Non-Relativistic Stochastic Hydrodynamics

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Hydrodynamics is an effective theory for many-body systems, operating on a scale somewhere in between system size and inter-particle distances, and assumes a local equilibrium. The degrees of freedom are conserved quantities such as the particle, total energy and momentum densities in a simple non-relativistic case. Conservation equations govern their dynamics, containing only the equation of state and transport coefficients as medium-dependent input parameters. One distinguishes between ideal hydrodynamics describing systems in equilibrium, and dissipative hydrodynamics, in which fluxes are coupled to non-equilibrium gradients and lead to entropy increase.

(Relativistic) hydrodynamics is used as a tool to describe heavy ion collisions, in which the system undergoes an in general non-equilibrium trajectory in the phase diagram of QCD, possibly coming close or passing through a proposed QCD critical endpoint. Due to being driven by stochastic fluctuations, the simulation of such dynamic second-order phase transitions makes a stochastic hydrodynamic description necessary. In the following, I will give an introduction to the concept of stochastic hydrodynamics as well as an outline of my talk.

Since stochastic fluctuations are the microscopic cause for dissipation, both are strongly connected. An example of this is the description of Brownian motion, in which a fluctuation-dissipation relation has to be fulfilled in order for the system to equilibrate correctly. The classic hydrodynamic equations of motion are deterministic, neglecting fluctuations, which should be present even in equilibrium, and are in fact only valid as a fluctuation average. Inconsistencies especially arise e.g. when evaluating two-point functions, as is the case with Brownian motion, because the underlying fluctuation-dissipation relations are violated.

In stochastic hydrodynamics, dissipative fluxes present in the gradient expansion are replaced by stochastic fluxes introducing fluctuations. However, the direct tuning of the stochastic terms is in general non-trivial. Our present numerical setup is able to simulate stochastic, non-relativistic hydrodynamics in a box and employs the well-established Kurganov-Tadmor scheme for the simulation of ideal hydrodynamics, with stochastic fluxes being sampled using a Metropolis-Hastings algorithm. The setup is particularly effective, since microscopic assumptions about the noises are not necessary and unphysical entropy generation is prevented. However, non-trivial correlations between the stochastic noises have to be introduced in order to attain the correct continuum limit.

In my talk, I will first give an introduction to non-relativistic stochastic hydrodynamics as well as the numerical setup and then present both macroscopic and microscopic test cases for equilibration properties and fluctuation-dissipation relations. I give an outlook on renormalization of transport coefficients and future directions, as well as relativistic hydrodynamics and applications in the context of heavy-ion collisions.