Anomalous thresholds in $B \to (P, V)\gamma^*$ form factors

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Apr 10, 2024

Beyond the Flavour Anomalies V, Siegen

in collaboration with Bastian Kubis[2]

$B \to (P, V)\gamma^*$ form factors

Matrix elements for $B \to (P, V)\ell^+\ell^-$

$$\mathcal{A}(\bar{B}(q+k) \to \begin{Bmatrix} K^{(*)}(k) \\ \pi(\rho)(k) \end{Bmatrix} \ell^+(q_1)\ell^-(q_2)) = \frac{G_F \alpha}{\sqrt{2}\,\pi} \begin{Bmatrix} V_{ts}^* V_{tb} \\ V_{td}^* V_{tb} \end{Bmatrix} \left[(C_9 L_V^\mu + C_{10} L_A^\mu)\mathcal{F}_\mu - \frac{L_V^\mu}{q^2}(2im_b C_7 \mathcal{F}_{T,\mu} + 16\pi^2 \mathcal{H}_\mu) \right]$$

with

$$L_V^\mu = \bar{u}_\ell(q_1)\gamma^\mu v_\ell(q_2) \qquad L_A^\mu = \bar{u}_\ell(q_1)\gamma^\mu \gamma_5 v_\ell(q_2)$$

- **Local form factors**: conventions from Gubernari, van Dyk, Virto 2021

$$\mathcal{F}_\mu^{B\to(P,V)}(k,q) = \langle (P,V)(k)| \begin{Bmatrix} \bar{s} \\ \bar{d} \end{Bmatrix} \gamma_\mu b_L |\bar{B}(q+k)\rangle$$

$$\mathcal{F}_{T,\mu}^{B\to(P,V)}(k,q) = \langle (P,V)(k)| \begin{Bmatrix} \bar{s} \\ \bar{d} \end{Bmatrix} \sigma_{\mu\nu} q^\nu b_R |\bar{B}(q+k)\rangle$$

$B \to (P,V)\gamma^*$ form factors

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- **Non-local form factors**: conventions from Gubernari, van Dyk, Virto 2021

$$\mathcal{H}_\mu^{B\to(P,V)}(k,q) = i\int d^4x\, e^{iq\cdot x}\langle (P,V)(k)|T\{j_\mu^{\text{em}}(x), \sum_{i,q=u,c} C_i^q \mathcal{O}_i^q(0)\}|\bar{B}(q+k)\rangle$$

with

$$j_\mu^{\text{em}} = \sum_q Q_q \bar{q}\gamma_\mu q \qquad \mathcal{O}_1^q = \begin{Bmatrix} \bar{s} \\ \bar{d} \end{Bmatrix} \gamma_\mu T^a q_L\, \bar{q}\gamma^\mu T^a b_L \qquad \mathcal{O}_2^q = \begin{Bmatrix} \bar{s} \\ \bar{d} \end{Bmatrix} \gamma_\mu q_L\, \bar{q}\gamma^\mu b_L \qquad \ldots$$

$\hookrightarrow$ **This talk**: analytic properties of the non-local form factors $\mathcal{H}_\mu(q^2)$

A primer on form factor singularities

- **Kinematic invariants**
  - **Meson masses** $(q+k)^2 = M_B^2$, $k^2 = (M_P^2, M_V^2)$

    $\hookrightarrow$ only defined on-shell, otherwise model dependence from choice of interpolating field
  - **Photon virtuality** q^2

    $\hookrightarrow$ can define analytic continuation for q^2 arbitrary in the complex plane
- **Singularities in q^2**
  - **Poles**: (infinitely) narrow states

    $\hookrightarrow q^2 = M_{J/\psi}^2, M_{\psi(2S)}^2$
  - **Normal thresholds**: branch points of $\gamma^* \to \{\pi^+\pi^-, D\bar{D}, \ldots\}$ cuts

    $\hookrightarrow q^2 = \{4M_\pi^2, 4M_D^2, \ldots\}$
  - **Anomalous thresholds**: anomalous branch points

    $\hookrightarrow$ can arise depending on left-hand-cut structure of $B \to (P, V)\gamma^*$ amplitude

Anomalous thresholds: where do they come from?

- **Landau equations**: singularities of general loop integral

$$\int \prod_{\ell=1}^{L} \frac{d^4 q_\ell}{(2\pi)^4} \prod_{i=1}^{n} \frac{i}{k_i^2 - m_i^2 + i\epsilon} \quad \text{singular when} \quad \begin{cases} \lambda_i(k_i^2 - m_i^2) = 0 & \text{for all } i = 1, \ldots, n \\ \sum\limits_{i=1}^{n} \lambda_i k_i \cdot \frac{\partial k_i}{\partial q_\ell} = 0 & \ell = 1, \ldots, L \end{cases}$$

$\hookrightarrow$ "leading singularity" $\Leftrightarrow$ all $\lambda_i \neq 0$

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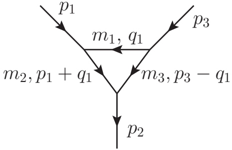
- **Triangle diagram**: $L = 1$, $n = 3$, Landau equations become

$$\lambda_i (k_i^2 - m_i^2) = 0 \qquad \sum_{i=1}^{3} \lambda_i k_i = 0$$



  - **Normal thresholds**: e.g., $\lambda_3 = 0 \Rightarrow p_1^2 = (m_1 \pm m_2)^2$

    [zeros of $\lambda(p_1^2, m_1^2, m_2^2)$, $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2(ab + ac + bc)$]

  - **Anomalous threshold**: all $\lambda_i \neq 0$

    $$\hookrightarrow p_2^2 = s_\pm \equiv p_1^2 \frac{m_1^2 + m_3^2}{2m_1^2} + p_3^2 \frac{m_1^2 + m_2^2}{2m_1^2} - \frac{p_1^2 p_3^2}{2m_1^2} - \frac{(m_1^2 - m_2^2)(m_1^2 - m_3^2)}{2m_1^2} \pm \frac{1}{2m_1^2} \sqrt{\lambda(p_1^2, m_1^2, m_2^2)\lambda(p_3^2, m_1^2, m_3^2)}$$

Anomalous thresholds: when do they matter?

Dispersion relation for scalar loop function $C_0(s) \equiv C_0(p_1^2, s, p_3^2, m_1^2, m_2^2, m_3^2)$

$$C_0(s) = \frac{1}{2\pi i}\int_{(m_2+m_3)^2}^{\infty} ds' \frac{\text{disc}\, C_0(s')}{s'-s}$$

- Discontinuity takes the form (modulo analytic continuations)

$$\text{disc}\, C_0(s) = \frac{2\pi i\theta(s-(m_2+m_3)^2)}{\sqrt{\lambda(s,p_1^2,p_3^2)}} \log\frac{a-b}{a+b}$$

$$a = s^2 - s(p_1^2+p_3^2+m_2^2+m_3^2-2m_1^2) + (p_1^2-p_3^2)(m_2^2-m_3^2) \qquad b = \sqrt{\lambda(s,m_2^2,m_3^2)\lambda(s,p_1^2,p_3^2)}$$

$\hookrightarrow$ $s_\pm$ correspond to the zeros of the logarithm

- For sufficiently small p_1^2, p_3^2, $s_\pm$ lie on the second sheet, but $p_1^2 = M_B^2$ large
- Anomalous branch point moves onto first sheet for

$$m_3 p_1^2 + m_2 p_3^2 - (m_2+m_3)(m_1^2 + m_2 m_3) > 0$$

$\hookrightarrow$ anomalous term in dispersion relation

Anomalous thresholds: when do they matter?

Dispersion relation for scalar loop function $C_0(s) \equiv C_0(p_1^2, s, p_3^2, m_1^2, m_2^2, m_3^2)$

$$C_0(s) = \frac{1}{2\pi i} \int_{(m_2+m_3)^2}^{\infty} ds' \frac{\text{disc}\, C_0(s')}{s' - s} + \theta\left[m_3 p_1^2 + m_2 p_3^2 - (m_2+m_3)(m_1^2 + m_2 m_3)\right] \frac{1}{2\pi i} \int_0^1 dx \frac{\partial s_x}{\partial x} \frac{\text{disc}_{\text{an}}\, C_0(s_x)}{s_x - s}$$

- Discontinuity takes the form (modulo analytic continuations)

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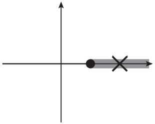
Anomalous thresholds: deformation of the integration contour

- Anomalous branch point on first sheet (can be either s_+ or s_-) requires **deformation of the integration contour**

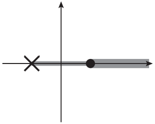
$$s_x = x\big(m_2 + m_3\big)^2 + (1 - x)s_\pm$$

- Three cases:

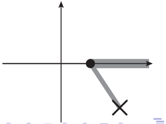
1. $s_\pm$ on normal cut
   ↪ analytic continuation of normal discontinuity



2. $s_\pm$ on negative real axis
   ↪ integration deformed along real axis



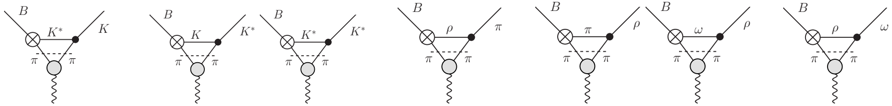
3. $s_\pm$ in complex plane
   ↪ integration deformed into complex plane

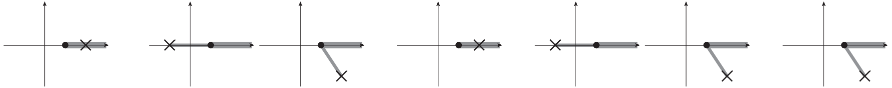


Anomalous thresholds for $B \to (P, V)\gamma^*$: strategy

- Start with ***u*-quark loop** and **$\pi\pi$ intermediate states**:
  - Pion form factor well known
  - Most branching fractions $B \to (P, V)\pi\pi$ known
  - Phenomenological knowledge of $B \to (PP, PV, VV)$
  - Sizable energy gap to next state $\pi\omega$
    $\hookrightarrow$ cf. various $D_{(s)}^{(*)}\bar{D}_{(s)}^{(*)}$ for hadronization of charm loop
- **CKM scaling**
  - $b \to s$: u-quark loop, $\mathcal{O}(\lambda^4)$, CKM suppressed compared to charm, $\mathcal{O}(\lambda^2)$
  - $b \to d$: both scale as $\mathcal{O}(\lambda^3)$
- Strategy/goals of this exercise:
  - Consider left-hand cuts that lead to anomalous thresholds
  - Fix parameters from $B \to (P, V)\pi\pi$ and $B \to (PP, PV, VV)$ input
  - Quantify the size of anomalous contributions to $B \to (P, V)\gamma^*$ form factors

Anomalous thresholds for $B \to (P, V)\gamma^*$: list of processes





$B \to K\gamma^*$	$B \to K^*\gamma^*$	$B \to \pi\gamma^*$	$B \to \rho\gamma^*$	$B \to \omega\gamma^*$
$s_+ = 18.6\,\text{GeV}^2$	-57.8 $0.5 - 4.2i$	26.4	-859.3 $0.7 - 4.8i$	$0.2 - 4.9i$
$\text{Br}[B \to K^*\pi]$	$\text{Br}[B \to K^{(*)}\pi]$	$\text{Br}[B \to \rho\pi]$	$\text{Br}[B \to \pi\pi, \pi\omega]$	$\text{Br}[B \to \rho\pi]$
$\text{Br}[B \to K\pi\pi]$	$\text{Br}[B \to K^*\pi\pi]$	$\text{Br}[B \to 3\pi]$	$\text{Br}[B \to \rho\pi\pi]$	$\text{Br}[B \to \omega\pi\pi]$

- $s_{\text{thr}} = 4M_\pi^2 = 0.08\,\text{GeV}^2$
- The branching fractions in the last line assume $\pi\pi$ in a P-wave.
- Consider K^*, ρ, ω narrow for now (could integrate over spectral functions).
- To disentangle helicity amplitudes, not only branching ratios, but polarization fractions are required.

Helicity amplitudes for $B \to (P, V)\gamma^*$ and $B \to (P, V)\pi\pi$

- Decompose **non-local form factors** $\mathcal{H}_\mu^{B\to(P,V)}(q^2)$ into covariant structures with scalar coefficients $\Pi_{P,\lambda}(q^2)$ free of kinematic singularities/zeros:

$$\mathcal{H}_\mu^{B\to P}(q^2) = \tilde{\mathcal{S}}_\mu \Pi_P(q^2)$$
$$\mathcal{H}_\mu^{B\to V}(q^2) = \eta^{*\alpha}\left[\tilde{\mathcal{S}}^{\perp}_{\alpha\mu}\Pi_\perp(q^2) + \tilde{\mathcal{S}}^{\parallel}_{\alpha\mu}\Pi_\parallel(q^2) + \tilde{\mathcal{S}}^{0}_{\alpha\mu}\Pi_0(q^2)\right]$$

  - For V: polarization vector η^α, helicity components $\lambda \in \{\perp, \parallel, 0\}$
  - Longitudinal component $\Pi_0(q^2)$

Helicity amplitudes for $B \to (P, V)\gamma^*$ and $B \to (P, V)\pi\pi$

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  - For V: polarization vector η^α, helicity components $\lambda \in \{\perp, \parallel, 0\}$
  - Longitudinal component $\Pi_0(q^2)$
- Also decompose $B \to (P, V)\pi\pi$ amplitude into helicity amplitudes:

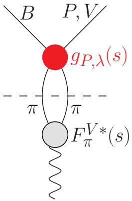
$$\mathcal{M}^{B\to P\pi\pi}(s,t,u) = \mathcal{F}_P(s,t,u)$$
$$\mathcal{M}^{B\to V\pi\pi}(s,t,u) = \eta^{*\alpha}\left[\tilde{T}^{\perp}_{\alpha}\mathcal{F}_\perp(s,t,u) + \tilde{T}^{\parallel}_{\alpha}\mathcal{F}_\parallel(s,t,u) + \tilde{T}^{0}_{\alpha}\mathcal{F}_0(s,t,u)\right]$$

- Polarization fractions $f_\lambda = |\mathcal{F}_\lambda|^2 / \sum_{\lambda'} |\mathcal{F}_{\lambda'}|^2$
  - Know longitudinal fraction $f_L \equiv f_0$ from experiment
  - Set $f_\perp = f_\parallel = (1 - f_L)/2$, motivated by QCD factorization Beneke, Rohrer, Yang 2007

Left-hand cuts in $B \to (P, V)\pi\pi$

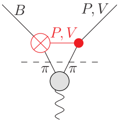
- Unitarity relations for $\pi\pi$ intermediate states FF ($s = q^2$)

$$\text{disc}\,\Pi_{P,\lambda}(s) = 2i\,\nu_{P,\lambda}(s)\,g_{P,\lambda}(s)\,F_\pi^{V*}(s)$$



  - Kinematic factors $\nu_{P,0}(s) \equiv \sigma_\pi^3(s)$, $\nu_{\perp,\parallel}(s) \equiv s\,\sigma_\pi^3(s)$, where $\sigma_\pi(s) = \sqrt{1 - 4M_\pi^2/s}$
  - $g_P(s)$, $g_\lambda(s)$ denote P-waves of $\mathcal{F}_P(s,t,u)$, $\mathcal{F}_\lambda(s,t,u)$
- Consider left-hand cuts from t, u-channel Born exchange
  - P-wave projection leads to triangle topology:

$$g_{P,\lambda}(s) \sim \text{disc}\,C_0(s)$$



  - Anomalous thresholds in FF dispersion relations:

$$\Pi_{P,\lambda}(s) = \underbrace{\frac{1}{\pi}\int_{4M_\pi^2}^{\infty} ds'\,\frac{\nu_{P,\lambda}(s')\,g_{P,\lambda}(s')\,F_\pi^{V*}(s')}{s'-s}}_{\equiv \Pi_{P,\lambda}^{\text{norm}}(s)} + \underbrace{\frac{1}{\pi}\int_0^1 dx\,\frac{\partial s_x}{\partial x}\,\frac{\nu_{P,\lambda}(s_x)\,\text{disc}\,g_{P,\lambda}(s_x)\,F_\pi^{V*}(s_x)}{s_x - s}}_{\equiv \Pi_{P,\lambda}^{\text{anom}}(s)}$$

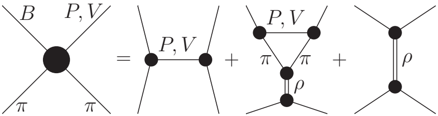
Unitarization of $B \to (P, V)\pi\pi$

- Simple Born amplitude violates unitarity (Watson's theorem):

$$\text{disc}\, g_{P,\lambda}(s) = 2i\, g_{P,\lambda}(s) \sin\delta(s)\, e^{-i\delta(s)} = 2i\, g_{P,\lambda}(s)\, \sigma_\pi(s)\, t^*(s)$$

$\hookrightarrow$ $\pi\pi$ elastic scattering phase shift $\delta(s)$ ($I = 1, L = 1$), $t(s) = \sin\delta(s)\, e^{i\delta(s)}/\sigma_\pi(s)$

- Unitarize via Muskhelishvili–Omnès representation:



$g_{P,\lambda}(s)$ $g^{\text{Born}}_{P,\lambda}(s)$ $g^{\text{MO}}_{P,\lambda}(s)$ $g^{\text{Omnès}}_{P,\lambda}(s)$

$$g^{\text{MO}}_{P,\lambda}(s) = \Omega(s)\left[\frac{s}{\pi}\int_{4M_\pi^2}^{\infty}\frac{ds'}{s'}\frac{g^{\text{Born}}_{P,\lambda}(s')\sin\delta(s')}{|\Omega(s')|(s'-s)} + \frac{s}{\pi}\int_0^1\frac{dx}{s_x}\frac{\partial s_x}{\partial x}\frac{\text{disc}\, g^{\text{Born}}_{P,\lambda}(s_x)\,\sigma_\pi(s_x)\, t(s_x)}{\Omega(s_x)(s_x - s)}\right]$$

$$g^{\text{Omnès}}_{P,\lambda}(s) = a_{P,\lambda}\,\Omega(s)$$

$$\Omega(s) = \exp\left[\frac{s}{\pi}\int_{4M_\pi^2}^{\infty}\frac{ds'}{s'}\frac{\delta(s')}{s'-s}\right] = |\Omega(s)|\, e^{i\delta(s)}$$

- One subtraction constant $a_{P,\lambda}$ needed due to $g_{P,\lambda}(s) \sim \mathcal{O}(s^{-1})$

Fixing the subtraction constant

- Use experimental values of branching fractions $\text{Br}[B \to \rho(P, V)]$ and polarization fractions $f_L[B \to \rho V]$ to fix the subtraction constants $a_{P,\lambda}$
  - P-wave contributions to differential decay rate

$$d\Gamma_{P,\lambda}[B \to (P, V)\pi\pi] = \phi_{P,\lambda}(s)|g_{P,\lambda}(s)|^2 ds$$

  ↪ Phase space factors $\phi_{P,\lambda}(s)$

  ↪ Remember $g_{P,\lambda}(s) = g^{\text{Born}}_{P,\lambda}(s) + g^{\text{MO}}_{P,\lambda}(s) + a_{P,\lambda}\Omega(s)$

  - Demand that ρ-band of width $\Delta = 2M_\rho\Gamma_\rho$ be saturated:

$$\Gamma[B \to \rho P] \stackrel{!}{=} \mathcal{N}_P^{-1} \int_{M_\rho^2-\Delta}^{M_\rho^2+\Delta} ds \frac{d\Gamma_P[B \to P\pi\pi]}{ds}$$

$$f_\lambda\, \Gamma[B \to \rho V] \stackrel{!}{=} \mathcal{N}_\lambda^{-1} \int_{M_\rho^2-\Delta}^{M_\rho^2+\Delta} ds \frac{d\Gamma_\lambda[B \to V\pi\pi]}{ds}$$

$$\text{with } \mathcal{N}_{P,\lambda} = \left(\int_{M_\rho^2-\Delta}^{M_\rho^2+\Delta} ds\, \phi_{P,\lambda}(s)|\Omega(s)|^2\right) \Big/ \left(\int_{4M_\pi^2}^{(M_B-M_{P/V})^2} ds\, \phi_{P,\lambda}(s)|\Omega(s)|^2\right)$$

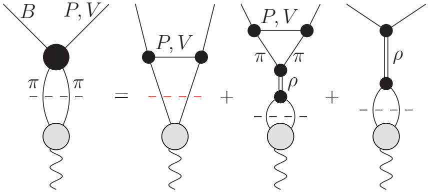
  ↪ Normalization factor $\mathcal{N}_{P,\lambda}$ as we only integrated over the ρ-band

  - Solve this for $a_{P,\lambda}$ assuming $a_{P,\lambda}$ is real (sign ambiguity!)

FF dispersion relations

- Plugging the unitarized P-waves into the FF unitarity relation:

$$\text{disc}\,\Pi_{P,\lambda}(s) = 2i\,\nu_{P,\lambda}(s)\,g_{P,\lambda}(s)\,F_\pi^{V*}(s)$$

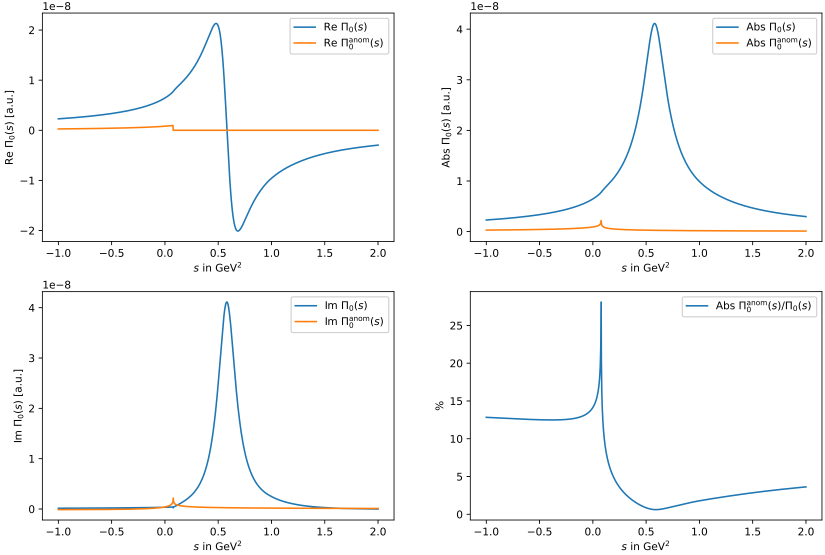


$\hookrightarrow$ Only gives an anomalous contribution $\sim \text{disc}\,g_{P,\lambda}(s) = \text{disc}\,g^{\text{Born}}_{P,\lambda}(s)$

- Approximate pion form factor simply as $F_\pi^V(s) = \Omega(s)$
- Obtain unsubtracted FF dispersion relation

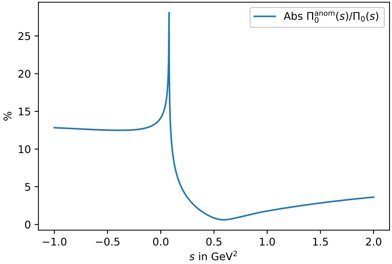
$$\Pi_{P,\lambda}(s) = \underbrace{\frac{1}{\pi}\int_{4M_\pi^2}^{\infty} ds' \frac{\nu_{P,\lambda}(s')\,g_{P,\lambda}(s')\,\Omega^*(s')}{s'-s}}_{\equiv \Pi^{\text{norm}}_{P,\lambda}(s)} + \underbrace{\frac{1}{\pi}\int_0^1 dx \frac{\partial s_x}{\partial x}\frac{\nu_{P,\lambda}(s_x)\,\text{disc}\,g^{\text{Born}}_{P,\lambda}(s_x)\,\Omega^*(s_x)}{s_x-s}}_{\equiv \Pi^{\text{anom}}_{P,\lambda}(s)}$$

Example: anomalous contribution to the longitudinal $B^+ \to K^{*+}\gamma^*$ FF



Example: anomalous contribution to the longitudinal $B^+ \to K^{*+}\gamma^*$ FF

- Anomalous fraction $|\Pi_0^{\text{anom}}(s)/\Pi_0(s)| \sim 10\ \%$ in this case
- Suppression around ρ-peak and singularity at threshold



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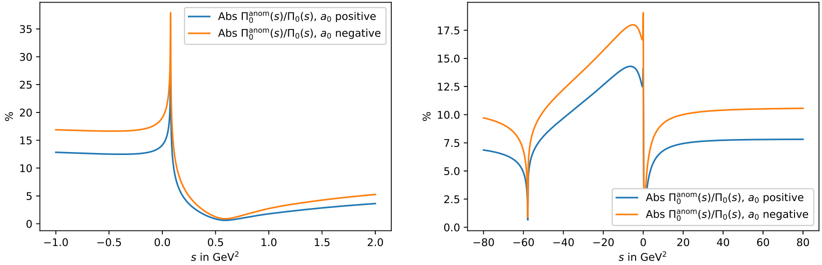
- Anomalous fraction $|\Pi_0^{\text{anom}}(s)/\Pi_0(s)| \sim 10$ % in this case
- Suppression around ρ-peak and singularity at threshold
- No qualitative difference due to sign ambiguity of $a_{P,\lambda}$
- Even more relevant for space-like s



- Results qualitatively similar in other cases, percentage (slightly) lower

Summary and outlook

- Investigated **anomalous contributions to $B \to (P, V)\gamma^*$** for u-quark loop

  $\hookrightarrow$ estimated their relevance on basis of experimental data

- Key finding: **anomalous contributions can be as large as 10% off resonance**

  $\hookrightarrow$ relevant for matching in the space-like region?

- Consequences for **u-quark loop**:
  - Phenomenological estimates of non-local contributions

    $\hookrightarrow$ combination of dispersion relations, data input, QCD factorization, LCSRs, ...
  - Extension to higher intermediate states

    $\hookrightarrow$ $4\pi \simeq \pi\omega$ could be feasible

- Generalization to **c-quark loop**:
  - Expect impact of anomalous thresholds to be qualitatively similar
  - More intermediate states in close proximity: $\bar{D}D$, $\bar{D}D^*$, $\bar{D}^*D^*$, ...
  - Phenomenology of form factors and amplitudes less well understood