2.1 Pion-like Higgs

The first and perhaps most obvious possibility to consider is whether the Higgs boson is just like the pions of QCD. This is something that has been considered for some time and it is a very interesting and well-motivated possibility.

When discussing how a scalar mass might be kept small I have frequently referred to the scalar enjoying a shift symmetry. There is in fact a natural setting in which such symmetries arise, and may even be generalised beyond to non-Abelian shift symmetries that include non-linear interactions. It is a deep and very beautiful theorem, proven by Jeffrey Goldstone and others [18, 19], that when an exact global symmetry is spontaneously broken this gives rise to massless scalar bosons. More specifically, if a global symmetry $\mathcal{G}$ is spontaneously broken to a smaller symmetry $\mathcal{H}$ then the theory will contain massless Nambu-Goldstone bosons living in the coset space, $\mathcal{G}/\mathcal{H}$.

Now, calling it a broken symmetry is actually a bit of a misnomer, because the entire symmetry $\mathcal{G}$, is actually always there, however in the Lagrangian we will see the remaining symmetry $\mathcal{H}$ very explicitly as a linearly realised symmetry, with fields transforming in the usual way, whereas the symmetry for the other generators of $\mathcal{G}$, described by generators living in $\mathcal{G}/\mathcal{H}$, will actually be less apparent, and we only see it by its 'non-linear' action on the Goldstone bosons, which to linear order will correspond to the shift symmetry we desire.

If the global symmetry is also explicitly broken then the fields are not true Goldstone bosons, but if this explicit breaking is small then they may still be much lighter than the other scales in the theory. Now, since the symmetry is explicitly broken, we call them 'pseudo-Nambu-Goldstone bosons' (pNGBs).

With this in mind, we can see why the pions were light. In the UV theory, which is QCD, the up and down quark masses are much smaller than the strong coupling scale, and these are the only parameters that explicitly break an $\mathrm{SU}(2)_A$ chiral symmetry, which acts as the chiral symmetry discussed for the electron case did, but as a non-Abelian transformation acting on the up and down quarks. If we assume these mass parameters are the same, then the approximate action is

$$\mathcal{S} = -\int d^4x \,\left[\mathcal{L}_{Kin} + M(\mathbf{q}_L^\dagger \cdot \mathbf{q}_R + h.c)\right] \quad . \tag{2.24}$$

where $M \ll \Lambda$ and the quarks are in doublets. This action respects an $\mathrm{SU}(2)_V$ vector flavour symmetry $\mathbf{q}_L \to U\mathbf{q}_L$, $\mathbf{q}_R \to U\mathbf{q}_R$, however in the limit $M \to 0$ the symmetry is doubled to two independent symmetries $\mathbf{q}_L \to U_L\mathbf{q}_L$, $\mathbf{q}_R \to U_R\mathbf{q}_R$. This means that the mass parameter explicitly breaks an $\mathrm{SU}(2)_A$ axial global symmetry. When the quarks condense due to QCD $\langle \mathbf{q}_L^\dagger \cdot \mathbf{q}_R \rangle \propto \Lambda^3_{QCD}$ the axial symmetry is spontaneously broken $\mathrm{SU}(2)_A \to 0$, hence we get $(2^2 - 1)$ Goldstone bosons. These Goldstone bosons are the three pion degrees of freedom we have been discussing all along! When the quark masses are turned on the symmetry is explicitly broken, thus the pions become massive, however they can be *naturally* lighter than the cutoff as the quark mass is the only parameter that breaks the shift symmetry.

So the obvious questions is: Could the Higgs be a pNGB and even perhaps composite, emerging from some strongly coupled gauge theory in the UV, just like the pions? This has been an extremely active area of investigation and the answer is yes, the Higgs could be just like the pion and, just as the charged pions have gauge interactions that break their shift symmetry, so too can a pseudo-Goldstone Higgs boson. The top quark interactions which explicitly break the global symmetry, and hence the shift symmetry, are very large however, so some work is required to have them not lead to very large Higgs mass corrections.

Let's see how this goes. Since the Higgs is not massless it is not really a Goldstone, but a pseudo-Goldstone boson, just like the pions, thus I will refer to models of this class as pNGB Higgs models, (pseudo-Nambu-Goldstone boson Higgs). The main classes of models of this class are composite Higgs models (just like pions), Little Higgs models (similar technology, but with machinery that can protect the Higgs mass to higher loop orders), and the more recently popular Twin Higgs models. In the next section I will sketch the main ideas common to composite and Little Higgs models. However, if you wish to know more the lectures by Contino [20] not only beautifully explain the field theory behind composite Higgs models, but also present the types of models more commonly considered for vanilla composite Higgs scenarios. The review by Schmaltz and Tucker-Smith on the Little Higgs models is a superb starting point for these models [21].

pNGB Higgs

The basic recipe is the following. Let us take some symmetry $\mathcal{G}$ with a gauged subgroup $\tilde{\mathcal{G}}$, spontaneously broken to $\mathcal{H}$ with a gauged subgroup $\tilde{\mathcal{H}}$. Now, we have have $N_G = \dim(\mathcal{G}) - \dim(\mathcal{H})$ Goldstone bosons, of which $N_g = \dim(\tilde{\mathcal{G}}) - \dim(\tilde{\mathcal{H}})$ are eaten by the gauge bosons, leaving $N = N_G - N_g$ massless scalars at tree-level.

Thus we see that in order to fit a Higgs doublet into this concoction we must have at least $N \geq 4$ and $\tilde{\mathcal{H}} \supset \mathrm{SU}(2) \times \mathrm{U}(1)$. A great deal of effort has gone into enumerating the possibilities, but let us study the absolute simplest one. This model has $\mathcal{G} = \mathrm{SU}(3)$, $\mathcal{H} = \tilde{\mathcal{H}} = \mathrm{SU}(2)$, thus the number of Goldstone bosons is $N = 8 - 3 = 5$. This model in fact does not respect custodial symmetry, which means that the dangerous operator

$$\mathcal{O}_T = \frac{1}{\Lambda^2}(H^\dagger D_\mu H)^2 \tag{2.25}$$

that modifies the SM prediction for the W to Z-boson mass ratio can be generated by the physics at the UV, so this model is actually very much disfavoured by the precision LEP measurements. Nonetheless, this model is so simple that it serves well as a straw man for pNGB scenarios, so we will study it here.

The low energy dynamics of the pNGBs are described in full generality by the CCWZ construction [22, 23], that I encourage you to read, however for these lectures it suffices that we may capture the relevant operators by considering what is generally known as a non-linear sigma model, with the field parameterisation

$$\Sigma = e^{i\Pi/f}\Sigma_0, \qquad \Pi = \pi^a T^a, \tag{2.26}$$

where π^a are the pNGBs, T^a are the broken generators of G, and $\langle\Sigma\rangle = |\Sigma_0| = f$. The global symmetry breaking is induced by a scalar field, Σ, transforming as a **3** under SU(3), which acquires a vacuum expectation value $\Sigma_0 = (0, 0, f)$. The pNGBs can thus be parameterized by the non-linear sigma field as in Eq. (2.26), with

$$\Pi = \pi^a T^a = \begin{pmatrix} 0 & 0 & h_1 \\ 0 & 0 & h_2 \\ h_1^\dagger & h_2^\dagger & 0 \end{pmatrix} + \ldots, \tag{2.27}$$

with T^a the broken generators of $\mathrm{SU}(3)_W$ and we have not included the additional singlet pNGB corresponding to the diagonal generator. We may write the sigma field explicitly as

$$\Sigma = \begin{pmatrix} ih_1 \dfrac{\sin(|h|/f)}{|h|/f} \\ ih_2 \dfrac{\sin(|h|/f)}{|h|/f} \\ f\cos(|h|/f) \end{pmatrix} , \tag{2.28}$$

where $|h| \equiv \sqrt{h^\dagger h}$.

Gauge Interactions

The gauge interactions can be added in the usual way. If we wish, we can add them in an SU(3)-invariant manner, with the covariant derivative

$$D_\mu \Sigma \quad , \quad D_\mu = \partial_\mu + ig \sum_a W_\mu^a \lambda^a \quad , \tag{2.29}$$

where λ^a are the SU(3) generators. Then we simply set all but the SU(2) gauge fields to zero. Note that after electroweak symmetry breaking the interaction strength of the physical Higgs boson with the SM gauge fields is suppressed by a factor[6]

$$\cos(v/f) \approx 1 - \frac{1}{2}\frac{v^2}{f^2} \quad , \tag{2.30}$$

thus we can test a pNGB scenario like this by looking for modified Higgs interactions!

Gauging a subgroup of the full global symmetry is an explicit breaking of the global symmetry, thus the pNGB Higgs mass is not protected against quadratic corrections in the gauge sector. Indeed, in analogy with the pion mass corrections from before, at one loop gauge interactions will generate a Higgs mass-squared proportional to

$$\delta m_h^2 \sim \frac{g^2}{16\pi^2}\Lambda^2 \tag{2.31}$$

where Λ is the UV cutoff. In a pNGB model where this is the full story then one must follow calculations such as for the pion mass splitting, which include a priori unknown form factors, in order to estimate the correct magnitude of the correction. Note that in order for these corrections not to be too large one requires that the cutoff is not too far away, and thus the full-blown dynamics of the composite sector, including heavy vector mesons, should/could be accessible at the HL-LHC.

We may also employ additional tricks to suppress these corrections. Imagine we didn't switch off the additional gauge bosons. Then we would have the full SU(3) symmetry, however we wouldn't have any leftover Goldstone bosons to play the role of the Higgs doublet. Then let us instead take two separate Σ fields, each with their own SU(3) global symmetry, but we gauge the diagonal combination of these symmetries, such that both fields are charged

[6] This may be found from the usual relation $c_{hVV} = \frac{1}{g m_V}\frac{\partial m_V^2}{\partial h}$.

under the SU(3) gauge symmetry. When both fields get a vev we get two sets of SU(3)/SU(2) Goldstone bosons. One set is eaten, but the other set remains light. Since the original theory was written in a fully SU(3)-invariant manner, no quadratic divergences arise. At worst, at one loop the gauge interactions will induce dangerous interactions such as $(\Sigma_1 \cdot \Sigma_2)^2$, but this is suppressed by a loop factor, such that the resulting correction to the Higgs mass is

$$\delta m^2 \sim \frac{g^4}{16\pi^2} \log\left(\frac{\Lambda^2}{\mu^2}\right) f^2 \tag{2.32}$$

which is significantly smaller than the correction in the simplest model! This is the essence of the Little-Higgs trick for the gauge sector, and it can be extended to a greater number of symmetries to further suppress these corrections.

Top Quark Interactions

We must also add the top quark Yukawa. The simplest way to do this is to work in analogy with the gauge sector. With the gauge sector we start with a full SU(3) gauge multiplet, and set some fields to zero, which explicitly breaks the symmetry. Here we may do the same, by introducing the incomplete SU(3) quark multiplet $Q = (t, b, 0)$, alongside the usual right-handed top t_R. Then the Yukawa interaction can be written in an SU(3)-invariant manner

$$\mathcal{L}_\lambda = \lambda_t Q \cdot \Sigma t_R \tag{2.33}$$

Of course, just as with the gauge sector, at one loop a quadratically divergent Higgs mass correction will be generated

$$\delta m^2 \sim \frac{3\lambda_t^2}{16\pi^2}\Lambda^2 \quad . \tag{2.34}$$

One may wish to simply tolerate this, and thus place strong limits on how large Λ can be for the solution to the hierarchy problem to really hold. Other options include adding 'top partner fields'. For composite Higgs scenarios there are numerous possibilities, thus I refer the interested reader to [24] for an overview. Let me just sketch a basic example showing how these extra fields may cancel quadratic divergences.

Let us really make the interaction SU(3)-invariant by putting the missing field back in $Q = (t, b, T)$, and also add a little bit of explicit breaking of SU(3) by coupling T to a conjugate fermion to give it a Dirac mass $M_T \ll \Lambda$. Thus we have

$$\mathcal{L}_\lambda = \lambda_t Q \cdot \Sigma t_R + M_T T^c T \quad , \tag{2.35}$$

where now the SM right-handed top quark will in general be a linear combination of t_R and T_c. At high energies M_T is just a small perturbation, and the Yukawa is fully SU(3) symmetric, thus based on symmetry reasons alone the largest quadratic correction we can generate for the Higgs mass can at most be of the order

$$\delta m^2 \sim \frac{3\lambda_t^2}{16\pi^2} M_T^2 \quad . \tag{2.36}$$

If $M_T \ll \Lambda$ then we have tamed, to some degree, the quadratic corrections to the Higgs from the top sector. One can show that this setup leads to a diagrammatic cancellation of the form shown in fig. 5. However, we now have an additional coloured fermion that we can search for at the HL-LHC.

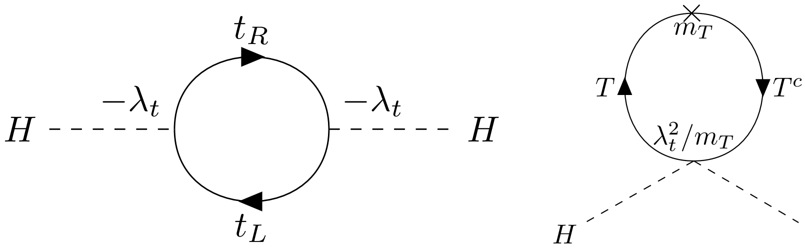

Figure 5: The cancellation between quadratic divergences from the top quark loop and fermionic top-partner loops.

Fine-Tuning

This concludes a summary of the basic features of pNGB Higgs models. The details are much more involved than I have sketched here, however these basic building blocks should provide enough coverage to delve into the literature! The last thing to consider is the fine-tuning in these theories. This stems from two angles. In any microscopic theory explaining the 'Why?' of EW symmetry breaking there are two observable parameters that must be *predicted* from within the UV theory. These are the scale of EW symmetry breaking (Higgs vev) and the Higgs mass. Let us focus on the former. An excellent reference for this discussion is [25].

Due to the nature of the global symmetry if one assumes all of the explicit symmetry breaking arises due to the gauge and Yukawa couplings then the leading contributions to the Higgs potential are of the form

$$V(h) \approx \beta f^2 \Lambda^2 \left(-r \sin^2 \frac{h}{f} + \sin^4 \frac{h}{f} \right) \quad , \tag{2.37}$$

where β and r are expected to be $\mathcal{O}(1)$ coefficients. The minimisation condition for this scalar potential doesn't care about the overall prefactor, thus we find that we require

$$r = 2 \sin^2 \frac{v}{f} \quad . \tag{2.38}$$

There is no symmetry in this theory that can suppress the coefficient of one term over the other, thus to have small v/f requires that parameters in the UV theory are fine-tuned so as to largely cancel and give a small value for r.

This fine-tuning issue is a generic problem for pNGB Higgs models and is known as 'v/f-tuning'. It exists independently of the tuning require to obtain a small Higgs mass, i.e. small β and so is, in some sense, a lower limit on the amount of fine-tuning that exists in this class of theories.[7]

Often we attempt to quantify the fine-tuning, not as an exact science, but as a means to understand how plausible a physical theory is, assuming nature doesn't arbitrarily fine-tune parameters just for our amusement. We typically call this parameter Δ and, if large, the theory is fine-tuned. So here we see that

$$\Delta \gtrsim \frac{1}{2} \frac{f^2}{v^2} \quad . \tag{2.39}$$

[7] There are exceptions if one relaxes the assumption that all explicit symmetry breaking comes from the gauge and Yukawa interactions [26, 27].

Note, however, that the corrections to the Higgs couplings scaled similarly, thus we have that

$$|\Delta| \gtrsim \frac{1}{4\delta c_{hVV}} \quad . \tag{2.40}$$

Hence in minimal incarnations of pNGB (or pion-like) Higgs models one find a direct connection between the magnitude of modifications to Higgs couplings and the amount of fine-tuning present in the theory. The more SM-like, the more fine-tuned! This will inform our perspective on these models at the HL-LHC, as we now discuss.

HL-LHC Prospects

We're here to discuss BSM at the HL-LHC. The prospects for a pNGB Higgs are interesting and come from three angles.

Higgs Couplings

As we saw, one expects modifications to Higgs couplings in pNGB scenarios and these modification are directly tied to the amount of fine-tuning, hence 'plausibility', by some metric, of the theory. Presently we have constraints on Higgs couplings with a precision of around $7-8\%$, or globally around the 6% level [7, 8]. In fact, in the vector couplings ATLAS and CMS both have a slight preference for an enhanced coupling, although this is not significant, placing even more stringent constraints on pNGB models at present, since coupling reductions are more difficult to accommodate with the measured data.

Either way, for sake of comparison with HL-LHC expectations I will assume a SM-like central value, and present constraints at around the 7% level. At HL-LHC we expect the coupling precision to increase significantly, down to the 1.5% level [10]. Note that this is consistent with a $\sqrt{\mathcal{L}}$ scaling. There are two ways to look at this. What is presently a $\sim 1.5\sigma$ fluctuation could grow to a 5σ discrepancy in Higgs couplings at the HL-LHC! There really is plenty of room left for surprises to show up in Higgs couplings.

For another perspective, a SM-like Higgs with present coupling sensitivity corresponds to fine-tuning around the 30% level at the least. On the other hand, if the Higgs coupling measurements persistently remain SM-like, to within precision, at the HL-LHC then in minimal pNGB-like Higgs scenarios the fine-tuning would grow considerably, to around the 6% level.

Vector Resonances

You will recall that in QCD a great number of resonances, beyond the lightest pions, arise as a consequence of confinement in QCD. Thus, similarly, were the Higgs to be pion-like, arising as a composite from some strongly-coupled high energy sector, we should also expect a slew of heavy resonances to come along for the ride. In particular, vector counterparts of the pions, such as the ρ-meson in QCD, ought to arise. Such resonances are thus characteristic, and largely unavoidable, expectations for a pion-like Higgs.

These resonances would couple to the EW gauge sector and essentially mix with the EW gauge bosons in the UV. As a result they would inherit couplings to fermions and could thus arise as very clear signatures such as dilepton or diboson resonances. Present limits on

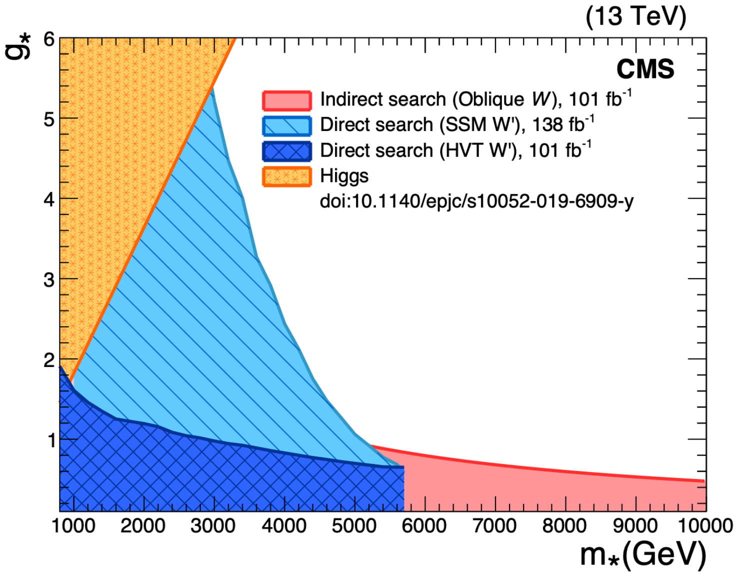

Figure 6: CMS limits, taken from [28], on the heavy vector resonances expected in pNGB-like Higgs scenarios.

dilepton resonances are coupling-dependent, however, under certain assumptions they can already exceed 5 TeV (see fig. 6).

What are the prospects for the HL-LHC? Since, as a function of mass, the production cross section for a resonance depends on the parton PDFs one can not perform an easy rescaling to determine the reach. However, at this point I would introduce you to the Collider Reach tool.[8] This tool makes simple assumptions and rescales according to pdfs. Using it one finds that the mass reach should extend from 5 TeV to above 6.5 TeV at HL-LHC, which is indeed consistent with more dedicated studies [29].

Coloured Resonances

We saw in sect. 2.1 that we required coloured (QCD-charged) fermionic state to complete the multiplets in our example pNGB Higgs scenario. It turns out that this is generically the case and so a typical prediction for a pNGB-like Higgs scenario is the existence of such coloured fermions around the TeV scale. Such states have a rich and interesting phenomenology [24].

At present, we have not observed such states at the LHC, up to around 1.5 TeV [30]. This is already starting to put fine-tuning pressure on vanilla pNGB models since $\frac{3}{16\pi^2}(1.5\text{ TeV})^2 \sim (200\text{ GeV})^2$, typically leading to fine-tuning contributions at the $\mathcal{O}(10\text{'s})$ % level. As we will soon see, this leads model builders to consider how 'generic' the requirement for such top partners is. In any case, how will things change at the HL-LHC?

For broad consideration of BSM resonances at HL-LHC I draw your attention to [32]. For the case at hand, one does not expect the reach to increase significantly as compared to the present day. The reason is made clear in fig. 7. The dashed line shows the pair production cross section for a particular type of heavy coloured fermionic resonance, alongside the projected reach. With present limits around 1.5 TeV, based on $\sqrt{\mathcal{L}}$ scaling we should expect

[8] Be careful to use only http, not https, which won't open.

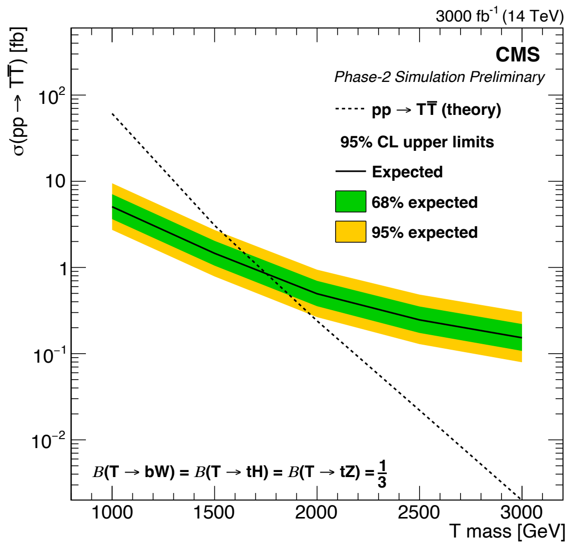

Figure 7: CMS projection, taken from [31], of the HL-LHC reach for vector-like T quarks. Note the slope of the cross section with mass.

to be able to access a cross section which is a factor ~ 4.6 or so smaller. From the dashed line of fig. 7 we see this corresponds to masses around $\lesssim 1.8$ TeV, which is indeed where the expected limit line for HL-LHC crosses the predicted cross section.

As a result, we see that, due to a production cross section which falls steeply with mass, largely because of the decrease of PDFs at larger x for fixed $\sqrt{S}$ we ought not to expect the limit on the coloured fermionic resonances typical of pNGB models to change significantly between now and the end of the HL-LHC.