5.1 The Relaxion

"The Relaxion" was proposed in 2015 [72].[14] This approach uses symmetries in the underlying model, but the Higgs mass itself is not protected by a symmetry. Instead, dynamical evolution of this Higgs mass in the early Universe halts at a point where it is tuned to be much smaller than the cutoff.

As described in [72], if the Higgs is a fundamental scalar then the hierarchy problem relates to the fact that if we keep the theory fixed but change the Higgs mass, the point with a small Higgs mass is not a point of enhanced symmetry. However, this may be a special point with regard to dynamics, since this is the point where the SM fields become light.

The structure of the theory is relatively simple to write down and we will, as always, rely on EFT arguments. Let us consider the SM as an effective theory at the scale M, which is the cutoff of the theory. Following the standard EFT rules we include all of the operators, including non-renormalizable ones, consistent with symmetries. All dimensionful scales are taken to the cutoff M. We add to this theory a scalar ϕ which is invariant under a continuous shift symmetry, $\phi \to \phi + \kappa$, where κ is some constant. This shift symmetry only allows for kinetic terms for ϕ. We then add a dimensionful spurion g which breaks this shift symmetry. As g is the only source of shift symmetry breaking then a selection rule may be imposed, such that any potential terms for ϕ will enter in the combination $(g\phi/M^2)^n$. Thus the theory is written

$$\mathcal{L} = \mathcal{L}_{SM} - M^2|H|^2 + g\phi|H|^2 + gM^2\phi + g^2\phi^2 + ... \tag{5.100}$$

where the ellipsis denote all of the other higher dimension terms and it should be understood that the coefficients of all the operators in eq. (5.100) could vary by $\mathcal{O}(1)$ factors and the negative signs have been taken for ease of presentation.

The next step is to add an axion-like coupling of ϕ to the QCD gauge fields

$$\frac{\phi}{32\pi^2 f} G\widetilde{G} \quad . \tag{5.101}$$

This coupling is very special. As $G\widetilde{G}$ is a total derivative, in perturbation theory eq. (5.101) preserves the shift symmetry on ϕ, thus it is consistent to include this operator without a factor of g in the coupling. Perturbatively this operator will not generate any potential for ϕ, thus all of the shift-symmetry breaking terms involving g remain radiatively stable and it is technically natural for them to be small. However, non-perturbatively the full topological structure of the QCD vacuum breaks the shift symmetry $\phi \to \phi + \kappa$ down to a discrete shift symmetry $\phi \to \phi + 2\pi f z$, where z is an integer. Thus the complete story behind the model is one of symmetries. ϕ enjoys a shift symmetry which is broken to a discrete shift symmetry by QCD effects. The discrete shift symmetry is then broken completely by g.

Let's see how this works. At first pass the field ϕ is massless, and enjoys a shift symmetry $\phi \to \phi + f$. This is the 'nonlinear' realisation of a U(1) symmetry, and so we identify ϕ as a Goldstone boson. Now let us return to the quarks and charge them under this symmetry, such that they cannot have a bare mass term, but can only have a Yukawa interaction with a complex scalar Φ, of which ϕ/f is the phase, as enforced by the U(1) symmetry. Once the

[14] A similar idea was considered much earlier for the cosmological constant problem [141], and alternative relaxation-based approaches to the hierarchy problem have also been explored [142, 143] more recently.

scalar obtains a vev, spontaneously breaking the symmetry, then we can see that the action for the quarks becomes

$$\begin{aligned} \mathcal{L} &= i\overline{\psi}\gamma^{\mu}D_{\mu}\psi + m_{\psi}e^{i\theta_q}\overline{\psi}\psi + h.c. + (\theta + \phi/f)\,\frac{g^2}{32\pi^2}\epsilon_{\mu\nu\alpha\beta}G^a_{\mu\nu}G^a_{\alpha\beta} && (5.102)\\ &\to i\overline{\psi}\gamma^{\mu}D_{\mu}\psi + m_{\psi}e^{i(\theta_q+\theta+\phi/f)}\overline{\psi}\psi + h.c. \quad , && (5.103)\end{aligned}$$

where in the last line we performed an anomalous chiral rotation to move the QCD angle into the quark mass term, and the hermitian conjugate is just an alternative way of writing action without the γ_5 matrix.

The important point is that the Goldstone boson enters the action in just the same way as the bare CP-violating angles. With only these terms this field would be the axion, and we will refer to this U(1) symmetry as $\mathrm{U}(1)_{PQ}$, after Roberto Peccei and Helen Quinn, who spotted that this global symmetry had very interesting implications for the strong-CP problem. Since the axion has a shift symmetry, we may happily shift away the angles $\phi \to \phi - f(\theta_q + \theta)$ such that the action is simply

$$\mathcal{L} = i\overline{\psi}\gamma^{\mu}D_{\mu}\psi + m_{\psi}e^{i\phi/f}\overline{\psi}\psi + h.c. \tag{5.104}$$

This is, of course, relating a shift of the axion field to a quark chiral field rotation! The overall background value of the axion field $\langle\phi/f\rangle$ is now the total physical strong-CP phase. For example, the neutron electric dipole moment is simply proportional to this value $n_{EDM} \propto \langle\phi/f\rangle$. What should this value be?

Lets see what happens when the quarks condense and work now within the SM. We will not include the neutral pion field, associated with the spontaneous breaking of the chiral SU(2) symmetry, however one should consult [144] for a clear and up-to-date treatment including the pions. The result in the SM for the approximation $m_u = m_d = m_q$ is that $m_q\langle\overline{\psi}\psi\rangle = f_\pi^2 m_\pi^2$, thus the action becomes

$$\begin{aligned} \mathcal{L} &= e^{i\phi/f}\langle m_{\psi}\overline{\psi}\psi\rangle + h.c. && (5.105)\\ &\to f_\pi^2 m_\pi^2 e^{i\phi/f} + h.c. && (5.106)\end{aligned}$$

Thus the potential generated for the axion, within QCD, is

$$V(\phi) = -f_\pi^2 m_\pi^2 \cos\left(\frac{\phi}{f}\right) \quad . \tag{5.107}$$

Note that this is a very non-trivial result. We started with a global symmetry which was spontaneously broken, leading to a massless Goldstone boson. However, this symmetry was *anomalous* at the quantum level, under QCD. This means that although in perturbation theory no mass would ever be generated for the axion, there was no obstruction to generating a mass non-perturbatively, and this is precisely what has happened: The $\mathrm{U}(1)_{PQ}$ symmetry was not a true quantum symmetry of the theory, and when QCD became strongly coupled non-perturbative effects become large. Since these effects need not respect the global symmetry, they need not respect the shift symmetry of the axion, and they can, and do, generate a potential and a mass for the axion.

This becomes the crucial insight for the relaxion, since the ϕ-potential generated by QCD effects depends on the light quark masses, which in turn depend on the Higgs vacuum

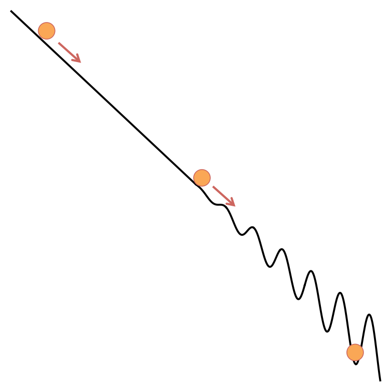

Figure 12: Evolution of the relaxion field in the early Universe from a point where the effective Higgs mass-squared is postive (left), passing through zero (middle), and negative (right).

expectation value and this will provide the dynamical back reaction. In practice this potential is

$$\begin{aligned} V_{QCD} &\sim f_\pi^2 m_\pi^2 \cos\phi/f && (5.108)\\ &\propto f_\pi^3 m_q \cos\phi/f && (5.109)\\ &\propto f_\pi^3 \lambda_{u,d}\langle|H|\rangle \cos\phi/f \quad . && (5.110)\end{aligned}$$

Let us now consider the vacuum structure of the theory for two values of ϕ, including also the effect of the g-terms. If $M^2 - g\phi > 0$ then the effective Higgs mass-squared is positive. QCD effects will break electroweak symmetry, and quark condensation will lead to a tadpole for the Higgs field, which will in turn lead to a very small vacuum expectation value for the Higgs. Thus in this regime the axion potential of eq. (5.110) exists but is extremely suppressed. If $M^2 - g\phi < 0$ the effective Higgs mass-squared will be negative and the Higgs will obtain a vacuum expectation value, so the height of the axion potential will grow proportional to the vev.

Cosmological Evolution

The general idea of the relaxion mechanism is sketched in fig. 12. Imagine at the beginning of a period of inflation the relaxion field begins at values far from the minimum of the scalar potential. We can, without loss of generality, take this to be at $\phi = 0$. Due to its potential it will roll, with Hubble friction providing the necessary dissipation for this to occur in a controlled manner. This Hubble friction can be understood from the equation of motion for a scalar in an inflating background

$$\partial_t^2\phi + 3H\partial_t\phi \approx gM^2 + \ldots \quad , \qquad (5.111)$$

where the ellipsis denotes higher order terms in g. During inflation $H =$ const, and this term provides a constant source of friction, and for large H, one has a non-accelerating solution to the equations of motion $\phi \sim (gM^2/3H)t$. All the while the effective Higgs mass-squared is evolving.

Once the effective mass-squared passes through zero the Higgs will obtain a vacuum expectation value and the axion potential of eq. (5.110) will turn on, growing linearly with the Higgs vev. If the gradient of this potential becomes locally great enough to overcome the gradient of the g-induced relaxion potential, i.e.

$$\frac{f_\pi^3}{f}\lambda_{u,d}\langle|H|\rangle > gM^2 \quad , \tag{5.112}$$

then the relaxion will stop rolling and become stuck. Once it has become stuck the effective Higgs mass-squared has also stopped evolving. If g is taken to be appropriately small, then this evolution will cease at a point where the Higgs vev is small $\langle|H|\rangle \ll M$. As g is a parameter which can take values that are naturally small, and g ends up determining the final Higgs vev, a naturally small value for the weak scale may be generated.

If it could be taken at face value, the picture painted above is quite a beautiful portrait involving SM and BSM symmetries and dynamics. QCD plays a crucial role in determining the weak scale and solving the hierarchy problem. Only an axion-like field, already motivated by the strong-CP problem, is added. Inflation, which is already required in cosmology, provides the dissipation required for solving the hierarchy problem. We even find an explanation for some other puzzles in the SM, such as why there are some quark masses determined by the weak scale which are nonetheless lighter than the QCD strong coupling scale. However, as we will see, some puzzles remain to be understood, presenting a number of interesting areas to explore on the theoretical front.

Parameter Constraints

To determine the viability of the relaxion mechanism it is necessary to consider any constraints on the theory. I will list them here.

- $\Delta\phi > M^2/g$: For the relaxion to scan the entire M^2 of Higgs mass-squared it must traverse this distance in field space.
- $H_I > M^2/M_P$: Inserting the previous $\Delta\phi$ into the potential we find that the vacuum energy must change by an amount $\Delta V \sim M^4$. For the inflaton to dominate the vacuum energy during inflation we require $V_I > M^4$, which corresponds to the aforementioned constraint on the Hubble parameter during inflation.
- $H_I < \Lambda_{QCD}$: For the non-perturbative QCD potential to form, the largest instantons, of size $l \sim 1/\Lambda_{QCD}$, must fit within the horizon.
- $H_I < (gM^2)^{1/3}$: Fluctuations in the relaxion field during inflation (due to finite Hubble) must not dominate over the classical evolution if the theory is to predict a small weak scale.
- $N_e \gtrsim H_I^2/g^2$: Inflation must last long enough for the relaxion to roll over the required field range.
- $gM^2 f \sim \Lambda_{QCD}^4$: It must be possible for a local minimum to form in the full relaxion potential whenever the Higgs vev is at the observed electroweak scale.

Combining these constraints it was found in [72] that the maximum allowed cutoff scale in the theory is

$$M < \left(\frac{\Lambda^4 M_{Pl}^3}{f}\right)^{1/6} \sim 10^7 \text{ GeV} \times \left(\frac{10^9 \text{ GeV}}{f}\right)^{1/6} \quad . \tag{5.113}$$

It is compelling that such a large hierarchy can be realised within the relaxion framework. Let us now saturate eq. (5.113) and take $f = 10^9$ GeV to explore the other parameters of the theory. In this limit we find

$$g \sim 10^{-26} \text{ GeV} \quad , \quad \Delta\phi \sim 10^{40} \text{ GeV} \quad , \quad 5 \times 10^{-5} \text{ GeV} \lesssim H_I \lesssim 0.2 \text{ GeV} \quad , \quad N_e \gtrsim 10^{43} \quad . \tag{5.114}$$

All of these features are quite puzzling or unfamiliar. As such they may represent interesting opportunities for continued theoretical investigation. The parameter g which explicitly breaks the shift symmetry is extremely small. Recent work has shed some light on this question [145]. On a related note, the required field displacement is not only large, it is 'super-duper Planckian' [146]. How such large field displacements can be accommodated by a story involving quantum gravity remains to be fully understood.

With regard to the inflationary aspects, the Hubble parameter is much smaller than is typical in inflationary models. The number of e-foldings is huge (remember the scale factor grows during inflation by a factor $\sim e^{N_e}$). Although not a problem in principle, it may be difficult to realise a natural inflationary model with the appropriate slow-roll parameters which reheats the Universe successfully and also accommodates the observed cosmological parameters.

A more tangible puzzle arises in the simplest QCD model presented above, as it is already excluded by experiment. In the electroweak breaking vacuum the full relaxion potential will be minimized whenever

$$\frac{\partial V_g}{\partial \phi} + \frac{\partial V_{QCD}}{\partial \phi} = 0 \quad , \tag{5.115}$$

where V_g is the scalar potential generated from the terms which explicitly break the shift symmetry, all originating from the parameter g, and V_{QCD} is the axion-like potential coming from the non-perturbative QCD effects. Since the relaxion is stopped by QCD effects before it reaches the minimum of V_g, the first term in eq. (5.115) is non zero. This then implies that the second term in eq. (5.115) must also be non-zero. By construction, V_{QCD} is minimised whenever the effective strong-CP angle is zero, thus if it is not minimised the effective strong-CP angle must be non-zero. In fact, it is typically expected to be close to maximal if the relaxion has stopped in one of the first minima that appears after the Higgs vev starts to grow. This is in clear contradiction with experimental bounds on the strong-CP angle and so the model must be extended, and a number of options have been proposed.

Summary

The relaxion is not yet a complete story, so it is perhaps premature to include it in a lecture course. However, it was the first major step towards a radically different perspective on the hierarchy problem, a perspective that may an important role in BSM theory for a long time to come, so it is included in these lectures.