ΔACP (experiment and theory)

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Beyond the Flavour Anomalies V
Siegen - April 10, 2024

Why charm is charming?

CKM suppression
$\sim V_{ub}V_{cb}\left(\frac{m_b}{m_W}\right)^2 \sim 10^{-6}$

GIM suppression
$\sim \frac{m_s^2 - m_d^2}{m_W^2}$
complete cancellation in the U-spin limit (m_d=m_s)

(SM *asymmetries* **~0.1% or below**)

- **CP violation** (CPV) and **mixing** are suppressed in charm
  - Room for **new physics** enhancement
- **Predictions** are difficult due to low-energy strong interaction effects [Phys.Lett. B222 (1989) 501]
- Experimental **measurements** are crucial
- ***LHCb*** is one of the main players:
  - **2013**, first observation* of **D0 mixing** in D0→Kπ
  - **2019**, first observation of **direct CPV** in D0→hh

$\sigma(pp \to c\bar{c}X)_{\sqrt{s}=13\,\text{TeV}} \cong 2.4$ mb
[JHEP 03 (2016) 159]

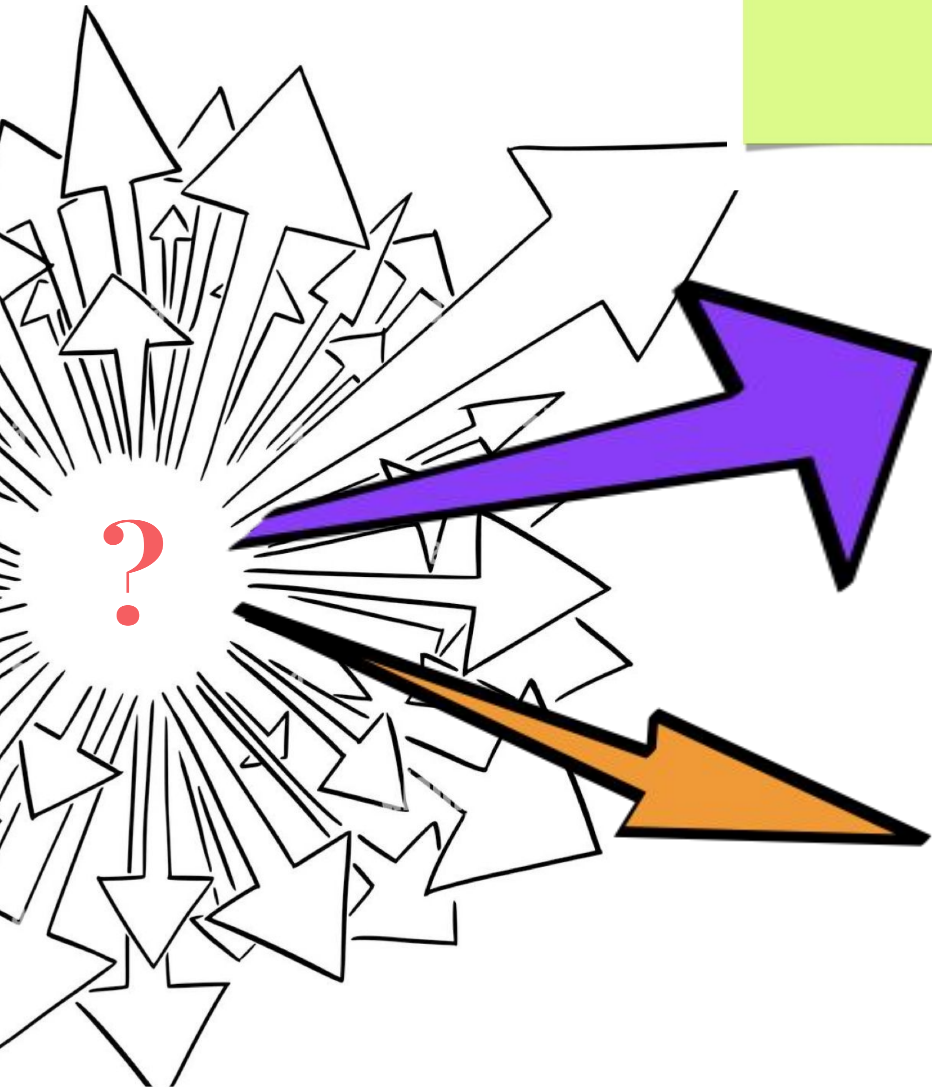
* from a single measurement

ΔACP

$$\Delta A_{CP} = A_{CP}(D^0 \to K^- K^+) - A_{CP}(D^0 \to \pi^- \pi^+)$$

$$= (-15.4 \pm 2.9) \times 10^{-4}$$

[Phys. Rev. Lett. 122, 211803]



- **E. Solominidi**: theoretical estimations using a data-driven approach

- **S. Maccolini**: measurement of the individual ACP(D0→hh)

Theory

based on *Phys.Rev.D* 108 (2023) 3, 036026
with Antonio Pich & Luiz Vale Silva
(& new preliminary results)

How CP violation arises

Generally: at least 2 interfering amplitudes

Can be parameterised as

$$\mathcal{A}(D^0 \to f) = A(f) + i r_{CKM} B(f)$$ where $$r_{CKM} = Im \frac{V_{cs}^* V_{us}}{V_{cd}^* V_{ud}} \approx -6.2 \cdot 10^{-4}$$

$$\mathcal{A}(\overline{D^0} \to f) = A(f) - i r_{CKM} B(f)$$

and consequently

$$\alpha_{CP}^{dir} \approx 2 \underbrace{r_{CKM}}_{\text{weak phases}} \frac{|B(f)|}{|A(f)|} \cdot \underbrace{\sin \arg \frac{A(f)}{B(f)}}_{\text{strong phases}}$$

At the scale of the charm quark mass:

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left[\Sigma_{i=1}^2 C_i(\mu) \left(\lambda_d Q_i^d(\mu) + \lambda_s Q_i^s(\mu) \right) - \lambda_b (\Sigma_{i=3}^6 C_i(\mu) Q_i(\mu) + C_{8g}(\mu) Q_{8g}(\mu)) \right]$$

current-current operators → affect branching ratios & aCP's

penguin operators → affect only aCP's

$$\lambda_q = V_{cq}^* V_{uq}\,, \quad q = d, s, b.$$
$$|\lambda_d| \approx |\lambda_s| = \mathcal{O}(\lambda)$$
$$\lambda_d + \lambda_s + \lambda_b = 0$$
$$|C_{3-6}| < 0.1 C_2,\ 0.03 C_1$$

Challenge: to calculate $\langle P^+ P^- | Q_i | D^0 \rangle,\ P = \pi, K$

A first estimate: Large number of colors

At the limit of a very large number of colors N_C, scattering between mesons is suppressed

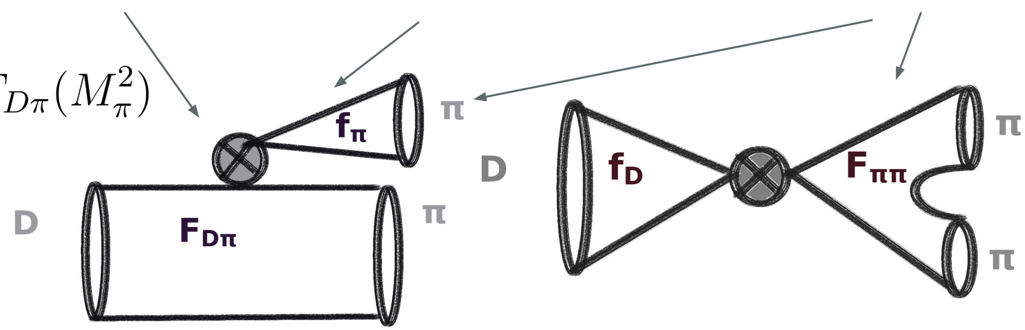
Large N_C leads to the factorization of hadronic matrix elements (*no use of large N_C for the Wilson coefficients*)

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left[\Sigma_{i=1}^{2} C_i(\mu) \left(\lambda_d Q_i^d(\mu) + \lambda_s Q_i^s(\mu)\right) - \lambda_b(\Sigma_{i=3}^{6} C_i(\mu) Q_i(\mu) + C_{8g}(\mu) Q_{8g}(\mu))\right]$$

yields $\mathcal{A}(D^0 \to \pi^+\pi^-) \propto C_1 \lambda_d \langle \pi^+\pi^- | Q_1^d | D^0 \rangle_{fac} - \lambda_b \left(C_4 \langle \pi^+\pi^- | Q_4 | D^0 \rangle_{fac} + C_6 \langle \pi^+\pi^- | Q_6 | D^0 \rangle_{fac}\right.$

where e.g.

$$\langle \pi^+\pi^- | Q_1^d | D^0 \rangle_{fac} = i f_\pi \left(M_D^2 - M_\pi^2\right) F_{D\pi}(M_\pi^2)$$



Results in:

✅ Correct estimation of *some* decay amplitudes

❌ No prediction for the strong phases → necessary for CP violation!

[See also Lenz, Piscopo, Rusov '23]

For decay constants & form factors: lattice (χPT for ππ scalar form factor)

How to incorporate strong phases: isospin & unitarity

Isospin is a good symmetry

of strong interactions - Use Wigner-Eckart theorem

$$A(D^0 \to \pi^+\pi^-) = -\frac{1}{\sqrt{6}}|A^{I=0}_{\pi\pi}|e^{i\delta_{\pi\pi,0}} - \frac{1}{2\sqrt{3}}|A^{I=2}_{\pi\pi}|e^{i\delta_{\pi\pi,2}}$$

$$A(D^0 \to \pi^0\pi^0) = -\frac{1}{\sqrt{6}}|A^{I=0}_{\pi\pi}|e^{i\delta_{\pi\pi,0}} + \frac{1}{\sqrt{3}}|A^{I=2}_{\pi\pi}|e^{i\delta_{\pi\pi,2}}$$

$$A(D^+ \to \pi^+\pi^0) = \frac{\sqrt{3}}{2\sqrt{2}}|A^{I=2}_{\pi\pi}|e^{i\delta_{\pi\pi,2}}$$

$$A(D^0 \to K^-K^+) = \frac{1}{2}\left(|A^{I=1}_{KK}|e^{i\delta_{KK,1}} - |A^{I=0}_{KK}|e^{i\delta_{KK,0}}\right)$$

$$A(D^0 \to \overline{K}^0K^0) = \frac{1}{2}\left(-|A^{I=1}_{KK}|e^{i\delta_{KK,1}} - |A^{I=0}_{KK}|e^{i\delta_{KK,0}}\right)$$

$$A(D^+ \to \overline{K}^0K^+) = |A^{I=1}_{KK}|e^{i\delta_{KK,1}}$$

Both ππ and KK have an isospin-zero component

[see also Gavrilova,Grossman, Schacht '23]

The S-matrix is unitary

In *isospin-zero*, *spin-zero*, the strong S-submatrix is also unitary

(assumption: no other channels leak to ππ and KK)

$$\begin{pmatrix} A(D\to\pi\pi) \\ A(D\to KK) \end{pmatrix} = \underbrace{\begin{pmatrix} S_0(\pi\pi\to\pi\pi) & S_0(\pi\pi\to KK) \\ S_0(KK\to\pi\pi) & S_0(KK\to KK) \end{pmatrix}}_{\text{strong-interaction-driven}} \cdot \begin{pmatrix} A^*(D\to\pi\pi) \\ A^*(D\to KK) \end{pmatrix}$$

If isospin-zero ππ and KK channels didn't communicate:

Watson's theorem

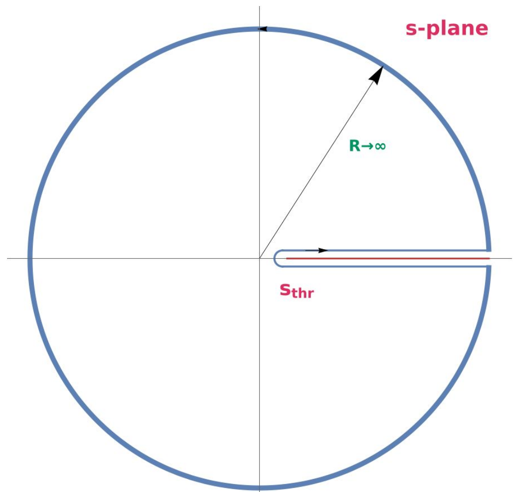
$$\arg A(D\to\pi\pi) = \arg(\pi\pi\to\pi\pi,\ \text{S-wave}) \mod \pi$$

Instead now the phases of D→PP depend on the magnitudes of D→PP + the strong S-submatrix

$$S_0 = \begin{pmatrix} \eta e^{i2\delta_1} & i\sqrt{1-\eta^2}e^{i(\delta_1+\delta_2)} \\ i\sqrt{1-\eta^2}e^{i(\delta_1+\delta_2)} & \eta e^{i2\delta_2} \end{pmatrix}$$

How the phases affect the amplitudes

Through analyticity by applying Cauchy's theorem



$$\mathrm{Re}A(s) = \frac{1}{\pi}\int_{s_{thr}}^{\infty} ds' \frac{\mathrm{Im}A(s')}{s'-s}$$

and ***if rescattering is elastic****, through unitarity we get the* ***dispersion relation***

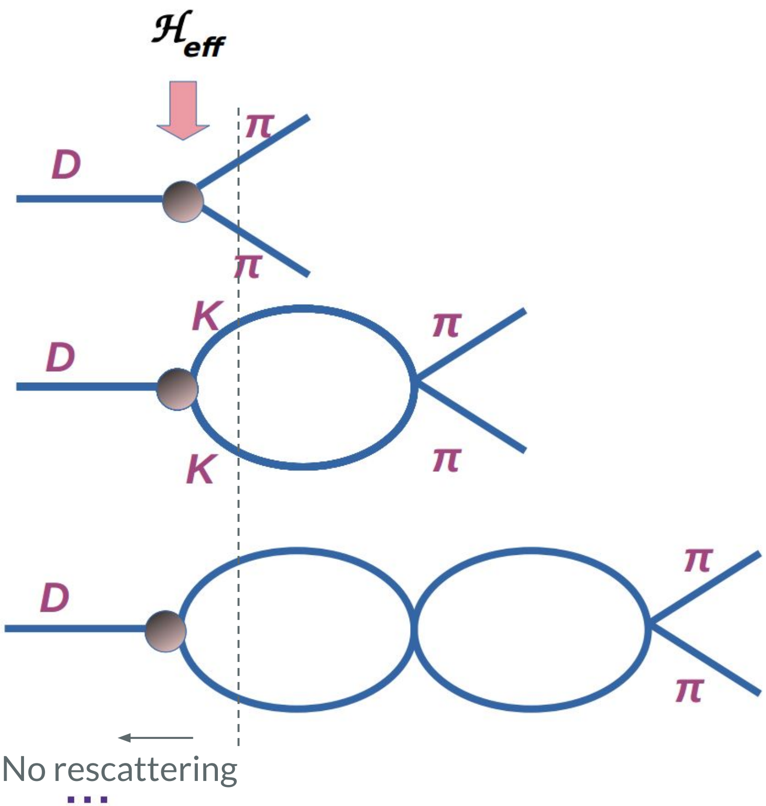
$$\mathrm{Re}A(D\to\pi\pi)(s) = \frac{1}{\pi}PV\int_{s_{thr}}^{\infty} ds' \frac{\tan\delta_1(s')}{s'-s}\mathrm{Re}A(D\to\pi\pi)(s')$$

which has the solution (Omnes)

$$|A(D\to\pi\pi)(s)| = A(s_0)\cdot exp\{\frac{s-s_0}{\pi}PV\int_{4M_\pi^2}^{\infty} dz \frac{\delta_1(z)}{(z-s_0)(z-s)}\}$$

limit of no rescattering → large N_C

Omnes factor Ω; ***Large phases modify amplitude***



We correct large N_C / factorization by incorporating s-channel rescattering of the final states

Two-channel case

In the isospin-zero block there are both ππ and KK :

$$|A(D \to \pi\pi)(s)| = A(s_0) \cdot exp\{\frac{s - s_0}{\pi} PV \int_{4M_\pi^2}^{\infty} dz \frac{\delta_1(z)}{(z - s_0)(z - s)}\}$$

now becomes

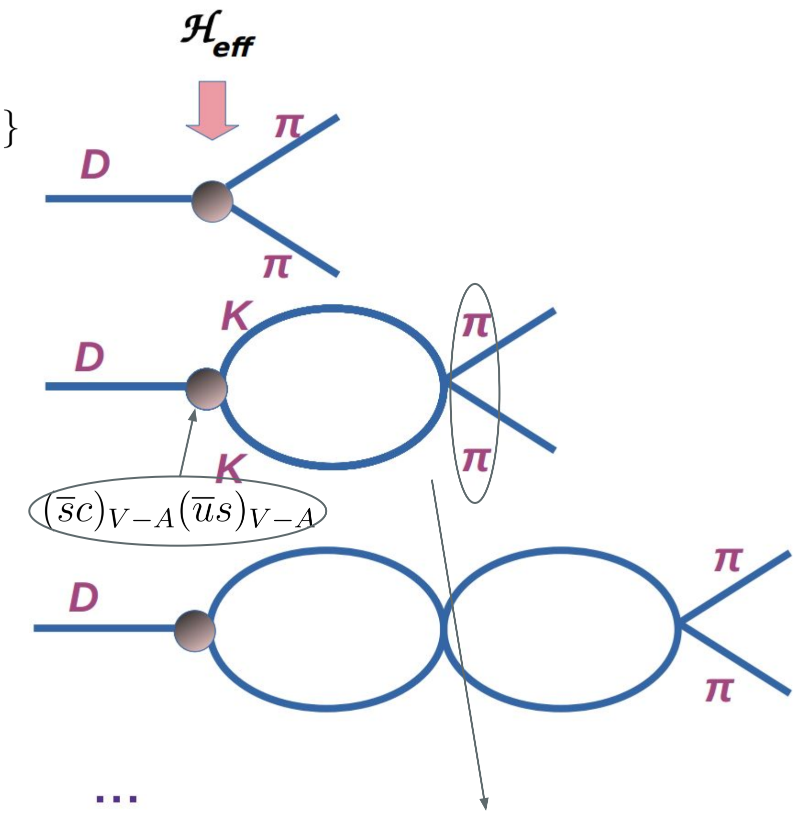
$$\begin{pmatrix} \mathcal{A}(D \to \pi\pi) \\ \mathcal{A}(D \to KK) \end{pmatrix} = \Omega \cdot \begin{pmatrix} \mathcal{A}_{(\text{large } N_C)}(D \to \pi\pi) \\ \mathcal{A}_{(\text{large } N_C)}(D \to KK) \end{pmatrix}$$

rescattering — no rescattering

where Ω is a 2-by-2 matrix

that has to be found **numerically**

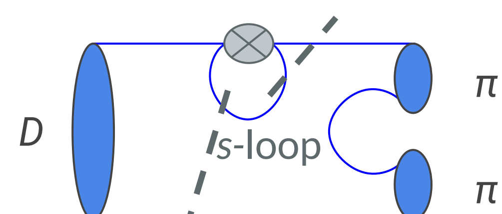
by solving the two-channel dispersion relation



- In the language of hadronic matrix elements:

  Non-diagonal Ω creates $\langle \pi\pi(I = 0)|Q_i^s|D\rangle \neq 0$

  $\langle KK(I = 0)|Q_i^d|D\rangle \neq 0$

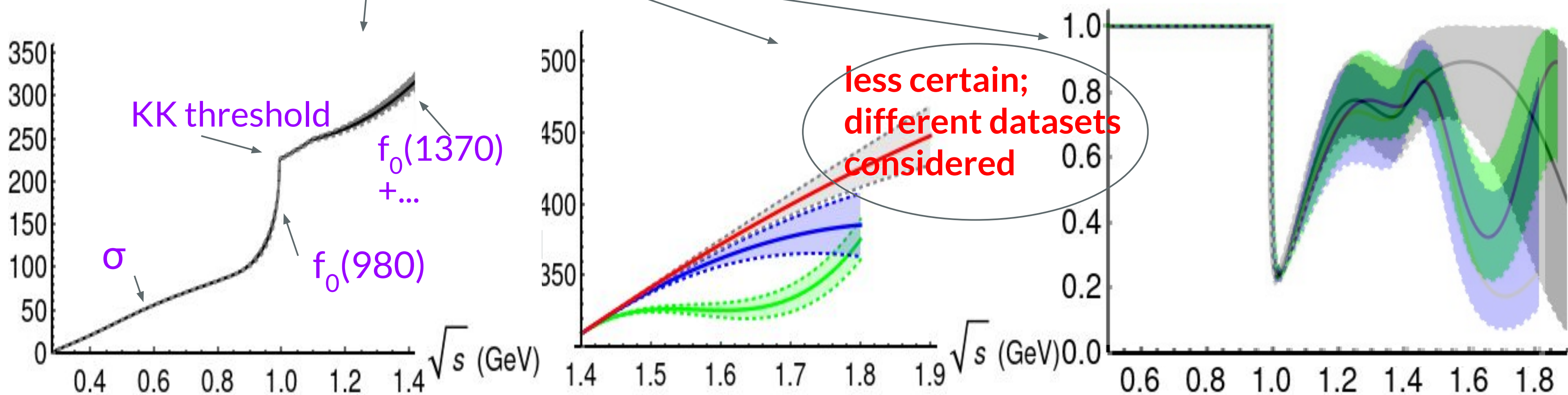


Long-distance penguins

Implementation of the strong rescattering

Isospin zero:

$$S_0 = \begin{pmatrix} \eta e^{i2\delta_1} & i\sqrt{1-\eta^2}e^{i(\delta_1+\delta_2)} \\ i\sqrt{1-\eta^2}e^{i(\delta_1+\delta_2)} & \eta e^{i2\delta_2} \end{pmatrix} = \begin{pmatrix} S_0(\pi\pi\to\pi\pi) & S_0(\pi\pi\to KK) \\ S_0(KK\to\pi\pi) & S_0(KK\to KK) \end{pmatrix}$$



[Kaminski, Pelaez, Yndurain '07, Garcia-Martin, Kaminski, Pelaez, Ruiz de Elvira, Yndurain '11, Pelaez, Rodas, Ruiz De Elvira '19]

- Data-driven parameterizations taking into account known resonances & other features
- Extrapolations for energies higher than 1.9 GeV

Isospins 1 and 2:

- Elastic ππ, KK rescattering
- No (enough) data available ⟶ use measured Br's of D+ decays

$$\begin{cases} A(D^+ \to \pi^+\pi^0) = \dfrac{\sqrt{3}}{2\sqrt{2}}|A_{\pi\pi}^{I=2}|e^{i\delta_{\pi\pi,2}} \\ A(D^+ \to \overline{K}^0 K^+) = |A_{KK}^{I=1}|e^{i\delta_{KK,1}} \end{cases}$$

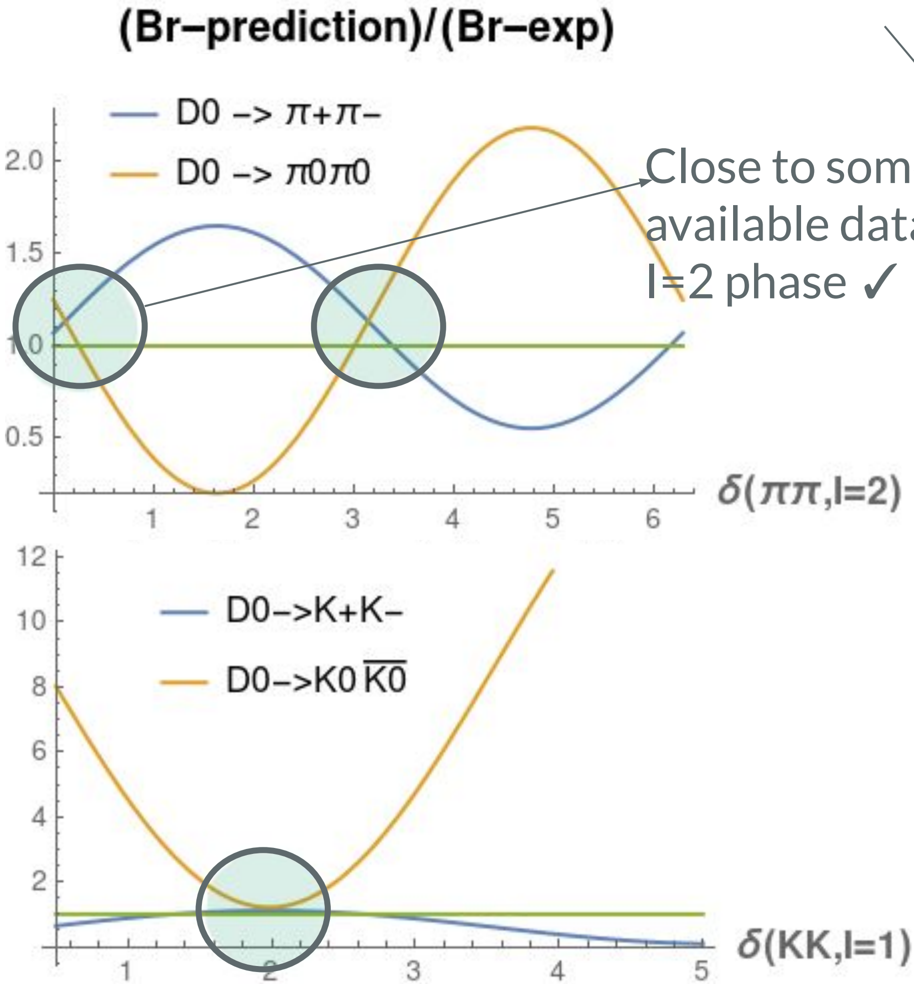
free

Sorting through the uncertainties

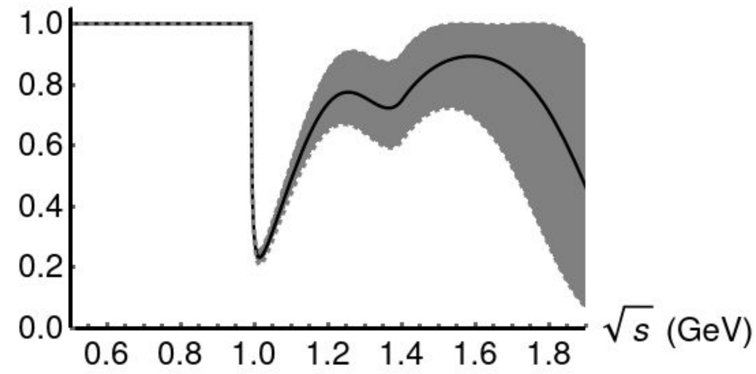
Solving the Omnes equations provides a full description of the decay amplitudes

→ *Select* among the strong rescattering input

the one that yields values close to exp. Br's *for all decay channels simultaneously*



Close to some available data for I=2 phase ✓



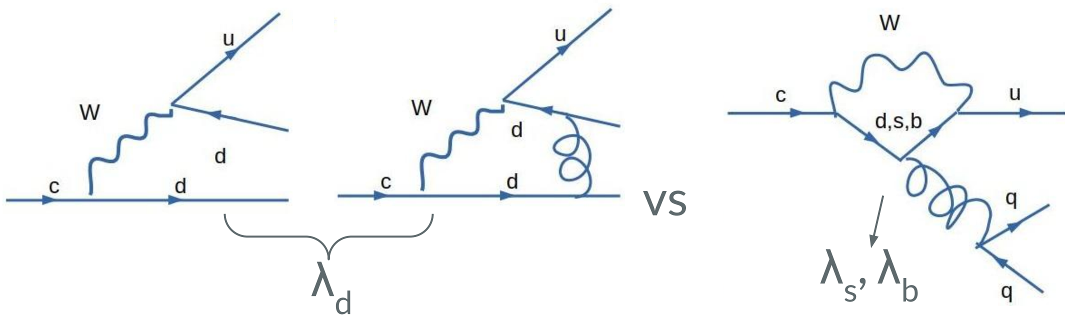
Only this I=0 inelasticity survives, giving an Omnes matrix like

$$\Omega_{I=0} = \begin{pmatrix} 0.58e^{1.8i} & 0.64e^{-1.7i} \\ 0.58e^{-1.4i} & 0.61e^{2.3i} \end{pmatrix}$$

→ **large rescattering between ππ and KK in the I=0 channel**

Sources of CP violation

At the quark level (full theory):



At the level of amplitudes:
Recall: ***different weak phases & strong phases needed***
For D→ ππ (similarly for D→KK):
One I=2 amplitude

$$\lambda_d \cdot \langle \pi\pi_{I=2} | (\bar{d}c)(\bar{u}d) | D \rangle$$

(current-current operators implied)

and several I=0 amplitudes

$$\lambda_d \cdot \langle \pi\pi_{I=0} | (\bar{d}c)(\bar{u}d) | D \rangle + \lambda_s \cdot \langle \pi\pi_{I=0} | (\bar{s}c)(\bar{u}s) | D \rangle - \lambda_b \cdot \langle \pi\pi_{I=0} | \text{penguin operators} | D \rangle$$

Long-distance penguin

Short-distance penguin
(**significant for Q_6**
operator-annihilation topology)

If ππ did not rescatter to KK:

$\langle \pi\pi_{I=0} | (\bar{s}c)(\bar{u}s) | D \rangle = 0$ AND

$\arg\langle \pi\pi_{I=0} | \text{penguin operators} | D \rangle = \arg\langle \pi\pi_{I=0} | (\bar{d}c)(\bar{u}d) | D \rangle$ (Watson's theorem)

→ Only source ***would be*** interference of I=2 vs I=0 short-distance penguin

Instead: more sources of CP violation now
BUT cancellations between different CPV sources turn out small

Results: CP asymmetry predictions

We find $\Delta a_{CP}^{dir} \approx -5 \cdot 10^{-4}$ ~⅓ of the measured value!

while $a_{CP}^{dir}(\pi^+\pi^-) \approx 3 \cdot 10^{-4}$ $a_{CP}^{dir}(K^+K^-) \approx -2 \cdot 10^{-4}$

and similar levels predicted for $\pi^0\pi^0,\ K^0\overline{K^0}$

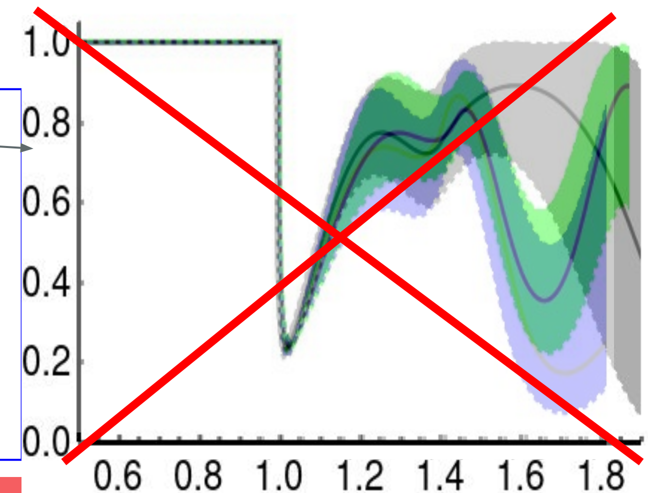
- SU(3) breaking manifested through differences in ππ, KK channels; comparable to known levels
- Expressed in terms of two amplitudes: $\mathcal{A}(D^0 \to f) = A(f) + i r_{CKM} B(f)$

$$a_{CP}^{dir} \approx 2 \underbrace{r_{CKM}}_{\sim 6\cdot 10^{-4}} \underbrace{\frac{|B(f)|}{|A(f)|}}_{\sim 1/3} \cdot \underbrace{\sin \arg \frac{A(f)}{B(f)}}_{\sim 1}$$

→ no sufficient enhancement of the CP-odd amplitudes

[preliminary]

Also: if we want to *bypass* the rescattering **inelasticity**, we still manage to constrain the aCP coming from the **interference of isospin-zero amplitudes** to a **few * 10^{-4}** while the aCP from isospin-2 interference with isospin-0 would require **unnaturally large Omnes matrix elements**



Experiment

based on *Phys. Rev. Lett. 131, 091802*

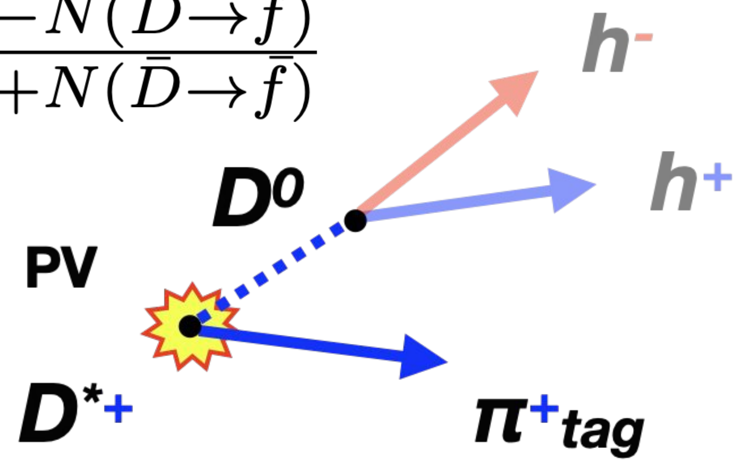
How can you measure ACP?

- Choose a flavour-specific decay such as **D*+→ D0π+** (prompt) to determine whether the meson is a D0 or D0bar
- The **raw asymmetry** (A) in **D0 → K-K+** decays

$$A(D \to f) = \frac{N(D\to f)-N(\bar{D}\to \bar{f})}{N(D\to f)+N(\bar{D}\to \bar{f})}$$

includes both physics and detector effects:

$$A = A_{CP} + A_P + A_D$$



NUISANCE ASYMMETRIES:

Production asymmetry of D^{*+}

Detection asymmetry of π^{+}_{tag}

$$\sigma(pp \to D^{*+}X) \neq \sigma(pp \to D^{*-}X) \qquad \epsilon(\pi^+) \neq \epsilon(\pi^-)$$

How can you measure ACP?

- Choose a flavour-specific decay such as **D*+→ D0π+** (prompt) to determine

$$N^{\pm} \propto \sigma^{\pm} \epsilon^{\pm} \Gamma^{\pm}$$

$$\sigma^{\pm} = \sigma(pp \to D^{*\pm}) \propto (1 \pm A_P(D^{*+}))$$

$$\epsilon^{\pm} = \epsilon(\pi^{\pm}) \propto (1 \pm A_D(\pi^{+}))$$

$$\Gamma^{\pm} = \Gamma(\overset{(-)}{D^0} \to K^-K^+) \propto (1 \pm A_{CP})$$

$$A = \frac{N^+ - N^-}{N^+ + N^-}$$

$$= \frac{(1+A_P)(1+A_D)(1+A_{CP}) - (1-A_P)(1-A_D)(1-A_{CP})}{(1+A_P)(1+A_D)(1+A_{CP}) + (1-A_P)(1-A_D)(1-A_{CP})}$$

$$= A_P + A_D + A_{CP} + \mathcal{O}(A_P A_D^2)$$

$$\sigma(pp \to D^{*+}X) \neq \sigma(pp \to D^{*-}X) \qquad \epsilon(\pi^+) \neq \epsilon(\pi^-)$$

Strategy for ACP(D0→K-K+)

particles with same color must have identical kinematic distributions
➔ weighting!

- **Prompt** D0→K-K+ collected during Run-2
- Two methods to cancel **nuisance** asymmetries:
  - **D+ decays**, same used in *Run-1 analysis* (CD+)
  - **Ds+ decays**, new! (CDs+)
- Correct raw asymmetry *A* using samples of *Cabibbo-favoured* (CF) *D0*, *D+* and *D(s)+* decays (where CPV can be neglected):

CD+:

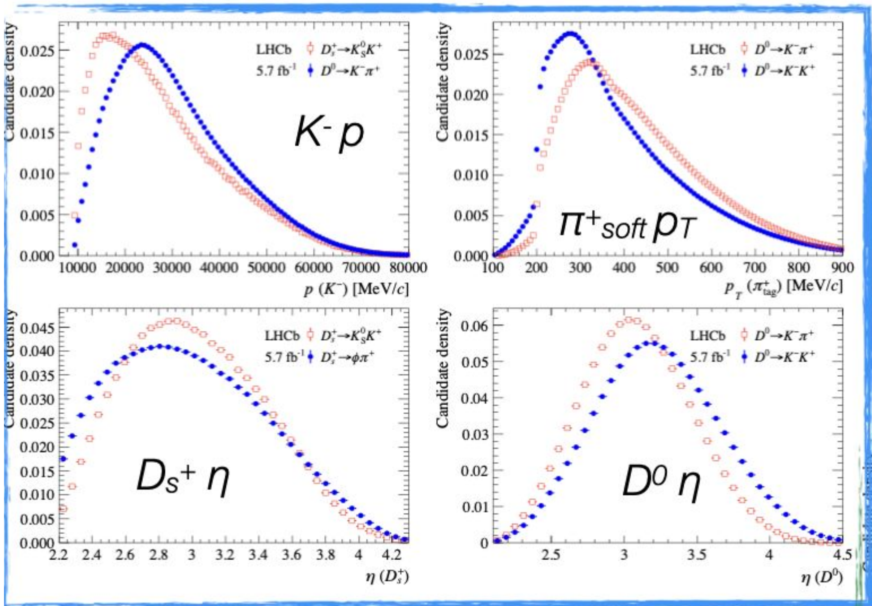
$$A_{CP}(D^0 \to K^- K^+) = A(D^{*+} \to (D^0 \to K^- K^+)\,\pi^+_{soft}) - A(D^{*+} \to (D^0 \to K^- \pi^+)\,\pi^+_{soft}) + A(D^+ \to K^- \pi^+ \pi^+) - \left[A(D^+ \to \bar{K}^0\,\pi^+) - A(\bar{K}^0)\right]$$

CDs+:

$$A_{CP}(D^0 \to K^- K^+) = A(D^{*+} \to (D^0 \to K^- K^+)\,\pi^+_{soft}) - A(D^{*+} \to (D^0 \to K^- \pi^+)\,\pi^+_{soft}) + A(D^+_s \to \phi \pi^+) - \left[A(D^+_s \to \bar{K}^0\,K^+) - A(\bar{K}^0)\right]$$

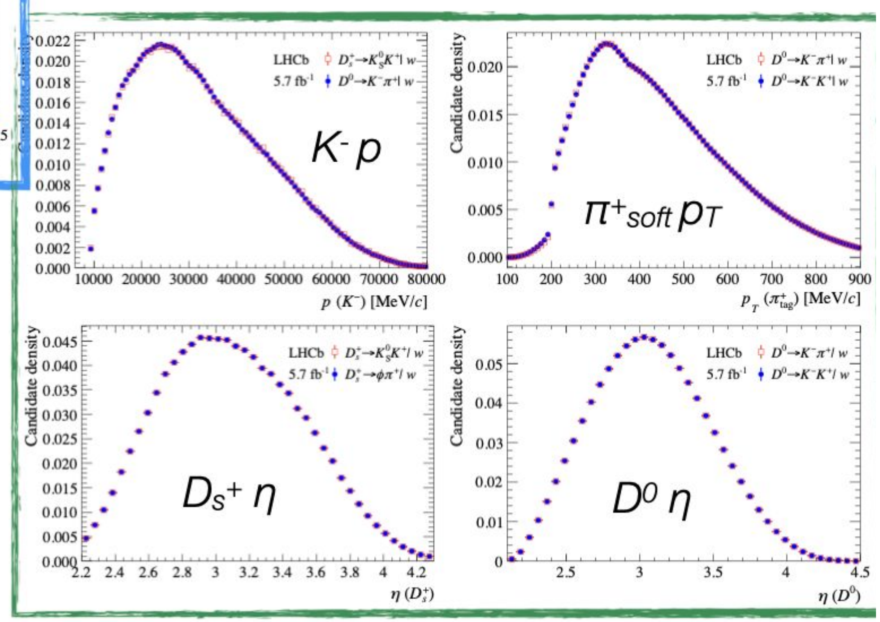
Weighting procedure

$$A_{det}(h^+) = \int A_{det}(\vec{p} \mid h^+) D(\vec{p}) d\vec{p}$$



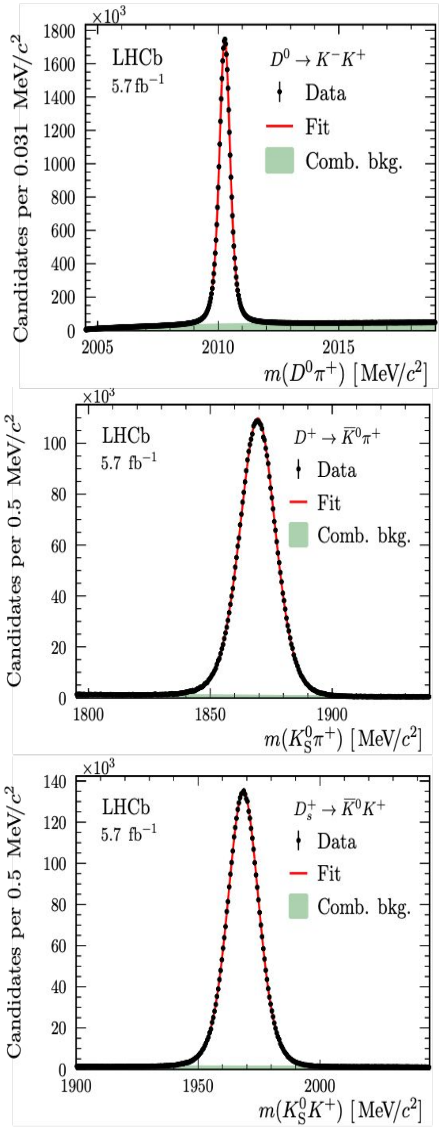
BEFORE

AFTER



Yields and impact of the weighting

Decay mode	Signal yield $[10^6]$		Red. factor	
	C_{D^+}	$C_{D_s^+}$	C_{D^+}	$C_{D_s^+}$
$D^0 \to K^- K^+$	37	37	0.75	0.75
$D^0 \to K^- \pi^+$	58	56	0.35	0.75
$D^+ \to K^- \pi^+ \pi^+$	188	–	0.25	–
$D^+ \to \overline{K}^0 \pi^+$	6	–	0.25	–
$D_s^+ \to \phi \pi^+$	–	43	–	0.55
$D_s^+ \to \overline{K}^0 K^+$	–	5	–	0.70



- Statistical precision on ACP limited by the **calibration samples** after the kinematic weighting, in particular by *D+→KSh+* decays

$\Delta A_{CP} = A_{CP}(D^0 \rightarrow K^- K^+) - A_{CP}(D^0 \rightarrow \pi^- \pi^+)$
$= (-15.4 \pm 2.9) \times 10^{-4}$
[Phys. Rev. Lett. 122, 211803]

Results

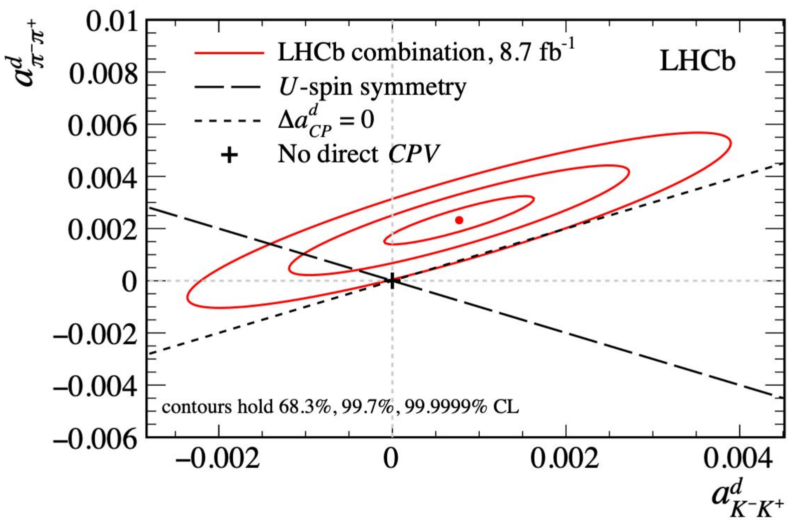
- The **combination** of the two approaches yields:

$$\mathcal{A}_{CP}(K^- K^+) = [6.8 \pm 5.4\,(\text{stat}) \pm 1.6\,(\text{syst})] \times 10^{-4},$$

- **Run1+Run2** measurements are combined and CP violation in D0→π-π+ is extracted considering the observed CPV in **ΔACP**

$$a^d_{K^-K^+} = (\ 7.7 \pm 5.7) \times 10^{-4}$$
$$a^d_{\pi^-\pi^+} = (23.2 \pm 6.1) \times 10^{-4}$$

with = 88%



Discussion

What is going on?

$$a^d_{\pi^-\pi^+} = (23.2 \pm 6.1) \times 10^{-4}$$ LHCb
$$\approx 3 \cdot 10^{-4}$$ Pich, ES, Vale Silva '23
$$\lesssim 5 \cdot 10^{-4}$$ &'24 (in preparation)

The discrepancy between theory and exp persists in $D^0 \to \pi^+\pi^-$

Independent theoretical determinations agree in this aspect: LCSRs [Khodjamirian, Petrov '17 + Lenz, Piscopo, Rusov '23] ✔, U-spin breaking arguments [Schacht '23] ✔

Could something be missing from the theory prediction?

- 3rd channel in isospin zero? e.g. $\rho\rho$, $a_1\,\pi$ ($\to 4\pi$)

  → rescattering *expected to be small* (but maybe important for aCP?)

  → no data available as required for dispersion relations - would require model dependence

- Could try to *understand better* I=2 (we don't *predict* the rescattering but use exp. Br's)

  → no known resonances & most likely elastic $\pi\pi \to \pi\pi$

Theoretical cross-checks:

- More theoretical determinations of related channels: **D→3π** (could highlight the *enhancement of aCP from some resonance*), **D→ππμμ**
- Address indirect CPV theoretically? (could shed light into underlying long-distance dynamics)

Experimental cross-checks of studied channels (see next slide):

- $\pi^0\pi^0,\ K^0\overline{K^0}$ already theoretically calculated [see also Nierste, Schacht '15]

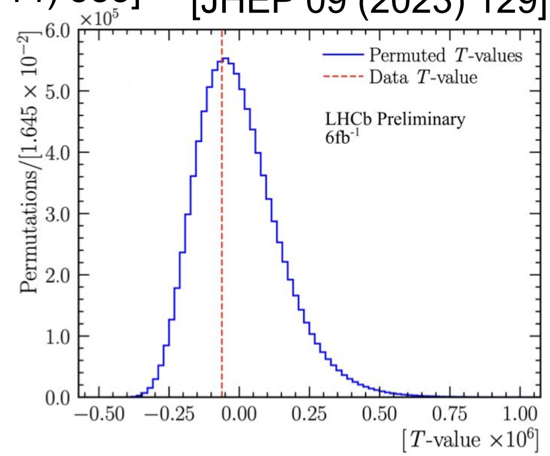
NP?

- Z' model breaking U-spin, see [Hiller et al. '23] also [Lenz, Rusov et al. '19]

Experimental “cross-checks”

- **D→3π**: statistic tests [PLB 728 (2014) 585] [JHEP 09 (2023) 129]
  - (2014-Run1) no evidence for CPV in **D+→π-π+π+** (3 M)
  - (2023-Run2) no evidence for CPV in **D0→πππ0** (2.4 M)

  ongoing work for D+→π-π+π+ with Run2 data…
  Upper limit on ACP? Not trivial!
  Several millions of decay candidates (precise results could be achieved) but experimentally challenging!



- **D0→KSKS:**
  no evidence for ACP so far (uncertainties ~ percent)
  measured by LHCb, Belle and recently CMS

[PRD 104 (2021) 3, L031102]
[CMS-BPH-23-005]

- **D0→π0π0:**
  Measurement from Belle [PRL 112 (2014) 211601]
  Uncertainty of 6x10-3, ~10 times bigger than ACP(D0→hh)
  D0 vertex non-reconstructible (not favourable for LHCb maybe in U2…)

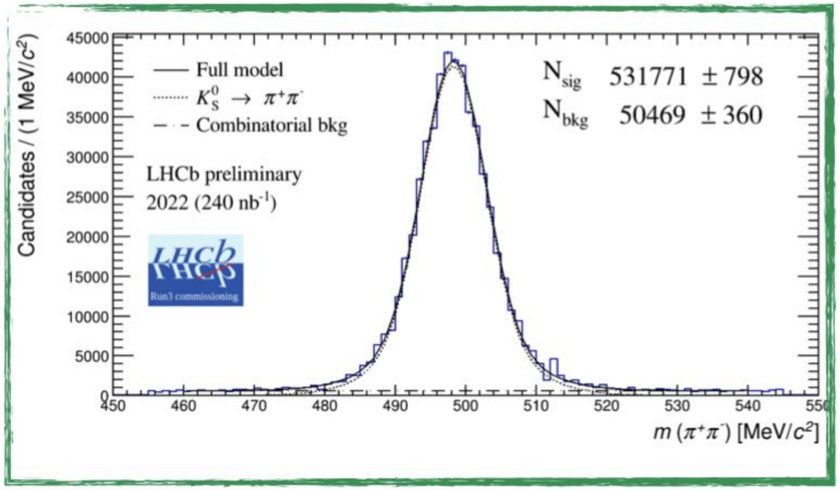
How can we do better?

NB: ACP(D0⟶hh) measurement limited by the calibration samples to control nuisance asymmetries

- **Run1+Run2** data:
  - ACP(D0->hh): *new* strategy *ongoing* with D0->KSpipi decays

$$A_{CP}(D^0 \to K^-K^+) = +A(D^{*+} \to (D^0 \to K^-K^+)\,\pi^+_{soft}) - [\,A(D^{*+} \to (D^0 \to \overline{K}^0\pi^-\pi^+)\,\pi^+_{soft}) - A(\overline{K}^0) - A_{det}(\pi^-\pi^+)\,]$$

- Upcoming **Run3** data:
  - **x4** statistics (luminosity)
  - *new* **online** selections, eg. Hlt1 on **Ks** decays
  - Studies ongoing for *new* **strategies**

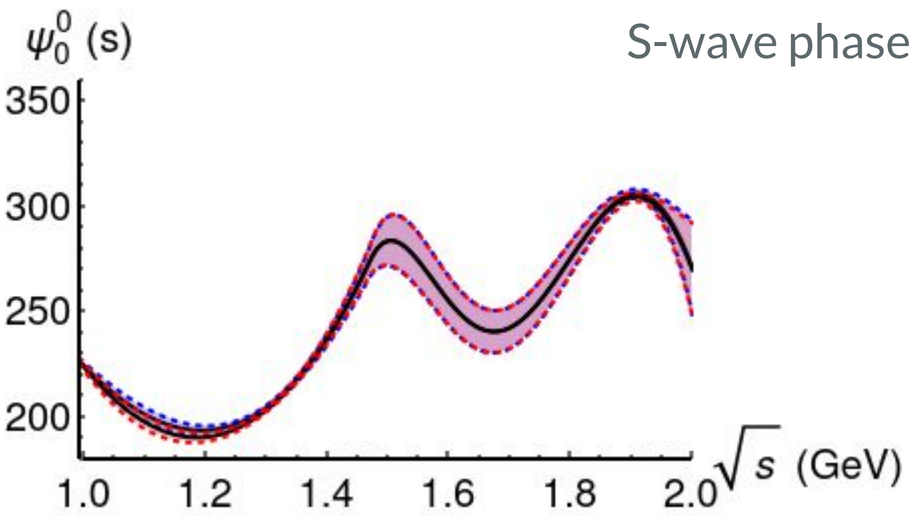


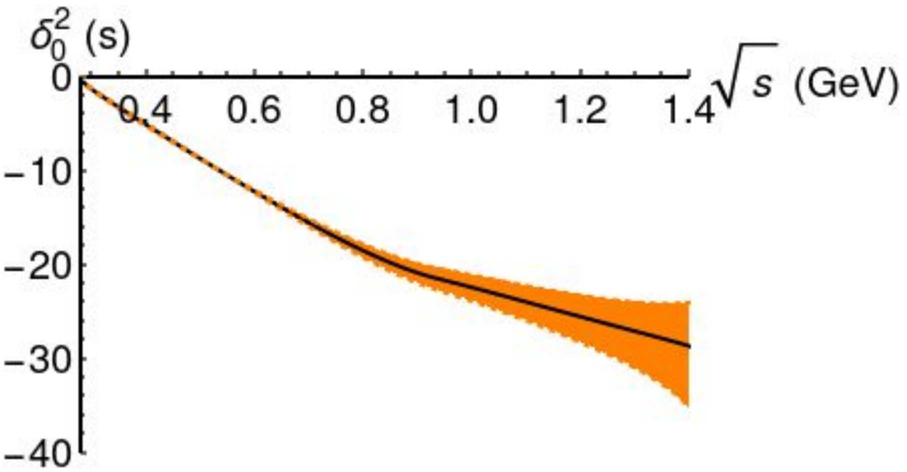
Thank you!

BACKUP (theory)

More rescattering

S-wave phase-shifts





Isospin-zero ππ+KK phase

Fewer uncertainties

Using only this results in

$$0 < a_{CP}(\pi\pi)(0-0) \lesssim 5 \cdot 10^{-4}$$

$$-3 \cdot 10^{-4} \lesssim a_{CP}(KK)(0-0) < 0$$

KK in I=1: not available

Isospin-two ππ phase

Elastic - admits Omnes solution

$$|A_{I=2}(D \to \pi\pi)(s)| = A_{I=2}(s_0) \times \underbrace{exp\{\frac{s-s_0}{\pi} PV \int_{4M_\pi^2}^{\infty} dz \frac{\delta_0^2(z)}{(z-s_0)(z-s)}\}}_{\text{Omnès factor } \Omega}$$

which at infinity behaves as

$$\Omega(s) \sim \frac{1}{s^n}, \quad n = \frac{\delta_0^2(\infty) - \delta_0^2(4m_\pi^2)}{\pi}$$

and has to go to zero

→ phase has to go to positive multiples of π

Naive estimate of final-state-interaction effects

We can write [Bauer, Stech, Wirbel '86]

$$\begin{pmatrix} A^{I=0}_{\pi\pi} \\ A^{I=0}_{KK} \end{pmatrix} = S_S^{1/2} \cdot \begin{pmatrix} A^{I=0}_{\pi\pi,\text{bare}} \\ A^{I=0}_{KK,\text{bare}} \end{pmatrix}$$

$$S_S = \begin{pmatrix} \eta e^{i2\delta_1} & i\sqrt{1-\eta^2} e^{i(\delta_1+\delta_2)} \\ i\sqrt{1-\eta^2} e^{i(\delta_1+\delta_2)} & \eta e^{i2\delta_2} \end{pmatrix}$$

where the bare amplitudes come from factorization (no strong phases)

This reproduces correctly Watson's theorem in the limit of elastic rescattering

What S-matrix unitarity gives:

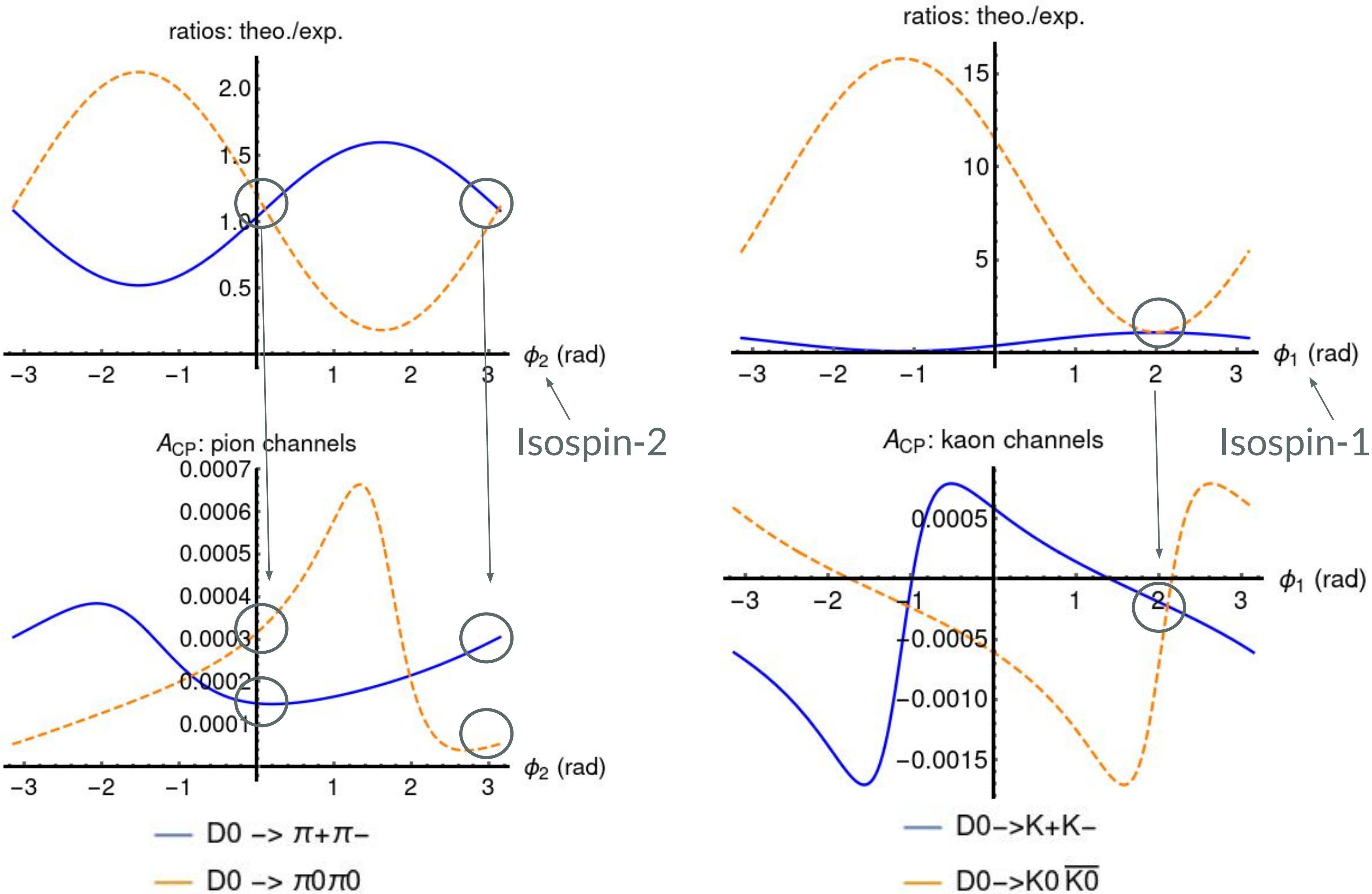
$$\begin{pmatrix} A^{I=0}_{\pi\pi} \\ A^{I=0}_{KK} \end{pmatrix} = S_S \cdot \begin{pmatrix} (A^{I=0}_{\pi\pi})^* \\ (A^{I=0}_{KK})^* \end{pmatrix}$$

→ No direct solution for the amplitudes; can relate them to the rescattering phases

$$arg A^{I=0}_{\pi\pi} = \delta_1 + arccos \sqrt{\frac{(1+\eta)^2 - \left(\frac{|A^{I=0}_{KK}|}{|A^{I=0}_{\pi\pi}|}\right)^2 (1-\eta^2)}{4\eta}}$$

$$arg A^{I=0}_{KK} = \delta_2 + arccos \sqrt{\frac{(1+\eta)^2 - \left(\frac{|A^{I=0}_{\pi\pi}|}{|A^{I=0}_{KK}|}\right)^2 (1-\eta^2)}{4\eta}}$$

Br's and ACP's as functions of the free phases

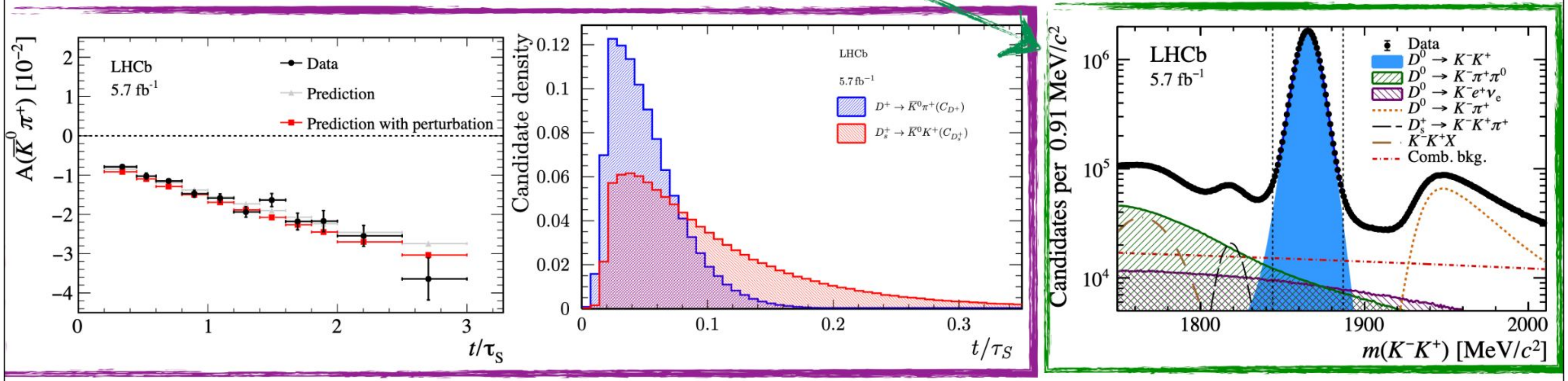


BACKUP (experiment)

ACP(Do→KK) - systematic uncertainties

Source	C_{D^+} $[10^{-4}]$	$C_{D_s^+}$ $[10^{-4}]$	ρ
Neutral kaon asym.	0.6	1.3	1.00
Secondary decays	0.6	0.3	/
Peaking backgrounds	0.3	0.4	0.74
Fit model	1.1	1.0	0.05
Kinematic diff.	0.8	0.4	/
Charged kaon asym.	/	1.0	/
Total systematic	1.6	2.0	0.28
Statistical	8.8	6.7	0.05

- neutral kaon asymmetry: accuracy tested with a data-driven approach
- secondary decays: presence of *D* meson from semi-leptonic *B* decays estimated
- peaking backgrounds: impact estimated by fits to the *m(KK)* invariant mass
- fit model: alternative signal and background shapes evaluated
- kinematic difference: residual difference from weighting procedure
- charged kaon asymmetry: K^-K^+ asymmetries from $D_s^+ \to \phi\pi^+$ decays



[Updated list]

Charm CPV + mixing results

CPV in decay

2-body

$\Delta A_{CP}(hh)$ and $A_{CP}(hh)$:
PRL 108 (2012) 111602
PLB 723 (2013) 33
JHEP 07 (2014) 041
PRL 116 (2016) 191601
PLB 767 (2017) 177
PRL 122 (2019) 211803
PRL 131 (2023) 091802

$D^0 \to K_S K_S$:
JHEP 10 (2015) 055
JHEP 11 (2018) 048
PRD 104 (2021) L031102

$D_{(s)}^+ \to \eta' \pi^+$:
PLB 771 (2017) 2
PAPER-2021-051

$D_{(s)}^+ \to h^+\pi^0, h^+\eta$:
JHEP 06 (2021) 019

$D_{(s)}^+ \to K_S h^+$:
JHEP 06 (2013) 112
JHEP 10 (2014) 025
PRL 122 (2019) 19180

multi-body

$D^0 \to K^-K^+\pi^-\pi^+, \pi^-\pi^+\pi^-\pi^+$:
PLB 726 (2013) 623 (SCP)
JHEP 10 (2014) 005 (T-odd)
PLB 769 (2017) 345 (ET)
JHEP 02 (2019) 126 (AmAn)

$\Xi_c^+ \to pK^-\pi^+$ (SCP, KNN):
PRD 102 (2020) 071101(R)

$D^+ \to K^-K^+\pi^+$:
PRD 84 (2011) 112008 (SCP)
JHEP 06 (2013) 112

$D_{(s)}^+ \to K^-K^+K^+$:
JHEP 07 (2023) 067

$D^+ \to \pi^+\pi^-\pi^+$:
PLB 728 (2014) 585 (SCP)

$D^0 \to \pi^+\pi^-\pi^0$:
PLB 740 (2015) 158 (ET)
JHEP 09 (2023) 129 (ET)

$\Lambda_c^+ \to ph^+h^-$:
JHEP 03 (2018) 182 (DACP)

rare

$D^0 \to h^+h^-\mu^+\mu^-$:
PRL 128 (2022) 221801

mixing + mixing-induced CPV

2-body

$A_\Gamma(hh)$:
JHEP 1204 (2012) 129
PRL 112 (2014) 041801
JHEP 04 (2015) 043
PRL 118 (2017) 261803
PRD 101 (2020) 012005
PRD 104 (2021) 072010

$y_{CP}(hh)$:
PRL 122 (2019) 011802
PRD 105, (2022) 092013

WS $D^0 \to K^+\pi^-$:
PRL 110 (2013) 101802
PRL 111 (2013) 251801
PRD 95 (2017) 052004
PRD 97 (2018) 031101

multi-body

$D^0 \to K^-\pi^+\pi^-\pi^+$:
PRL 116 (2016) 241801

$D^0 \to K_S\pi^+\pi^-$:
JHEP 04 (2016) 033
PRL 122 (2019) 231802
PRL 127 (2021) 111801
LHCB-PAPER-2022-020