Mixing and CP violation

Time evolution of neutral mesons
CP violation
Constraining the CKM triangle: the angle γ

Neutral mesons states

$$|\psi\rangle = a(t)\begin{pmatrix} |M^0\rangle \\ 0 \end{pmatrix} + b(t)\begin{pmatrix} 0 \\ |\overline{M}^0\rangle \end{pmatrix}$$

- Consider states of neutral mesons
  - These are orthogonal flavour eigenstates
- Time evolution follows Schrödinger equation
- Effective Hamiltonian is a combination of two Hermitian 2×2 matrices
  - Mass and decay matrices
  - Can interpret H_0 as strong interaction
  - H_{int} represents weak interaction (think W boson)

$$i\hbar\frac{d}{dt}|\psi\rangle = \mathcal{H}|\psi\rangle$$

$$\mathcal{H} = \mathcal{M} - \frac{i}{2}\Gamma$$

$$\frac{d}{dt}\langle\psi|\psi\rangle = -\frac{1}{\hbar}\langle\psi|\Gamma|\psi\rangle$$

Show!

$$\mathcal{H} = H_0 + H_{\mathrm{int}} + \sum_n \frac{H_{\mathrm{int}}|n\rangle\langle n|H_{\mathrm{int}}}{m_0 - E_n}$$

Hamiltonian eigenstates

$$\mathcal{H}|M_{1,2}\rangle = \lambda_{1,2}|M_{1,2}\rangle$$

- Hamiltonian eigenstates follow time evolution according to eigenvalue equation

$$|M_{1,2}(t)\rangle = e^{-im_{1,2}t-\Gamma_{1,2}t/2}|M_{1,2}(0)\rangle$$

- Hamiltonian eigenstates can be expressed as linear combinations of flavour eigenstates with normalised coefficients
- CPT requires the coefficients to be the same for both eigenstates
- Time evolution follows from $M_{1,2}$ equations (next slide)

$$\begin{aligned} |M_1\rangle &= p_1|M^0\rangle + q_1|\overline{M}^0\rangle \\ |M_2\rangle &= p_2|M^0\rangle - q_2|\overline{M}^0\rangle \end{aligned}$$

$$|p_1|^2 + |q_1|^2 = 1 = |p_2|^2 + |q_2|^2$$

?

- Exercise: Evaluate the orthogonality of the Hamiltonian eigenstates and interpret the result: $\langle M_1|M_2\rangle$

$$|M_{1,2}\rangle = p|M^0\rangle \pm q|\overline{M}^0\rangle$$

Time evolution

$$|M^0(t)\rangle = \frac{1}{2p}\Big[\left(e^{-im_1t-\Gamma_1t/2} + e^{-im_2t-\Gamma_2t/2}\right) p|M^0\rangle + \left(e^{-im_1t-\Gamma_1t/2} - e^{-im_2t-\Gamma_2t/2}\right) q|\overline{M}^0\rangle\Big]$$

- Time evolution can be expressed with functions, $f_\pm(t)$, which encapsulate time dependence
  - Uses average mass, m, and decay width, Γ
  - Uses dimensionless quantities x and y, which link to the difference in the eigenvalues for $m_{1,2}$ and $\Gamma_{1,2}$

$$|M^0(t)\rangle = f_+(t)|M^0\rangle + \frac{q}{p}f_-(t)|\overline{M}^0\rangle$$

$$|\overline{M}^0(t)\rangle = \frac{p}{q}f_-(t)|M^0\rangle + f_+(t)|\overline{M}^0\rangle$$

$$f_\pm(t) = \frac{1}{2}e^{-imt-\Gamma t/2}\left(e^{(ix+y)\Gamma t/2} \pm e^{-(ix+y)\Gamma t/2}\right)$$

?
- Exercise: derive this
- Various other variants exist in literature

$$m \equiv (m_1 + m_2)/2 \qquad \Gamma \equiv (\Gamma_1 + \Gamma_2)/2$$

$$x \equiv \frac{\Delta m}{\Gamma} \qquad y \equiv \frac{\Delta\Gamma}{2\Gamma}$$

Meson oscillations

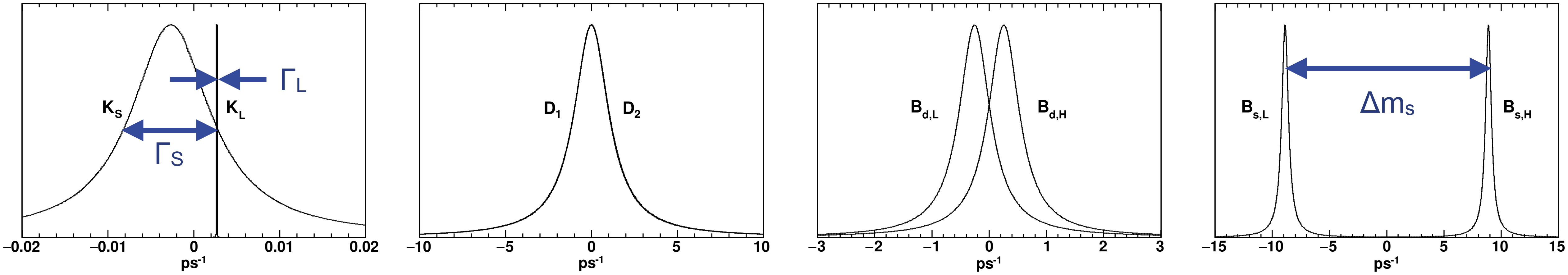
- Observable transition probabilities follow directly from previous equations
- Mass difference, x, related to sinusoidal oscillations
- Width difference alters exponential decay
- Difference in meson-antimeson and antimeson-meson transition solely linked to |*q*/*p*|

$$
\begin{aligned}
P(M^0 \to M^0, t) = P(\overline{M}^0 \to \overline{M}^0, t) &= |f_+(t)|^2 \\
&= \frac{1}{2} e^{-\Gamma t}[\cosh(y\Gamma t) + \cos(x\Gamma t)] \\
P(M^0 \to \overline{M}^0, t) &= \left|\frac{q}{p}\right|^2 |f_-(t)|^2 \\
&= \frac{1}{2}\left|\frac{q}{p}\right|^2 e^{-\Gamma t}[\cosh(y\Gamma t) - \cos(x\Gamma t)] \\
P(\overline{M}^0 \to M^0, t) &= \left|\frac{p}{q}\right|^2 |f_-(t)|^2 \\
&= \frac{1}{2}\left|\frac{p}{q}\right|^2 e^{-\Gamma t}[\cosh(y\Gamma t) - \cos(x\Gamma t)]
\end{aligned}
$$

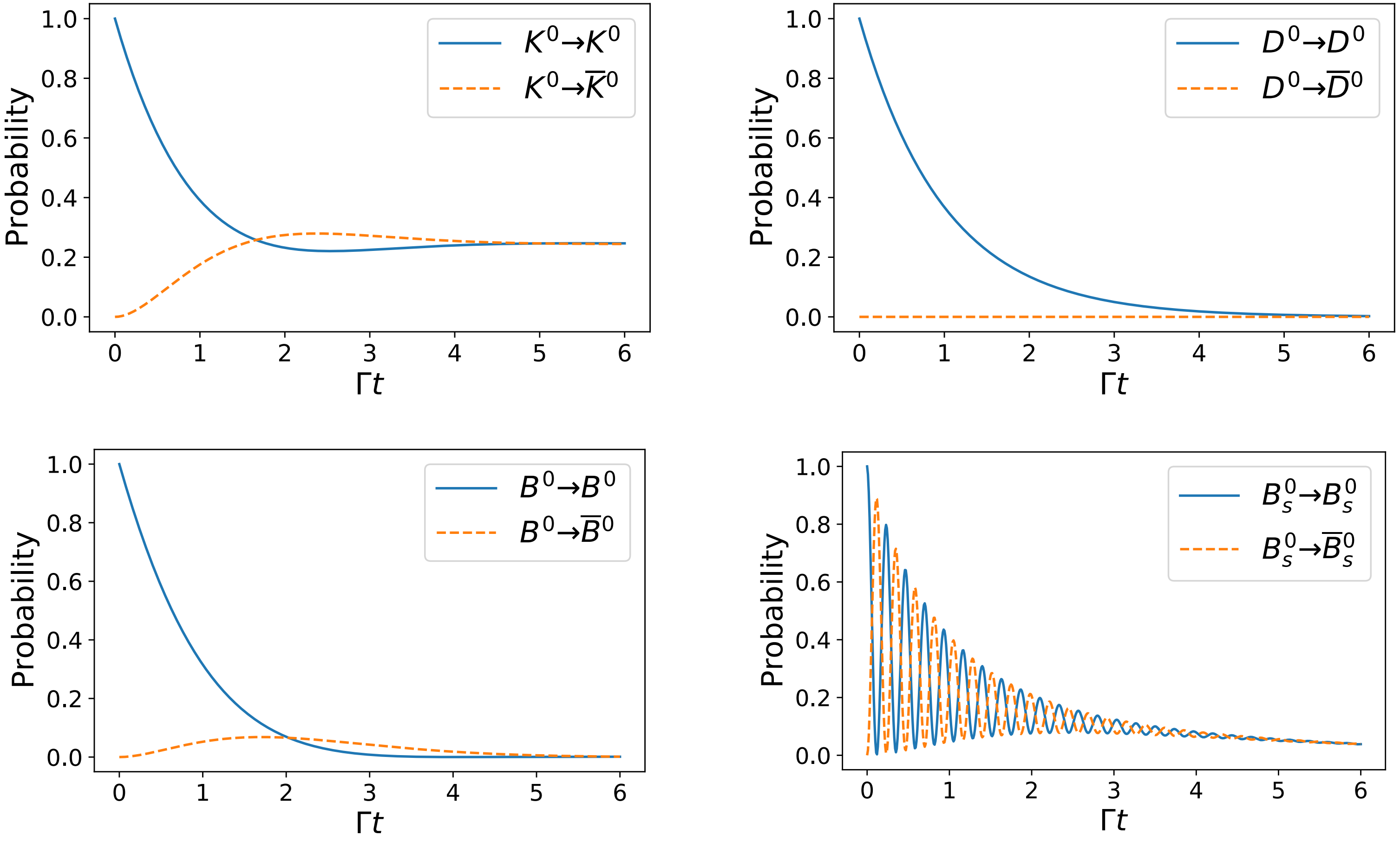
Mixing and mesons

$$P(M^0 \to \overline{M}^0, t) = \frac{1}{2}\left|\frac{q}{p}\right|^2 e^{-\Gamma t}[\cosh(y\Gamma t) - \cos(x\Gamma t)]$$

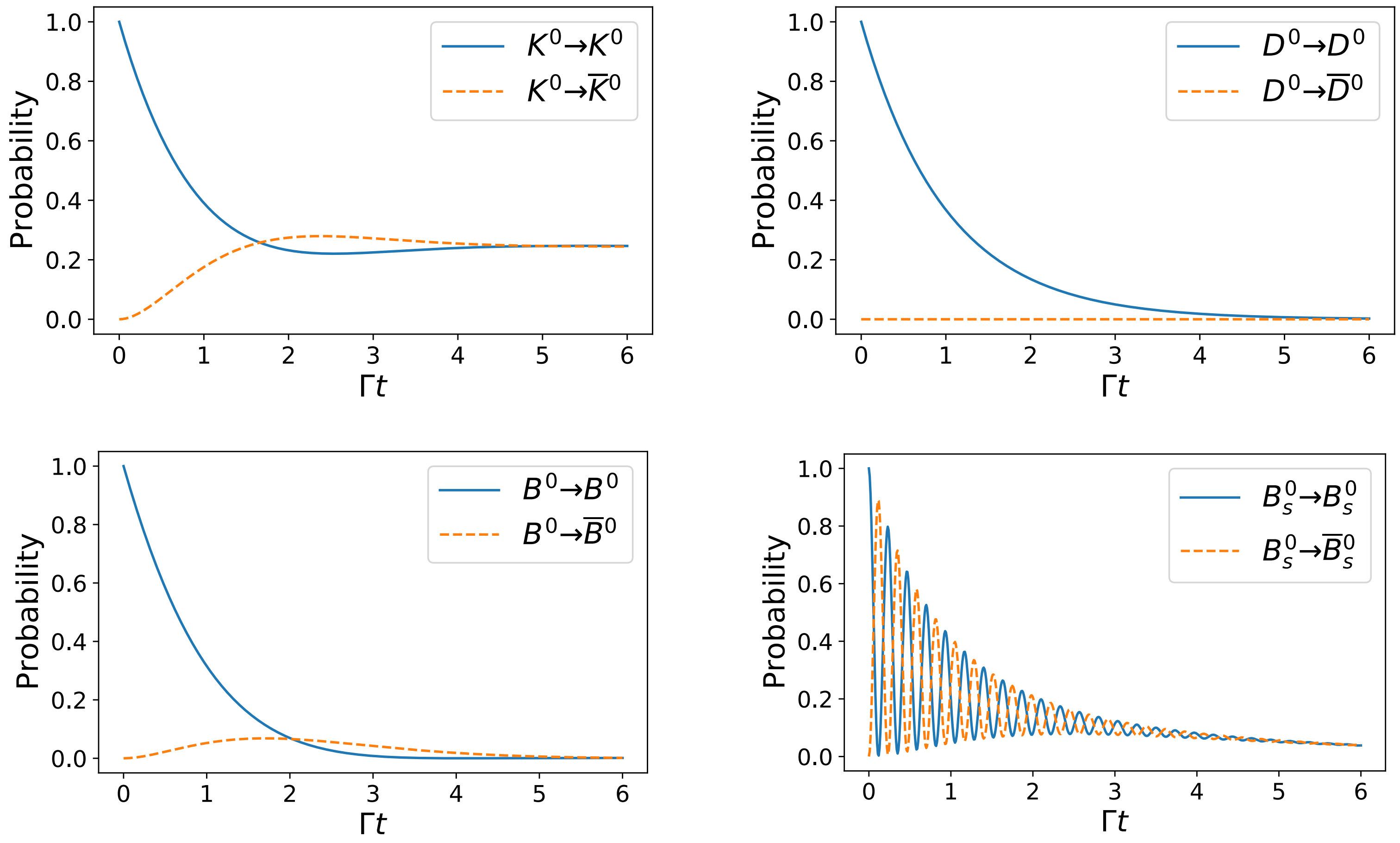
Meson	Mass in GeV	Width in ps^{-1}	Lifetime in ps	x	y
K^0	0.49761(1)	0.005594(2)	89.54(4) 51160(210)	0.946(2)	−0.996(1)
D^0	1.86484(5)	2.438(9)	0.4101(15)	0.0041(4)	0.0065(2)
B^0	5279.66(12)	0.658(2)	1.519(4)	0.769(4)	0.001(5)
B_s^0	5366.92(10)	0.658(2)	1.527(11)	27.01(10)	−0.064(4)



The four neutral mesons oscillating

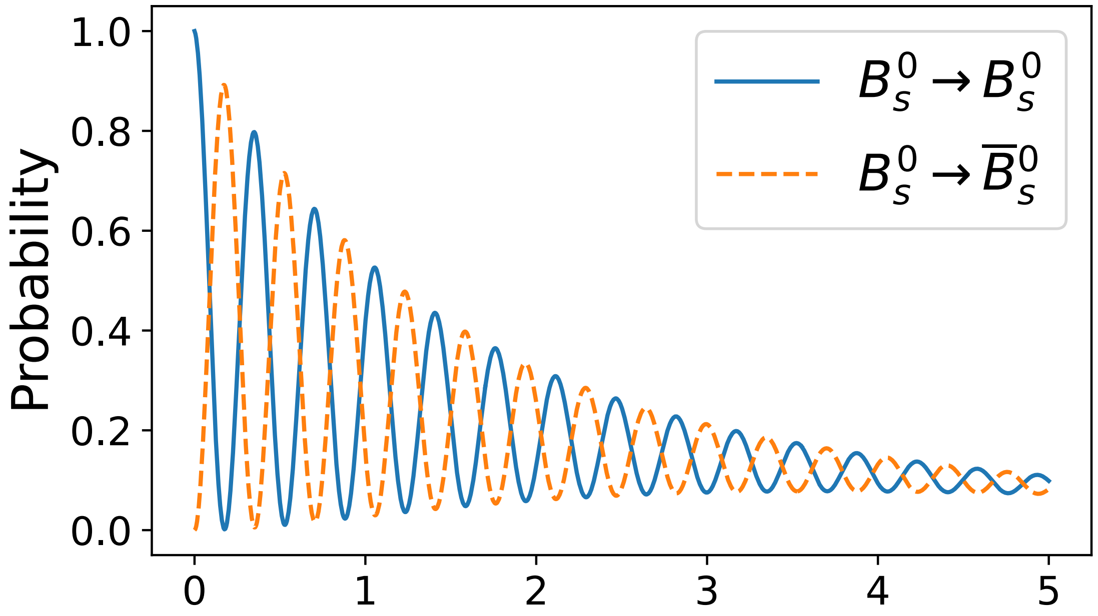


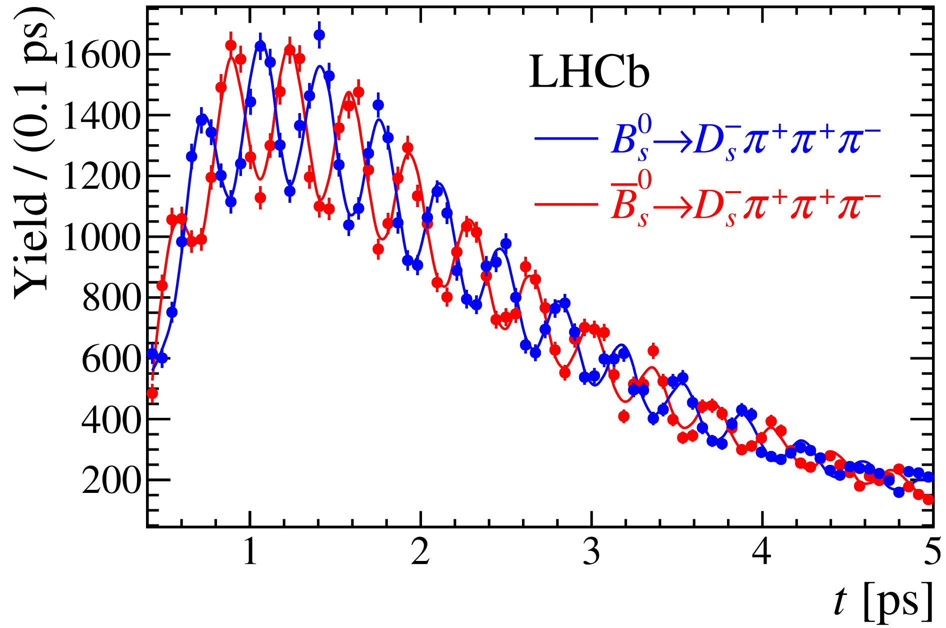
The four neutral mesons oscillating



- All mesons shown over 6 average lifetimes (technically avg. widths)
- Kaons split in extremely different lifetimes with long-lived K_L visible as apparently constant state
- D^0 mesons have strongly suppressed oscillations, not visible on this scale
- B^0 and $B_s{}^0$ are separated by frequency of oscillation
  - Factor of 35 difference in x

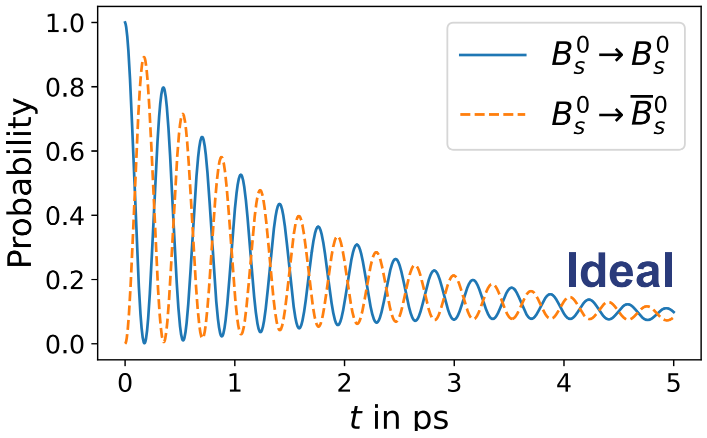
Oscillation measurements

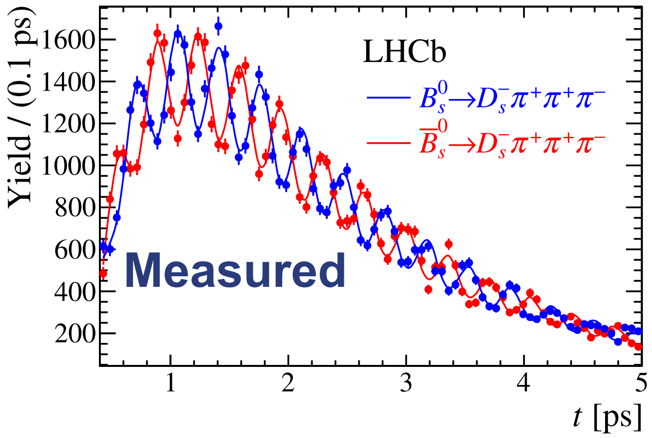


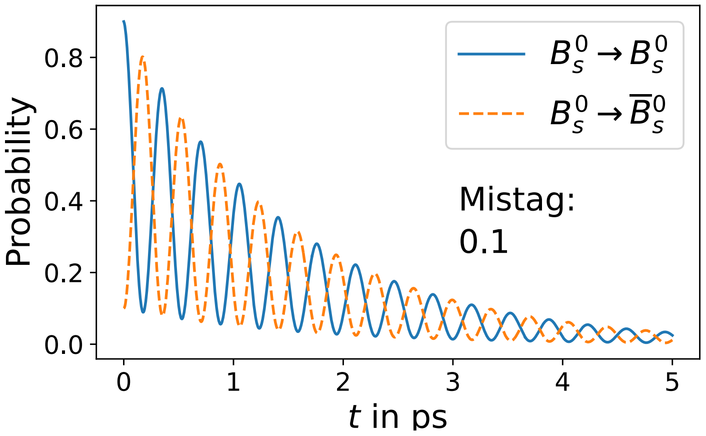


- Oscillation measurements look very different to theoretical curve
- Three significant effects at play…

Experimental effects







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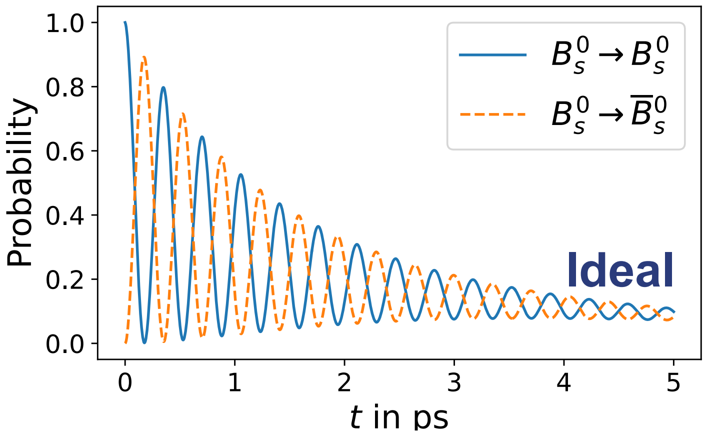

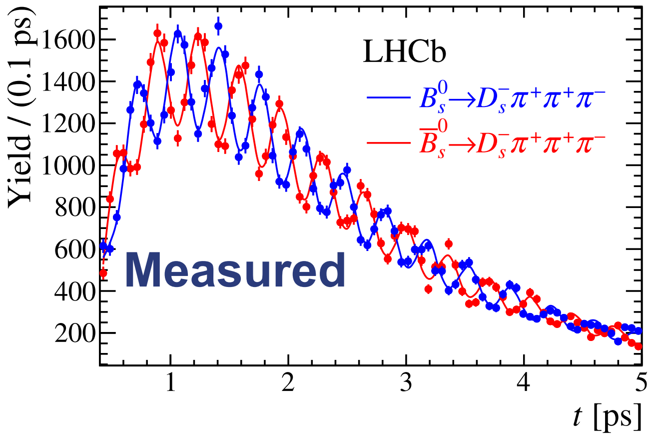

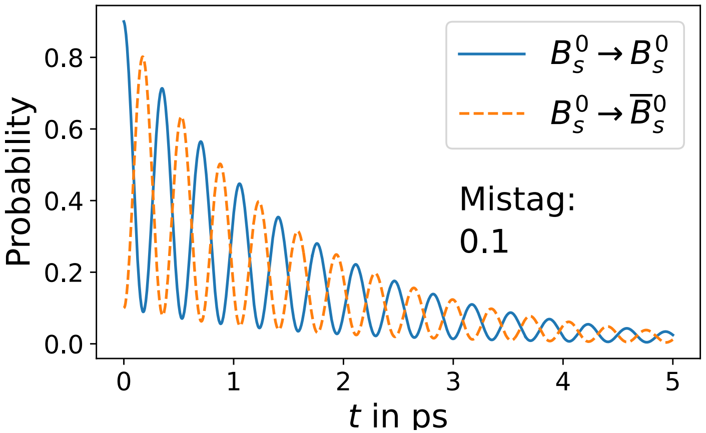

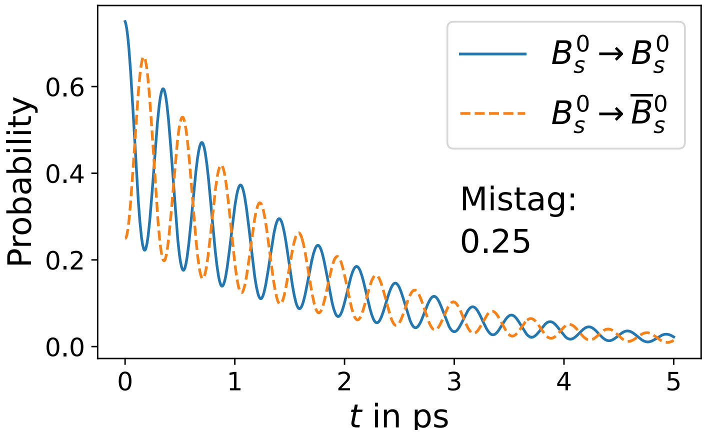

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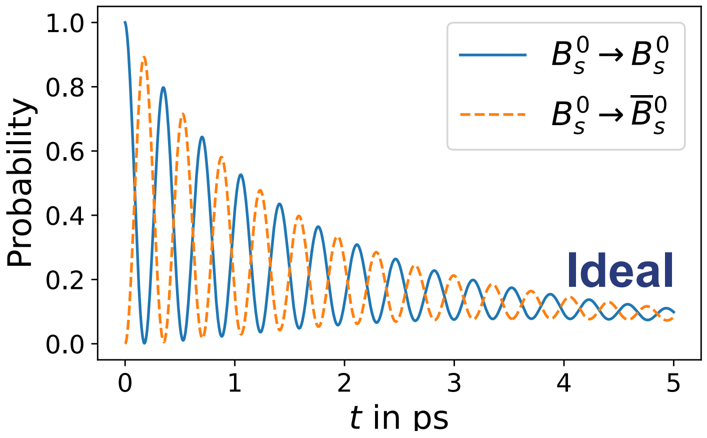

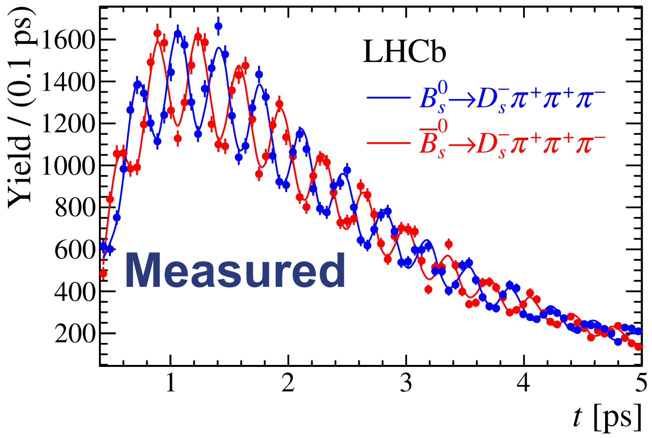

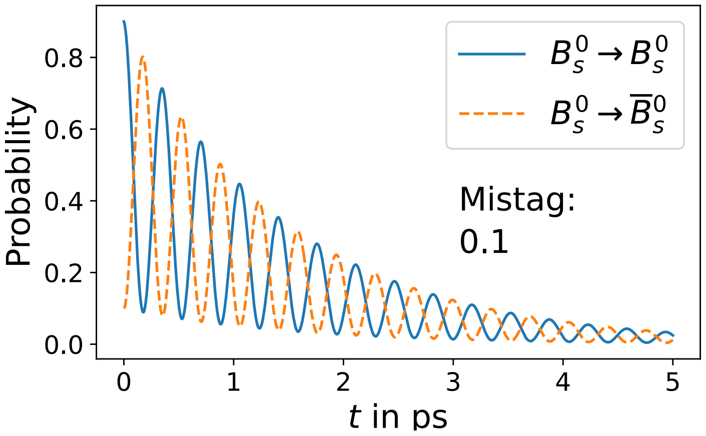

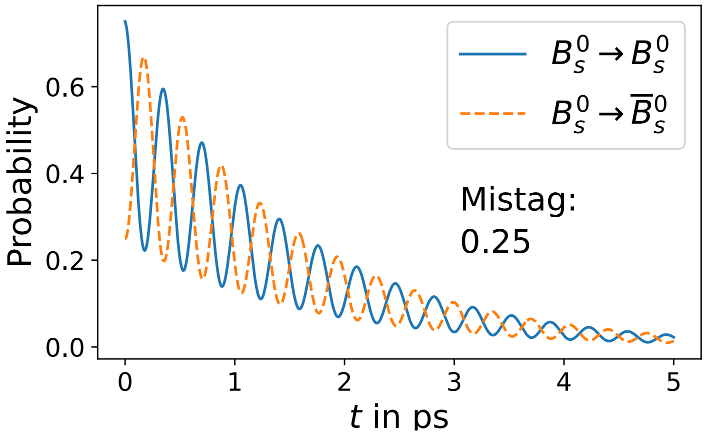

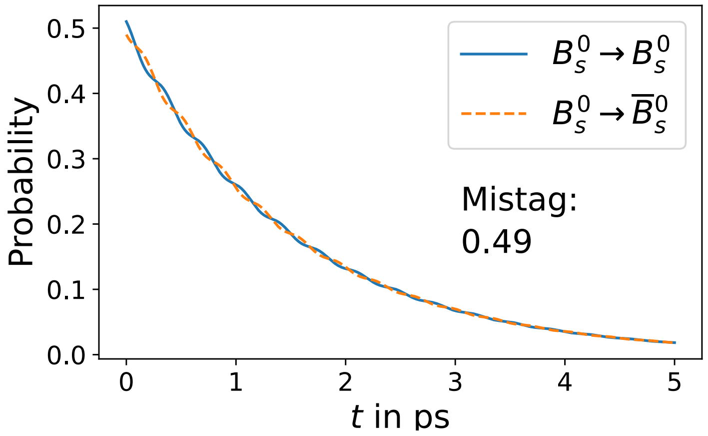

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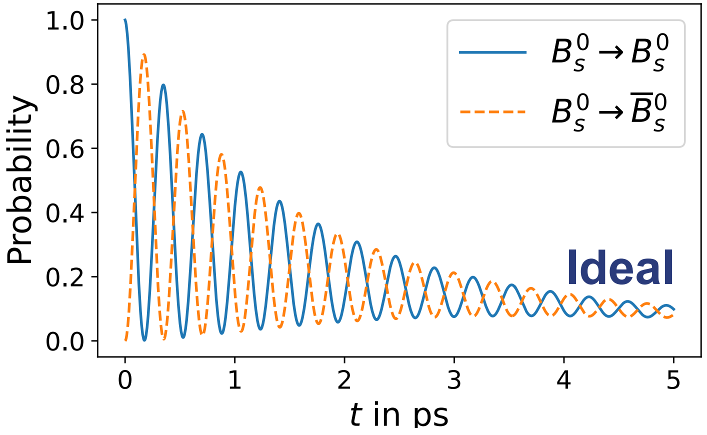

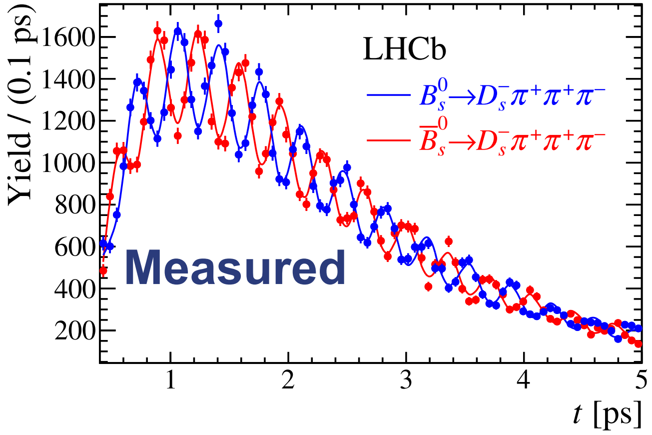

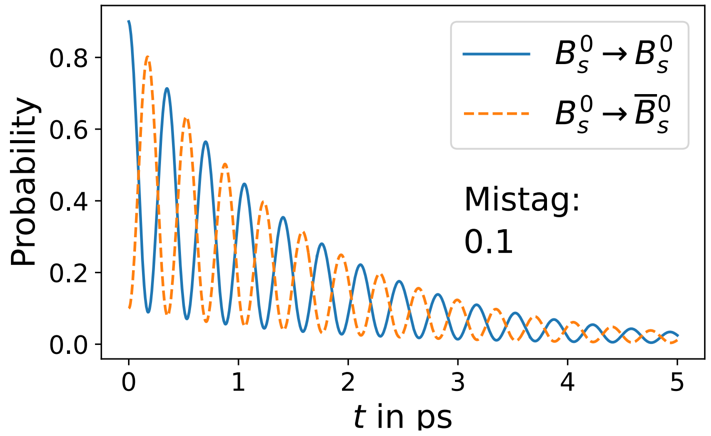

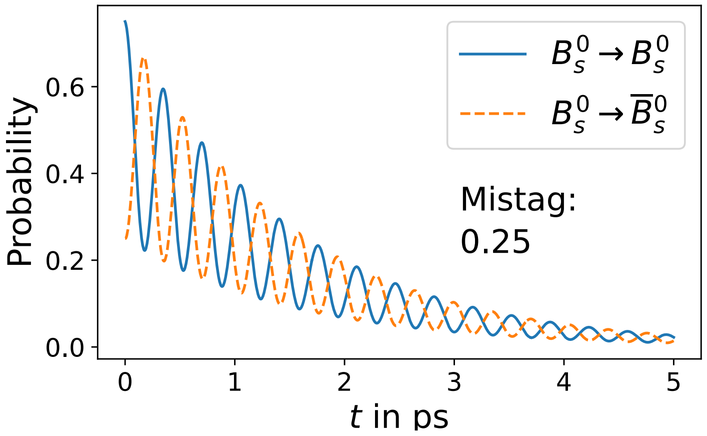

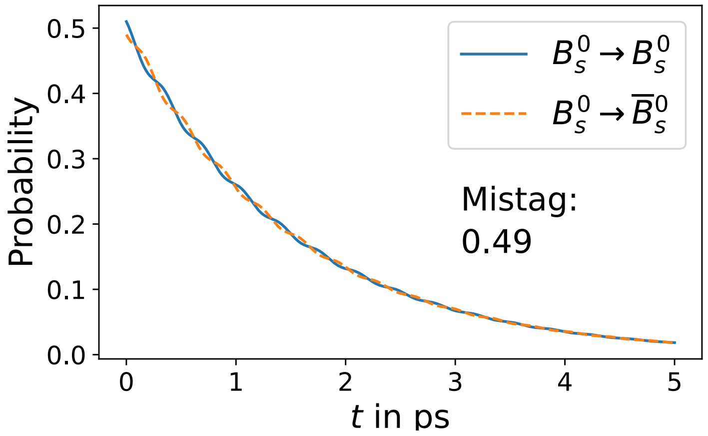

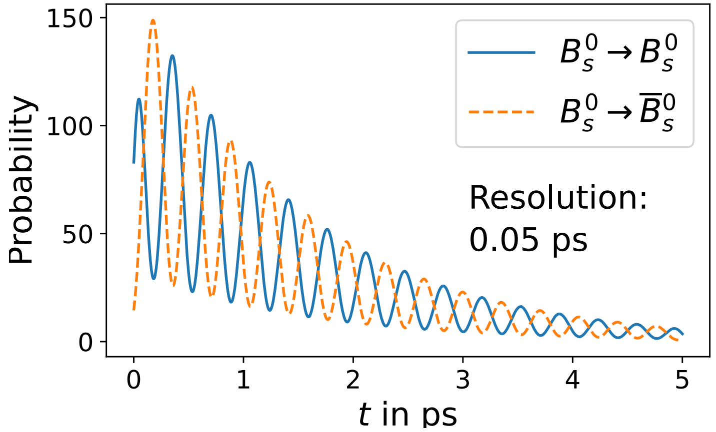

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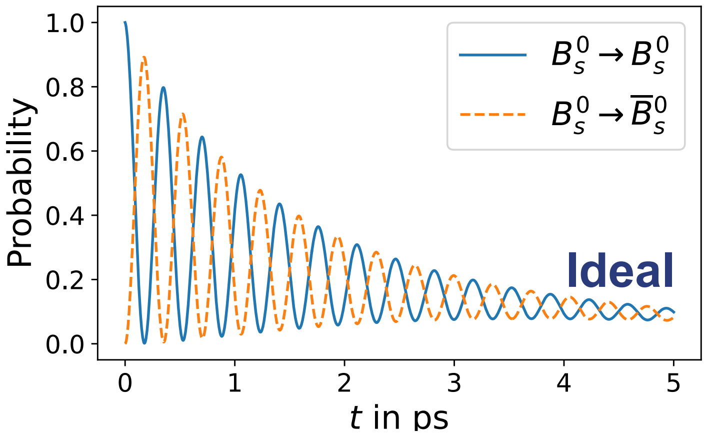

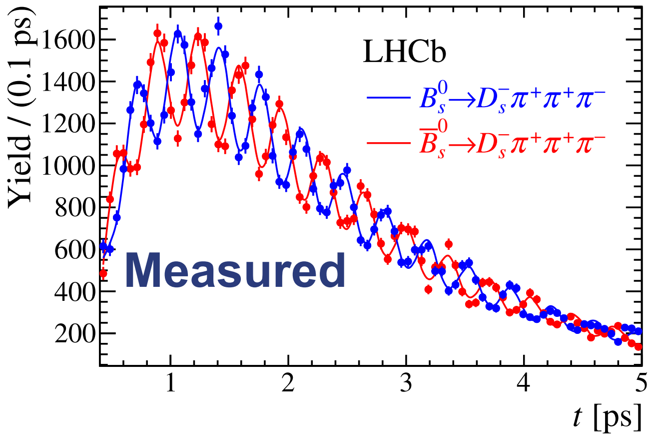

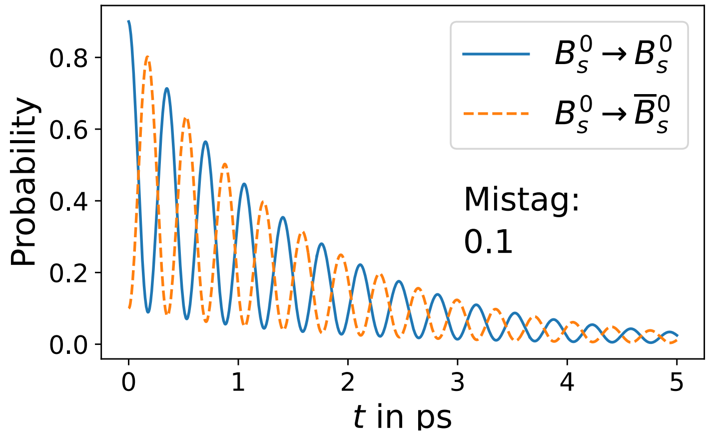

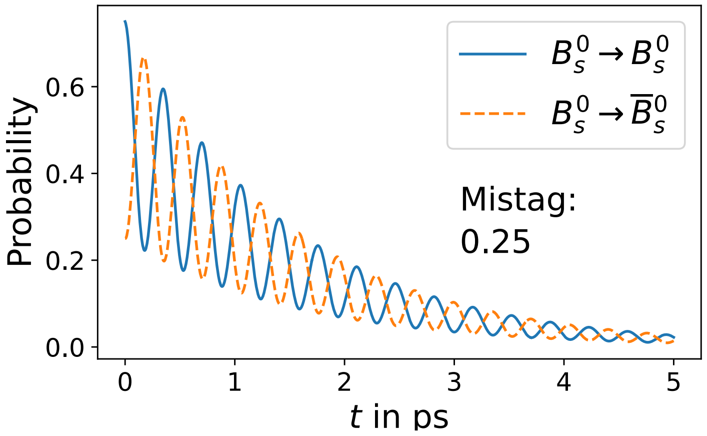

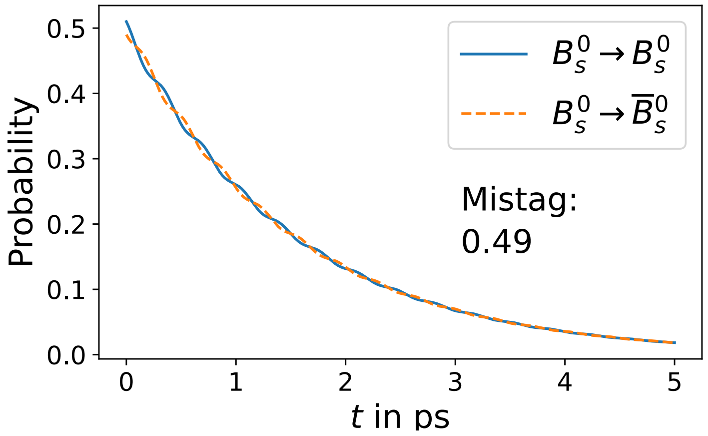

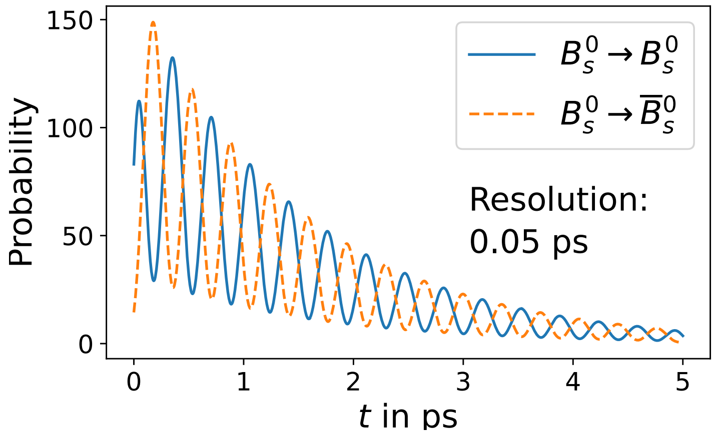

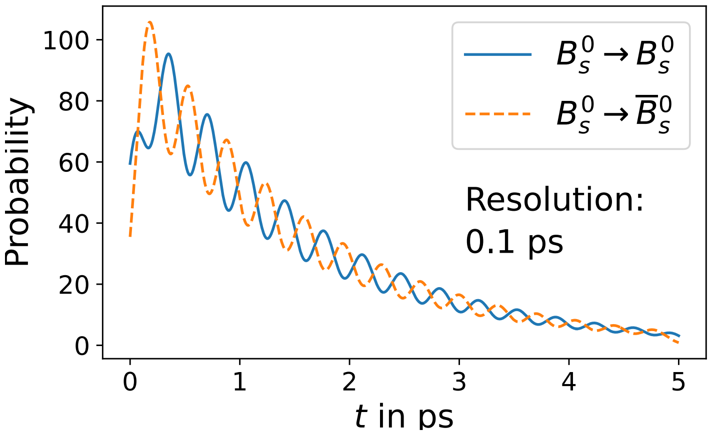

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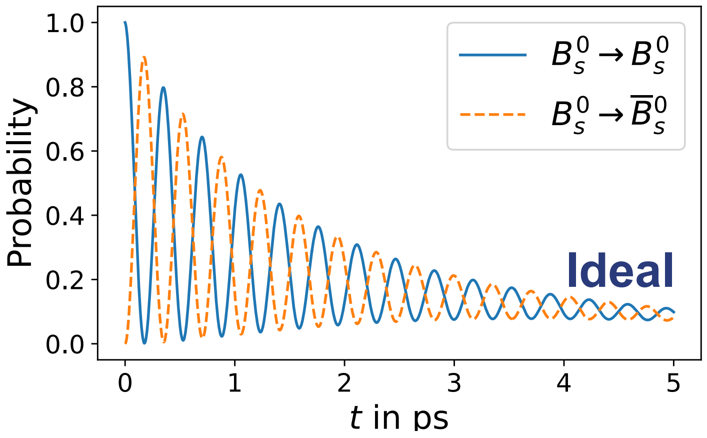

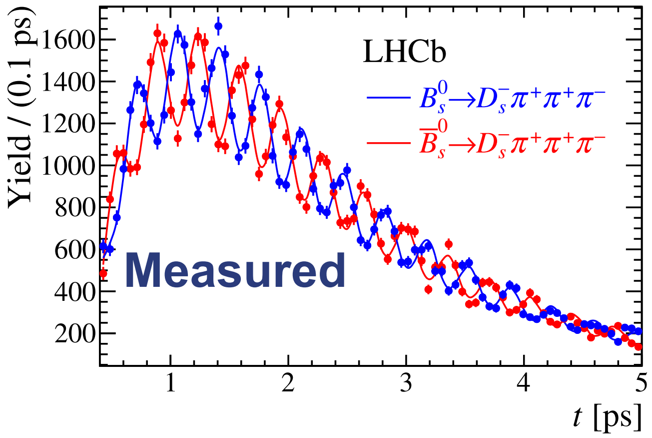

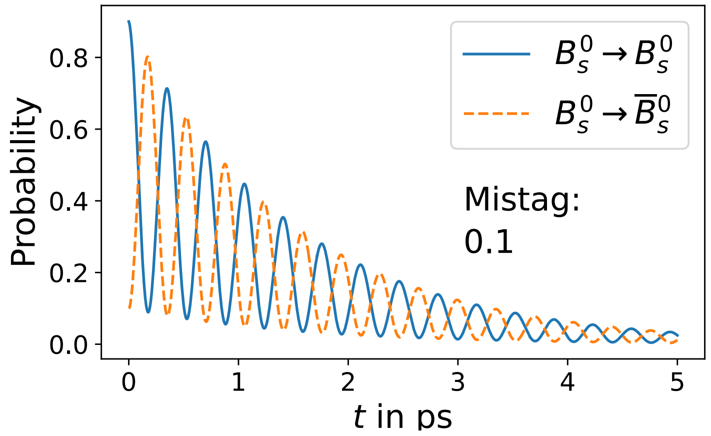

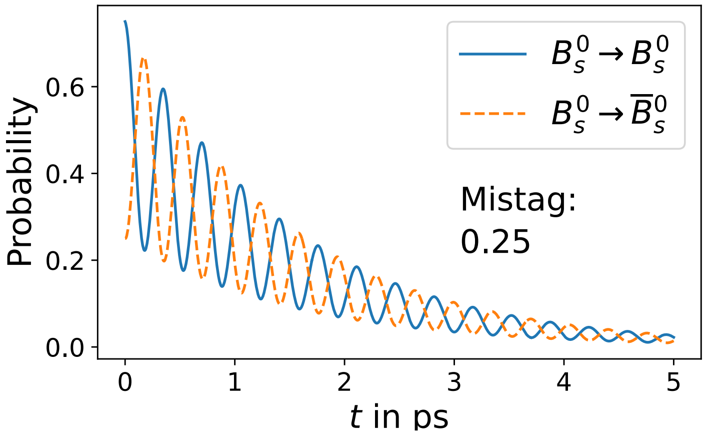

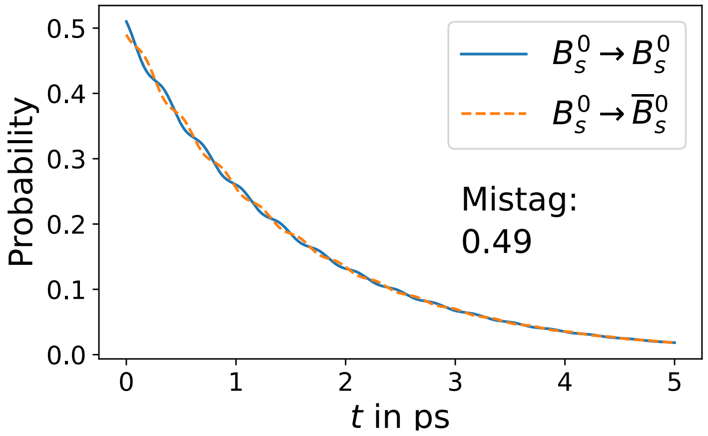

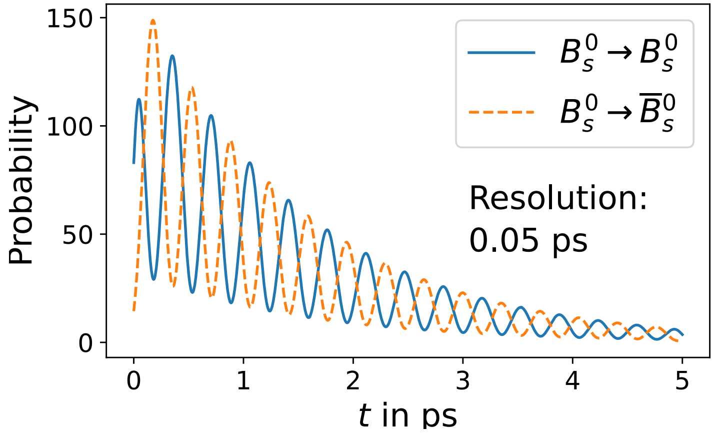

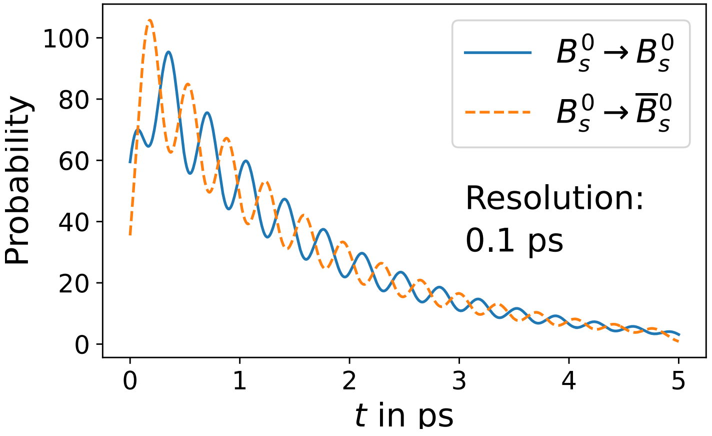

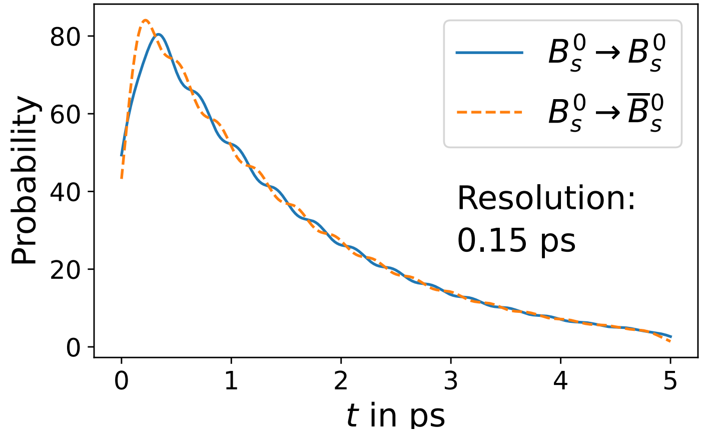

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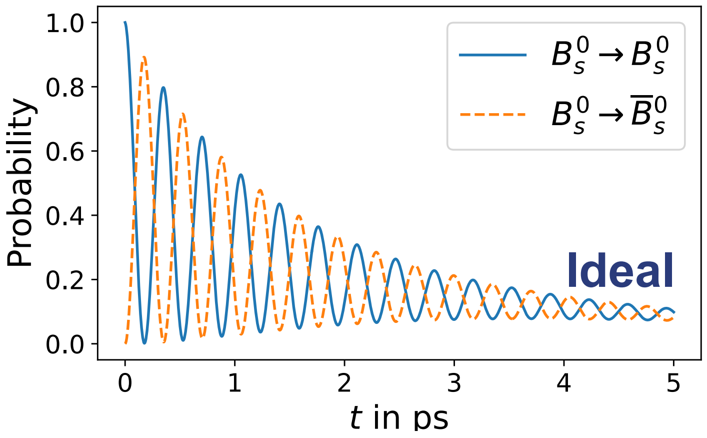
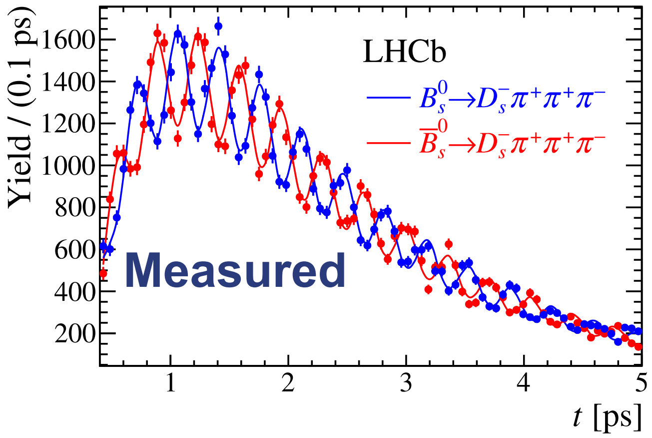
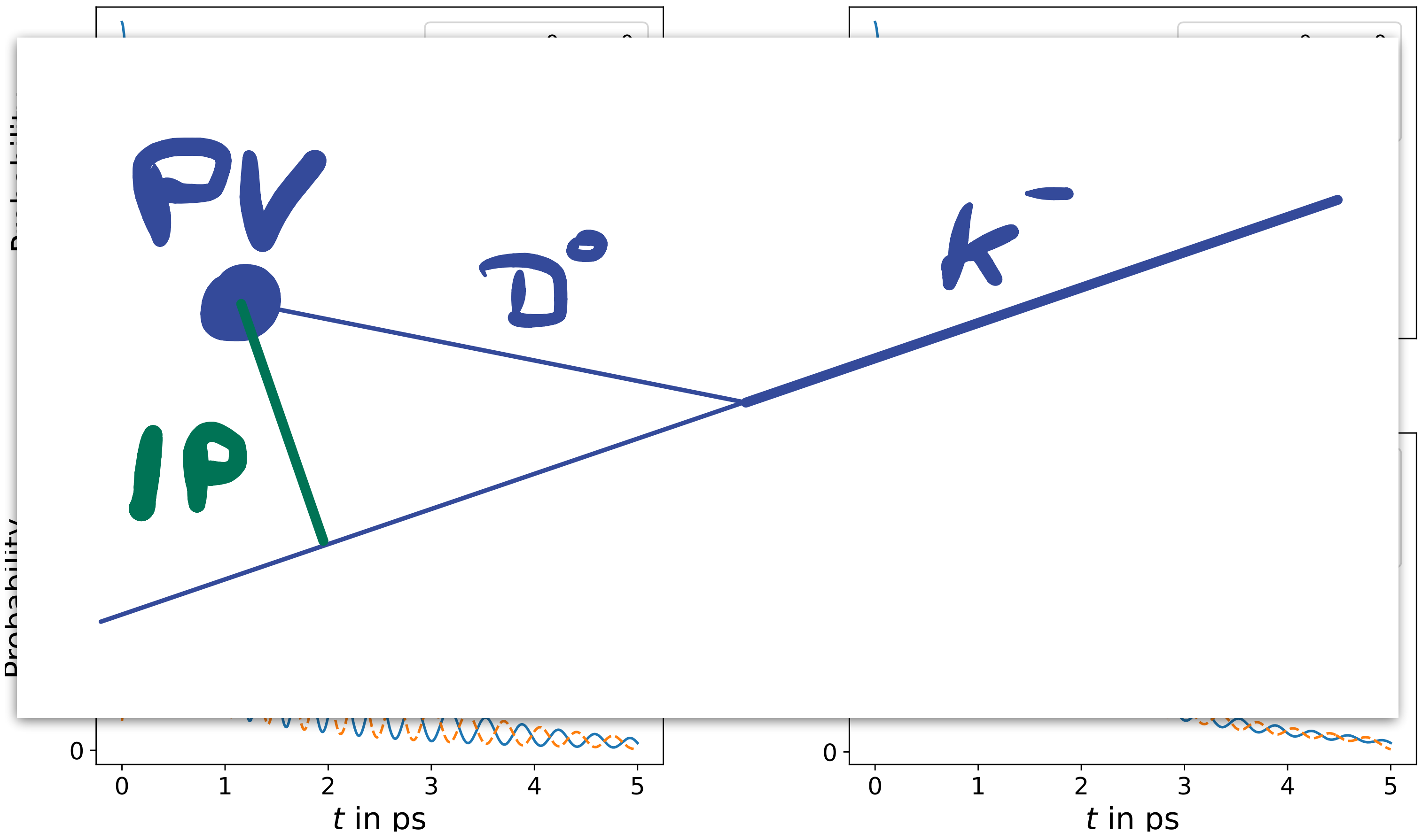
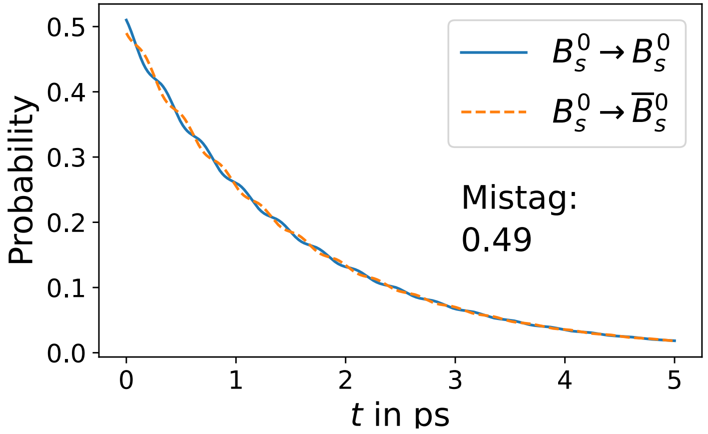
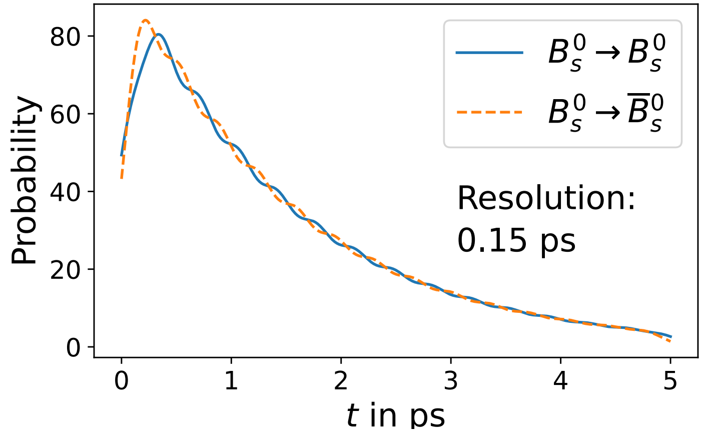










CP eigenstates

$$CP|M_\pm\rangle = \pm|M_\pm\rangle$$

$$|M_\pm\rangle = \frac{1}{\sqrt{2}}(|M^0\rangle \pm |\overline{M}^0\rangle)$$

- CP eigenstates are combinations of flavour eigenstates
  - They overlap with Hamiltonian eigenstates if the magnitudes of *q* and *p* are equal.
- CP asymmetry is measured as decay rate asymmetry of CP-conjugate decays
- A non-zero asymmetry requires
  - Two contributing amplitudes with differing strong and weak phases
- ? Exercise: Derive the last equation

$$A_{CP}(P\to f) = \frac{\Gamma(P\to f) - \Gamma(\overline{P}\to \overline{f})}{\Gamma(P\to f) + \Gamma(\overline{P}\to \overline{f})}$$

$$\Gamma(P\to f) = |\mathcal{A}(P\to f)|^2$$

$$\mathcal{A}(P\to f) = \mathcal{A}_1(1 + re^{i(\delta+\phi)})$$

$$A_{CP}(P\to f) \approx 2r\sin\delta\sin\phi$$

Time-dependent CP asymmetries

- Time-dependent CP asymmetries are relevant for neutral mesons
  - Separate different types of CP violation

$$A_{CP}(M^0 \to f, t) = \frac{\Gamma(\overline{M}^0(t) \to f) - \Gamma(M^0(t) \to f)}{\Gamma(\overline{M}^0(t) \to f) + \Gamma(M^0(t) \to f)}$$

- For B mesons can assume |q/p|=1
  - Introducing $\lambda_f \equiv \frac{q}{p}\frac{\overline{A}_f}{A_f}$

$$A_{CP}(B \to f, t) = \frac{2\Im(\lambda_f)\sin(x\Gamma t) - (1 - |\lambda_f|^2)\cos(x\Gamma t)}{2\Re(\lambda_f)\sinh(y\Gamma t) + (1 + |\lambda_f|^2)\cosh(y\Gamma t)}$$

- For B^0 mesons (as opposed to B_s^0) can assume y=0

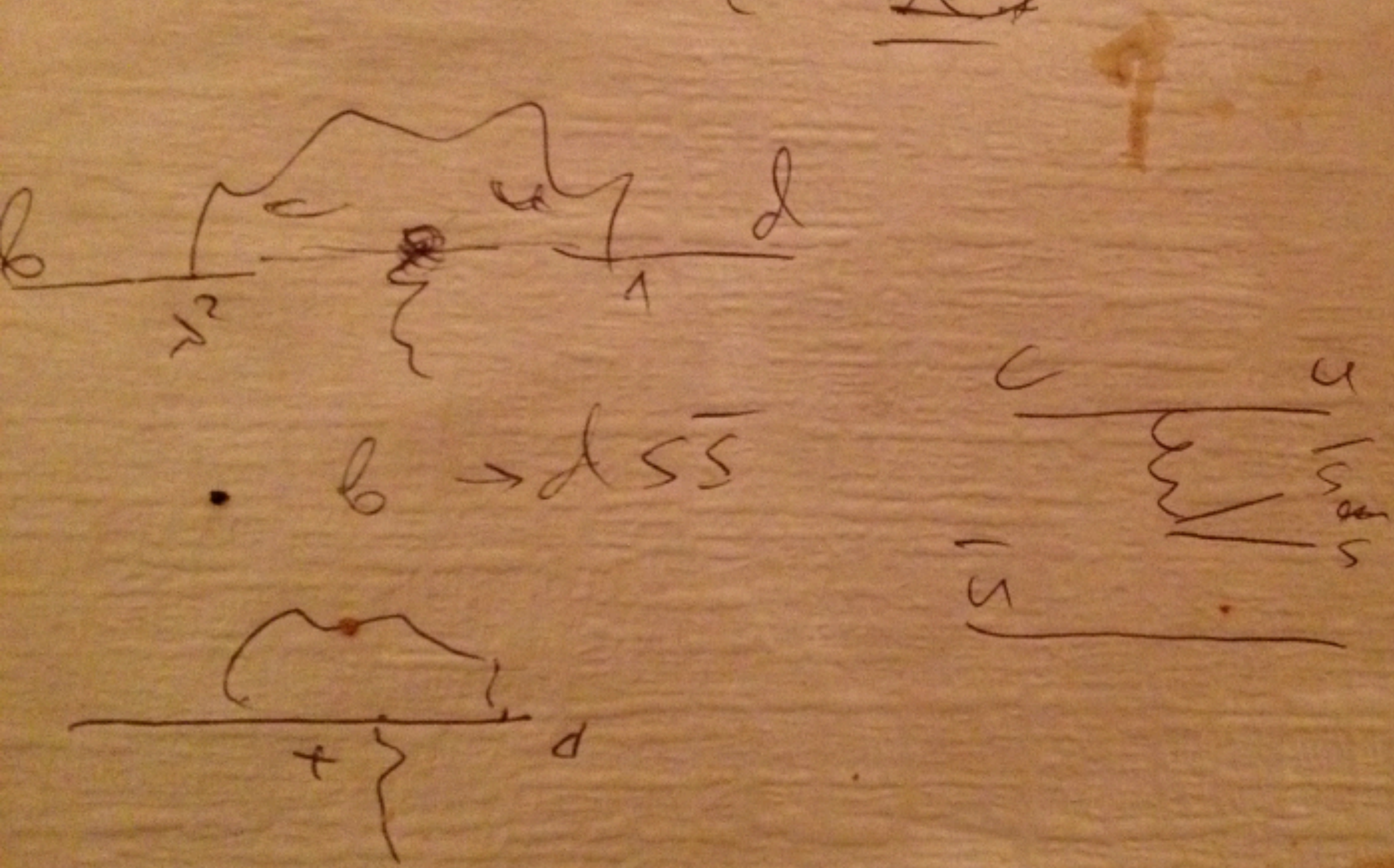
$$A_{CP}(B^0 \to f, t) = S\sin(x\Gamma t) - C\cos(x\Gamma t),$$

$$S \equiv \frac{2\Im(\lambda_f)}{1 + |\lambda_f|^2}, \quad \text{and} \quad C \equiv \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2},$$

- For D^0 mesons use Taylor expansion

$$\Gamma(D^0(t) \to f, t) = |A_f|^2 e^{-\Gamma t}\left(1 + [-\Im(\lambda_f)x + \Re(\lambda_f)y]\Gamma t + |\lambda_f|^2 \frac{x^2 + y^2}{2}(\Gamma t)^2\right)$$

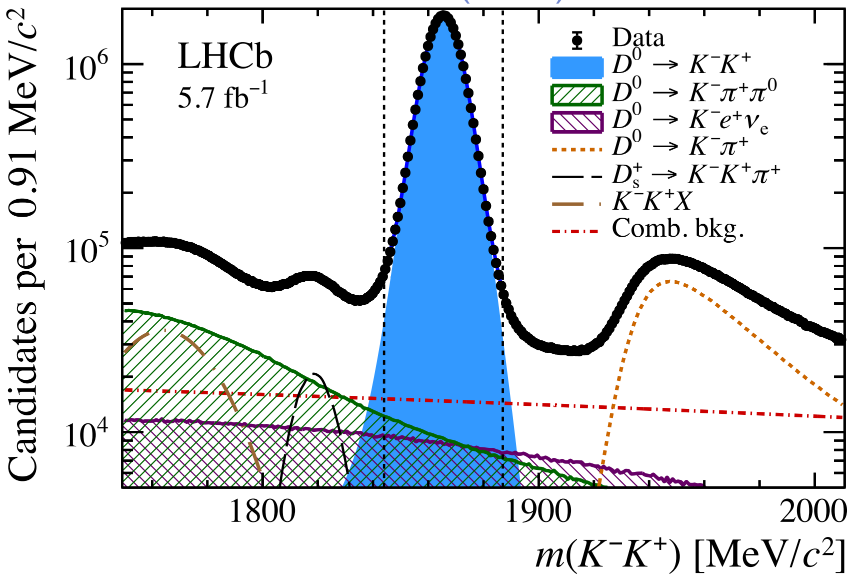
Credit: Zoltan Ligeti

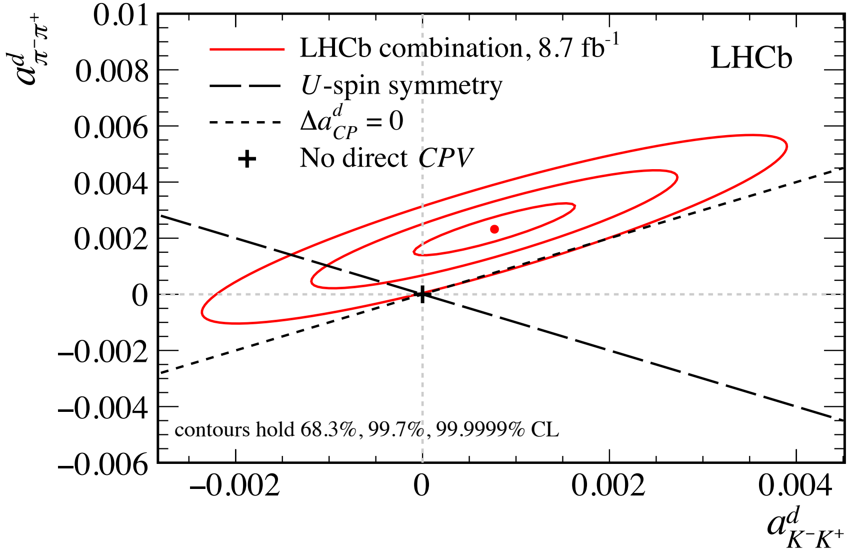


(November 9, 2011, Pizzeria de la Place, Meyrin)

CP violation in decays

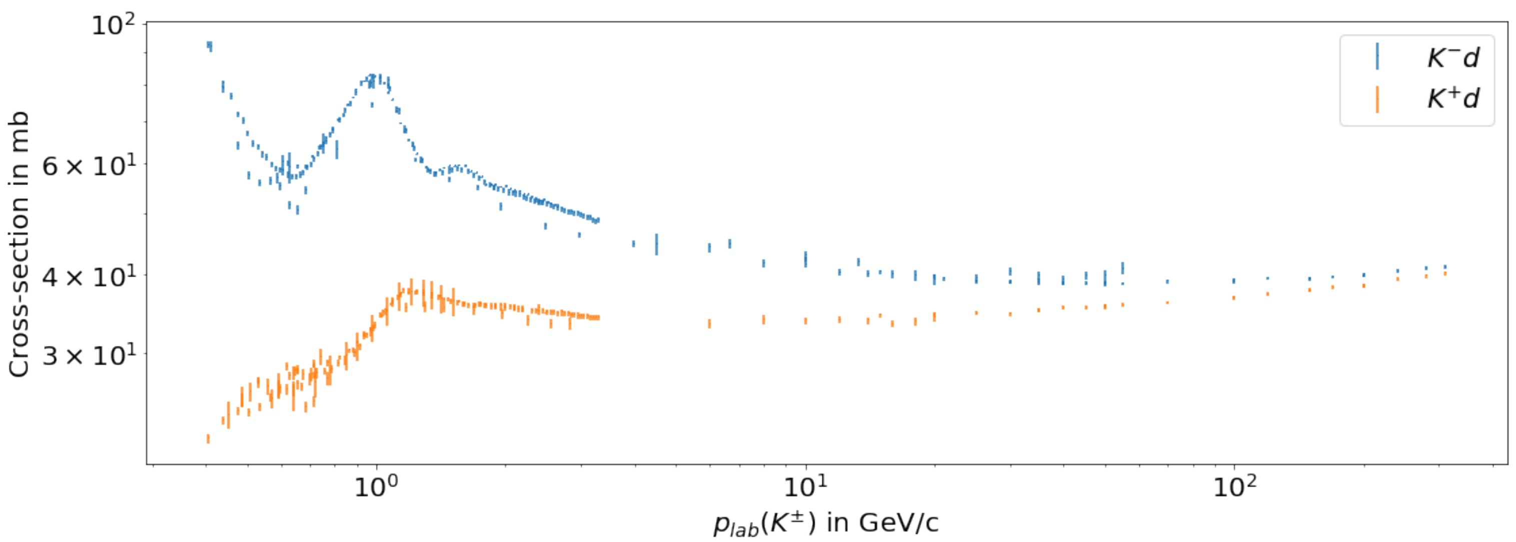
LHCb, PRL 131 (2023) 091802



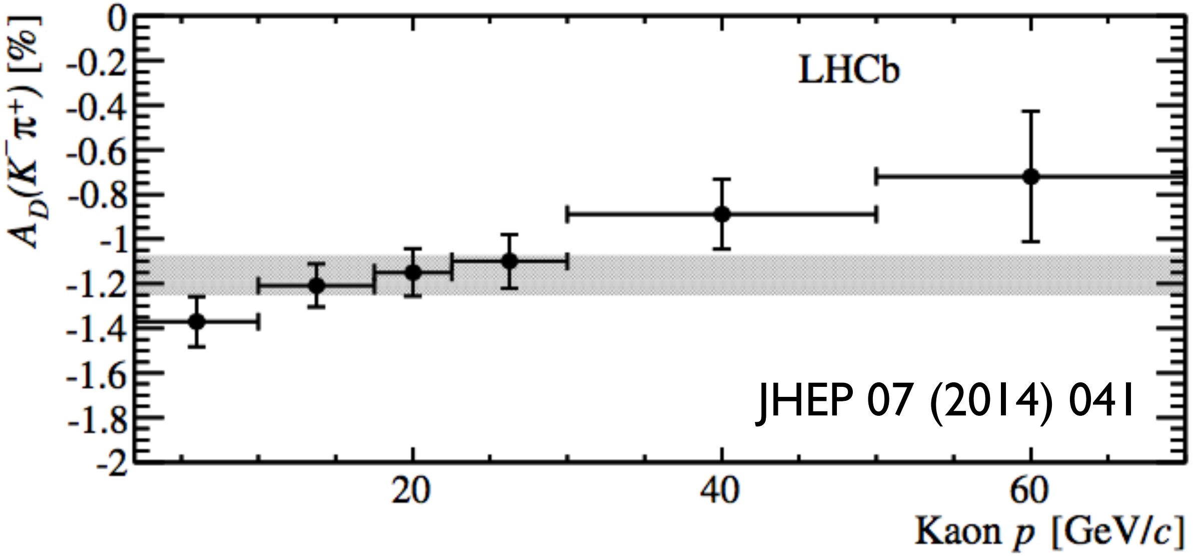


- 2019 discovery of CP violation in charm decays:
  $\Delta A_{CP} \equiv A_{CP}(KK) - A_{CP}(\pi\pi) = (-0.182 \pm 0.033)\%$ LHCb, PRL 122 (2019) 211803
  - Prompted investigation of individual decay modes
- Use control channels $D^0 \to K^-\pi^+$, $D^+ \to K^+\pi^+\pi^-$, $D^+ \to K_S\pi^+$ and $A_{CP}(K_S)$ to constrain production and detection asymmetries
- Result indicates ΔA_{CP} largely driven by $A_{CP}(\pi\pi)$
  - In tension with U-spin symmetry expectation

Detection asymmetries



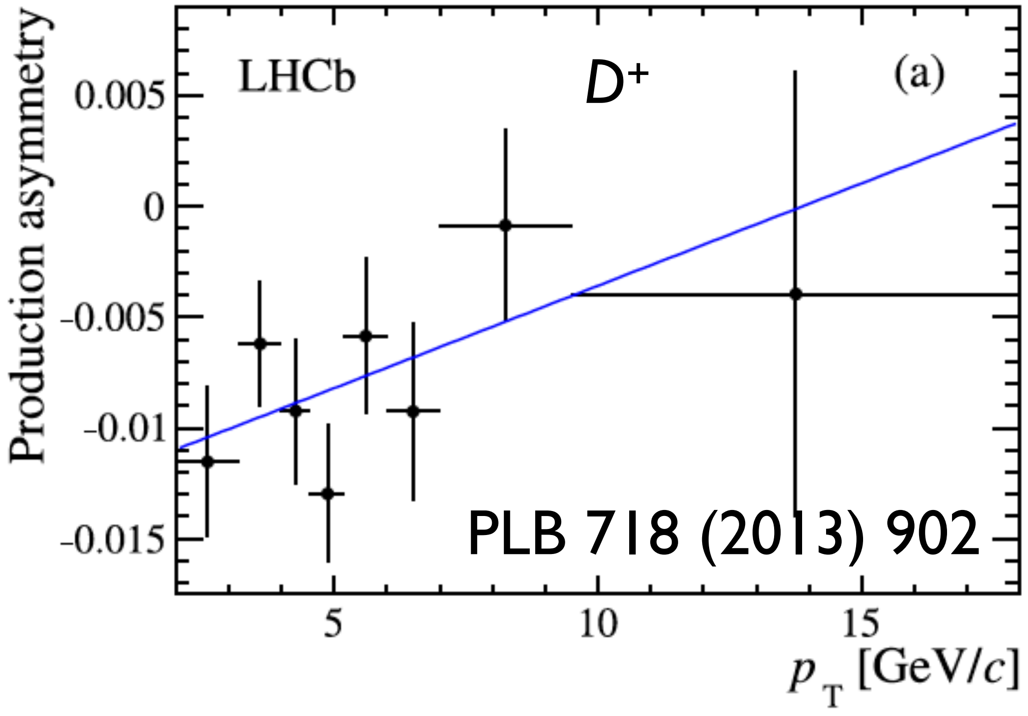
- Two contributions
  - Detector asymmetries
    - Can come from asymmetric construction or different performance, with respect to a magnetic field to lead to a "nuisance" asymmetry
    - May be in part mitigated if field polarity can be reversed
  - Interaction asymmetry
    - Inherent difference of CP-conjugate particle pairs interacting with (hence diasppearing in) detector material



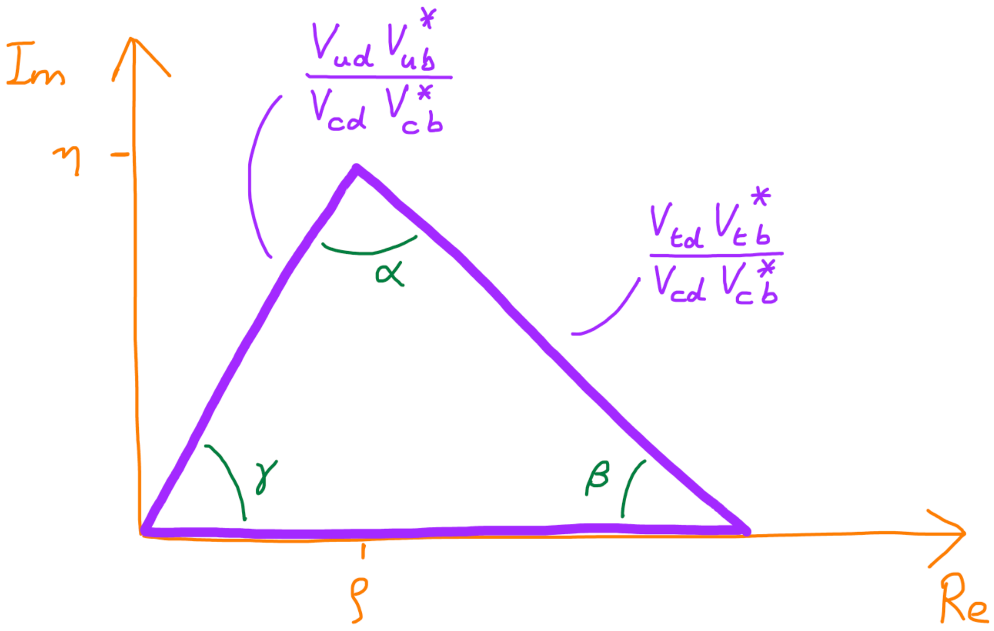
Production asymmetries

- Particular to pp collider
- Valence quarks favour production of matter baryons
  - Hence expect surplus of antimatter mesons
  - More complicated mechanisms are at play too
- Production asymmetry can depend on kinematics

- At symmetric colliders (e^+e^-, $p\bar{p}$)
  - Can have forward-backward asymmetry due to interaction with CP conjugate beam particles

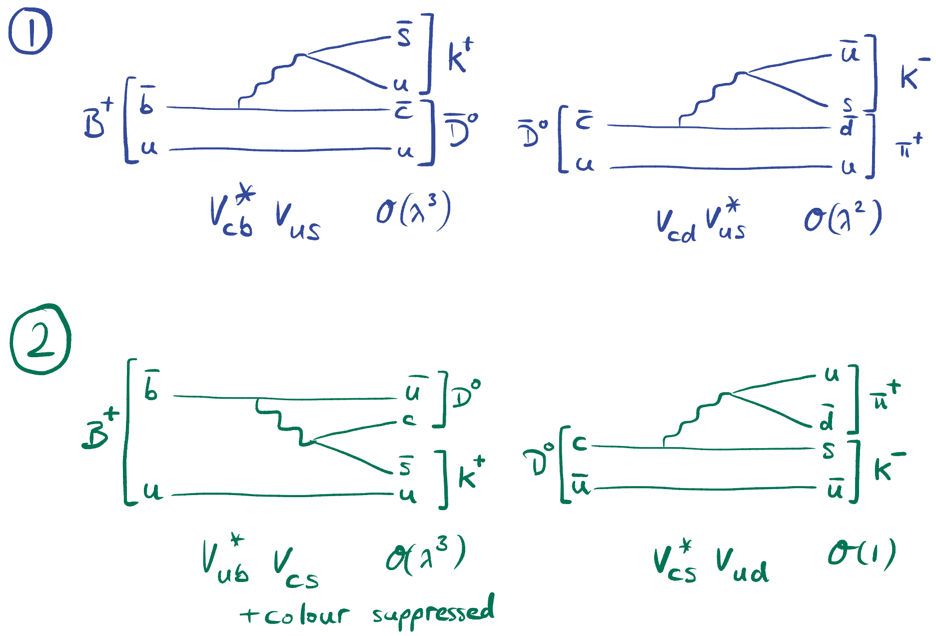


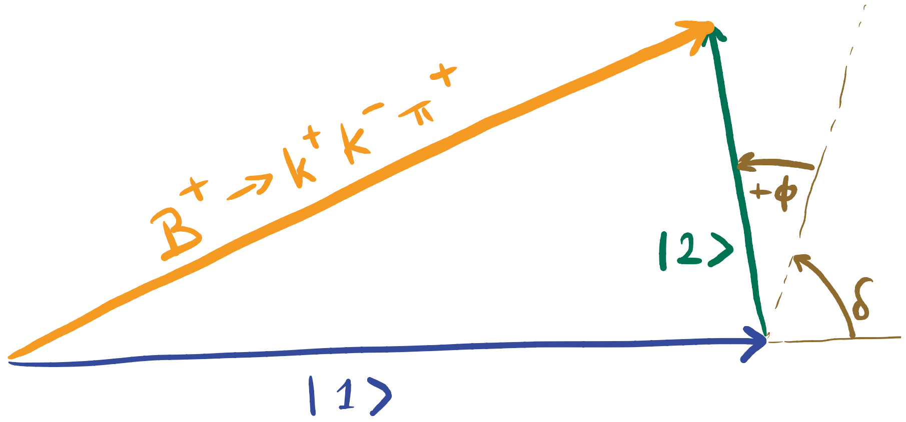
CKM metrology



- CKM matrix can be represented by unitarity triangles
  - Off-diagonal elements of unitarity relations depicted in complex plane: $VV^\dagger=\mathbb{1}$
- Complex phases are related to angles in triangle
  - Phases arise largely through smallest elements: V_{ub} and V_{td}
- Sides are determined by magnitude ratios
  - Uncertainties typically driven by uncertainty of smallest elements: V_{ub} and V_{td}

Measuring γ



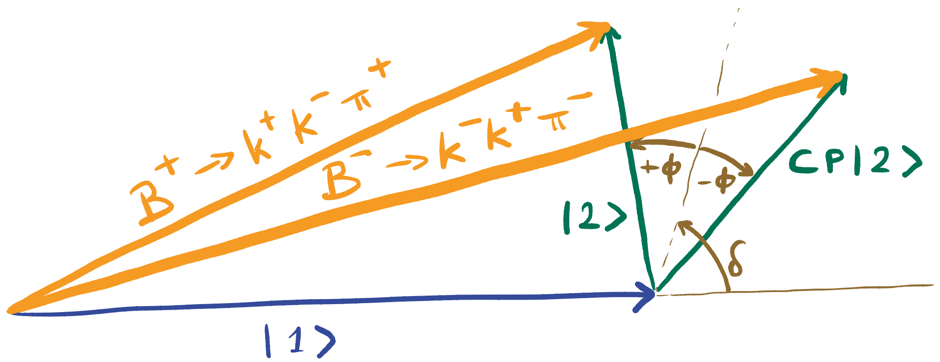


- Many methods to measure γ
  - Here illustrated with Atwood, Dunietz, Soni (ADS)
- All based on analysing two contributing amplitudes
  - 1: $\bar{D}^0$ decay doubly Cabibbo-suppressed
  - 2: B^+ decay colour-suppressed, D^0 decay Cabibbo-favoured
- Similar amplitude magnitudes give sensitivity to phase difference:

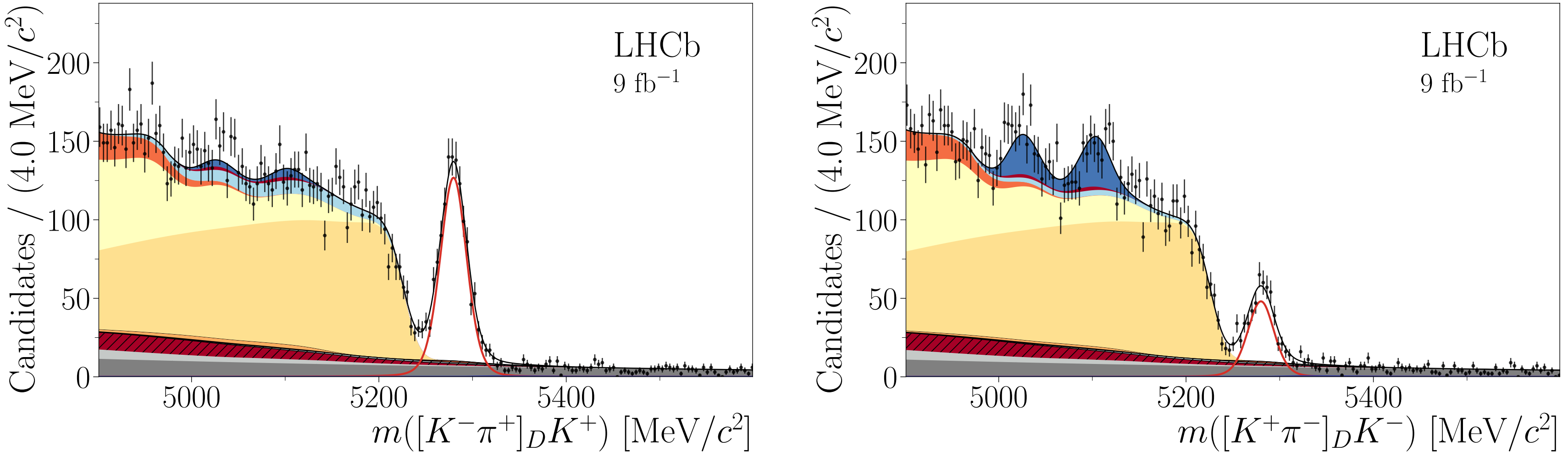
$$
\begin{aligned}
\phi &= \arg(V_{cb}^* V_{us} V_{cd} V_{us}^*) - \arg(V_{ub}^* V_{cs} V_{cs}^* V_{ud}) \\
&= \arg\left(\frac{V_{cb}^* V_{cd}}{V_{ub}^* V_{ud}}\right) \\
&= \arg\left(-\frac{V_{ub}^* V_{ud}}{V_{cb}^* V_{cd}}\right) \\
&= \gamma.
\end{aligned}
$$

Measuring γ — Part 2

- Measurable rates are combinations of two amplitudes with different strong and weak phases
- Weak phases flip sign under CP conjugation
- CP asymmetries give access to weak phase differences

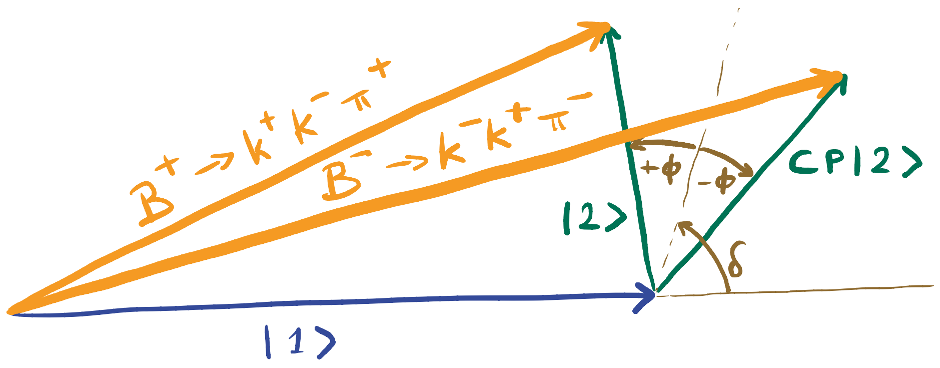


LHCb, JHEP 04 (2021) 081



Measuring γ — Part 2

- Measurable rates are combinations of two amplitudes with different strong and weak phases
- Weak phases flip sign under CP conjugation
- CP asymmetries give access to weak phase differences
- Requires input on strong phase difference and amplitude ratios
  - Can lead to ambiguities
    - Exploit external constraints



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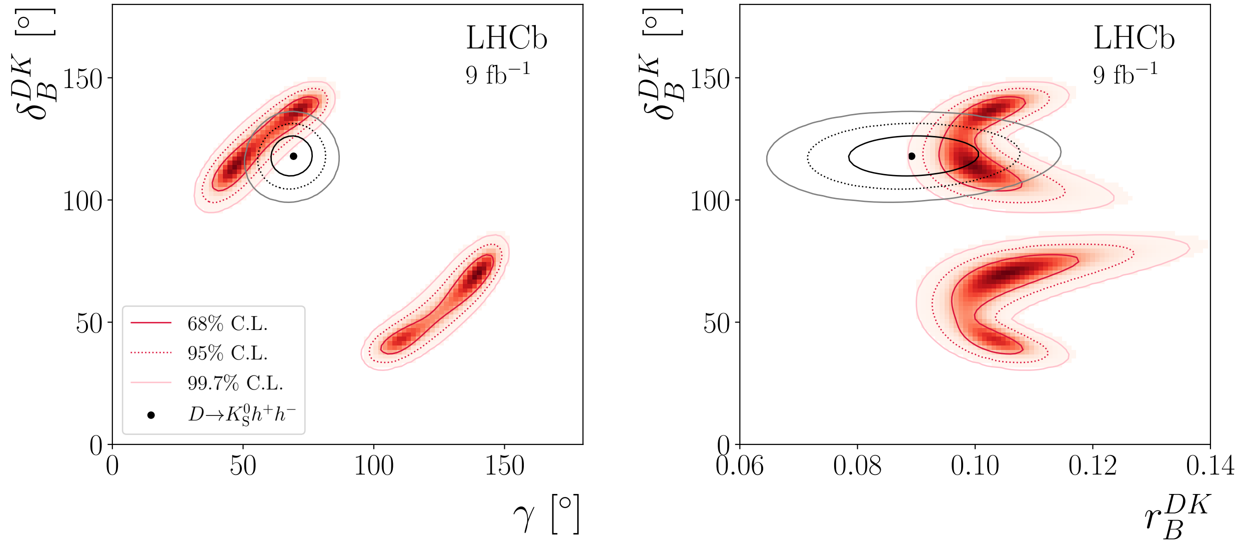

Measuring γ — Part 2

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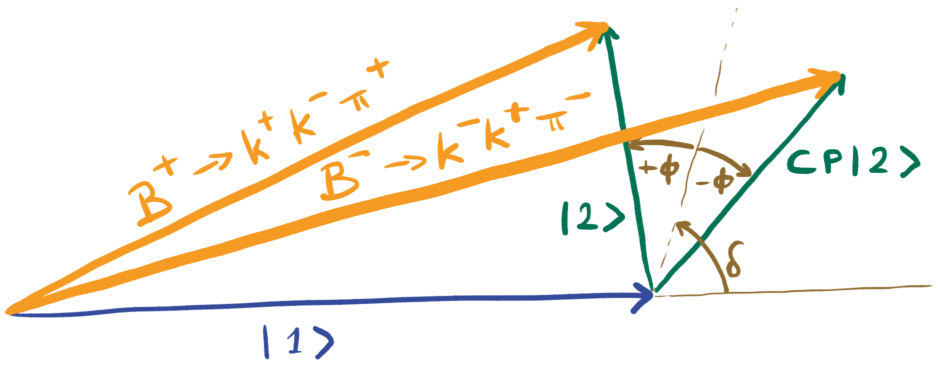
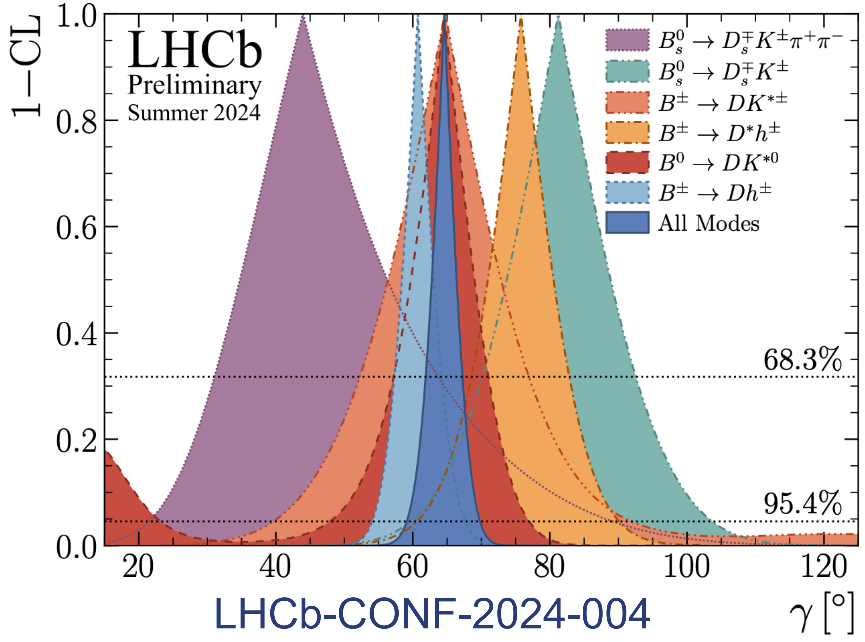
  - Can lead to ambiguities
    - Exploit external constraints
  - Extract γ with global fit of many decays





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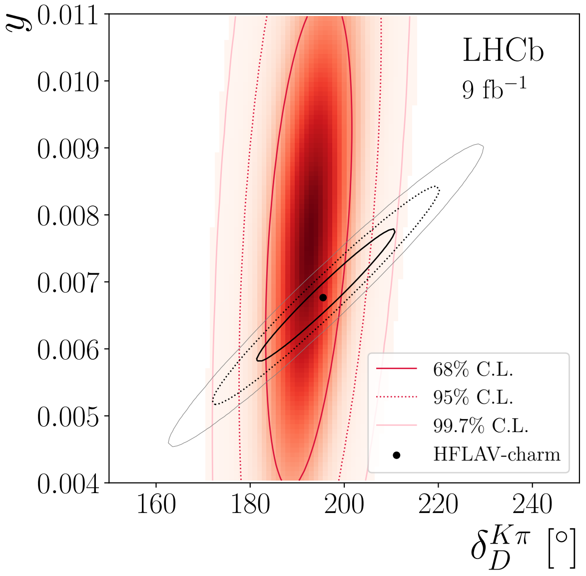
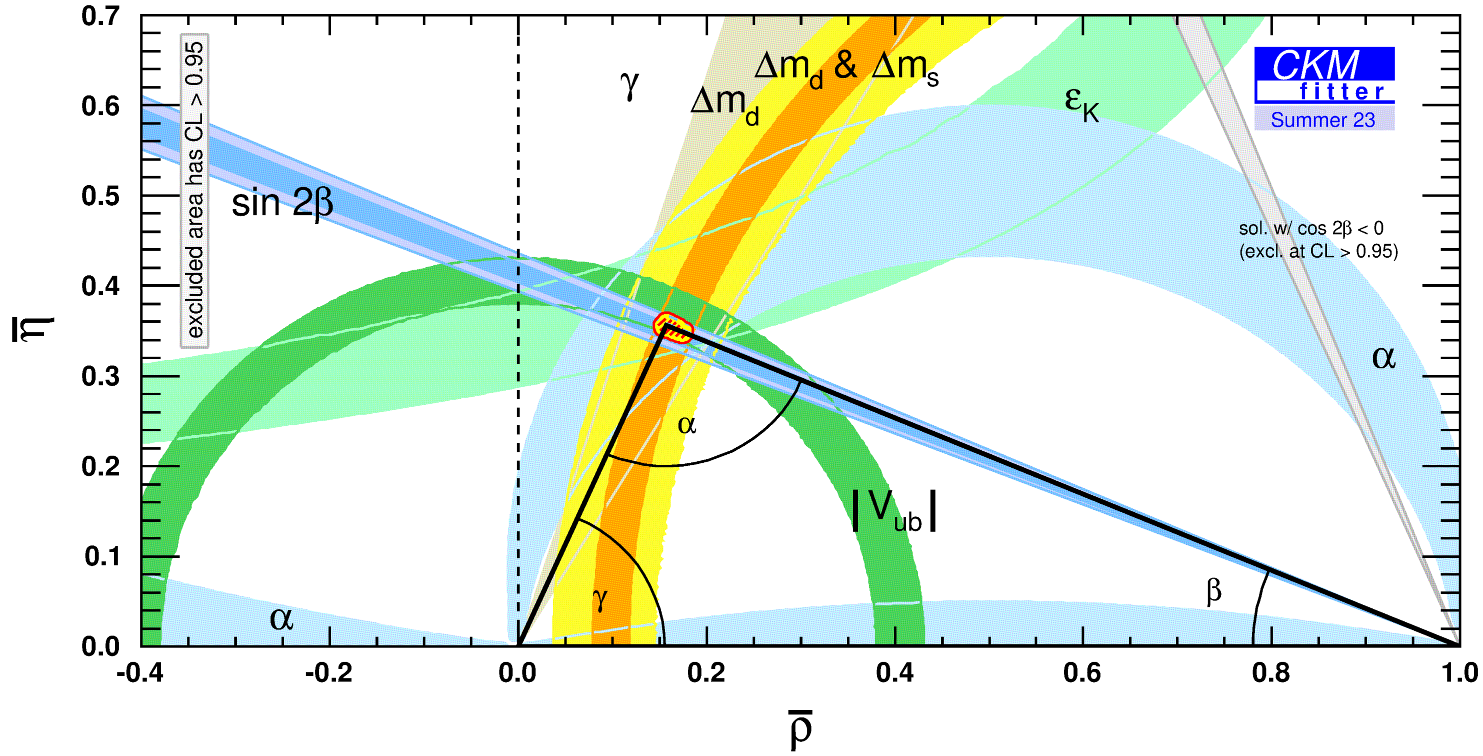
LHCb, JHEP 04 (2021) 081

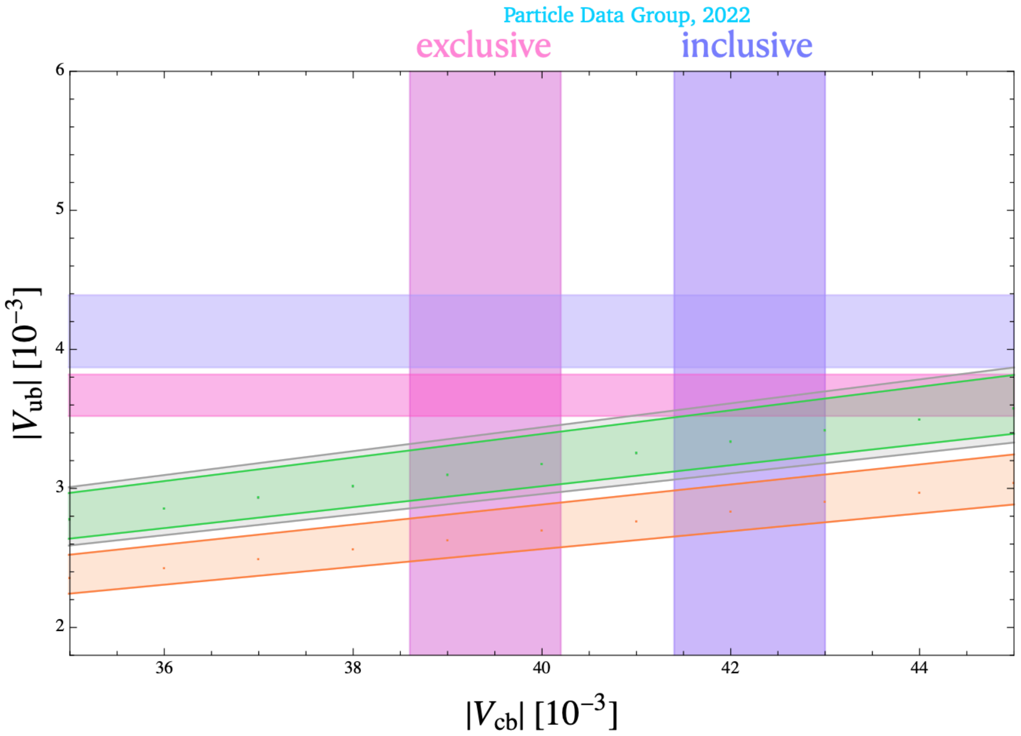


CKM measurements



- Statistical combination of many measurements with theory input
- Apex of triangle in agreement with all inputs
- First determination of angle β was main goal of B-factories BaBar and Belle
  - Nobel Prize 2008
- Angle γ now known to about 1° in large parts due to LHCb input
- Much more precise tests expected especially from Belle II and LHCb in future

Aside: V_{ub} & V_{cb}



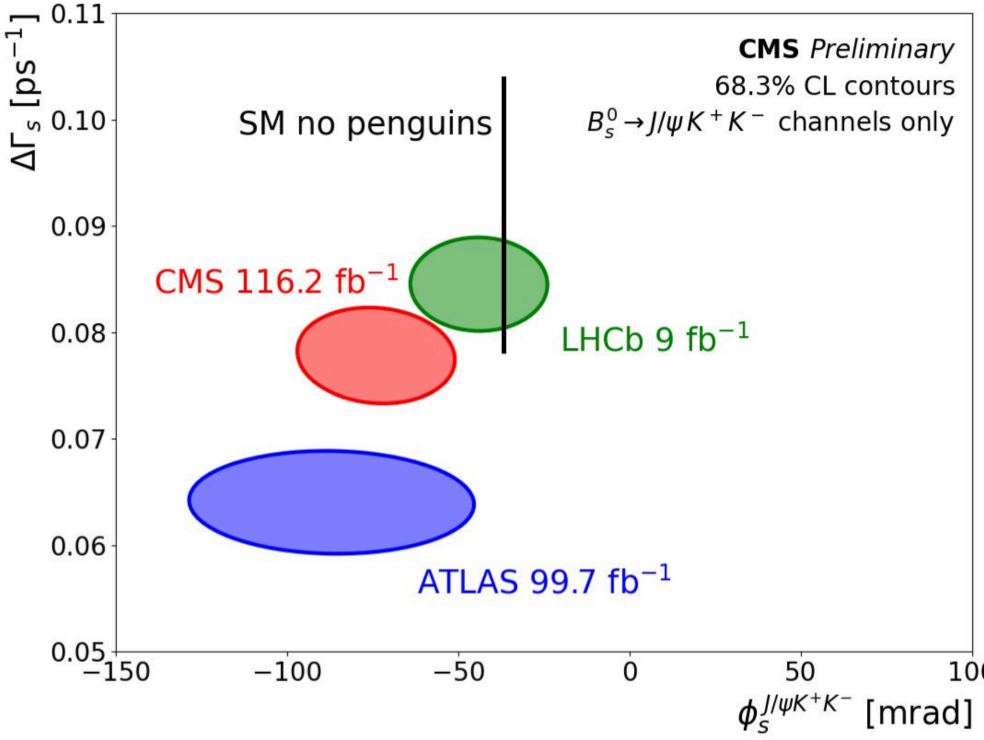
From: Carolina Bolognani, CKM 2023

- Leptonic and semi-leptonic hadron decays used to extract CKM element magnitudes
- Leptonic decays
  - Two-body decays to lepton-neutrino pair:
    - Theoretically clean, but
    - Experimentally difficult to reconstruct and to determine efficiency
- Semi-leptonic decays
  - Decays to Hadron and lepton-neutrino pair:
    - Theoretically challenging due to dependence on decay kinematics with additional challenge for exclusive decays
    - Experimentally easier but still challenging due to neutrino and still more challenging for inclusive decays
- Bottom line: lots of theoretical and experimental work ongoing

The angle $\beta_{(s)}$

- Measured in $B^0_{(s)} \to J/\psi K_S (J/\psi\phi)$ and related final states
- ? Exercise:
  - Determine the angle β in terms of CKM elements
  - Convince yourself that this is accessible through the decay $B^0 \to J/\psi K_S$
    - Hint: Consider B oscillations, the B decay and K oscillations
- The angle β_s remains to be measured to be non-zero
  - Latest measurements approaching required sensitvity
  - Some tension between experiments in $\Delta\Gamma_s$ and Γ_s

Comparison with other LHC experiments



From: Alberto Bragagnolo, Moriond EW 2024
And: CMS BPH-23-004

Recap

- Neutral mesons oscillate with very different parameters for the different mesons
- CP asymmetry requires (at least) two interfering amplitudes with different strong and weak phases
- Time-dependent asymmetry can use different approximations for different mesons

- CP violation observed in kaons, charm and beauty mesons
- Production and detection asymmetries need to be accounted for
- CKM angle γ measured with $B \rightarrow DK$ decays
- CKM element magnitudes measured with (semi-)leptonic decays
- CP violating phase β_s emerging around SM expecation