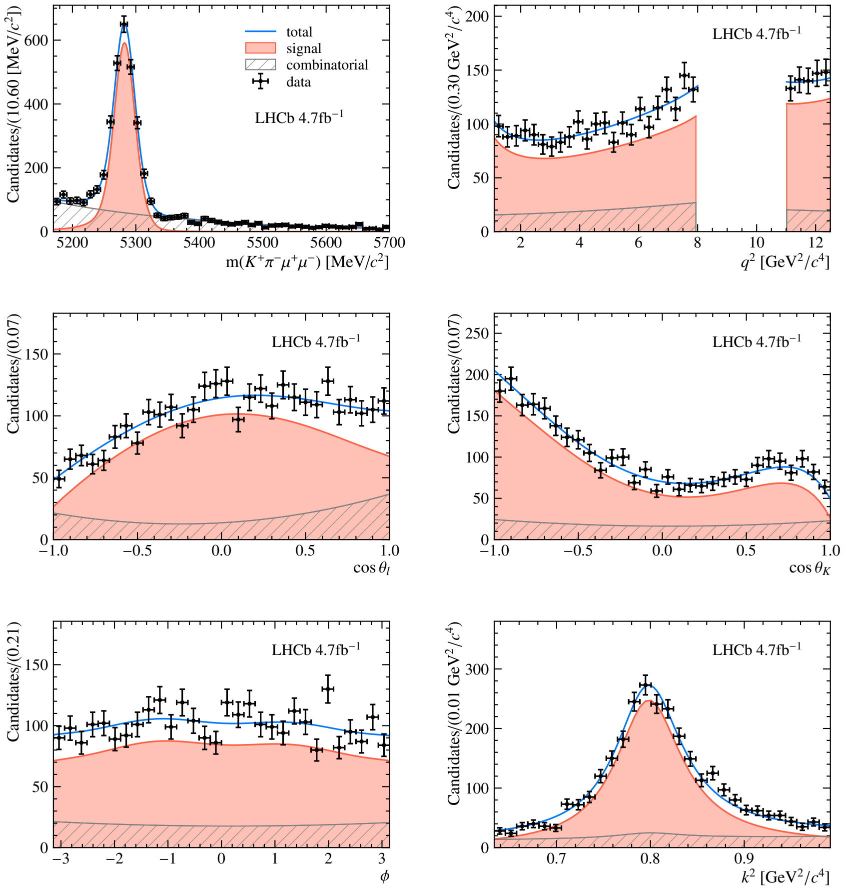
- $\mathscr{R}(C_9), \mathscr{R}(C_{10}), \mathscr{R}(C_9'), \mathscr{R}(C_{10}')$
- Real+Imag non-local parameters α_λ^n (18-30 pars)
- Form factors

dispersion relation

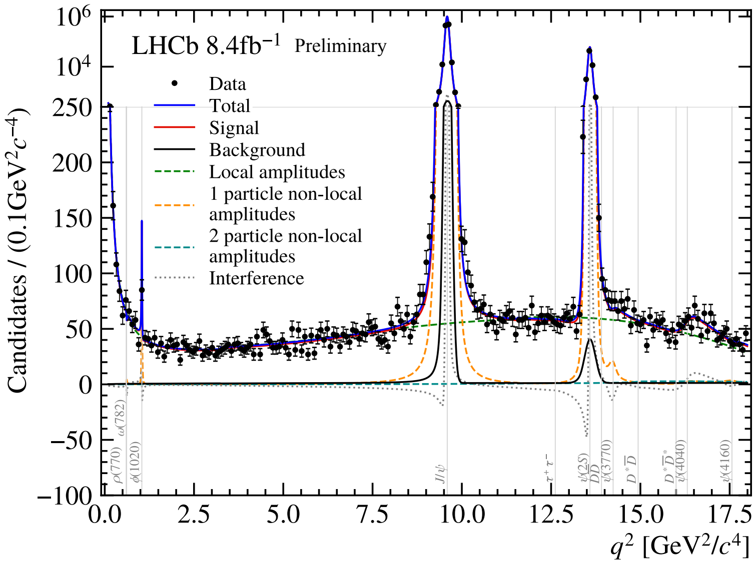
- $\mathfrak{R}(C_9), \mathfrak{R}(C_{10}). \mathfrak{R}(C_9'), \mathfrak{R}(C_{10}'), \mathfrak{R}(C_9^\tau)$
- Mag. and Phase of 1-particle resonances
- Real+Imag $D^{(*)}\overline{D}^{(*)}$ per helicity
- ΔC_7 per helicity
- Form factors

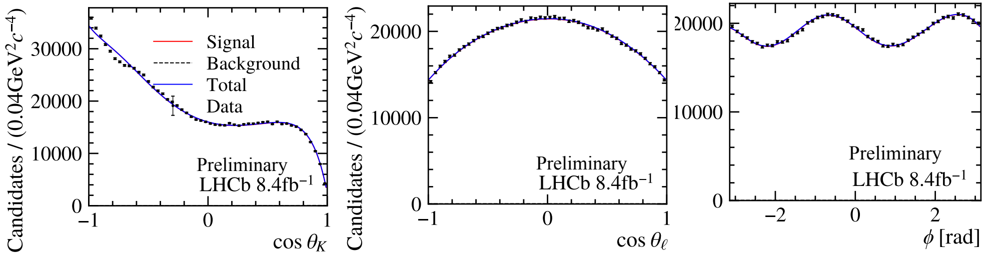
Fit projections

z-expansion



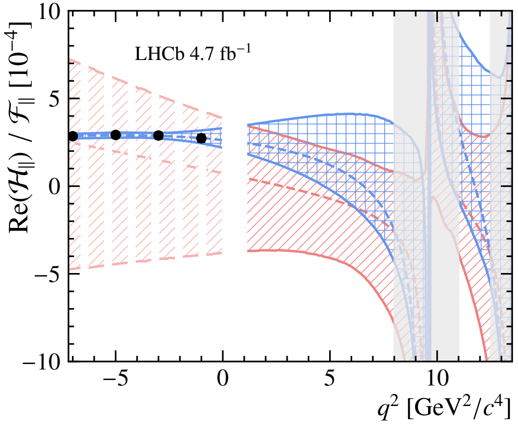
dispersion relation

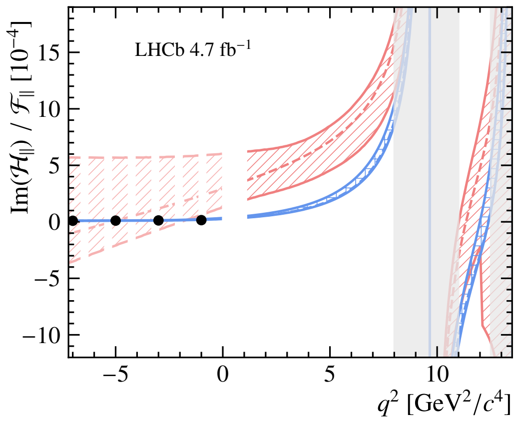




Non-local result

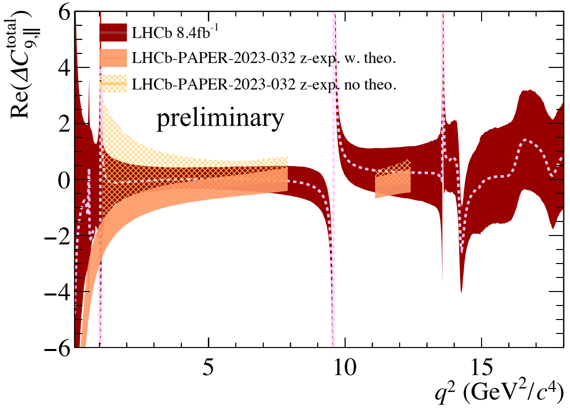
z-expansion

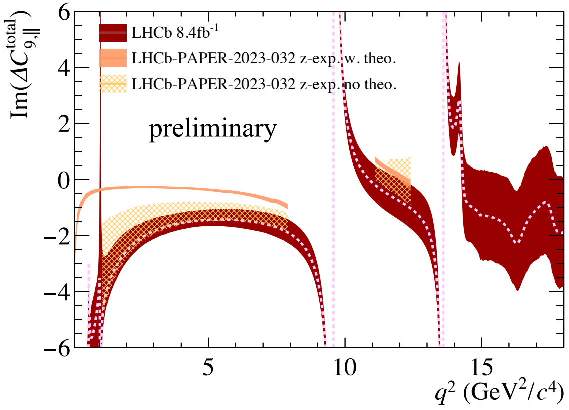




- Good agreement in the real part between the two fit configurations
- Small discrepancy in the imaginary part

dispersion relation



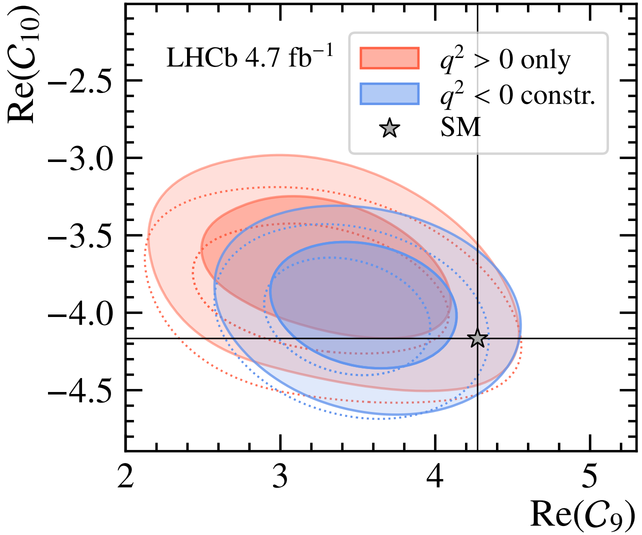


In general, good agreement between the two analyses!

Wilson coefficients 2D

- Results consistent with current global analyses of $b \to s\mu^+\mu^-$ decays

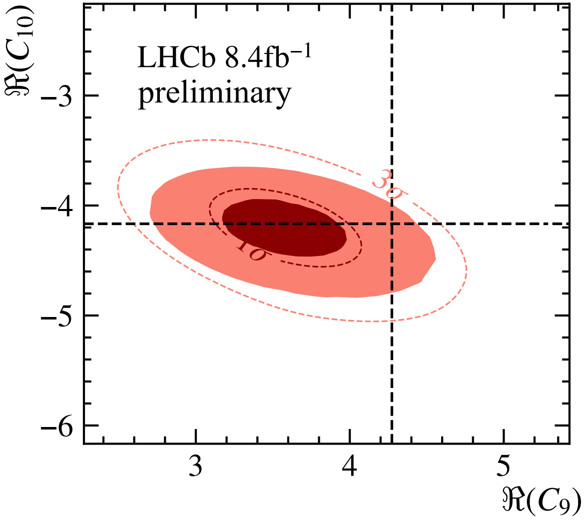
z-expansion



$\mathcal{C}_9$	$3.34^{+0.53}_{-0.57}$	$(3.59^{+0.33}_{-0.46})$	$1.9\,(1.8)\,\sigma$
$\mathcal{C}_{10}$	$-3.69^{+0.29}_{-0.31}$	$(-3.93^{+0.27}_{-0.28})$	$1.5\,(0.9)\,\sigma$
$\mathcal{C}'_9$	$0.48^{+0.49}_{-0.55}$	$(0.26^{+0.40}_{-0.48})$	$0.9\,(0.5)\,\sigma$
$\mathcal{C}'_{10}$	$0.38^{+0.28}_{-0.25}$	$(0.27^{+0.25}_{-0.27})$	$1.5\,(1.0)\,\sigma$

Global significance $1.3\,(1.4)\,\sigma$

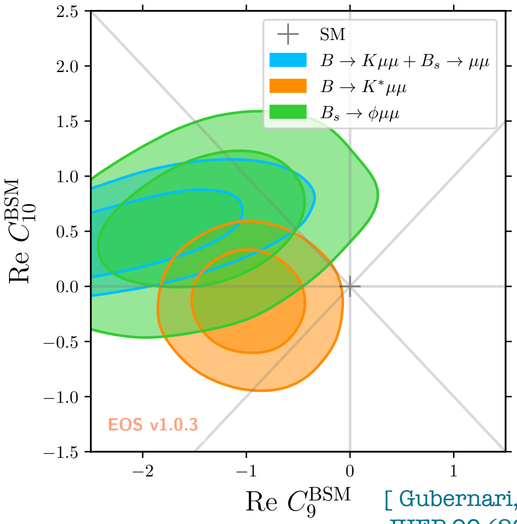
dispersion relation



$\mathcal{C}_9$	$3.56 \pm 0.28 \pm 0.18$	2.1σ
$\mathcal{C}_{10}$	$-4.02 \pm 0.18 \pm 0.16$	0.6σ
$\mathcal{C}'_9$	$0.28 \pm 0.41 \pm 0.12$	0.7σ
$\mathcal{C}'_{10}$	$-0.09 \pm 0.21 \pm 0.06$	0.4σ
$\mathcal{C}^{\tau}_9$	$-116 \pm 264 \pm 98$	0.4σ

Global significance $1.5\,\sigma$

global fit from binned data

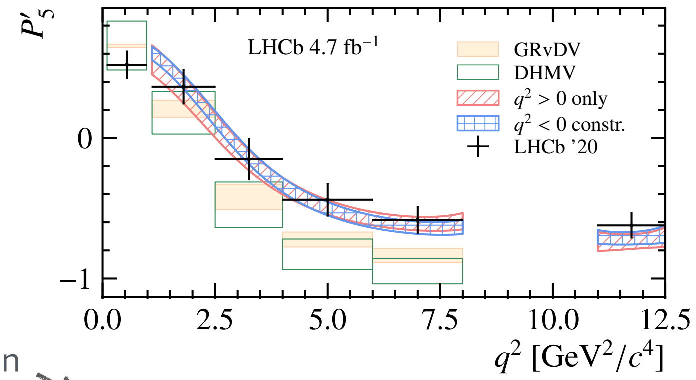


[Gubernari, et al.; JHEP 09 (2022) 133]

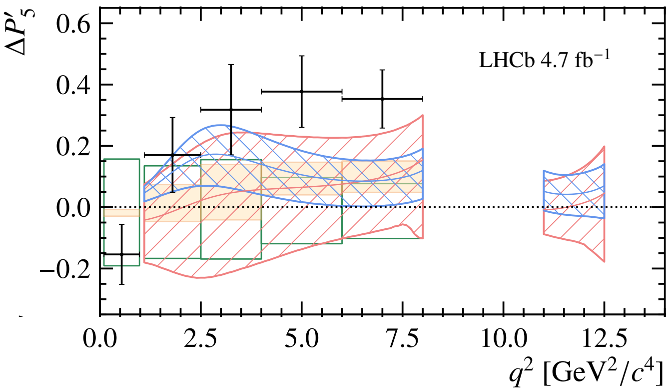
$\mathcal{B}(B^0 \to J/\psi K^{*0})$ **dominates systematic uncertainty**

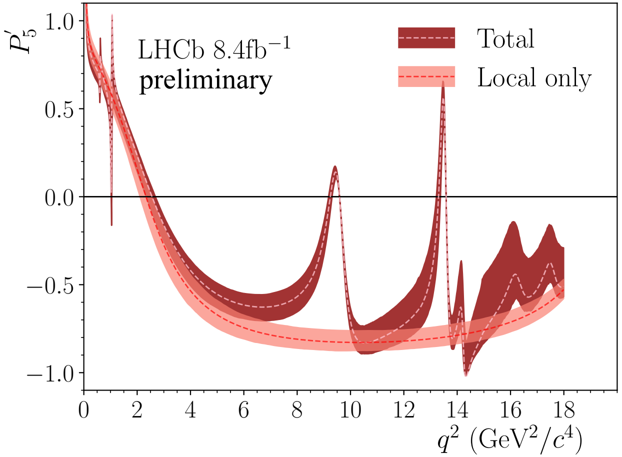
Non-local contribution to P5’

- From the fit result we can reproduce the classic angular observables

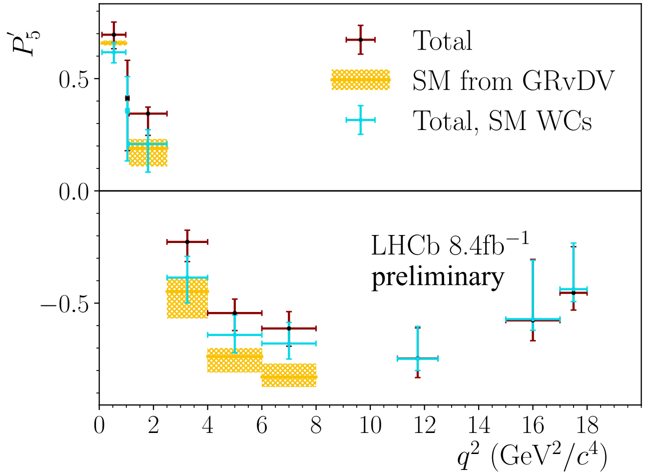


isolating contribution due to $c\bar{c}$





integrating in bins

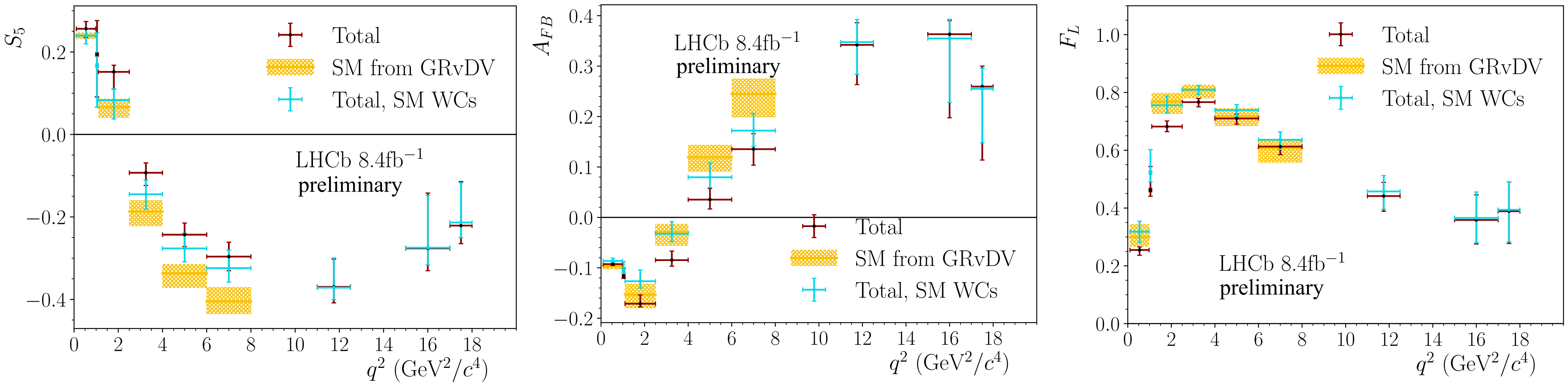


Red vs. Cyan
Impact of allowing NP

Cyan vs. Yellow
Impact of nonlocal modelling

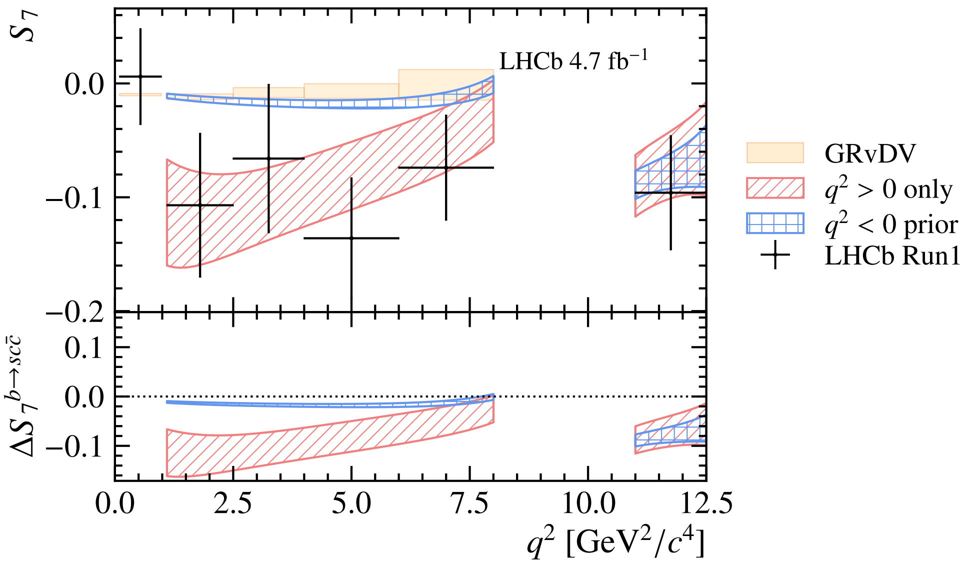
Note: bins are correlated

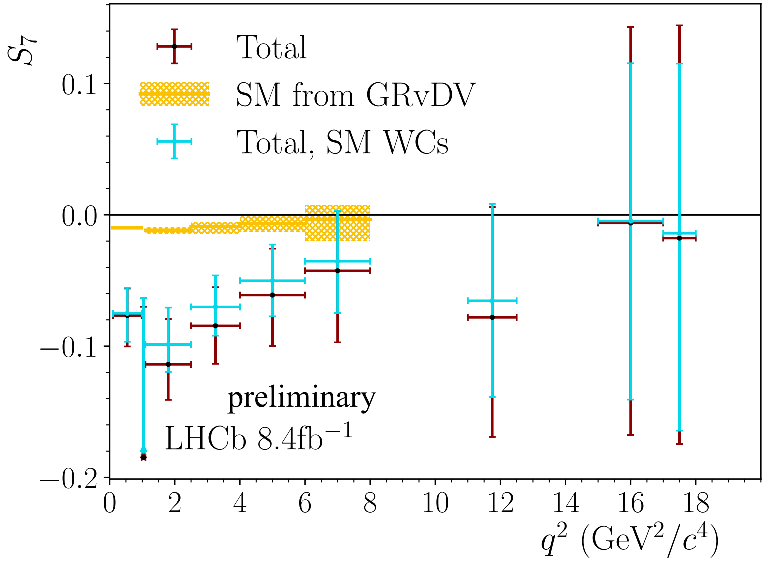
More angular observables (I)



We should NOT think in terms of bin by bin discrepancy in a single observable, but in terms of global significance

More angular observables (II)





Residual q^2 dependence

- Cross check:
  - Dispersion analysis check for possible residual q^2 dependence by allowing linearly varying C_9 and C_{10} in the fit:

$$C_9^{q^2} = C_9 + \alpha(q^2 - 8.95)$$
$$C_{10}^{q^2} = C_{10} + \beta(q^2 - 8.95)$$

$$\alpha = 0.029 \pm 0.082$$
$$\beta = -0.058 \pm 0.026 \longrightarrow$$

2.2σ deviation from zero is observed in C_{10}

tension in the FFs between low vs high-q2…?

What did we learn…

Take home message #1

- **Very compatible results between the 2 analyses**
- alternative/complementary q^2 model
- shift in C_9 of order -0.7

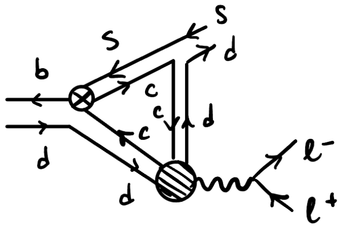
Take home message #2

- **Still something to understand**
- Im part of non-local contribution larger than expected
- SM postdictions differ from formal SM predictions in different observables
- Let's not forget about the form factors…

And what about:

- fit of C_9 in bins of q^2
- $D^{-(*)}D_s^{+(*)}$ rescattering…?

Let's hear from Arianna…



University of Zurich[UZH]

Tests of Short-Distance Dynamics in $b \to s\bar{\ell}\ell$

Based on 2401.18007 in collaboration with M. Bordone, G. Isidori, S. Mächler and a work in progress in collaboration with G. Isidori and Z. Polonsky

Motivation

- There is a long-standing **tension** with the SM in the exclusive $B \to K^{(*)}\bar{\ell}\ell$ in rates and angular distributions, especially in the low-q^2 region.
- The difficulty of performing precise SM tests lies in the difficulty of estimating **non-perturbative contributions**: form factors for the local $b \to s$ operators and non-local hadronic matrix elements of four-quark operators related to charm rescattering.
- Our goal is to try to disentangle a possible short-distance effect from long-distance dynamics.
- We parametrize the long-distance effects from charm resonances using dispersion relations in combination with data. After parametrizing these effects, we can determine a residual amplitude that can describe missing long-distance dynamics and possible short-distance dynamics.

Theoretical Framework

$$\mathcal{L}_{\rm eff}^{b\to s\bar{\ell}\ell} = -\frac{4G_F}{\sqrt{2}}\frac{\alpha_e}{4\pi}\left(V_{ts}^*V_{tb}\sum_{i=1}^{10} C_i\mathcal{O}_i + \text{h.c.}\right) + \mathcal{L}_{\rm QCD\times QED}^{N_f=5}$$

$$\mathcal{O}_1 = \frac{4\pi}{\alpha_e}(\bar{s}_L\gamma_\mu T^a c_L)(\bar{c}_L\gamma^\mu T^a b_L) \qquad \mathcal{O}_2 = \frac{4\pi}{\alpha_e}(\bar{s}_L\gamma_\mu c_L)(\bar{c}_L\gamma^\mu b_L)$$

$$\mathcal{O}_3 = \frac{4\pi}{\alpha_e}(\bar{s}_L\gamma_\mu b_L)\sum_q(\bar{q}_L\gamma^\mu q_L) \qquad \mathcal{O}_4 = \frac{4\pi}{\alpha_e}(\bar{s}_L^a\gamma^\mu T^a b_L^b)\sum_q(\bar{q}_L{}^b\gamma^\mu T^a q_L^a)$$

$$\mathcal{O}_5 = \frac{4\pi}{\alpha_e}(\bar{s}_L\gamma_\mu\gamma_\nu\gamma_\rho b_L)\sum_q(\bar{q}_L\gamma^\mu\gamma^\nu\gamma^\rho q_R) \qquad \mathcal{O}_6 = \frac{4\pi}{\alpha_e}(\bar{s}_L^a\gamma_\mu\gamma_\nu\gamma_\rho T^a b_L^b)\sum_q(\bar{q}_L{}^b\gamma^\mu\gamma^\nu\gamma^\rho T^a q_R^a)$$

$$\mathcal{O}_7 = \frac{m_b}{e}(\bar{s}_L\sigma^{\mu\nu}b_R)F_{\mu\nu} \qquad \mathcal{O}_8 = \frac{g_s}{e^2}m_b(\bar{s}_L\sigma^{\mu\nu}T^a b_R)G^a_{\mu\nu}$$

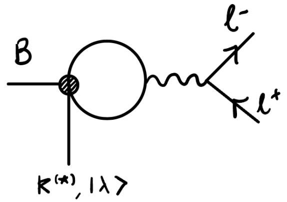
$$\mathcal{O}_9 = (\bar{s}_L\gamma_\mu b_L)(\bar{\ell}\gamma^\mu\ell) \qquad \mathcal{O}_{10} = (\bar{s}_L\gamma_\mu b_L)(\bar{\ell}\gamma^\mu\gamma_5\ell)$$

We want to extract information on the non-local matrix elements of the four-quark operators $\mathcal{O}_{1-6}$ from data.

Note that to all orders in α_s, and to first order in $\alpha_{\rm em}$, these matrix elements have the same structure as the matrix elements of $\mathcal{O}_7$ and $\mathcal{O}_9$:

$$\mathcal{M}(B\to K\ell\ell)|_{C_{1-6}} = -i\frac{32\pi^2\mathcal{N}}{q^2}\bar{\ell}\gamma^\mu\ell\int d^4x e^{iqx}\langle H_\lambda|T\{j_\mu^{\rm em}(x),\sum_{i=1,6}C_i\mathcal{O}_i(0)\}|B\rangle = \left(\Delta_9^\lambda(q^2)+\frac{m_B^2}{q^2}\Delta_7^\lambda\right)\langle H_\lambda\ \ell^+\ell^-|\mathcal{O}_9|B\rangle$$

The (regular for $q^2\to 0$) contributions of the non-local matrix elements of the four-quark operators can be effectively taken into account by the shift:



$$C_9 \to C_9 + Y^\lambda(q^2) \qquad \lambda = K, \perp, //, 0$$

Theoretical Framework

$$C_9 \to C_9 + Y^{\lambda}(q^2)$$

$$\lambda = K, \perp, //, 0$$

More precisely, this shift includes:

$$C_9 \to C_9^{\lambda}(q^2) + Y_{q\bar{q}}^{[0]}(q^2) + Y_{b\bar{b}}^{[0]}(q^2) + Y_{c\bar{c}}^{\lambda}(q^2)$$

encodes (factorizable) perturbative contributions from 4-quark operators

encodes the perturbative charm-loop contributions and $c\bar{c}$ resonances

To estimate the non-perturbative contributions generated by the $c\bar{c}$ resonances, we use dispersive relations in combination with data:

$$Y_{c\bar{c}}^{\lambda}(q^2) = Y_{c\bar{c}}^{\lambda}(q_0^2) + \frac{16\pi^2}{\mathcal{F}_{\lambda}(q^2)}\Delta\mathcal{H}_{c\bar{c}}^{\lambda}(q^2),\ q_0^2 = 0$$

$$\Delta\mathcal{H}_{c\bar{c}}^{\lambda,1P} = \sum_V \eta_V^{\lambda} e^{i\delta_V^{\lambda}} \frac{q^2}{m_V^2} A_V^{\mathrm{res}}(q^2) \qquad A_V^{\mathrm{res}}(q^2) = \frac{m_V\Gamma_V}{m_V^2 - q^2 - i m_V \Gamma_V}$$

$B \to K\bar{\ell}\ell$

V	η_V	δ_V
J/ψ	32.3 ± 0.6	-1.50 ± 0.05
$\psi(2s)$	7.12 ± 0.32	2.08 ± 0.11
$\psi(3770)$	$(1.3 \pm 0.1) \times 10^{-2}$	-2.89 ± 0.19
$\psi(4040)$	$(4.8 \pm 0.8) \times 10^{-3}$	-2.69 ± 0.52
$\psi(4160)$	$(1.5 \pm 0.1) \times 10^{-2}$	-2.13 ± 0.33
$\psi(4415)$	$(1.1 \pm 0.2) \times 10^{-2}$	-2.43 ± 0.43

$B \to K^*\bar{\ell}\ell$

V	Polarization	η_V	δ_V
J/ψ	$\perp$	26.6 ± 1.1	1.46 ± 0.06
	$\parallel$	12.3 ± 0.5	-4.42 ± 0.06
	longitudinal	13.9 ± 0.5	-1.48 ± 0.05
$\psi(2s)$	$\perp$	3.0 ± 0.9	3.2 ± 0.4
	$\parallel$	1.11 ± 0.30	-3.32 ± 0.22
	longitudinal	1.14 ± 0.06	2.10 ± 0.11

Extraction of C_9

We extract the residual contribution to C_9:

$$C_9 \to C_9^{\lambda}(q^2) + Y_{q\bar{q}}^{[0]}(q^2) + Y_{b\bar{b}}^{[0]}(q^2) + Y_{c\bar{c}}^{\lambda}(q^2)$$

↓

extract from data

$$C_9^{\lambda}(q^2) = C_9^{\mathrm{SM}} + C_9^{\mathrm{LD},\lambda}(q^2) + C_9^{\mathrm{SD}}$$

Long-distance, no reason to assume it is independent of λ or q^2

Short-distance, independent of λ and q^2

Can we find this contribution from data? ⟶ Fit from data for every bin in q^2 and every polarization

$B \to K\bar{\ell}\ell$

- We perform a fit of C_9 bin by bin using the measured branching ratio by LHCb + CMS — 2014 LHCb, 2023 CMS
- Form factors from lattice QCD — Parrott, W. G., et al., arXiv:2207.13371 and 2207.12468

$B \to K^*\bar{\ell}\ell$

- We perform a fit of C_9 from the branching ratio and from the angular observables $F_L, S_3, S_4, S_5, A_{FB}, S_7, S_8, S_9$ measured by LHCb — 2016 and 2020 LHCb
- Form factors from light-cone sum rules + lattice — Bharucha, Aoife, David M. Straub, and Roman Zwicky, arXiv:1503.05534

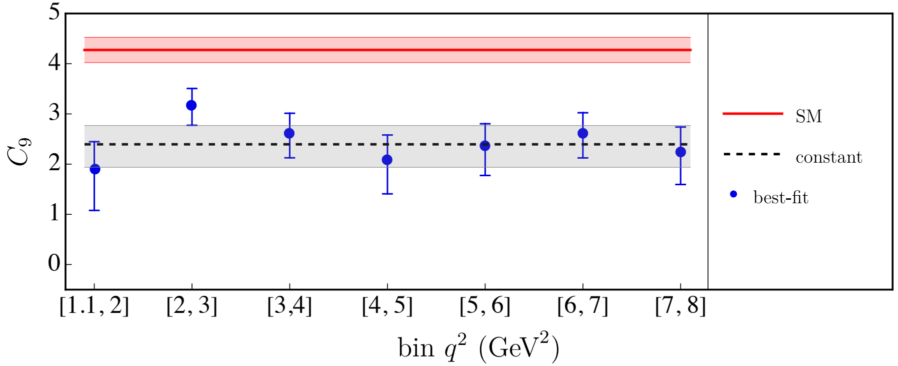
Results

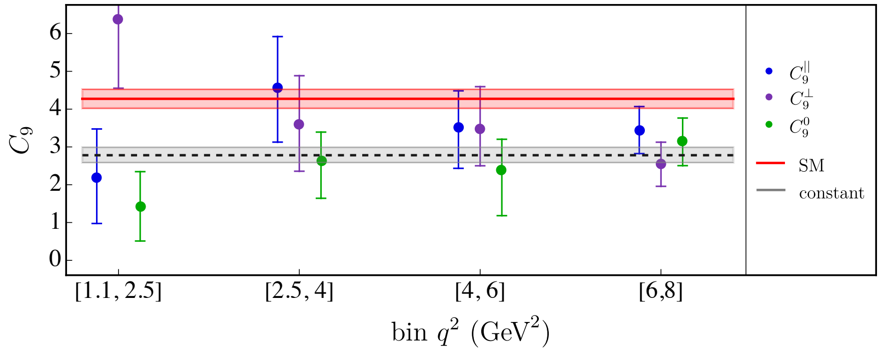
Compatible with LHCb analysis!

$B \to K \bar{\ell} \ell$

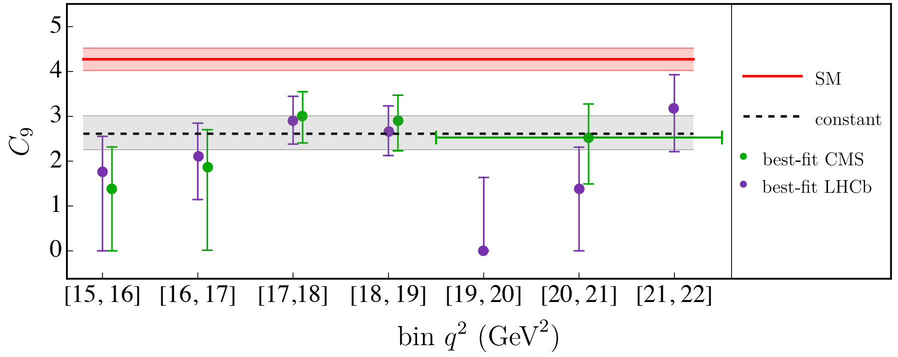
$B \to K^* \bar{\ell} \ell$

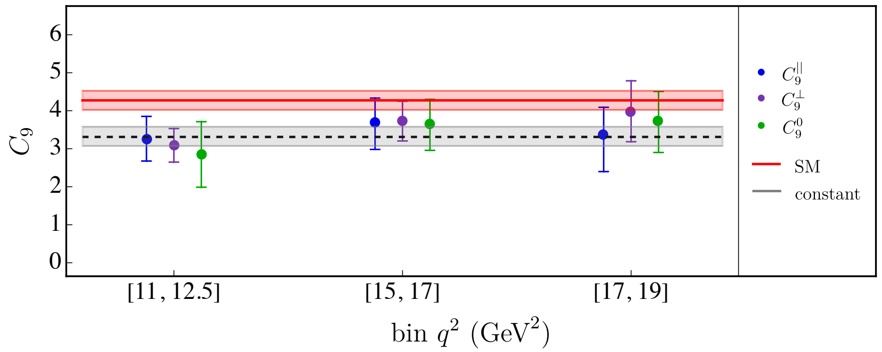
Low q^2





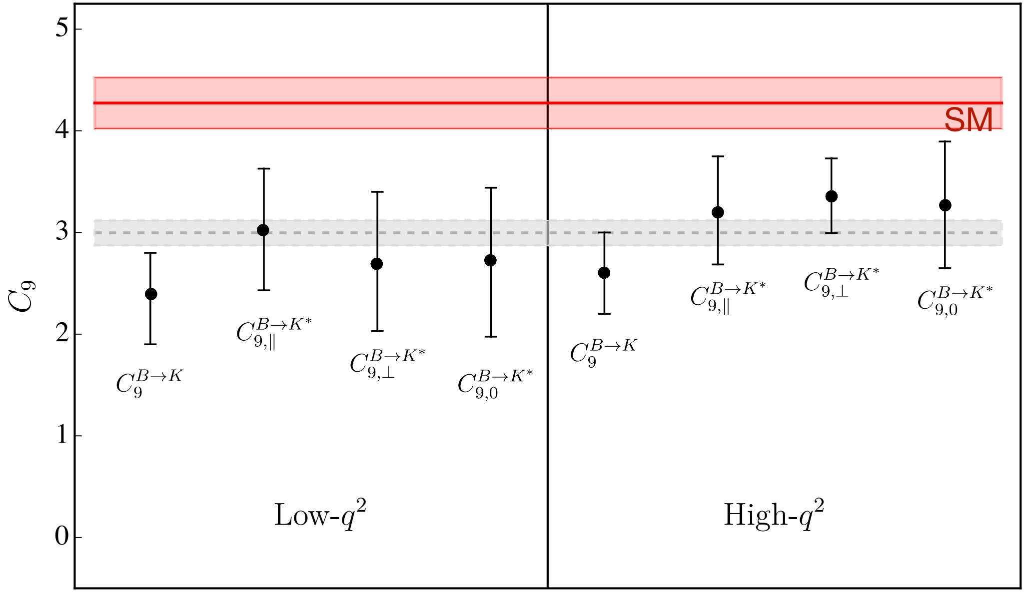
High q^2





Results

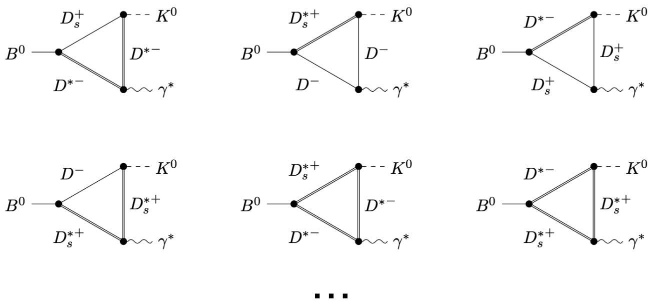
Independent determinations of C_9 assuming it to be constant:

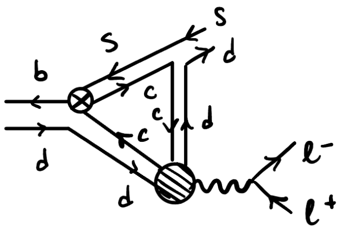


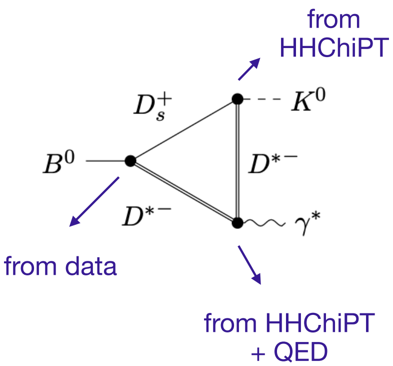
- We find that the values of C_9 are **consistent** throughout the different modes and polarizations, and that there is no significant q^2-dependence.
- This is in opposition to the expected behavior in the case of long-distance contributions beyond those already included -> Data provide no evidence of sizable unaccounted-for long-distance contributions.
- The discrepancy in the experimentally-determined C_9 value is consistent with a short-distance effect of non-SM origin.

Charm rescattering in $B \to K\bar{\ell}\ell$

- We cannot exclude a sizable long-distance contribution with a reduced q^2- or λ- dependence which would mimic a short-distance effect.
- For this reason, we tried to estimate the rescattering contribution from the leading two-body intermediate state $D_s D^*$ and $D_s^* D$.



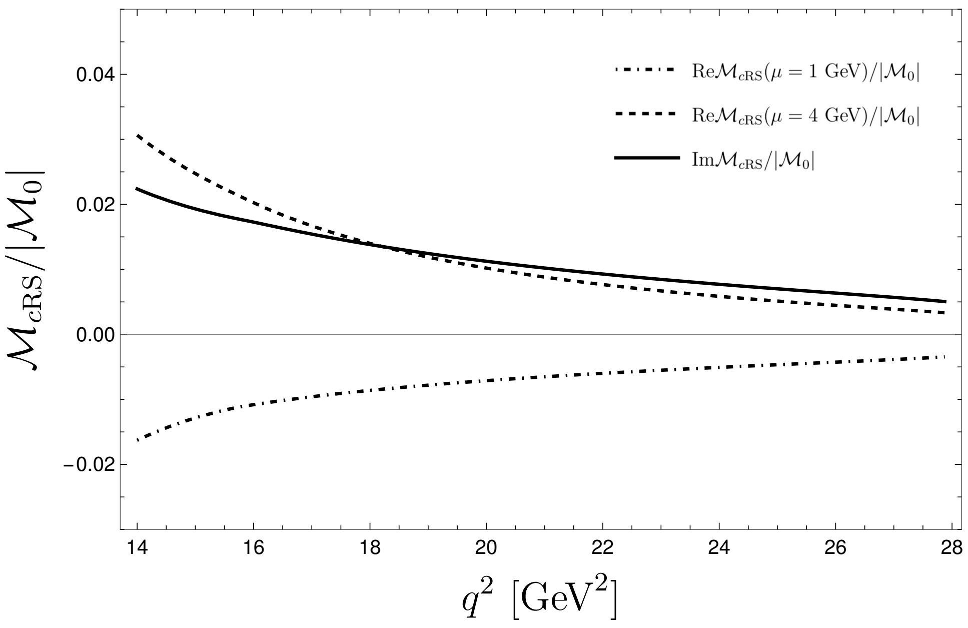




- We estimate this diagram using data on $B \to DD^*$ and Heavy Hadron Chiral Perturbation Theory (valid for soft kaons).
- Our result is most reliable close to the q^2 end-point (small kaon momentum), and satisfies constraints from gauge invariance.
- The absorptive part is finite and “exact” (no approximations) at the end-point.

Charm rescattering in $B \to K\bar{\ell}\ell$

(Preliminary)



We find that these contributions are not large enough to explain the bulk of the tension on the value of C_9.

$$\Rightarrow \left|\frac{\Delta C_9}{C_9}\right| \leq 3\,\%$$

Not enough to explain the tension with the SM value (the shift needed is of order $\approx 25\,\%$)

Conclusions

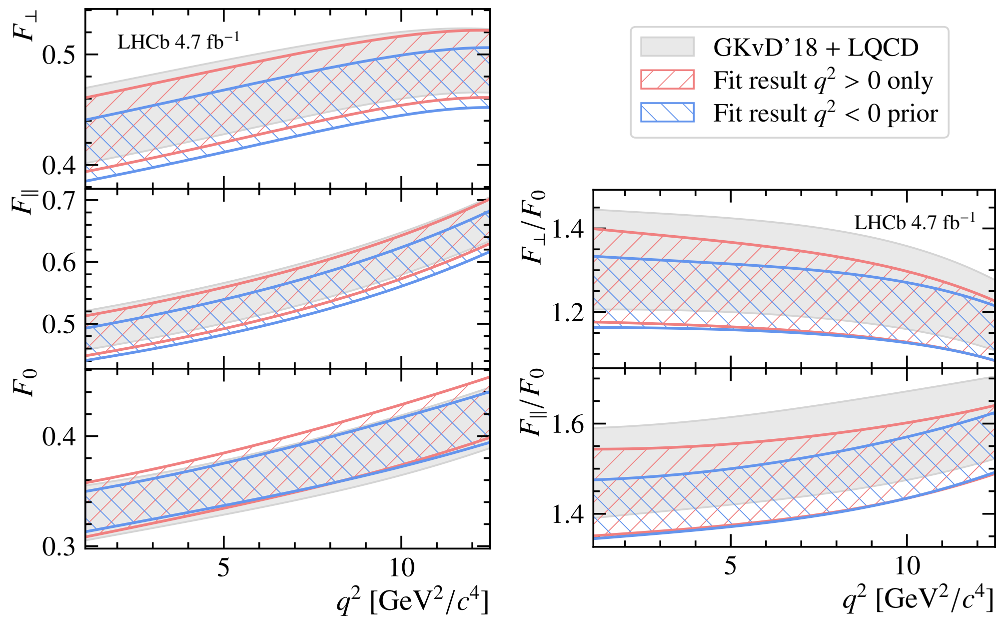
- Hard to explain the bulk of the tension with **only long-distance** QCD effects.
- Data provide no evidence of sizable unaccounted-for long-distance contributions, and our estimate of charm-rescattering contributions that mimic short-distance effects cannot explain all the tension.
- The discrepancy in the experimentally-determined C_9 value is consistent with a short-distance effect of non-SM origin.
- The uncertainties of the independently-determined C_9 are still large.
- The method presented here has no theoretical limitations ➔ with more precise data we can get more precise results (more accurate description of charm rescattering, as in recent LHCb analysis).
- If the absence of q^2- and λ- dependence survives with smaller uncertainty, the presence of long-distance unaccounted-for contributions would be more plausible.

Thanks for your attention!

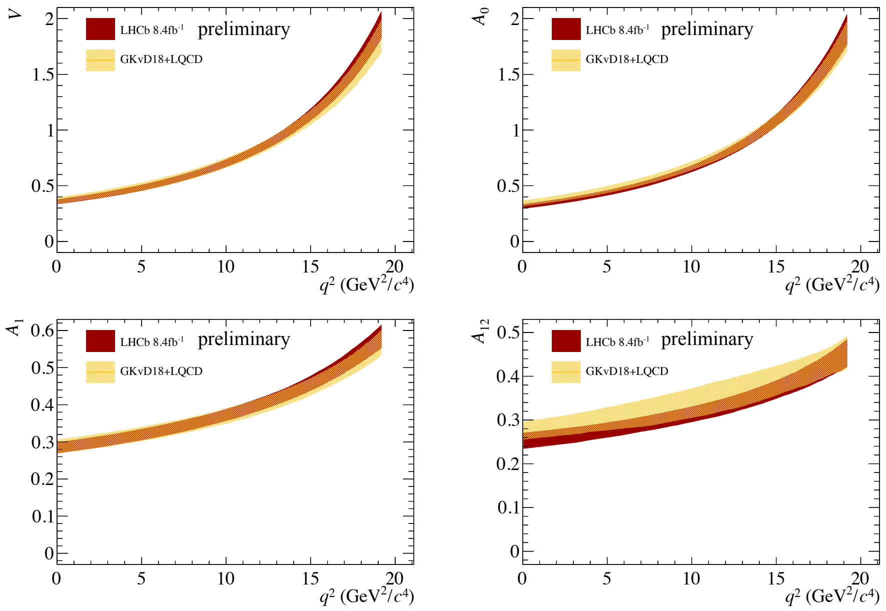
Backup

Form factor results

z-expansion



dispersion relation



- Fit results are found to require small adjustment in $\mathcal{F}_{\perp,\parallel}/\mathcal{F}_0$ ratio

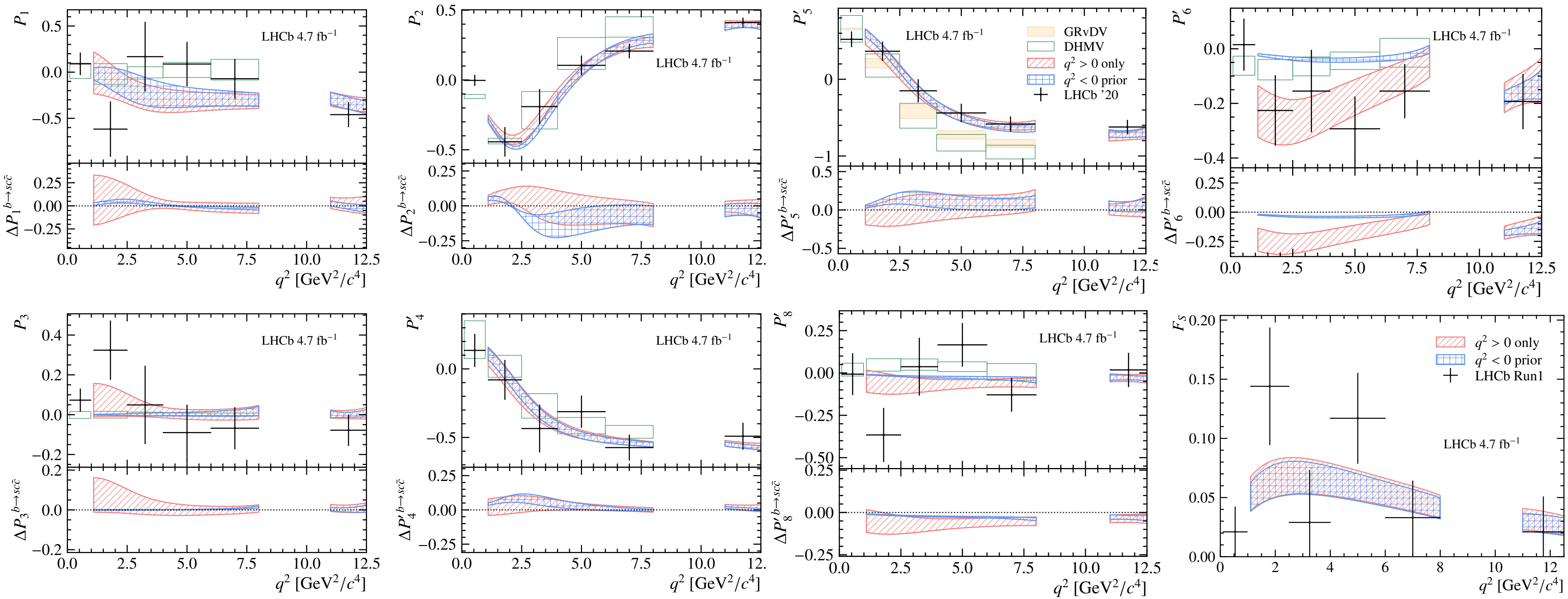
Use alternative local $B \to K^*$ form factors - different LCSR inputs

Bharucha, Straub, & Zwicky [JHEP 08 (2016) 098]

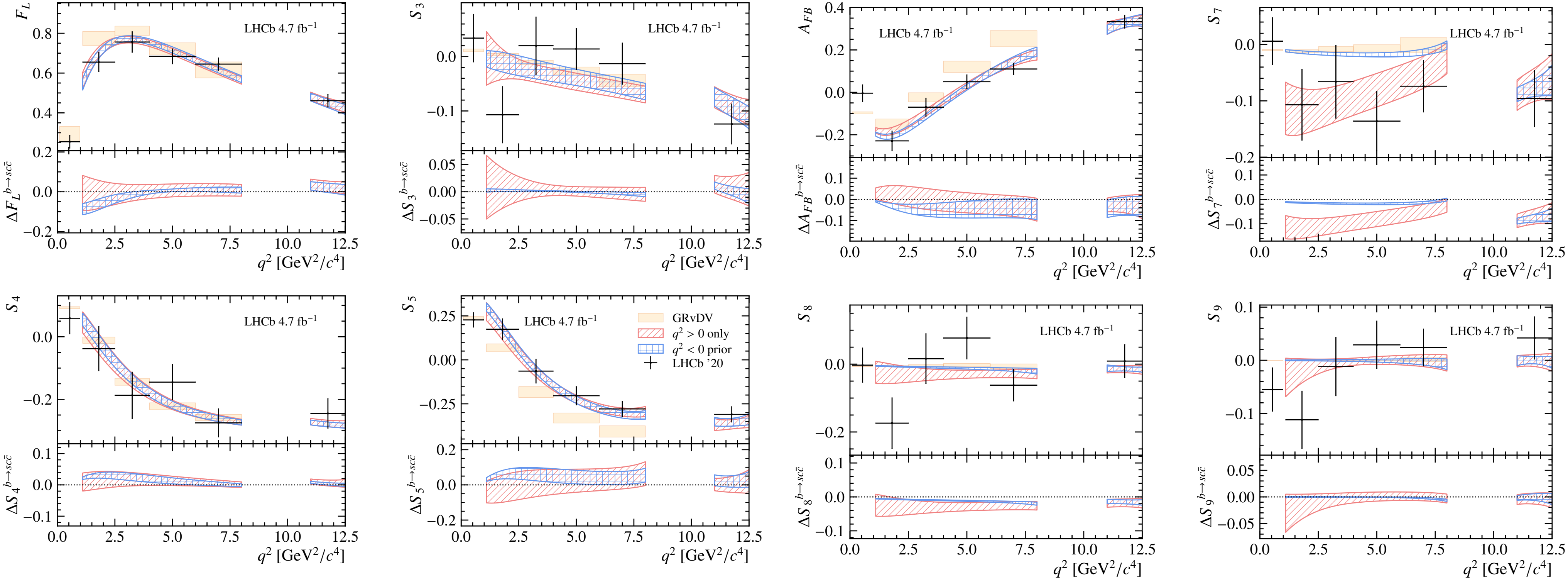
C_9 changes by 35% σ_{stat}

C_{10} changes by 90% σ_{stat}

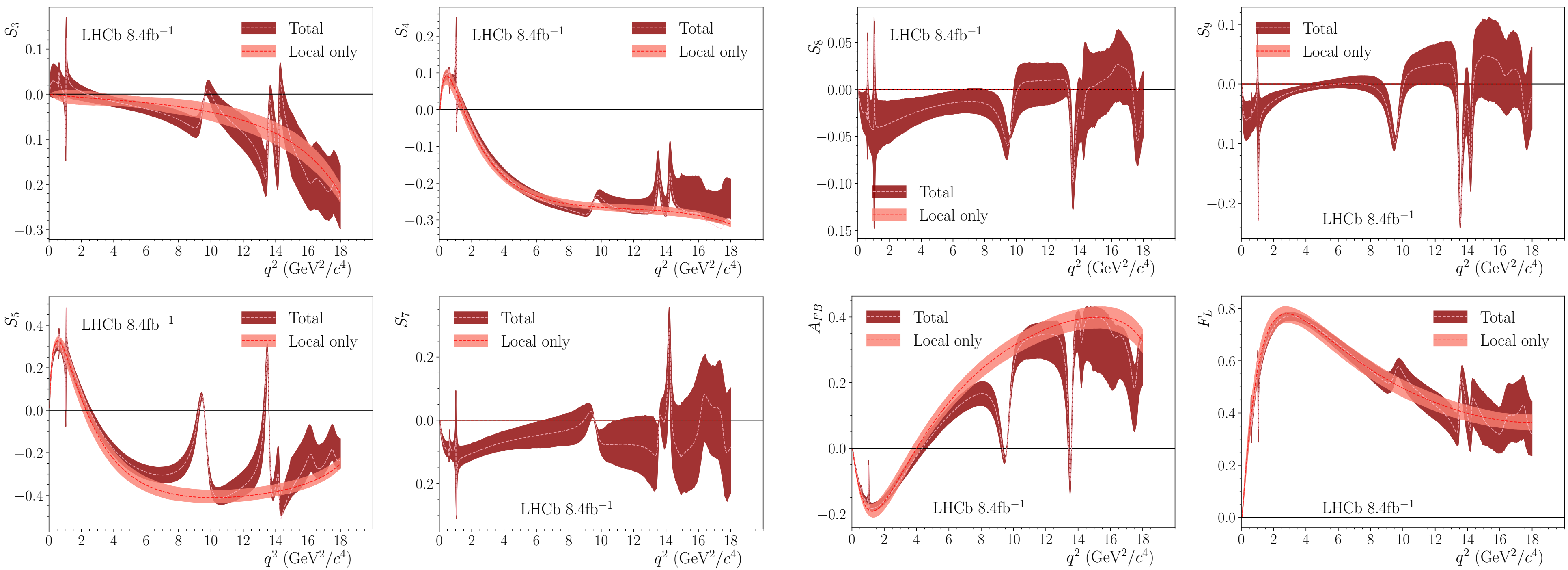
Ang. obs (P-basis)



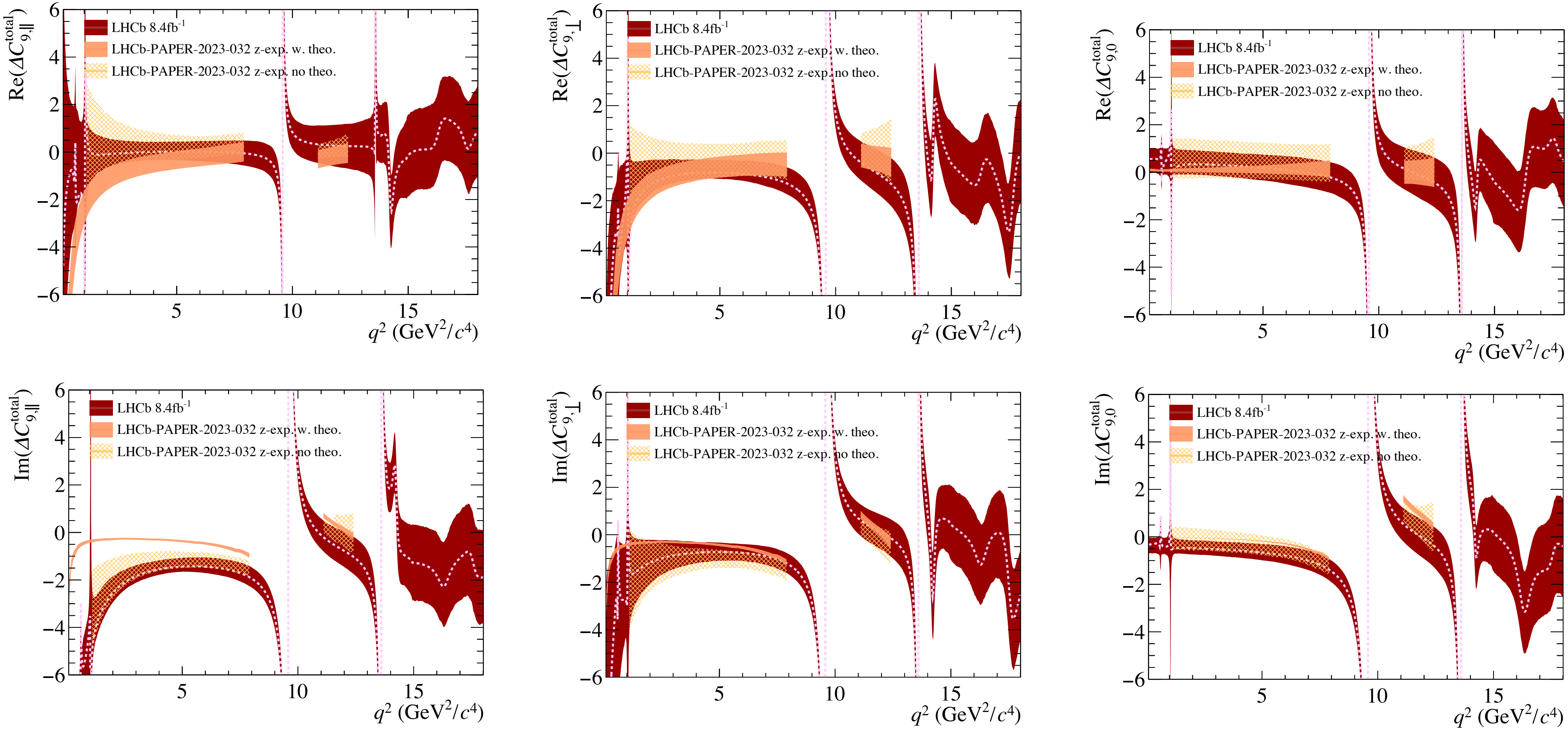
Ang. obs (S-basis)



Ang. obs (S-basis)



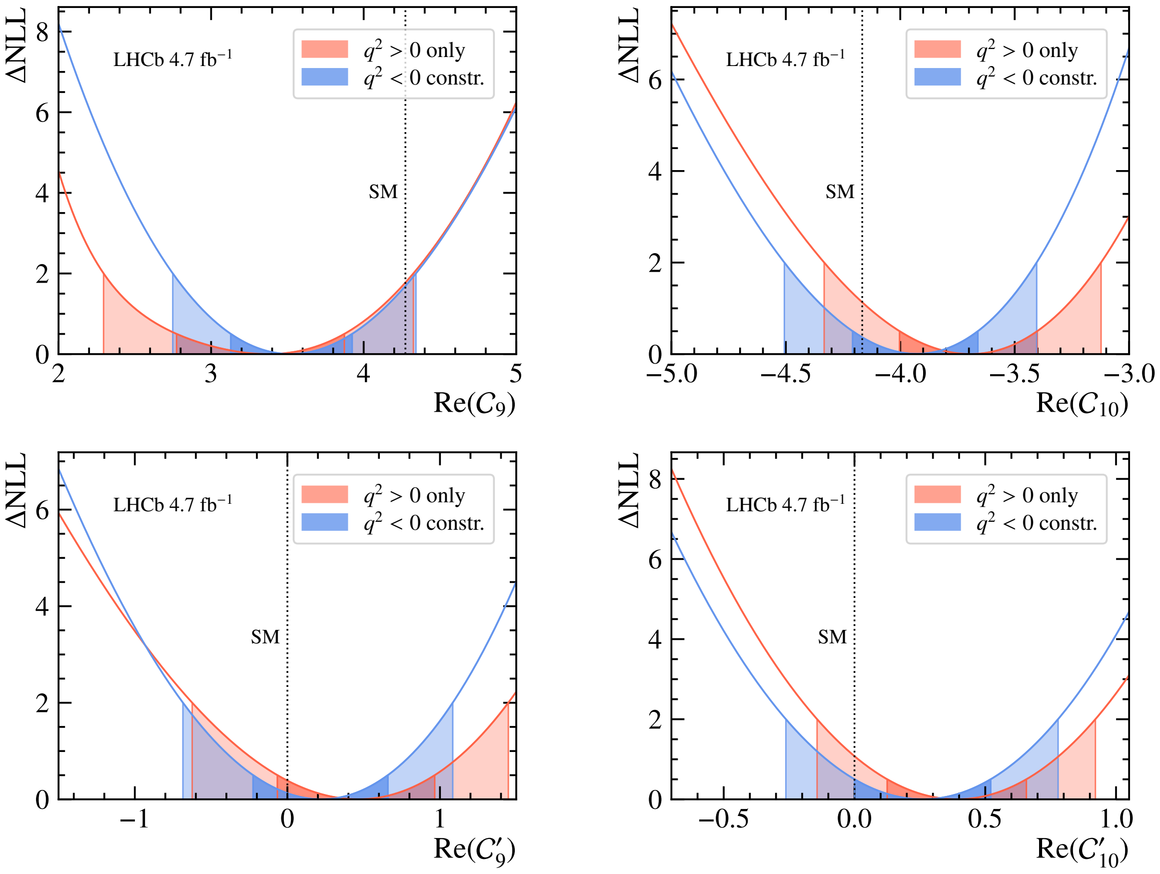
Non-local amplitudes



Wilson coefficients 1D

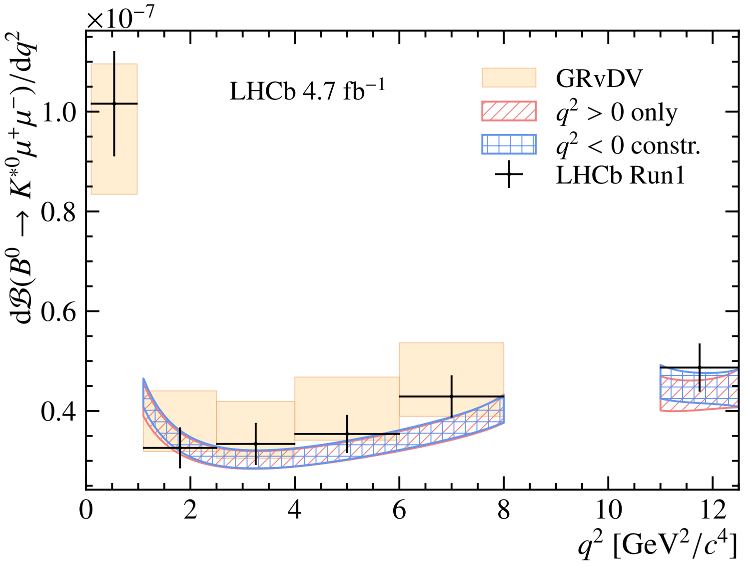
- Uncertainty obtained from neg. log-likelihood profile

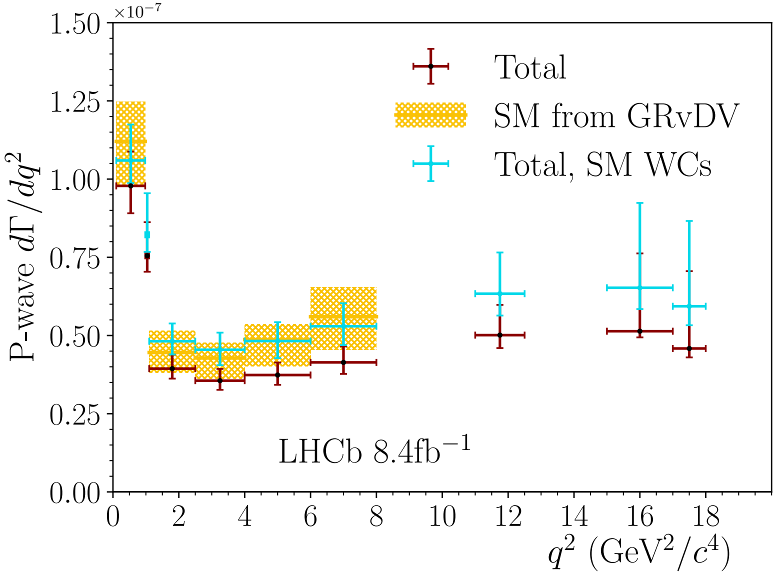
	$q^2 > 0$ only	
	Fit result	deviation from SM
$\Delta\mathcal{C}_9$	$-0.93^{+0.53}_{-0.57}$	$1.9\ \sigma$
$\Delta\mathcal{C}_{10}$	$0.48^{+0.29}_{-0.31}$	$1.5\ \sigma$
$\Delta\mathcal{C}'_9$	$0.48^{+0.49}_{-0.55}$	$0.9\ \sigma$
$\Delta\mathcal{C}'_{10}$	$0.38^{+0.28}_{-0.25}$	$1.5\ \sigma$
	$q^2 < 0$ prior	
$\Delta\mathcal{C}_9$	$-0.68^{+0.33}_{-0.46}$	$1.8\ \sigma$
$\Delta\mathcal{C}_{10}$	$0.24^{+0.27}_{-0.28}$	$0.9\ \sigma$
$\Delta\mathcal{C}'_9$	$0.26^{+0.40}_{-0.48}$	$0.5\ \sigma$
$\Delta\mathcal{C}'_{10}$	$0.27^{+0.25}_{-0.27}$	$1.0\ \sigma$



Br

- From the fit result we can reproduce the classic binned observables

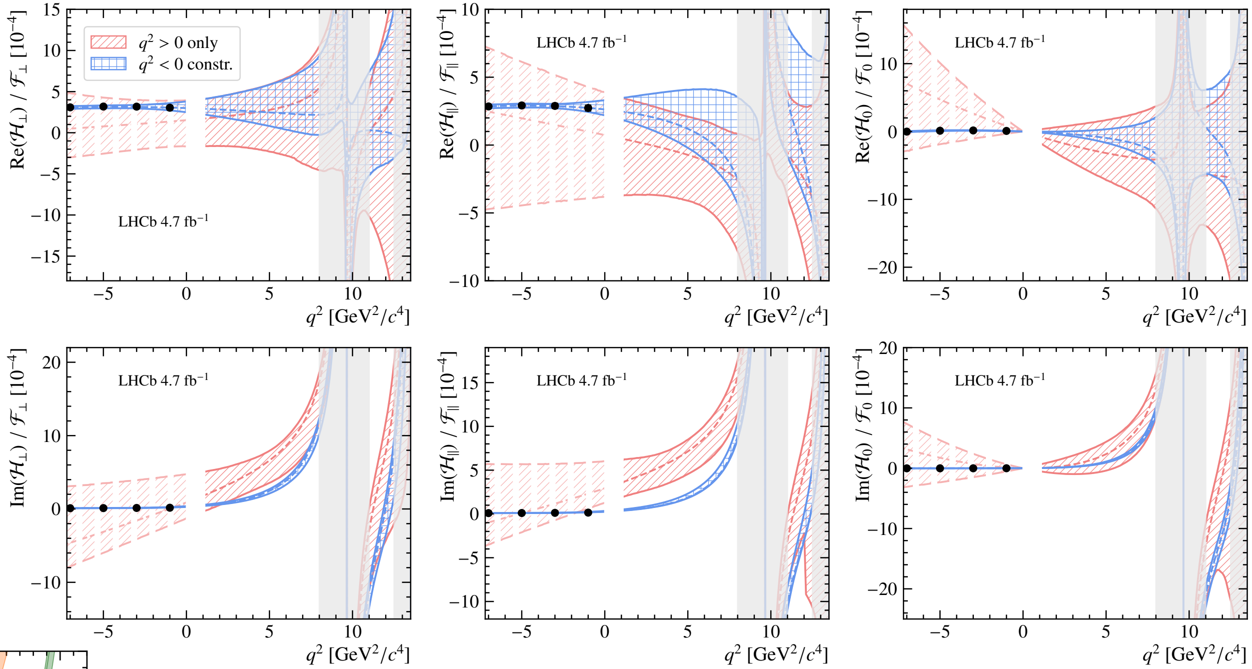




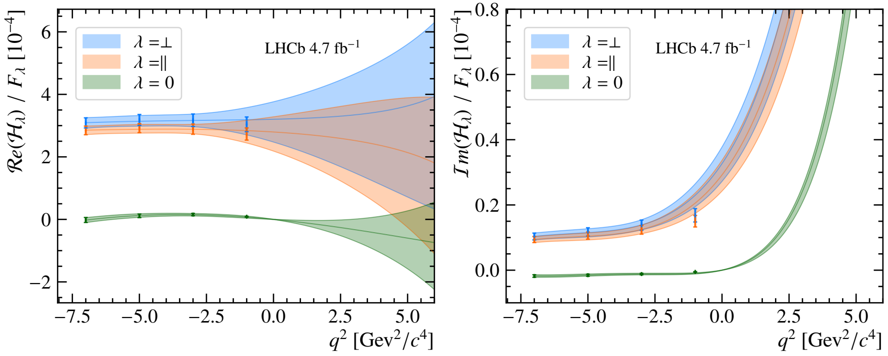
  - Lower BR compared to LHCb Run1 due to updated normalisation inputs

Non-local result : z-expansion

- Good agreement between the two configurations
  - Small discrepancy in $Im\,\mathcal{H}_{\parallel}(q^2)$



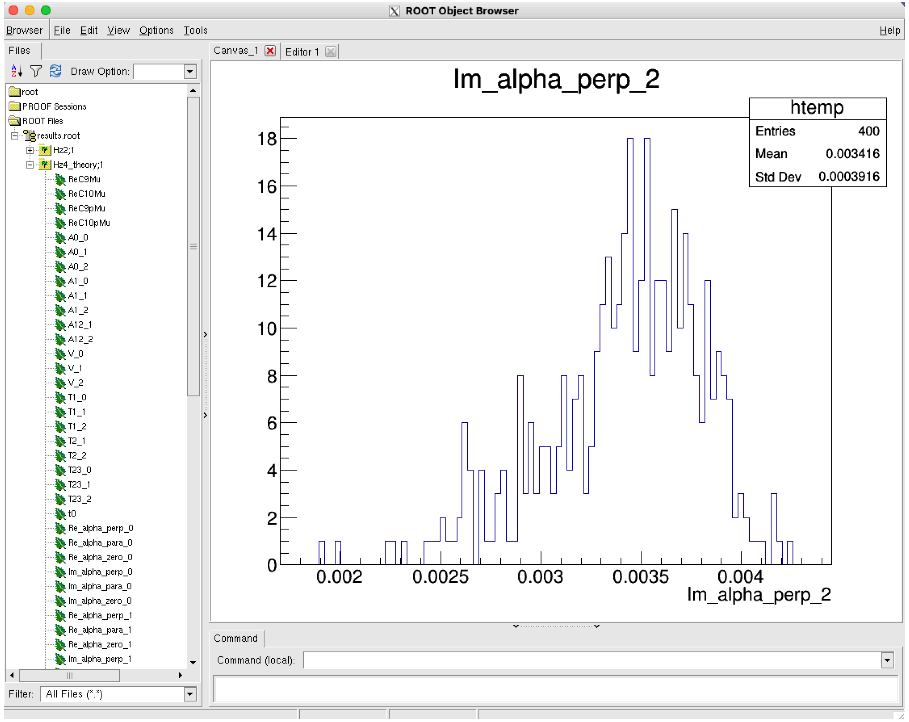
Zoom at $q^2 < 0$



- Sharp variation in $Im\,\mathcal{H}_\lambda(q^2)$ between $q^2 < 0$ and $q^2 > 0$
  - require high polynomial order $\mathcal{H}_\lambda[z^4]$

Auxiliary files

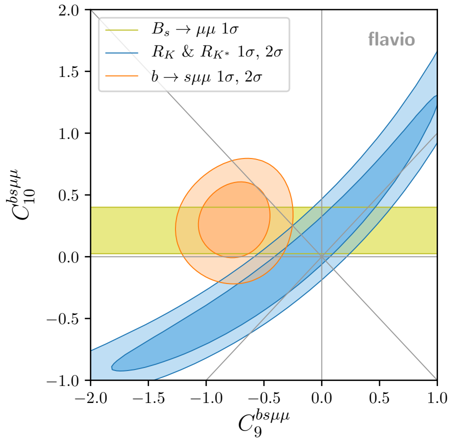
- Analysis offers a large set of results
- Strong interplay between theory and experiment
- Publish set of bootstrapped fit parameters to favour future reinterpretation of the analysis
  - non-trivial correlations
  - allow to reproduce confidence intervals for any desired quantity
  - can transform fit results to different models



Branching ratio constraint

- Differential decay rate can only access the relative size of the Wilson coefficients
  - Scale of Wilson coeff. set by branching ratio
- Extended fit allows to link the observed yield to the signal branching fraction

[Greljo, Salko, Smolkovic, Stangl; JHEP 05 (2023) 087]



$$\mathcal{B}(B^0 \to K^{*0}\mu^+\mu^-) = \frac{\tau_B}{\hbar}\int_{q^2_{\min}}^{q^2_{\max}}\int_{k^2_{\min}}^{k^2_{\max}} \frac{\mathrm{d}^2\Gamma}{\mathrm{d}q^2\mathrm{d}k^2}\mathrm{d}q^2\mathrm{d}k^2$$

Wilson coefficients enter here

$$N_{sig} = N_{J/\psi K\pi} \times \frac{\mathcal{B}(B^0 \to K^{*0}\mu^+\mu^-) \times \frac{2}{3}}{\mathcal{B}(B^0 \to J/\psi K^+\pi^-) \times f^{J/\psi K\pi}_{\pm 100\mathrm{MeV}} \times \mathcal{B}(J/\psi \to \mu^+\mu^-)} \times R_\varepsilon$$

Normalised to $B^0 \to J/\psi K^+\pi^-$ control channel to reduce systematic

Input for BR determination

- BR determination requires several external inputs:

$K^{*0} \to K^+\pi^-$

$$N_{sig} = N_{J/\psi K\pi} \times \frac{\mathcal{B}(B^0 \to K^{*0}\mu^+\mu^-) \times \frac{2}{3}}{\mathcal{B}(B^0 \to J/\psi K^+\pi^-) \times f^{J/\psi K\pi}_{\pm 100\mathrm{MeV}} \times \mathcal{B}(J/\psi \to \mu^+\mu^-)} \times R_\varepsilon$$

from mass fit to control channel (include exotica contribution)

from Belle dedicated $B^0 \to J/\psi K^+\pi^-$ amplitude analysis

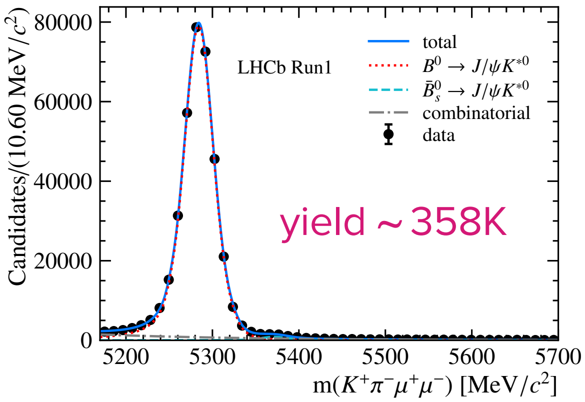
[PRD 90 (2014) 1122009]

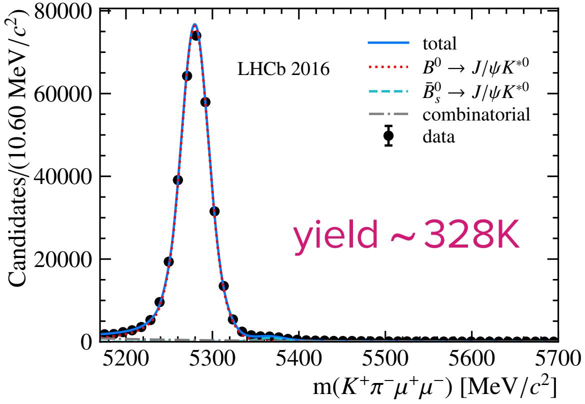
PDG

Ratio of efficiency: from simulations

- $\mathcal{B}(B^0 \to J/\psi K^+\pi^-) = (1.15 \pm 0.01 \pm 0.05) \cdot 10^{-3}$ → inclusive norm. BR
- $f^{B^0 \to J/\psi K\pi}_{\pm 100\mathrm{MeV}} = 0.644 \pm 0.010$ → fraction of events in the $m(K^+\pi^-)$ window of the analysis (determined from Belle model assuming *conservative* uncorrelated uncertainties)

$B^0 \to J/\psi K^+\pi^-$





Systematic uncertainties

Systematics due to the amplitude model

Largest systematic for $\mathcal{C}_9$, $\mathcal{C}_{10}$ comes from BR external inputs

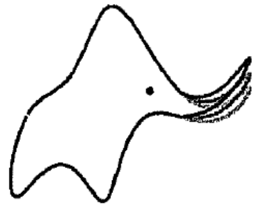
Systematics related to exp. effects are in common with binned BR/angular analyses

Total syst. negligible w.r.t. statistical uncertainty

	$\mathcal{C}_9$	$\mathcal{C}_{10}$	$\mathcal{C}'_9$	$\mathcal{C}'_{10}$
Amplitude model				
S-wave form factors	< 0.01	< 0.01	< 0.01	< 0.01
S-wave non-local hadronic	0.02	0.02	0.14	0.04
S-wave k^2 model	< 0.01	< 0.01	0.05	0.03
Subtotal	0.02	0.02	0.15	0.05
External inputs on BR				
$\mathcal{B}(B^0 \to J/\psi K^+\pi^-)$	0.05	0.08	0.02	0.01
$f^{B^0\to J/\psi K\pi}_{\pm 100\mathrm{MeV}}$	0.03	0.03	0.01	< 0.01
Others (R_ε)	0.03	0.04	0.03	0.01
Subtotal	0.07	0.09	0.04	0.01
Background model				
Chebyshev polynomial order	0.01	0.01	0.01	< 0.01
Combinatorial shape in k^2	0.02	< 0.01	0.02	< 0.01
Background factorisation	0.01	0.01	0.01	0.01
Peaking background	0.01	< 0.01	0.02	0.01
Subtotal	0.03	0.02	0.03	0.01
Experimental effects				
Acceptance parametrisation	< 0.01	< 0.01	< 0.01	< 0.01
Statistical uncertainty on acceptance	0.02	< 0.01	0.02	< 0.01
Subtotal	0.02	< 0.01	0.02	< 0.01
Total systematic uncertainty	0.08	0.10	0.16	0.05
Statistical uncertainty ($q^2 < 0$ constr.)	0.40	0.28	0.40	0.24

Choice of the z order

- Data driven determination of the truncation order:
  - fit repeated with increasing polynomial order $\mathcal{H}_\lambda[z^2, z^3, z^4, \dots]$
  - till no significant improvement in the likelihood is found



"With four parameters I can fit an elephant, and with five I can make him wiggle his trunk." J. von Neumann

$$2\Delta \log \mathcal{L} > 2\Delta N_{\text{pars}}$$

(each z-order brings six additional parameters)

	$2\Delta \log \mathcal{L}$	
	$q^2 < 0$ constr.	$q^2 > 0$ only
$\mathcal{H}_\lambda[z^3] - \mathcal{H}_\lambda[z^2]$	-	3.6
$\mathcal{H}_\lambda[z^4] - \mathcal{H}_\lambda[z^3]$	21.22	-
$\mathcal{H}_\lambda[z^5] - \mathcal{H}_\lambda[z^4]$	8.64	-

- $\mathcal{H}_\lambda[z^2]$ for $q^2 > 0$ only fit
- $\mathcal{H}_\lambda[z^4]$ for $q^2 < 0$ constr. fit

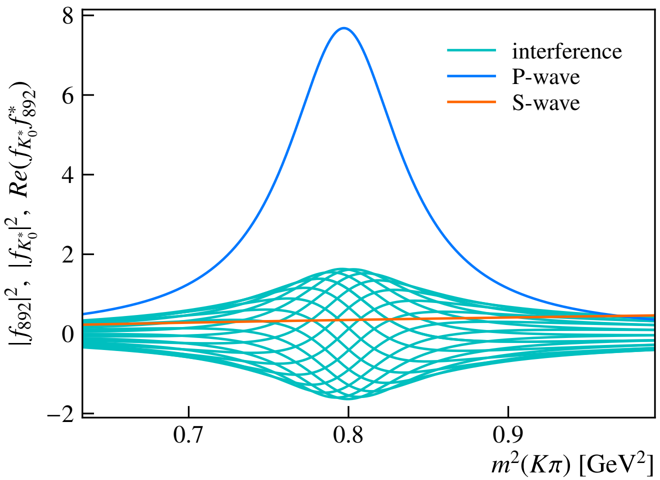
S-wave

- $B^0 \to K^+\pi^-\mu^+\mu^-$ decays can also proceed though a scalar $K^+\pi^-$ configuration (S-wave)
  - require additional scalar amplitudes $\mathcal{A}^{L,R}_{S\,0} = -\mathcal{N}\frac{\sqrt{\lambda(M_B^2,q^2,k^2)}}{M_B\sqrt{q^2}}\left\{\left[(\mathcal{C}_9-\mathcal{C}_9')\mp(\mathcal{C}_{10}-\mathcal{C}_{10}')\right]f_+(q^2,k^2)+\frac{2m_bM_B}{q^2}(\mathcal{C}_7-\mathcal{C}_7')f_T(q^2,k^2)\right\}$
  - extend the fit to $k^2 = m^2(K^+\pi^-)$ [Descontes-Genon, Khodjamirian, Virto; JHEP 12 (2019) 083]

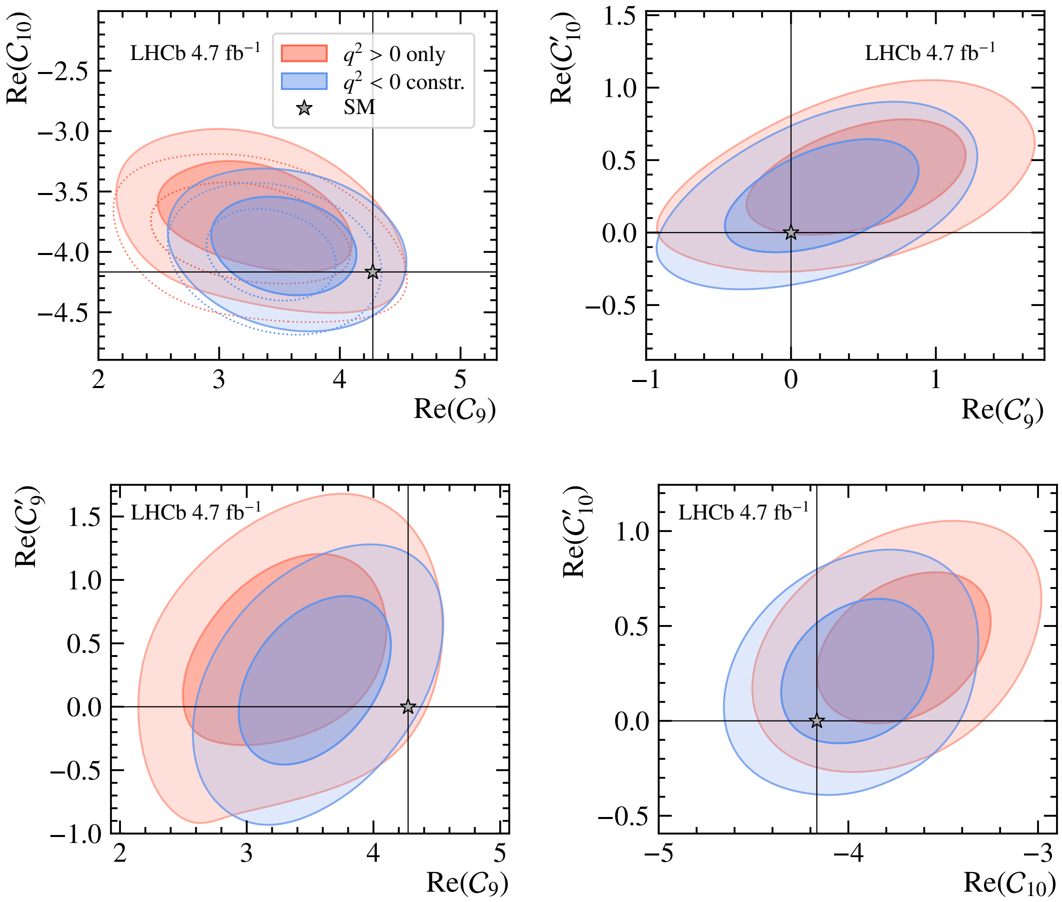
P-wave: $\mathcal{A}^{L,R}_{0,\perp,\parallel,t} \mapsto \mathcal{A}^{L,R}_{0,\perp,\parallel,t} \times \hat{f}_{\mathrm{BW}}(k^2)\,,$

S-wave: $\mathcal{A}^{L,R}_{S\,0,\,S\,t} \mapsto \mathcal{A}^{L,R}_{S\,0,\,S\,t} \times \boxed{|g_S|\,e^{i\delta_S}}\,\hat{f}_{\mathrm{LASS}}(k^2)$

relative magnitude and phase between P and S-wave

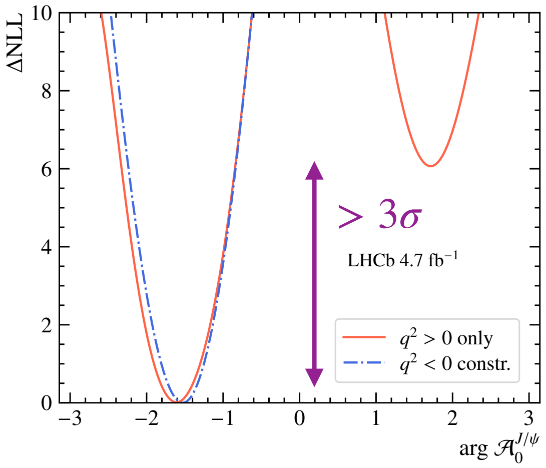


Wilson coefficients 2D



Non-local hadronic results (J/ψ)

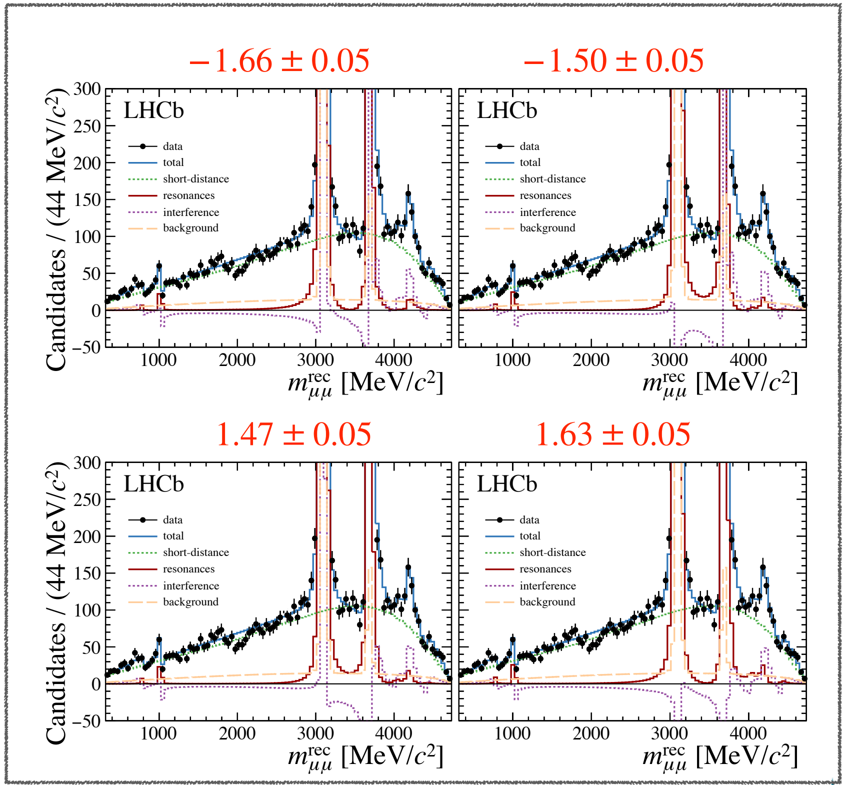
- Phase difference between rare mode and $B^0 \to J/\psi K^{*0}$ decays
  - $\arg \mathcal{A}_0^{J/\psi} = \begin{cases} -1.55^{+0.22}_{-0.18} & [q^2 < 0] \\ -1.61^{+0.22}_{-0.20} & [q^2 > 0] \end{cases}$



  - No sensitivity to $\arg \mathcal{A}_0^{\psi(2S)}$

Compatible with what measured in $B^+ \to K^+\mu^+\mu^-$ decays

$B^+ \to K^+\mu^+\mu^-$ EPJ C77 (2017) 161



Form factors

$$
\begin{aligned}
\mathcal{F}_\perp &\mapsto \frac{\sqrt{2\lambda(M_B^2, q^2, k^2)}}{M_B(M_B + M_{K^{*0}})} V\,, \\
\mathcal{F}_\parallel &\mapsto \frac{\sqrt{2}(M_B + M_{K^{*0}})}{M_B} A_1\,, \\
\mathcal{F}_0 &\mapsto \frac{(M_B^2 - q^2 - M_{K^{*0}}^2)(M_B + M_{K^{*0}})^2 A_1 - \lambda(M_B^2, q^2, k^2) A_2}{2 M_{K^{*0}} M_B^2 (M_B + M_{K^{*0}})}\,, \\
\mathcal{F}_\perp^T &\mapsto \frac{\sqrt{2\lambda(M_B^2, q^2, k^2)}}{M_B^2} T_1\,, \\
\mathcal{F}_\parallel^T &\mapsto \frac{\sqrt{2}(M_B^2 - M_{K^{*0}}^2)}{M_B^2} T_2\,, \\
\mathcal{F}_0^T &\mapsto \frac{q^2(M_B^2 + 3M_{K^{*0}}^2 - q^2)}{2M_B^3 M_{K^{*0}}} T_2 - \frac{q^2 \lambda(M_B^2, q^2, k^2)}{2M_B^3 M_{K^{*0}}(M_B^2 - M_{K^{*0}}^2)} T_3\,, \\
\mathcal{F}_t &\mapsto \frac{\sqrt{\lambda(M_B^2, q^2, k^2)}}{M_B \sqrt{q^2}} A_0\,.
\end{aligned}
$$

Back-up

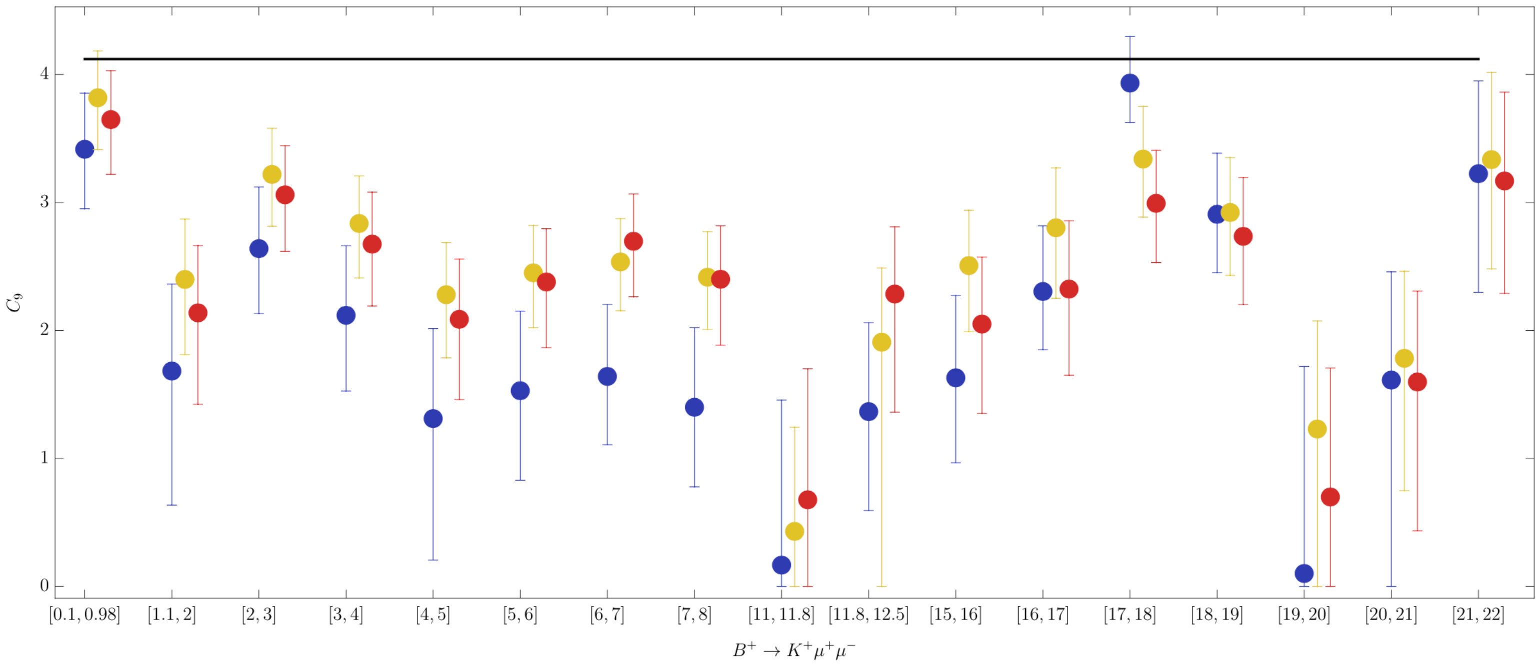


Figure 7.5.2: Results of the fits for $B^+ \to K^+\mu^+\mu^-$ in the whole q^2 range. The blue dots correspond to scenario (i), that is purely perturbative contributions, while the red and yellow dots correspond the the scenarios (ii) and (iii) respectively, which take into account the resonance contributions. The horizontal black line corresponds to the SM value of C_9.

(i) with C_9^{eff} defined as in eq. (7.2.4)

(ii) with C_9^{eff} defined in eq. (7.3.9)

(iii) with eq. (7.3.9) with $\tilde{Y}(q^2)$ replaced by $Y(q^2)$.

Back-up

$$C_7^{\text{eff}} = C_7 - \frac{1}{3}\left(C_3 + \frac{4}{3}C_4 + 20C_5 + \frac{80}{3}C_6\right)$$

and an effective C_9

$$C_9^{\text{eff}} = C_9 + Y\left(q^2\right)$$

with

$$\begin{aligned} Y\left(q^2\right) = \frac{4}{3}C_3 + \frac{64}{9}C_5 + \frac{64}{27}C_6 - \frac{1}{2}h\left(q^2, 0\right)\left(C_3 + \frac{4}{3}C_4 + 16C_5 + \frac{64}{3}C_6\right) \\ + h\left(q^2, m_c\right)\left(\frac{4}{3}C_1 + C_2 + 6C_3 + 60C_5\right) \\ - \frac{1}{2}h\left(q^2, m_b\right)\left(7C_3 + \frac{4}{3}C_4 + 76C_5 + \frac{64}{3}C_6\right), \end{aligned}$$

$$h\left(q^2, m\right) = -\frac{4}{9}\left(\ln\frac{m^2}{\mu^2} - \frac{2}{3} - x\right) - \frac{4}{9}(2+x)\begin{cases} \sqrt{x-1}\,\arctan\frac{1}{\sqrt{x-1}}, & x > 1 \\ \sqrt{1-x}\left(\ln\frac{1+\sqrt{1-x}}{\sqrt{x}} - \frac{i\pi}{2}\right), & x \geq 1 \end{cases}$$

Back-up

$$
\begin{aligned}
\mathcal{M}\left(B \to K\ell^+\ell^-\right)\big|_{C_{7,9}} &= 2\mathcal{N}\left[C_9\langle K|\bar{s}_L\gamma_\mu b_L|B\rangle - \frac{2m_b}{q^2}C_7\langle K|\bar{s}_L i\sigma_{\mu\nu}q^\nu b_R|B\rangle\right]\ell\gamma^\mu\ell \\
&= \mathcal{N}\,C_9\left[f_+(q^2)(p_B+p_K)^\mu + f_-(q^2)q^\mu\right]\ell\gamma^\mu\ell \\
&+ \mathcal{N}\,C_7\,\frac{f_T(q^2)}{(m_B+m_K)}\left[q^2(p_B+p_K)^\mu - (m_B^2-m_K^2)q^\mu\right]\left(\frac{2m_b}{q^2}\right)\ell\gamma^\mu\ell \qquad (2.5)
\end{aligned}
$$

and

$$
\begin{aligned}
\mathcal{M}\left(B \to K^*\ell^+\ell^-\right)\big|_{C_{7,9}} &= \mathcal{N}\,C_9\Bigg[-2i\epsilon_{\mu\nu\rho\sigma}(\epsilon^*)^\nu p_B^\rho p_{K^*}^\sigma\frac{V(q^2)}{m_B+m_{K^*}} \\
&+ q_\mu\,(\epsilon^*\cdot q)\,\frac{2m_{K^*}}{q^2}A_0(q^2) + \left(\epsilon_\mu^* - q_\mu\frac{\epsilon^*\cdot q}{q^2}\right)(m_B+m_{K^*})\,A_1(q^2) \\
&- \left((p_B+p_{K^*})_\mu - q_\mu\frac{m_B^2-m_{K^*}^2}{q^2}\right)\frac{\epsilon^*\cdot q}{m_B+m_{K^*}}A_2(q^2)\Bigg]\bar{\ell}\gamma^\mu\ell \\
&+ \mathcal{N}\,C_7\Bigg[-2i\epsilon_{\mu\nu\rho\sigma}(\epsilon^*)^\nu p_B^\rho p_{K^*}^\sigma T_1(q^2) + (\epsilon^*\cdot q)\left(q_\mu - \frac{q^2}{m_B^2-m_{K^*}^2}(p_B+p_{K^*})_\mu\right)T_3(q^2) \\
&+ \left(\epsilon_\mu^*(m_B^2-m_{K^*}^2) - (\epsilon^*\cdot q)\,(p_B+p_{K^*})_\mu\right)T_2(q^2)\Bigg]\left(\frac{2m_b}{q^2}\right)\ell\gamma^\mu\ell\,, \qquad (2.6)
\end{aligned}
$$

where

$$
q^\mu = p_B^\mu - p_{K^{(*)}}^\mu\,, \qquad \mathcal{N} = \sqrt{2}G_{\mathrm{F}}\alpha_{\mathrm{em}}V_{tb}V_{ts}^*/(4\pi)\,, \qquad (2.7)
$$

Back-up

$$\mathcal{M}\left(B \to K\ell^+\ell^-\right)\Big|_{C_{7,9}} = \mathcal{N}\left[C_9 + \frac{2m_b}{m_B + m_K}\frac{f_T(q^2)}{f_+(q^2)}C_7\right] f_+(q^2)(p_B + p_K)_\mu\,\bar{\ell}\gamma^\mu\ell$$

$$\begin{aligned}
\mathcal{M}\left(B \to K^*\ell^+\ell^-\right)\Big|_{C_{7,9}} = \mathcal{N}\Bigg\{ \\
&-\left[C_9 + \frac{2m_b(m_B + m_{K^*})}{q^2}\frac{T_1(q^2)}{V(q^2)}C_7\right]\frac{2V(q^2)}{m_B + m_{K^*}} i\epsilon_{\mu\nu\rho\sigma}(\epsilon^*)^\nu p_B^\rho p_{K^*}^\sigma \\
&-\left[C_9 + \frac{2m_b(m_B + m_{K^*})}{q^2}\frac{T_2(q^2)}{A_2(q^2)}C_7\left(1 + O\Big(\frac{q^2}{m_B^2}\Big)\right)\right]\frac{A_2(q^2)}{m_B + m_{K^*}}(\epsilon^*\cdot q)(p_B + p_{K^*})_\mu \\
&+\left[C_9 + \frac{2m_b(m_B^2 - m_{K^*})}{q^2}\frac{T_2(q^2)}{A_1(q^2)}C_7\right] A_1(q^2)\,(m_B + m_{K^*})\,\epsilon^*_\mu\Bigg\}\,\bar{\ell}\gamma^\mu\ell\,, \qquad (2.8)
\end{aligned}$$

Back-up

q^2 region	Amplitude	C_9 values			
Low q^2	$B \to K$	$2.4^{+0.4}_{-0.5}$		$2.7^{+0.2}_{-0.2}$ (χ^2/dof=1.27 (0.10))	$3.1^{+0.1}_{-0.1}$ ($\chi^2/\text{dof} = 1.33$ (0.02))
	$B \to K^*(\epsilon_\parallel)$	$3.1^{+0.6}_{-0.6}$	$2.8^{+0.2}_{-0.2}$		
	$B \to K^*(\epsilon_\perp)$	$2.8^{+0.7}_{-0.7}$			
	$B \to K^*(\epsilon_0)$	$2.7^{+0.7}_{-0.8}$			
High q^2	$B \to K$	$2.6^{+0.4}_{-0.4}$		$3.1^{+0.2}_{-0.2}$ (χ^2/dof=1.04 (0.40))	
	$B \to K^*(\epsilon_\parallel)$	$3.3^{+0.5}_{-0.5}$	$3.4^{+0.3}_{-0.3}$		
	$B \to K^*(\epsilon_\perp)$	$3.5^{+0.4}_{-0.4}$			
	$B \to K^*(\epsilon_0)$	$3.5^{+0.6}_{-0.6}$			

Table 4.1: Best-fit points assuming constant C_9 values in the low- and high-q^2 regions, separating or combining the different decay amplitudes, or considering the same value over the full q^2 spectrum for all the decay amplitudes (last column).

Back-up

$$Y^{\lambda}(q^2)\big|_{\alpha_s^0} = Y^{[0]}_{q\bar{q}}(q^2) + Y^{[0]}_{c\bar{c}}(q^2) + Y^{[0]}_{b\bar{b}}(q^2)\,, \tag{2.14}$$

where

$$\begin{aligned}
Y^{[0]}_{q\bar{q}}(q^2) &= \frac{4}{3}C_3 + \frac{64}{9}C_5 + \frac{64}{27}C_6 - \frac{1}{2}h(q^2,0)\left(C_3 + \frac{4}{3}C_4 + 16C_5 + \frac{64}{3}C_6\right)\,,\\
Y^{[0]}_{c\bar{c}}(q^2) &= h(q^2,m_c)\left(\frac{4}{3}C_1 + C_2 + 6C_3 + 60C_5\right)\,,\\
Y^{[0]}_{b\bar{b}}(q^2) &= -\frac{1}{2}h(q^2,m_b)\left(7C_3 + \frac{4}{3}C_4 + 76C_5 + \frac{64}{3}C_6\right)\,,
\end{aligned}$$

with

$$h(q^2,m) = -\frac{4}{9}\left(\ln\frac{m^2}{\mu^2} - \frac{2}{3} - x\right) - \frac{4}{9}(2+x)\begin{cases}\sqrt{x-1}\,\arctan\frac{1}{\sqrt{x-1}}\,, & x = \frac{4m^2}{q^2} > 1\,,\\ \sqrt{1-x}\left(\ln\frac{1+\sqrt{1-x}}{\sqrt{x}} - \frac{i\pi}{2}\right), & x = \frac{4m^2}{q^2} \le 1\,.\end{cases}$$