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Precision physics of the Standard Model

Outline

- 1) The Standard Model: Lagrangian, fields, interactions and all that 2) Input parameters of the Standard Model and what does it take to get them right 3) The muon decay and the Fermi constant "problem" 4) The muon anomalous moment saga 5) How to check the unitarity of the CKM matrix 6) Precision physics at the LHC: the anatomy of Higgs production in gluon fusion, electroweak Sudakov logarithms and why the interpretation of the top quark mass measurement is
- challenging

The Standard Model

laboratory where this theory can be studied.

• the gauge principle, which forces us to describe electromagnetic, weak and strong interactions by introducing ``compensating'' fields that allow us to turn the global symmetries of the theory

• the idea that the gauge symmetry is broken by the vacuum expectation value of an elementary scalar field: it makes electroweak gauge bosons massive and allows us to provide masses to quarks and leptons without explicitly breaking the gauge invariance in the Lagrangian.

• the requirement of the renormalisability which strongly restricts the number of admissible terms

- into the local ones.
-
- and free parameters in the Lagrangian and, eventually, makes the Standard Model an absolutely predictive theory (at least in principle).

The Standard Model is based on three principal ideas:

The theory contains matter fields (quarks and leptons). The gauge symmetry is broken by an elementary scalar field (the Higgs field). The theory is renormalisable.

The gauge group of the Standard Model is $SU(3) \times SU(2)$ _{LX}U(1) γ . The first gauge group is responsible for the physics of strong interactions; the second and third ones — for weak and electromagnetic ones. The subscript L means "left" and the subscript Y means "hypercharge".

- the kinetic terms of the gauge fields: they are fully-fixed by the selected gauge groups;
- •the kinetic terms of the matter fields: they are fully fixed once quantum numbers of the matter fields (i.e. the way they change under gauge transformations) are specified;
- field are fixed;
- the scalar field to be non-vanishing. Its form is fixed by the requirement of the renormalisability of the theory and the gauge invariance.
- Yukawa terms is fixed by the gauge invariance of the Lagrangian and by the renormalisability of the theory.

•the kinetic term of the scalar field: it is also fixed once the quantum numbers of the scalar

•the symmetry breaking term of the scalar field that drives the vacuum expectation value of

•the Yukawa terms: they describe the interaction of the scalar field with the matter fields. They give masses to matter fields after the spontaneous symmetry breaking. The form of the

With this, the Lagrangian of the Standard Model is the sum of the following terms:

⁼ @*^µB*⌫ @⌫*B^µ* ⌫ @⌫*^W* ⌫ @⌫*^G* relations Ge \overline{M} \overline{a} *I*
I alised in a standard way and satisfy the standard Lie algebra commutation The quantities *Tⁱ* and *T^a ^s* are generators of the *SU*(2) and *SU*(3) algebra in the community of the sentegration of the standard representations, respectively. The standard community standard com-Generators are normalised in a standard way and satisfy the standard Lie algebra commutation 1

Bµ⌫*B^µ*⌫*.*

$$
\text{Tr}[T^a, T^b] = \frac{1}{2} \delta^{ab} \qquad [\mathcal{T}^i, \mathcal{T}^j] = i \epsilon^{ijk} \mathcal{T}^k \qquad [\mathcal{T}_s^a, \mathcal{T}_s^b] = i f^{abc} \mathcal{T}^c
$$

 Ω \vec{l} \sim \sim \sim \sim $\theta^{\mu}B^{\nu}$ – *B
∂
∂ <i>B µ B D D B D D D D D* $\partial^{\mu}\hat{W}^{\nu} - \partial^{\nu}\hat{W}^{\mu} - ig[\hat{W}^{\mu},\hat{W}^{\nu}] \qquad B^{\mu\nu} = \partial^{\mu}B^{\nu} - \partial^{\nu}B^{\mu}$ $-i g[\hat{W}^{\mu}, \hat{W}^{\nu}]$ $B^{\mu\nu} = \partial^{\mu} B^{\nu}$ $B^{\nu}-\partial^{\nu}B^{\mu}$ $O^{\mu\nu}VV^{\nu} - O^{\nu}$ $= \partial^{\mu}\hat{W}^{\nu} - \partial^{\nu}\hat{W}^{\mu} - ig[\hat{W}^{\mu},\hat{W}]$ $\hat{W}^{\mu\nu}=\partial^{\mu}\hat{W}^{\nu}-\partial^{\nu}\hat{W}^{\mu}-ig[\hat{W}^{\mu},\hat{W}^{\nu}] \qquad B^{\mu\nu}=0$

The kinetic terms for the gauge fields read: The Kinetic terms for the gauge fields read: kinetic terms for gauge fields. We write *LITE KINELIC LEITIIS IOI LITE GUUGE HEIUS IEUU:* $\mathcal{L} = \mathcal{L}$ *he* k ˆ \overline{I} etic terms for the go $\overline{}$ *<u>Iuge</u> f* ˆ $\ddot{}$ *ield* ds *,*

$$
\hat{G}^{\mu} = \sum_{1}^{8} G^{a,\mu} T_{s}^{a}
$$

$$
\hat{W}^{\mu} = \sum_{1}^{3} W^{i,\mu} T^{i}
$$

$$
\mathcal{L}_{kin} = \mathcal{L}_{QCD}^{kin} + \mathcal{L}_{SU(2)}^{kin} + \mathcal{L}_{U(1)}^{kin}
$$
\n
$$
\mathcal{L}_{QCD}^{kin} = -\frac{1}{2} \text{Tr} \left[\hat{G}_{\mu\nu} \hat{G}^{\mu\nu} \right] \qquad \mathcal{L}_{SU(2)}^{kin} = -\frac{1}{2} \text{Tr} \left[\hat{W}_{\mu\nu} \hat{W}^{\mu\nu} \right] \qquad \mathcal{L}_{U(1)}^{kin} = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu}
$$

$$
\hat{G}^{\mu\nu} = \partial^{\mu}\hat{G}^{\nu} - \partial^{\nu}\hat{G}^{\mu} - ig_{s}[\hat{G}^{\mu}, \hat{G}^{\nu}] \qquad \hat{W}^{\mu\nu} = \partial^{\mu}
$$

*J*₂
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 U(1) DOTENTIALS AIF $r \in$ $\frac{1}{2}$ The field-strength tensors reads \Box *Inear* combinations of f generators of the corresponding **g** algebra *B***ar** combinations of general \blacksquare

nerators of the corresponding al and potentials are mathods in the conceptionity the argebra option of the corresponding algebra and
and snr ˆ $\frac{1}{2}$ *G n*ding LIE digebra space, v
anding glashra ror potentials are matrices in the corresponding - Lie digebra-space, written as the top of the top of the corresponding - Lie digebra-space, written as the top of the The vector potentials are matrices in the corresponding "Lie algebra" space, written as the linear combinations of generators of the corresponding algebra vector potentials ar *G* rating of the correspondin *,G* ine vootor poterities are matrices in the concepcitanty Lie argebra space, with
linear combinations of aenerators of the corresponding algebra and are macriced in the correspondence on the correspondence of generators of the correspondence

$$
\mathcal{L}_{kin} = \mathcal{L}_{QCD}^{kin} + \mathcal{L}_{S}^{ki}
$$

"right" fields are involved in the electromagnetic ones. re involved in the electromognatic ones Is are involved in the electromagnetic ones.

 δ_{γ} 1 γ 2 γ ³ $structe$ \overline{C} $\psi_L =$ $1 - \gamma_5$ $\frac{1}{2}$ ψ $\psi_R =$ matrix $\gamma_5 = i \gamma^0 \gamma^1 \gamma^2 \gamma^3$

position that die that the same differently under gauge transfermations made numbers. Note that the precedulation of any mass term in the appearance of any mass term in the angle of any ma
The angle of any mass term in the angle of any mass term in the angle of any mass term in the angle of any mas SM Lagrangian since mass terms involve products of left and right fields oright-holdo danoionn amoronay andor gaugo danoionnadono, i singlets (which means that they are not a↵ected under *SUL*(2) transforma- $\mathsf m$ Since left and right fields transform differently under gauge transformations, mass terms for matter fields are forbidden. Hence, all matter particles in the SM Lagrangian are originally massless.

$$
m\bar{\psi}\psi=\frac{m}{2}\left(\bar{\psi}_{R}\psi_{L}+\bar{\psi}_{L}\psi_{R}\right)
$$

It is the parameter of the obvious that is the interestion of the interest in the set of the set of the set of t the mutual included into the theory left-handed $\frac{1}{2}$ of the group *SUL*(2) and all right-handed fields are considered to be *SUL*(2) singlets (which means that they are not a↵ected under *SUL*(2) transforma-We will only consider left-handed neutrinos. Three generations of leptons and of up-type and down-type quarks are included into the theory. Left-handed fields are SUL(2) doublets; A fields are singlets of the SLI(2), aguae aroup. Both left fields and rightneutrino field, and we write for the *j*-th generation of leptons and quarks right-handed fields are singlets of the SU(2)L gauge group. Both left fields and right fields transform under $U(1)_Y$.

$$
L_{j,L} = \begin{pmatrix} V_j \\ I_j \end{pmatrix}_L, \quad I_{j,R}, \quad \Psi_{j,L} = \begin{pmatrix} U_j \\ D_j \end{pmatrix}_L, \quad U_{j,R}, \quad D_{j,R}
$$

To add matter terms, we need to distinguish between left and right fermion fields. This is necessary since "left" fields participate in the (charged) weak interactions and both "left" and ce "left" fields participate in the (charaed) weak interactions and bot itter terms, we need to distinguish between l<mark>eft and right fermion fie</mark>ld since "left" fields participate in the (charged) weak interactions and

tields are constructed using the projection operators that involve $\frac{1}{2}$ *R* $\frac{1}{2}$ *R* The left and right fields are constructed using the projection operators that involve the Dirac

$$
\psi_L = \frac{1 - \gamma_5}{2} \psi \qquad \psi_R = \frac{1 + \gamma_5}{2} \psi \qquad \gamma_5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \qquad \psi = \begin{pmatrix} \xi \\ \eta \end{pmatrix}
$$

¯ *j,LiDµ^µ j,L*

Uj,R, Y D SU(3)

$$
\vec{\mathcal{T}}\Psi_{j,L}=\frac{\vec{\mathcal{T}}}{2}\Psi_{j,L},\quad \vec{\mathcal{T}}U_{j,R}=\vec{\mathcal{T}}D_{j,R}=0,
$$

$$
U_Y(1) \quad \hat{Y}_{L_{j,L}} = -\frac{1}{2} L_{j,L}, \quad \hat{Y}_{J_{j,R}} = -I_{j,R}, \ \hat{Y}_{\Psi_{j,L}} = \frac{1}{6} \Psi_{j,L}, \quad \hat{Y}_{U_{j,R}} = \frac{2}{3} U_{j,R}, \quad \hat{Y}_{D_{j,R}} = -\frac{1}{3} D_{j,R},
$$

\n
$$
SU(3) \qquad \vec{T}_s L_{j,L} = \vec{T}_s I_{j,R} = 0, \quad \vec{T}_s \Psi_{j,L} = \frac{\vec{\lambda}}{2} \Psi_{j,L}, \quad \vec{T}_s U_{j,R} = \frac{\vec{\lambda}}{2} U_{j,R}, \quad \vec{T}_s D_{j,R} = \frac{\vec{\lambda}}{2} D_{j,R}.
$$

$$
SU_L(2) \qquad \vec{\tau}_{L_{j,L}} = \frac{\vec{\tau}}{2} L_{j,L}, \quad \vec{\tau}_{l_{j,R}} = 0, \qquad \vec{\tau}_{\Psi}
$$

The way the covariant derivative acts on the matter fields follows from the formulas below where $\vec{\tau}$ are the P Wriele *T* are the Pauli matrices and *A* are the Gelf-Mann ma \overline{a} *J*UICES. where $\vec{\tau}$ are the Pauli matrices and $\vec{\lambda}$ are the Gell-Mann matrices. Generators of the *SU*(2)*L*, *U*(1)*^Y* and *SU*(3) groups act in the following way acts on the matter fields it \overline{a} The way the cov aric \overline{a} int derive ˆ $\overline{1}$ Wriere τ are the Pauli matrices and λ are the Gelf-Mann matrices. ds follows from the formuld *where* $\vec{\tau}$ are the P α B ˆ $matrixes$ ˆ where $\vec{\tau}$ are the Pauli matrices and $\vec{\lambda}$ are the Gell-Mann matrices \overline{a} ovariant derivative acts on the matter fields follows from the formulas be \sim the Pauli matrices and $\vec{\lambda}$ are the Gell $\frac{1}{2}$ *j*–Mar $\frac{1}{2}$ *matrices* $\vec{\tau}$ are the Pauli matrices and $\vec{\lambda}$ are the Gell-Mann matrice *j,R* = 0*, Y L j,L* ⁼ ¹ 3

The kinetic term for leptons and quarks reads Lh $\frac{1}{2}$ ¯*lj,RiDµ^µlj,R* ⁺^X The kinetic term for leptons and quarks, reads are then *^L*kin ⁼ ^X The kinetic term for leptons and quarks, reads *^L*kin ⁼ ^X *^L*¯*j,LiDµ^µLj,L* ⁺^X ¯*lj,RiDµ^µlj,R* ⁺^X L leptons and quarks reads The field-strength tensors reads the field-strength tensors reads to the field-strength tensors reads to the f
The field-strength tensors reads to the field-strength tensors reads to the field-strength tensors reads to th

$$
L_{kin} = \sum_j \bar{L}_{j,L} i D_\mu \gamma^\mu L_{j,L} + \sum_j \bar{I}_{j,R} i D_\mu \gamma^\mu I_{j,R} + \sum_j \bar{\Psi}_{j,L} i D_\mu \gamma^\mu \Psi_{j,L} + \sum_j \bar{U}_{j,R} i D_\mu \gamma^\mu U_{j,R} + \sum_j \bar{D}_{j,R} i D_\mu \gamma^\mu D_{j,R}
$$

where the covariant derivatives +X *^U*¯*j,RiDµ^µUj,R* + +^X *D*¯*j,RiDµ^µDj,R,* where the covariant derivative reads $x \in \mathbb{R}^n$ *Where the covariant derivati j j* ariant derivative reads

where the covariant derivative reads
\n
$$
D_{\mu} = \partial_{\mu} - ig\hat{W} - ig'\hat{Y}B_{\mu} - ig_{s}\hat{G}
$$
\n
$$
\hat{W}^{\mu} = \sum_{1}^{3} W^{i,\mu}T^{i} \qquad \hat{G}^{\mu} = \sum_{1}^{8} G^{a,\mu}T_{s}^{a}
$$

Dj,R. \overline{a} $D_{j,R}$, *Dj,R, T* マ, ^s *.* (1.5) ^s *.* (1.5)

For reasons that become clear later, we will take Using the gauge symmetry of the parts of the SM Lagrangian that we described begin before the can always doubled into the following the Gauge transformations allow us to remove three real fields from the Higgs doublet U sing the gauge symmetry of the parts of the SM Lagrangian that we have been controlled by U diom de foreitione riliee leal lieids fiorit rile Higgs doubler into the following the following the following tions allow us to remove three real fields from the Higgs doublet Iree real fields from *,* (1.16) $iggs$ $\overline{\mathcal{A}}$ *<u>Jublet</u>*

$$
m_h^2 = 2\lambda v^2
$$

$$
D_{\mu} = \partial_{\mu} - ig \hat{W}_{\mu}^{i} - ig' B_{\mu} \hat{Y} \qquad \qquad \vec{\tau} \phi = \frac{\vec{\tau}}{2} \phi, \quad \hat{Y} \phi = \frac{1}{2} \phi
$$

becomes
$$
L_{\text{Higgs}} = (D_{\mu}\varphi)^{\dagger} (D^{\mu}\varphi) - \lambda v^2 h^2 \left(1 + \frac{h}{2v}\right)^2
$$

$$
\varphi \to U(x)\varphi(x), \quad U(x) = e^{iT^i\theta^i(x)} \qquad \varphi = \begin{pmatrix} \varphi_1 + i\varphi_2 \\ \varphi_3 + i\varphi_4 \end{pmatrix} \implies \varphi(x) = \begin{pmatrix} 0 \\ \frac{\nu + h(x)}{\sqrt{2}} \end{pmatrix}
$$

For L reasons the contract L reasons L reasons L \rightarrow L \rightarrow L \rightarrow L 1 becomes The Lagrangian becomes $L_{\text{Higgs}} = (D_{\mu}\varphi)$

The mass of the Higgs boson reads $\sin \theta$ hoson rands $m^2 - 2\lambda v^2$

 U sing the gauge symmetry of the parts of the parts of the parts of the parts of the SM Lagrangian that we have artic Higgs boson couplings are fully determined once the vo ϵ Hig '(*x*) = ⁰ *L*Higgs = (*Dµ*') value and the Higgs mass are known. Triple and quartic Higgs boson couplings are fully determined once the vacuum expectation and, after spontaneous symmetry breaking, Higgs boson triple- and quartic compositive and the sense couplings are runy actuallimica once the vacuum cxpcctution.
In the second compositions of the sense Hence, once the Higgs boson mass is measured and the vacuum expectation It determined once the vacuum expectation

The kinetic term of the scalar field and the symmetry breaking term read '³ + *i*'⁴ and etic term of the scalar field and the symmetry breaking term etry: ✓ '¹ + *i*'² $\overline{3}$ a read and the set of th '³ + *i*'⁴ *D^µ* = @*^µ igW ^µ ig*⁰ *BµY .* For extremely that become clear later, we will take the commonly terms

$$
L_{\text{Higgs}} = (D_{\mu}\varphi)^{\dagger} (D^{\mu}\varphi) - \lambda \left(\varphi^{\dagger}\varphi - \frac{v^2}{2}\right)^2 \qquad \varphi = \left(\begin{array}{c} \varphi_1 + i\varphi_2 \\ \varphi_3 + i\varphi_4 \end{array}\right)
$$

$$
- ig\hat{W}^i_\mu - ig'B_\mu \hat{Y} \qquad \qquad \vec{\tau}_\phi = \frac{\vec{\tau}}{2}\phi, \quad \hat{Y}_\phi = \frac{1}{2}\phi
$$

From the kinetic term, we From the kinetic term, we find

$$
\varphi(x) = \left(\begin{array}{c} 0 \\ \frac{v + h(x)}{\sqrt{2}} \end{array}\right) \longrightarrow
$$

$$
\cos\theta_W = \frac{g}{\sqrt{g^2 + {g'}^2}}, \quad \sin\theta_W = \frac{g}{\sqrt{g^2 + {g'}^2}}
$$

*m*² *^h* = *y* 2 ² *,* (1.18) $\overline{\text{ir}}$ *v* ²(*g*² + *g*0² \overline{a} of *the weak mixing c* $\overline{2}$ *.* where where the contract of the cont agu the gauge coup α ^{*l*} $\overline{8}$ constants where cosine and sine of the weak mixing angle are defined through the gauge coupling

The Higgs boson kinetic term plays a very important role in the Standard Model as it allows us to generate masses for gauge fields. and The Higgs boson kinetic term plays a very important role in the Standard Model as it allows us is the dependent masses for day defields.
In dependent masses for day a scalar *SU(2) download the vacuum expectation* reads to a scalar *SU(2) download to b* ✓ '¹ + *i*'² ira ivioae
' i t allows us

$$
L_{\text{Higgs}} = (D_{\mu}\varphi)^{\dagger} (D^{\mu}\varphi) - \lambda \left(\varphi^{\dagger}\varphi - \frac{v^2}{2}\right)^2 \qquad D_{\mu} = \partial_{\mu} - ig\hat{W}_{\mu}^{i} - ig'B_{\mu}\hat{Y}
$$

22 (22) 222 (22) 222 (22) 232 (22) Using the gauge symmetry of the gauge symmetry of the symmetry of the symmetry of the SM Lagrangian that we show that we have the symmetry of the symmetry of the symmetry of the symmetry of the SM Lagrangian that we have t de guuge neius, we neeu to repiuce the Higgs neiu with its By definition, mass terms are quadratic in the corresponding fields; to find the mass terms of the gauge fields, we need to replace the Higgs field with its vacuum expectation value. \overline{a} For reasons that become clear later, we will take J \mathcal{C} ,uun
. ϵ *, Y* \overline{C} $\overline{\text{E}}$ $\sqrt{6}$ *a* 145.

$$
\begin{array}{ccc}\n & \rightarrow & \varphi_{\text{vac}} = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}\n\end{array}
$$

$$
(D_{\mu}\varphi)^{\dagger} (D^{\mu}\varphi) \rightarrow \frac{v^{2}g^{2}}{8} (W_{\mu}^{1}W^{1,\mu} + W_{\mu}^{2}W^{2,\mu}) + \frac{v^{2}(g^{2} + g^{\prime 2})}{8} (\cos \theta_{W}W_{\mu}^{3} - \sin \theta_{W}B_{\mu})^{2}
$$

$$
\frac{g}{\sqrt{g^2+g^{\prime 2}}}, \quad \sin \theta_W = \frac{g'}{\sqrt{g^2+g^{\prime 2}}}
$$

The fields *W*¹*,*² and the field *Z* obtain masses *m^W* = *v g* 2 <u>CICOLIOITIO</u> *v g ,* (1.23) \mathcal{S}, I **U** IS (\overline{C} α *zandidate* to *v g* the electromagnetic field. We will now rewrite the covariant derivative in terms of the above fields. This the field A is mussless, it is a canalation are convenient the electromagnetic h P *ibe the electre* Since the field A is massless, it is a candidate to describe the electromagnetic field.

 $y = w - w - w^2$ *a W* and *Z* masses and the weak mixing angle $m_W = m_Z \cos \theta_W$ *,* (1.23) $m_{\text{HZ}} = m_B \cos \theta_{\text{HZ}}$ To this end, it is convenient to introduce linear combinations of *W*¹*,*² fields $m_W = m_Z \cos \theta_W$

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and after some manipulations find (*Dµ*') *†* (*D^µ*') ! *v* 2*g W*₁, THIS IS INCONVENIENT (*Dµ*') 2 and D field h *µ*
µ^{*W*}_W2 + *W*2
*W*2 + *W2* + *W2 v* ²(*g*² + *g*0² INCON Writter 110100, CHO HTUOS CONTINUTIO p_2 p_3 p_4 p_5 p_6 p_7 p_8 p_9 p_9 Written in terms of the W 3 and B fields, the mass term is not diagonal. This is inconvenient.

$$
(D_{\mu}\varphi)^{\dagger} (D^{\mu}\varphi) \rightarrow \frac{v^{2}g^{2}}{8} (W_{\mu}^{1}W^{1,\mu} + W_{\mu}^{2}W^{2,\mu}) + \frac{v^{2}(g^{2} + g^{\prime 2})}{8} (\cos\theta_{W}W_{\mu}^{3} - \sin\theta_{W}B_{\mu})^{2}
$$

$$
\cos\theta_{W} = \frac{g}{\sqrt{g^{2} + g^{\prime 2}}}, \quad \sin\theta_{W} = \frac{g^{\prime}}{\sqrt{g^{2} + g^{\prime 2}}}
$$

J LUKE UL
J *g g*² + *g*₂ *g* $\overline{\partial}$ ^p*g*² ⁺ *^g*0² *,* (1.21) are comme the sine of the sine of the so-called when we have mixing and a take care of this problem, we define new fields which diac To take care of this problem, we define new fields which diagonalize the mass matrix. They read

$$
Z_{\mu} = \cos \theta_{W} W_{\mu}^{(3)} - \sin \theta_{W} B_{\mu}, \quad A_{\mu} = \sin \theta_{W} W_{\mu}^{(3)} + \cos \theta_{W} B_{\mu} \qquad W_{\mu}^{\pm} = \frac{1}{\sqrt{2}} \left(W_{\mu}^{1} \mp i W_{\mu}^{2} \right)
$$

The masses of the gauge bosons are: $m_{W} = \frac{vg}{2}, \quad m_{Z} = \frac{vg}{2 \cos \theta_{W}}$, $m_{A} = 0$

The r

 $m_{\rm F}$ There is an important relation between W and Z masses and the weak mixing angle $m_W = m_Z \cos$

^µ ^T ⁺

$$
D_{\mu} = \partial_{\mu} - \frac{ig}{\sqrt{2}} \left(W_{\mu}^{-} T^{-} + W_{\mu}^{+} T^{+} \right) - i Z_{\mu} (g \cos \theta_{W} T^{3} - g' \sin \theta_{W} Y) - ig \sin \theta_{W} A_{\mu}
$$

$$
T^{\pm} = T_{1} \pm i T_{2}
$$

 SS eigenstate) (*W ^µ T* + *W*⁺ We rewrite the covariant derivative, in terms of the physical (mass eigenstate) gauge fields We rewrite the covariant derivative in terms of the physical (mass eigenstate) gauge fields. on the various matter fields

 $-$ *ig* sin $\theta_W A_\mu$ (*T*₃ + *Y*) μ , μ μ μ σ coupling on σ and σ *i* $\frac{1}{\sqrt{2}}$ $\frac{1}{\sqrt{2$ $iZ_{\mu}(g\cos\theta_{W}T^{3}-g^{\prime}\sin\theta_{W}Y)$ $\frac{1}{2}$ \overline{a} *Lj,L, T l* ~ ˆ *j,R* = 0*, Y L* $\mathsf{I}\ \sigma_{\mathsf{W}}\mathsf{A}_{\mathsf{\mu}}$ *Lj,L, Y l*

 $=T_1 \pm iT_2$ ~

 $\mathsf{v}_\mathsf{W} = \mathsf{v}_3 + \mathsf{v}_1$ τ $\mathcal{L} = \mathcal{L} \left(\mathcal{L} \right)$ are the three Pauli matrices and the three Pauli $\mathsf{Q} = I_3 + \mathsf{Y}_1$

as the operator of the operation of the electric charge (in units or electric charge (in 1/2).
It follows that the electric charge the contraction of the theoretic charge of the second charge of the second o Standard results for electric charges of neutrinos (Q = 0), leptons (Q=-1), up-quarks (Q=+2/3) $triangleright$ At the mean toophil and independent matter

Comparing this expression with the above covariant derivative, we find Comparing the two expressions, we find that we can associate *g* sin *theta^W* Comparing this e and interpretation with the comparticity *n* with the above covariant derivative, we find *i*
Tipuri \overline{Q} *LIIIS* EXPIESSION WILLI LITE UDOVE \overline{C} *j,L,* 2 2

$$
e = g \sin \theta_W \qquad Q = T_3 + Y_1
$$

and interpret the combination of the computation of the computation of the computation of the combination of Standard results for ele α *rier.* α neutrino electric charges is and the trick of the left- and right- and right-handed.
Support carliar electrons are 1, as it should be. We also find that electric charges of up to the up to up to up to up to up to and down-quarks $(Q = -1/3)$ easily follow from the weak isospin and hypercharge assignments discussed earlier. $\sqrt{2}$ *n*
al alaus !!
- Θ \overline{C} $J = \frac{1}{2}$ $\overline{\Gamma}$ 2 *S*
*M*_{α} *S*_{*n*}, *E*_{*s*}*D* = *J*, *E*_s*D* = *J*, *E*_s $\ddot{}$ \bigcap f *Dj,R.* USSIGNINGERS UISCUSSEU EUNIER.

Interactions of matter fields with gauge bosons are hidden in the kinetic term for the matter fields, more precisely in the covariant derivatives. the kine Furthermore, we need an inverse transformation for *W*³ and *B* to *Z* and *A*. Figlde The community of leptons and the kinetic terms and the summer three generations and three generations and thre
Control to the control of the control over the generations and the control of the control of the control of th fields, more precisely in the covariant derivatives. $\frac{1}{2}$

$$
L_{kin} = \sum_{j} \bar{L}_{j,L} i D_{\mu} \gamma^{\mu} L_{j,L} + \sum_{j} \bar{I}_{j,R} i D_{\mu} \gamma^{\mu} I_{j,R} + \sum_{j} \bar{\Psi}_{j,L} i D_{\mu} \gamma^{\mu} \Psi_{j,L} \qquad D_{\mu} = \partial_{\mu} - ig \hat{W} - ig' \hat{Y} B_{\mu} - ig \hat{S}
$$

In QED, the coupling of a fermion with the charge Qe (e > 0) to a photon field reads IFD, the coupling of a fermion with the $i\bar{\psi}\gamma^{\mu}$ ($\partial^{\mu} - iQeA_{\mu}$) ψ *i* σ α (σ σ) *...* (2) *a* σ σ σ σ σ σ $\frac{1}{2}$ Comparing that the charge $\bigcap_{i=1}^n (a_i > 0)$ to a photon field reads with the position charge charge
Service charge ψ In QED, the coupling of a fermion with the charge Qe (e > 0) to a photon fie $\overline{P}_{\mathsf{PQ}}\mu$ ($\partial\mu$ in $\Omega_{\mathsf{Q}}\Lambda$) also *j* ψ *j* $\overline{2}$ \overline{e} $\overline{1}$ *j,R* ⁼ ¹

) *.* (1.53)

is a search and renormalize and renormalize the direction of the search the search the search the search the search of the s point and upontumous up in moury wrough my, we up grangian involves coupling between the Higgs fields and *di*↵*erent* lepton fam-Is symmetry breaking, we are left with terms that are quadratic in fermion fields, but where different "generations" of leptons and quarks mix *S SU(1)* μ *U*(1) μ *U*(1) μ *U*(1) μ *U*(1) μ *U*(1) μ *U*(1) μ *V* (*S*) μ *U*(1) μ After the spontaneous symmetry breaking, we are left with terms that are quadratic in $\overline{}$ After the spontaneous symmetry breaking, we are left with terms that are quadratic in Terrinon Heids, but where different generations of leptor Once the Higgs field develops field develops and develops a value of the state of the state of the state of the mass of the quark mass mass of the mas the derivations of repressions it where different "generation termion fields, but where different generations \overline{C} cur
1en ing. *µ* We are left with terms that are quadratic in *jk*

Since for us neutrinos are massless, a similar m **THE PROVE TO PROVE THE CONSIDER ASSERTING** $inton_o$ Since for us neutrinos are massless, a similar matrix does not appear in the lepton sector.

Before we continue with the discussion of weak interactions of the matter fields, we need to discuss the Yukawa interactions. The most general Yukawa Lagrangian reads WE WE CONTINUE WILL LITE CHOEGOOD TO WE THE LITE OF LEPTONS. SINCE THE LITE OF LITE OF LITE OF LITE OF LITE OF LO **QISCUSS** LITE YUKUWU INLEIC ˆ cuons. The most gen the discussion of weak interactions of the matter fields, we need Using the properties of the most appearal Vulkawa Lagrangian reads $\frac{1}{2}$ can be down for down-type quarks but for the up-type quarks, we need to use properties of the *SU*(2) and construct another doublet out of the Higgs to discuse the Yukawa interactions The most general Yu quires the value of value of value \sim USUSUSIUT OF WUUN THULUUUS ULUTU THULUU HURUS, WU HUUU
Turkama Lagrangian for August 1 UULIUIS. IIIU IIIUSL e with the discussion of weak interactions of the matter fields w Using the choodool these properties, we can write a gauge-invited theorem. We can write a gauge-invited theorem
Using the contract of the cont LO AISCUSS LNE YUKAWA INLEFACLIONS. with the discussion of weak interactions of the matter fields, we per EWILLE CONSCUSSION OF WEAK THEFACTIONS OF LITE THALLEL HEIAS, WE HEE Refore we continue with the discussion of weak interactions of t DEIOIE WE CONTINUE WILLENTE CHOCUSSION OF WEAR THEFIACLIONS OF L rtions Th \mathfrak{f} \overline{p} *W*⁺ *µ U*¯*j,L^µDj,L* + *h.c.* (1.58) $\overline{111}$

$$
L_{y,L} = -f_{jk}\bar{L}_{j,L}\phi I_{k,R} + h.c. - \left(f_{jk}^{(d)}\bar{\Psi}_{L,j}\phi D_{k,R} + h.c\right) - \left(f_{jk}^{(u)}\bar{\Psi}_{L,j}\tilde{\phi} U_{k,R} + h.c\right) \qquad \tilde{\phi} = (i\tau_2\phi^*)
$$

$$
L_{\text{mass}} = -\frac{v}{\sqrt{2}} \left(f_{jk} \bar{I}_{j,L} I_{k,R} + f_{j,k}^* \bar{I}_{k,R} I_{j,L} \right) - \frac{v}{\sqrt{2}} \left(\bar{D}_{L,j} f_{jk}^{(d)} D_{R,k} + h.c. \right) - \frac{v}{\sqrt{2}} \left(\bar{U}_{L,j} f_{jk}^{(u)} U_{R,k} + h.c. \right)
$$

nder These terms have to be diagonalised. This requires different "inter-generational" rotations p of left-handed U and D fields which, however, are part of the same left doublet. *Lerm* that induces transitions from U to D and v offected by the 3 x 3 unitary mixing matrix which up-type a *fjk*¯*lj,Llk ,R* + *f* ⇤ *j,k*¯*lk ,Rlj,L* $\frac{1}{2}$ ⇣ *P* versa become ⌘ $\ddot{}$ gauge-interaction term that induces transitions from U to D and vice versa becomes affected by the 3 x 3 unitary mixing matrix, which descri Comparing to leptons, we see that diagonalization of the mass term for quarks ids writch, nowever, are part of the same left doublet. The $1e$ will require to perform independent rotations of the performance α These terms have to be diagonalised. This requires different "inter-generational" rotations of left-handed U and D fields which however are part of the same left doublet The ry mixing matrix which dog of left-handed U and D fields which, however, are part of the same left doublet. The affected by the 3 x 3 unitary mixing matrix, which describes the "mismatch" in rotations of up-type and down-types left-handed fields and down-types left-handed fields

$$
V^{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 0.9743(1) & 0.2253(6) & 0.0035(1) \\ 0.2252(6) & 0.9734(1) & 0.041(1) \\ 0.0087(3) & 0.040(1) & 0.99915(3) \end{pmatrix}
$$

*i*¹ eptons *g*(*^f*) where we sum over all quarks and leptons.

The current reads
$$
J_Z^{\mu} = \frac{1}{2} \sum_{\psi \in l,q} \bar{\psi} \left[(T_L^3 - Q \sin^2 \theta_W) \gamma_{\mu} - T_L^3 \gamma^{\mu} \gamma_5 \right] \psi
$$

It is convenient to re-write the Z-boson contribution to the covariant derivative through the third component of weak isospin T₃ and the electric charge. We find

$$
-i Z_{\mu} \frac{g}{\cos \theta_{W}} (T_3 - Q \sin \theta_{W}^2)
$$

tribution to the Lagrangian is written as
$$
\mathcal{L}_Z = \frac{g}{\cos \theta_{W}} J_Z^{\mu} Z_{\mu}
$$

$$
\mathcal{L}_W = \frac{g}{\sqrt{2}} W^+_\mu J^{\mu,+}_W + h.c. \qquad J^{\mu,+}_W = \frac{1}{2} \sum_l \bar{\nu}_l \gamma^\mu (1 - \gamma_5) l + \frac{1}{2} \sum_{i,j} \bar{u}_i \gamma^\mu (1 - \gamma_5) V_{ij} d_j
$$

The contribution to the Lagrangian is written as

The W-bosons contribution reads

Note the CKM matrix in the quark current.

 \int μ

In quantum field theory, one has to distinguish between parameters that appear in the Lagrangian and physical quantities observed in Nature since they may not be the same. Physical quantities, such as masses, couplings etc. have to be defined through physical observables and related to the Lagrangian parameters through (perturbative) computations.

Such computations may lead to poorly defined quantities that we refer to as divergent. These divergencies affect relations between physical and Lagrangian parameters, but this is a technical issue that, by itself, has nothing to do with the need for the renormalization.

In renormalisable theories, this problem of divergencies is taken care of by fixing a few physical parameters to their experimental values. If this is done and the results of the calculation are written in terms of physical parameters, all predictions become finite (i.e. independent of the "divergencies problem").

The Standard Model was proven to be a renormalisable theory. This implies that by fixing the (finite) number of input SM parameters from experimental measurements, we get a theory with an absolute predictive power (provided that our computational prowess is sufficient).

The input parameters include masses of quarks and leptons, CKM matrix elements, the Higgs self-coupling constant, the Higgs field vacuum expectation value and the gauge couplings for SU(3), SU(2) and U(1) gauge groups.

A useful alternative to fixing two gauge couplings (SU(2) and U(1)), the Higgs self-coupling and the Higgs vacuum expectation value, is to fix masses of W and Z bosons, the Higgs boson mass and the value of the electromagnetic coupling constant.

Remarks on precision SM physics

Because of this, the original goal of the SM precision physics was to determine the missing input parameters of the SM — the Higgs mass and the top quark mass — from their indirect effects on observables that can be precisely measured. However, with the discovery of the top quark mass and, later, the Higgs boson, and very precise measurements of their masses, the theory became fully determined.

For this reason, the current goal of the precision SM physics program is to systematically compare predictions of the SM with the results of measurements for many observables. We hope that by doing this, we will be able to establish a credible need for physics beyond the Standard Model.

Historically, after the SM was formulated as a theory of weak and electromagnetic interactions, and the first bunch of particles that one needed was discovered (charm quark, tau-neutrino, bottom quark, W and Z bosons), masses of the top quark and the Higgs boson remained unknown.

This simple idea is behind all high-precision low-energy experiments; it is also becoming the dominant philosophy behind the many LHC measurements. However, it has important limitations because getting to higher and higher precision forces us to dive deeper and deeper into complicated physics leading to uncertain outcomes.

- not all observables are equal; there are observables that are easier to understand theoretically than the other ones; simple observables must carry more weight in the comparison with the SM.
- "lack of perturbativity" by pushing to higher and higher precision.
- SM predictions at high energies is typically sufficient to probe for New Physics.

• SM physics is not the same as "perturbative SM physics"; in some cases, we start feeling

•observables that can be studied at the highest energies are important because effects of heavy New Physics at high energies are more pronounced. Thus, lower relative precision of

When discussing precision physics one has to remember that

Masses of the Standard Model particles (and some other things)

An important class of required SM inputs is comprised by the masses of elementary particles. Electron and muon masses are know from atomic physics, light-quark masses are known poorly but do not really matter, c-quark and b-quark masses are derived from D and B meson masses and the rest comes from collider physics measurements. Note that the (relative) precision of the top quark mass measurement is extraordinary (a few per mille).

The Z-boson mass was measured at LEP. Z-bosons were produced at LEP in collisions of electrons and positrons, and their decays into various final states were studied. An amplitude to describe the electron-positron annihilation into a pair of muons reads

However, now there is an apparent problem since the above matrix element blows up at the most interesting kinematic point $s \approx m_Z^2$. What happens there?

$$
10^{\circ} = 10^{\circ}
$$

$$
i\mathcal{M}=\tilde{J}^{\rho}_{Z,e}\frac{g_{\rho\sigma}}{s-m^{2}_{Z}}\tilde{J}^{\sigma}_{Z,\mu}+\tilde{J}^{\rho}_{\gamma,e}\frac{g_{\rho\sigma}}{s}\tilde{J}^{\sigma}_{\gamma,\mu}
$$

$$
= \quad \ \ \mathcal{N} \mathcal{N} \mathcal{N} \ + \ \ \mathcal{N} \mathcal{N} \ + \ \ \mathcal{N} \mathcal{N} \mathcal{N} \ \ + \ \ \cdots
$$

$$
\frac{-ig^{\mu\nu}}{s-m_Z^2} \Rightarrow
$$

$$
\Rightarrow \frac{-ig_{\mu\nu}}{s - m_Z^2 + \Pi_{ZZ}(s)}
$$

$$
J_Z^{\mu} = \frac{1}{2} \sum_{\psi \in l,q} \bar{\psi} \left[(T_L^3 - Q \sin^2 \theta_W) \gamma_{\mu} - T_L^3 \gamma^{\mu} \gamma_5 \right] \psi
$$

At $s \approx m_Z^2$ the non-resonant (photon) term can be neglected. $\qquad \qquad e^+$ $i\mathcal{M} =$ $R(m_Z^2)$ $s - m_Z^2$

$$
\psi \qquad \mathcal{L}_Z = \frac{g}{\cos \theta_W} J_Z^{\mu} Z_{\mu}
$$

$$
\frac{1}{2}
$$

 $\mu^{\bm{\mu\nu}}\Pi_{ZZ}(s)$

l, q

The Z-boson mass was measured at LEP. Z-bosons were produced at LEP in collisions of electrons and positrons, and their decays into various final states were studied. An amplitude
to describe the electron positron appibilation inte a pair of muons reads to describe the electron-positron annihilation into a pair of muons reads to describe the electron-positron annihilation into a pair c *R*(*m*² *Z*)

$$
i{\cal M}=\tilde{J}^\rho_{Z,e}\frac{g_{\rho\sigma}}{s-m_Z^2}\tilde{J}^\sigma_{Z,\mu}+\tilde{J}^\rho_{\gamma,e}\frac{g_{\rho\sigma}}{s}\tilde{J}^\sigma_{\gamma,\mu}
$$

At $s \approx m_Z^2$ the non-resonant (photon) term can be neglected. $s \approx m_Z^2$) the non-resonant (photon) term can be neg $s \approx m_Z^2$ the non-resonant (photon) term can be

$$
i{\cal M}=\frac{R(m^2_Z)}{s-m^2_Z}
$$

The residue R is computed from the product of the electro To understand when we show the self the vacuum polarization contributions in the self energy constants. $s \approx m_Z^2$. The residue R is computed from the product of the electron and the muon currents. The infinity at $\,s=m_Z^2\,$ is avoided by the resummation of the vacuum polarization contributions in the vicinity of $\left\{\qquad \right.$ $\left. \right.$ $\left. \right.$ $\left. \right.$ $\left. \right.$

$$
i\mathcal{M} = \frac{R}{s - m_Z^2 + \prod_{zz}(s)} \qquad m_Z^2 - \text{Re} \left[\Pi_{ZZ}(m_{z,\text{phys}}^2) \right] = m_{z,\text{phys}}^2
$$

Once this is done the matrix element is defined for all values of s, and $m_{z,\text{phys}} \to m_Z$ is the physical Z-boson Im⇧*zz* (*m*² Once this is done the matrix element is defined for all mass.

$$
\frac{\sigma}{\cdot} \widetilde{J}^{\boldsymbol{\sigma}}_{\gamma,\mu}
$$

 d

^z , we turn the above $\frac{q}{2}$

e+

 ϵ ⁺

 μ^+

The cross section for mu-pair production in the electron-positron annihilation in the vicinity of *x* is computed as follows. The starting point is the familiar expression that involves the Z-pole is computed as follows. The starting point is the familiar expression that involves squared amplitude, the normalization factor and the phase space. *R* oduction in the electron-positr JSILI VI I UI III IIIULIVI I II I LI *m*² *z* ⁺ *^Bq^µq*⌫ *m*² *z* If which the phose space. (*s m^z*)² + *m*² *z*² *z* the Z-pale is computed as follows. The starting point is the familiar expression that involves the phase space. $\frac{1}{2}$ $\$ amplitude, the normalization factor and formulation factors and formulation factors and formulation $\frac{1}{\rho\sigma}$

$$
d\sigma_Z = \mathcal{N} \frac{L_{z,e}^{\sigma} L_{z,\mu}^{\sigma}}{(s - m_z)^2 + m_z^2 \Gamma_z^2} d\Phi \qquad \mathcal{N} = \frac{1}{8s} \qquad L_{z,e(\mu)}^{\sigma} = \sum_{\text{pol}} \tilde{J}_{z,e(\mu)}^{\rho} \tilde{J}_{z,e(\mu)}^{\sigma,*}
$$

\n
$$
d\Phi = (2\pi)^4 \delta^{(4)} (p_e + p_e - p_\mu - p_\mu +) \frac{d^3 \vec{p}_\mu - d^3 \vec{p}_\mu + \frac{d^3 \vec{p}_\mu - d^3 \vec{p}_\mu + \cdots}{(2\pi)^3 2E_{\mu} - (2\pi)^3 2E_{\mu} - (2\pi)^3 2E_{\mu} - \cdots} d\Phi = \frac{1}{8\pi}
$$

\nWe integrate over the phase space of the final-state
\nmuons.....
\n
$$
q_\rho L_\rho^{\sigma \sigma} = 0 \qquad A = -2m_Z \Gamma_{z,\mu} \qquad \qquad f_{z,e}^{\sigma \sigma} = \sum_{\text{pol}} \epsilon_{z}^{\rho} \epsilon_{z}^{\sigma,*}
$$

\n
$$
d\sigma = \frac{2m_Z}{8s} \frac{\Gamma_{z,\mu}}{((s - m_Z)^2 + m_Z^2 \Gamma_Z^2)} \left(-g_{\rho\sigma} + \frac{q^\rho q^\sigma}{m_Z^2} \right) L_{z,e}^{\rho \sigma} \qquad 6m_z \Gamma_{z,e} = \left(-g_{\rho\sigma} + \frac{q^\rho q^\sigma}{m_Z^2} \right) L_{z,e}^{\rho \sigma} \Phi
$$

\n... and find the following (Breit-Wigner) result
\n
$$
\sigma = \frac{12\pi \Gamma_{z,e} \Gamma_{z,f}}{(s - m_z)^2 + m_z^2 \Gamma_Z^2} \qquad \mathcal{N} \qquad \mathcal{Z} \qquad \mathcal{Z}
$$

 $\overline{}$

$$
d\sigma_Z = \mathcal{N} \frac{L_{z,e}^{\rho\sigma} L_{z,\mu}^{\rho\sigma}}{(s - m_Z)^2 + m_Z^2 \Gamma_Z^2} d\Phi \qquad \mathcal{N} = \frac{1}{8s} \qquad L_{z,e(\mu)}^{\rho\sigma} = \sum_{\text{pol}} \tilde{J}_{z,e(\mu)}^{\rho} \tilde{J}_{z,e(\mu)}^{\sigma,*}
$$

\n
$$
d\Phi = (2\pi)^4 \delta^{(4)}(p_{e^-} + p_{e^-} - p_{\mu^-} - p_{\mu^+}) \frac{d^3 \vec{p}_{\mu^-}}{(2\pi)^3 2E_{\mu^-}} \frac{d^3 \vec{p}_{\mu^+}}{(2\pi)^3 2E_{\mu^+}} \qquad \Phi = \frac{1}{8\pi}
$$

\nWe integrate over the phase space of the final-state
\nmuons...
\n
$$
q_{\rho} L_{e^{\sigma}}^{\rho\sigma} = 0 \qquad A = -2m_Z \Gamma_{Z,\mu} \qquad -g^{\rho\sigma} + \frac{q^{\rho}q^{\sigma}}{m_Z^2} = \sum_{\text{pol}} \epsilon_{Z}^{\rho} \epsilon_{Z}^{\sigma,*}
$$

\n
$$
d\sigma = \frac{2m_Z}{8s} \frac{\Gamma_{Z,\mu}}{((s - m_Z)^2 + m_Z^2 \Gamma_Z^2)} \left(-g_{\rho\sigma} + \frac{q^{\rho}q^{\sigma}}{m_Z^2} \right) L_{z,e}^{\rho\sigma} \qquad 6m_z \Gamma_{z,e} = \left(-g_{\rho\sigma} + \frac{q^{\rho}q^{\sigma}}{m_Z^2} \right) L_{z,e}^{\rho\sigma} \qquad 6m_z \Gamma_{z,e} = \left(-g_{\rho\sigma} + \frac{q^{\rho}q^{\sigma}}{m_Z^2} \right) L_{z,e}^{\rho\sigma} \qquad \Phi
$$

\n... and find the following (Breit-Wigner) result
\n
$$
\sigma = \frac{12\pi \Gamma_{z,e} \Gamma_{z,f}}{(s - m_Z)^2 + m_Z^2 \Gamma_Z^2}
$$

$$
\sigma = \frac{12\pi \Gamma_{z,e}\Gamma_{z,f}}{(s-m_z)^2 + m_z^2\Gamma_z^2}
$$

Hence, this formula makes it very clear that from measurements of *Z*

muons…..

$$
q_{\rho}L_{e}^{\rho\sigma} = 0 \qquad \qquad A = -2m_{Z}\Gamma_{Z,\mu}
$$

0

Performing the Performing the integral, we find very large radiative corrections: $\gamma = \frac{m_Z}{m_Z}$ Performing the integral, we find very large radiative corrections:

The measured cross section looks very different from a simple formula that we derived. The reason is the radiative corrections; chief among them is the initial state radiation as it distorts the shape of the Breit-Wigner distribution. *m*² *z* $\frac{1}{2}$ and the the the the the the termula that we derived. The uve conections, chier annong them is the initial state radiation as it aistorts
roit Wigner distribution $\frac{1}{2}$ a y different from a simple formula that we denved. The requirement of having a state of having and the requirement of having and th The measured erese section looke very different from a simple fermulathat we derived mo moderation aloco accurations for your and them a ample formate that the darked.
Teason is the radiative corrections: chief amona them is the initial state radiation as it a the shape of the Breit-Wigner distribution. $12\pi \Gamma$

the shape of the Breit-Wigner distribution.
\n
$$
\sigma = \frac{12\pi \Gamma_{z,e}\Gamma_{z,f}}{(s-m_z)^2 + m_z^2\Gamma_z^2}
$$
\n
$$
\sum_{\substack{\epsilon \vdash n \\ \epsilon + n' \ge 0}}^{\infty} \frac{z}{\sqrt{n}} \left[\lim_{\substack{\vec{k} \mid \vec{p}_{e-} \\ \vec{k} \mid \vec{p}_{e-}} |M(p_{e-}, p_{e+}, k)|^2 \to \frac{e^2}{(p_{e-}k)} P_{ee}(x) |M(xp_{e-}, p_{e+})|^2}\right]
$$
\n
$$
\sum_{\substack{\epsilon \vdash n \\ \epsilon + n' \ge 0}}^{\infty} \frac{z}{\sqrt{n}} \left[\lim_{\substack{\mu \vdash n \\ \vec{k} \mid \vec{p}_{e-} \\ \vec{k} \mid \vec{p}_{e-} \le 0}} |M(p_{e-}, p_{e+}, k)|^2 \to \frac{e^2}{(p_{e-}k)} P_{ee}(x) |M(xp_{e-}, p_{e+})|^2
$$
\n
$$
\frac{e^2}{(2\pi)^3 2\omega} \frac{1}{(p_{e-}k)} \approx \frac{e^2}{4\pi^2} dx \frac{P_{ee}(x) |M(xp_{e-}, p_{e+})|^2}{E_z^2 + \theta_{\gamma}^2} \frac{1}{4\pi^2} dx \ln \frac{m_Z^2}{m_e^2}
$$
\n
$$
\frac{\delta \sigma}{\sigma} = \frac{\alpha}{\pi} \ln \frac{m_Z^2}{m_e^2} \int_0^1 dx (1 + x^2) \frac{(1 - x)}{(1 - x)^2 + \gamma^2} dx
$$

$$
\sigma = \frac{12\pi \Gamma_{z,e}\Gamma_{z,f}}{(s-m_z)^2 + m_z^2\Gamma_z^2}
$$
\n
$$
\lim_{\vec{k}||\vec{p}_{e^-}} |\mathcal{M}(p_{e^-}, p_{e^+}, k)|^2 \to \frac{e^2}{(p_{e^-}k)} P_{ee}(x) |\mathcal{M}(xp_{e^-}, p_{e^+})|^2
$$
\n
$$
P_{ee}(x) = \frac{1+x^2}{1-x} \qquad x = \frac{E_{e^-} - \omega}{E_{e^-}}
$$
\n
$$
\frac{d^3k}{(2\pi)^3 2\omega} \frac{1}{(p_{e^-}k)} \approx \frac{1}{4\pi^2} dx \frac{\theta_{\gamma} d\theta_{\gamma}}{\frac{m_e^2}{E_e^2} + \theta_{\gamma}^2} \to \frac{1}{4\pi^2} dx \ln \frac{m_Z^2}{m_e^2}
$$
\n
$$
\frac{f(xs)}{\sigma(s)} - 1\Big|_{s=m_Z^2} \Rightarrow \frac{\delta\sigma}{\sigma} = -\frac{\alpha}{\pi} \ln \frac{m_Z^2}{m_e^2} \int_0^1 \frac{dx}{(1+x^2)} \frac{(1-x)}{(1-x)^2 + \gamma^2} dx
$$
\nNow have random solutions.

$$
\frac{\delta\sigma}{\sigma} = -\frac{2\alpha}{\pi} \ln \frac{m_z^2}{m_e^2} \ln \frac{m_z}{\Gamma_z} \approx -0.4
$$

 $\gamma =$

m^Z

The mass of the Z boson is obtained from the peak position of the measured distribution; the total width — from its broadness, the hight at the peak gives access to partial decay widths. To properly extract all these quantities from the experimental measurements, radiative corrections are extremely important, as we have seen from the previous estimate. 6*mzz ,e* = e Z boson is obtaine

om its broadness, t *z* **L**
List rom the peak position of the mi
hight at the neak gives access 1 The mass of the Z boson is obtained from the peak position of the total width - from its broadness, the hight at the peak gives acce SItion of the mer
. $\frac{1}{2}$ ve ▏└
| the m
ccess *z m*² *e* \overline{a} I partial decay widths. ed distribution;
. run ut.
Itsl...rr s we have seen from the pre

$$
\sigma = \frac{12\pi \Gamma_{z,e}\Gamma_{z,f}}{(s-m_z)^2 + m_z^2\Gamma_z^2}
$$

$$
\frac{\delta \sigma}{\sigma} = -\frac{2\alpha}{\pi} \ln \frac{m_z^2}{m_e^2} \ln \frac{m_z}{\Gamma_z} \approx -0.4
$$

The Higgs boson, discovered at the LHC, is produced in the gluon fusion and observed in twophoton and four-lepton final states. Higgs boson is a very narrow resonance; for this reason measuring its line shape at the LHC is impossible.

$$
\sigma_{gg\to H} = \frac{4\pi^2}{9} \frac{\Gamma_{H\to gg}}{m_H} \delta(s - m_H^2) \frac{\Gamma_{H\to f}}{\Gamma_H}
$$

$$
\sigma_{pp\to H} = \int \mathrm{d}x_1 \mathrm{d}x_2 \ g(x_1)g(x_2) \ \sigma_{gg\to H}(x_1x_2s)
$$

 $m_H = 125.25(17) \text{ GeV}$

The W-boson mass measurement requires a somewhat different approach because hadronic

decays of the W-boson are buried in backgrounds, and neutrino is not observable, so reconstructing the invariant mass is not an option.

$$
M_{\perp}=\sqrt{2p_{e,\perp}p_{\nu,\perp}-2\vec{p}_{e,\perp}\cdot\vec{p}_{\nu,\perp}}
$$

It is possible to study two variables with pronounced kinematic boundaries the transverse W-mass and the transverse momentum of the charged lepton.

The W-boson mass measurement requires a somewhat different approach because W hadronic decays are buried in backgrounds, and neutrino is not observable. It is possible to study two variables with pronounced kinematic boundaries — the transverse W-mass and the transverse momentum of the charged lepton.

$m_W = 80.377(12) \text{ GeV}$ PDG

The precision of the measurement is extraordinary given the fact that QCD radiative corrections to processes at the LHC are typically of the order of 10 percent.

The strong coupling constant

The strong coupling constant is another input parameter of the Standard Model. We will

 0.5

The main problem with the strong coupling constant determination is that strong interactions are indeed strong, in which case we do not know how to relate the ↵*s* to observables. $\alpha_s(Q)$

discuss different ways used to determine it.

The idea is then to use the asymptotic freedom of QCD, $\begin{bmatrix} 0.3 & | & 1 \ \end{bmatrix}$ which implies that the strong coupling constant becomes $\begin{pmatrix} \phantom{\frac{\partial^2}{\partial x}} & \phantom{\frac{\partial^2}{\partial y}} \end{pmatrix}$ smallish at large energies/momenta (short distances), and to extract its value from short-distance processes that can be studied in QCD perturbation theory. which implies that the strong coupling constant becomes
concilieb at large aperaies/mementa (sbort distances) and is problem and the constant of the constant of the constant of the constant of the coupling constant becomes the constant \sim ↵*^s* (*µ*) = ⇡ sses tho ⇤QCD

$$
\alpha_{s}(\mu) = \frac{\pi}{\beta_0 \ln \frac{\mu}{\Lambda_{\text{QCD}}}} \qquad \beta_0 = \frac{11}{6} C_A - \frac{2}{3} n_f T_R
$$

Data

Theory

coupling constants are evaluated using the discussion of *Z* couplings to lepto the control of the control of the set of th 2 at the strong coupling col *v a* ² = (1 + $\overline{)}$ *w*) ² + 1 ⇡ ¹*.*4789*.* weak mixing angle. The value of the strong coupling constant allows $\frac{1}{2}$ $\frac{1}{2}$ + *g*(*d*) r_{6} weak mixing angle. The value of the strong coupling cons energy scale where QCD becomes strongly-coupled (i.e. i energy scale where QCD becomes strongly-coupled (i.e. non-perturbative). The QCD coupling constant depends on the distance scale at which it We obtain $\alpha_s(m_z)=$ 0.108 , but the result depends s weak mixing angle. The value of the strong coupling o energy scale where QCD becomes strong e result deper

^W)

2

$$
\alpha_s(\mu) = \frac{\pi}{\beta_0 \ln \frac{\mu}{\Lambda_{\text{QCD}}}}
$$
\n
$$
\beta_0 = \frac{11}{6} C_A - \frac{2}{3} n_f T_R
$$
\n
$$
\Lambda_{\text{QCD}} = m_Z e^{-\frac{\pi}{\beta_0 \alpha_s(m_Z)}} \approx 140 \text{ MeV}
$$

roan many angle. The value of the etteng eduping conetant allevie up to determine the
energy scale where QCD becomes strongly-coupled (i.e. non-perturbative). 3 ⇠*^d* = *g*(*d*) ing constant allow \overrightarrow{C} + *g*(*d*) gly on th
Tant allo \tilde{e} *s*2 *w*) ² + 1 ⇡ ¹*.*4789*.* \overline{z} ² + 1 ⇡ ¹*.*1475*,* suit depends stron 35 $\frac{1}{2}$ *z* e of t USING SUSING THESE RESULTS IN EQ. (3.25), where we include the well were two upeneray scale where QCD becomes strongly-coupled (*i.e.* non-perturbative). it the result depends strongly on the assumed value of the *w*) ² + 1 ⇡ ¹*.*4789*.* We obtain $\alpha_s(m_z) = 0.108$, but the result depends strongly on the assumed value of the weak mixing angle. The value of the strong coupling constant allows us to determine the aepend:
P \lim_{Ω} ult depends stronaly on the assumed ya cale where QCD becomes strongly-coupled (i.e. non-perturbative). S US nodi varab
0 determir
1) *n e* the is the leading order QCD -function. It follows that

One option is to consider decays of Z-bosons to hadrons and to charged leptons. Z decays occur at distances that are about 100 times smaller than the Compton wave length of the proton (which is a distance of where strong force becomes strong). For this reason, we can use quarks and QCD perturbation theory. ³ ⇥ ⁸³*.*984(86) MeV = 6*.*92*,* (3.24) z-woonio to hadrono dita to diarged ieptons. Z dedayo odda dita.
The lepton flavour is a lepton miniscule. Theoretically, we find of where strong force becomes strong). For this reason, we where we have used that the fact that the fact that the fact that the dependence on the lepton flavour is α minister dece
Inister $\bigcap_{n=1}^{\infty}$ = re strong force b *Nc* 00 times smaller than the C *q* e strong force becomes strong). For this reason, we can use quarks and *Z* zet the Compton wave lead $\frac{1}{2}$ Θ *Nc* \overline{a} (*g*))

and option is to consider deeque of 7 become to badrone and to obarged leptone 7 deeque oo r the proton (which
real OOD is subject $\overline{\mathbf{a}}$ and $\overline{\mathbf{a}}$ \bigcap e becomes stron *Nc* ler than the Compto ⁻
(can use quarks a and to consider decays of 7-bosons to hadrons and to coupling constances that are about 100 times smaller than the Compton with of whe

$$
\frac{\Gamma_{Z \to \text{hadr}}}{\Gamma_{Z \to \text{ch. lept}}} = \frac{1744(2) \text{ MeV}}{3 \times 83.984(86) \text{ MeV}} = 6.92 \quad \frac{\Gamma_{Z \to \text{hadr}}}{\Gamma_{Z \to \text{ch. lept}}} = \left(1 + \frac{\alpha_s(m_Z)}{\pi}\right) N_c \frac{\sum_q (g_V^{(q)})^2 + (g_a^{(q)})^2}{\sum_l (g_V^{(l)})^2 + (g_a^{(l)})^2} \quad \text{for } l \neq j
$$
\n
$$
J_Z^{\mu} = \frac{1}{2} \sum_{\psi \in l, q} \bar{\psi} \left[(T_L^3 - Q \sin^2 \theta_W) \gamma_{\mu} - T_L^3 \gamma^{\mu} \gamma_5 \right] \psi = \frac{1}{4} \sum_{\psi \in l, q} \bar{\psi} \left[g_V^{\psi} \gamma_{\mu} + g_A^{\psi} \gamma_{\mu} \gamma_5 \right] \psi
$$
\n
$$
\text{PDG} \quad S_W^2 = 0.231 \qquad g_V^{(l)^2} + g_A^{(l)^2} = (-1 + 4s_W^2)^2 + 1 \approx 1.0056
$$
\n
$$
g_V^{(u)^2} + g_A^{(u)^2} = (1 - \frac{8}{3} s_W^2)^2 + 1 \approx 1.1475 \qquad g_V^{(d)^2} + g_A^{(d)^2} = (-1 + \frac{4}{3} s_W^2)^2 + 1 \approx 1.4789
$$

$$
\beta_0 = \frac{11}{6} C_A - \frac{2}{3} n_f T_R \qquad \left(\Lambda_{\text{QCD}} = m_Z e^{-\frac{\pi}{\beta_0 \alpha_s(m_Z)}} \approx 140 \text{ MeV} \right)
$$

n contributions are quite small, thoy seed as $\frac{14}{2}$ and thorofore prod s sonichould be discussed. They searced to $\frac{1}{2}$ and, chercions, productions to $\frac{1}{2}$ is fruction of percent). The very important reature of the above form as $\sqrt{4}$ and therefore produce is the context of the solicity product of that to first concentration $\overline{}$ $\left(\begin{array}{cc} 0 & \lambda & \lambda & \lambda \\ 0 & \lambda & \lambda & \lambda \end{array} \right)$ Non-perturbative contributions are quite small; they scale as Λ_QCD^4 and, therefore, produce small corrections (fraction of a percent). The very important feature of the above formula is that there are no non-perturbative effects that are proportional to first, second or third power of $\Lambda_{\rm QCD}$, ² quite small they scale as Λ^4 and therefore n + *·* of perturbative QCD corrections that we consider next. \int in the Since the street of the process of the first concerned of the individual to determine to determine to determine to determine to determine the second of Λ

and this makes the non-perturbative corrections small.
\n
$$
\frac{\Gamma_{\tau,sl}}{\Gamma_{\tau,l}}\bigg|_{\exp} = 1.8393 \qquad \frac{\Gamma_{\tau,sl}}{\Gamma_{\tau,l}} = \frac{N_c}{2} \left(1 + \frac{\alpha_s(m_\tau)}{\pi} + 5.2 \left(\frac{\alpha_s(m_\tau)}{\pi}\right)^2 + 26.4 \left(\frac{\alpha_s(m_\tau)}{\pi}\right)^3 + \cdots \right)
$$

 \mathbf{t} , re suive the above equations for the strong coupling
execute and and a computation of the strong coupling $\begin{aligned} \text{compute} \quad \alpha_i \end{aligned}$ $\left(\frac{m_Z}{m_Z}\right)$ r the strong coupling constant and use the RG equation to We solve the above equations for the strong coupling constant and use the RG equation to $\alpha_{s}(m_{Z})$

 $\alpha_{\epsilon}(m_{\tau})$ $=$ $0.363(20)$

An alternative determination of the strong coupling constant involves decays of the tau lepton. The idea is identical to what we did with Z decays, but the energy scale (the tau mass) is smaller and the sensitivity to the strong coupling constant is (potentially) larger. The price is larger nonperturbative contributions that need to be analysed. arpoination of ⁺ he strong equipling espatant involves desays of ⁺ he to leptons. These decays include leptonic An alternative determination of the strong coupli In variance to the case of the case of the case of the case of the component of the decay of the side lower the sensitivity to the strong coupling constant is (potentially) larger Th show the sensitivity to the strong seaphly constant to \potentiamy, ranger. The pr \mathcal{L} include leptons. These decays include leptons include leptonic lepto nass) is smaller and semi-leptonic modes in the price of QCD, we can use the $\frac{1}{\sqrt{N}}$

$$
\tau \to \nu_{\tau} + l + \bar{\nu}_{l}, \quad l = e, \mu, \qquad \tau \to \nu_{\tau} + \text{hadrons}
$$

$$
\Gamma_{\tau,sl} \approx \Gamma_{\tau,0} \left(\text{pert} - \frac{c_q}{m_{\tau}^{\frac{1}{4}}} \langle \sum_{q \in \{u,d\}} m_q \bar{q} q \rangle + \frac{c_g}{m_{\tau}^{\frac{1}{4}}} \langle G_{\mu\nu}^a G^{a,\mu\nu} \rangle \right)
$$

Non-perturbative contributions are quite small; they scale as small corrections (fraction of a pe there are no non-perturbative effects that are proportional to and this makes the non-perturbative corrections small. the conclusion that the conclusion to the conclusion of the conclusion term of the conclusion term of the concl
The conclusion term of the conclusion term of the conclusion term of the conclusion term of the conclusion ter \overline{C} 25 and this makes the non-perturbative corrections small. EULO LI UITUIT UUITUULIUITU (ITUULIUITUI U PUTUUTIL). TIIU VUI JIIT
there ere no non-perturbetive offecto thet ere propert *Nc* is

$$
\alpha_s(m_\tau) = 0.363(20) \qquad \alpha_s(m_z) = \frac{\alpha_s(m_\tau)}{1 + \frac{\alpha_s(m_\tau)}{\pi} \beta_0 \ln \frac{m_z}{m_\tau}} \approx 0.118
$$

⌧*,sl*

Rates and "shapes" of higher-multiplicity processes are sometimes proportional to the strong coupling constant. A good example is the thrust variable T that is often used for the determination of α_{s} . D_{α} and "shapes" of h ⇡ 0*.*118*.* (3.37) α_s

1

 $T = max_{\vec{n}}$ *i* $|\vec{n} \cdot \vec{p}|$ *i|* $\frac{P_{11}}{Q}$ $_{14}$ $\frac{F_{11}}{P}$

<u>」</u>
つ

 \bigcup

 $2p_2k$

*Q*²

$$
\frac{d\sigma}{\sigma_0 d\tau} = C_F g_s^2 \int \frac{d^3 \vec{k}}{(2\pi)^3 \omega_k} \frac{2p_1 p_2}{(p_1 k)(p_2 k)} \delta\left(\tau - \min\left[\frac{2p_1 k}{Q^2}\right]\right)
$$

To see how this works, we consider the case where the emitted gluon is soft $k \ll Q$ shapes. Let us consider one example. The shape-variable thrust variable *T* is IO SEE HOW LITIS WORD, WE CONSIDER LITE CUSE WITH With this, To see how this works, we consider the case where the emitted gluon is sof *^Q ,* 2*p*2*k* we consider the case *^T* = 1 min $\frac{1}{2}$ To *s* Z d³~ *k* (2⇡)³!*^k* 2*p*1*p*² (*p*1*k*)(*p*2*k*) ϵ consider the he c *Q*² the case where the ϵ $k \ll Q$

$$
T = \max_{\vec{n}} \sum_{i} \frac{|\vec{n} \cdot \vec{p}_{i}|}{Q} \bigg|_{k \ll Q} \Rightarrow T = 1 - \min \left[\frac{2p_{1}k}{Q}, \frac{2p_{2}k}{Q} \right]
$$

where $\sum_{i} \frac{p_{1}}{Q}$ is the distance given to be the order $\vec{n} \ll \vec{e}$.

$$
\frac{d\sigma}{\sigma_0 d\tau} = \frac{\alpha_s}{2\pi} 4C_F \frac{1}{2} \int \frac{d\alpha \ d\beta}{\alpha\beta} \ \delta (\tau - \min(\alpha, \beta))
$$

$$
k_{\perp} \cdot p_{1,2} = 0 \qquad \frac{2kp_1}{Q^2} = \beta \qquad \frac{2kp_2}{Q^2} = \alpha
$$

$$
\frac{d\sigma}{d\sigma_0} = \frac{d\sigma}{2\pi} 4C_F \frac{1}{2} \int \frac{d\alpha}{\alpha\beta} \delta(\tau - \min(\alpha, \beta))
$$

$$
\int \frac{d\alpha d\beta}{\alpha\beta} \delta(\tau - \min(\alpha, \beta)) = 2 \int_{\tau}^{1} \frac{d\alpha}{\alpha} \int_{0}^{\alpha} \frac{d\beta}{\beta} \delta(\tau - \beta) = \frac{2}{\tau} \ln \frac{1}{\tau}
$$

$$
\frac{d^3k}{(2\pi)^3 2\omega_k} = \frac{d^4k}{(2\pi)^3} \delta(k^2) = \frac{\pi}{(2\pi)^3} \frac{Q^2}{2} d\alpha \ d\beta dk_\perp^2 \ \delta\left(Q^2\alpha\beta - k_\perp^2\right)
$$

*Q*² The requirt is proportional to the stropa coupling constant. n the c \overline{C} \overline{a} \overline{C} ↵*s* 2⇡ 4*C^F* $\overline{1}$ ⌧ ld fl trom the comparison w d ↵*s* 1 \overline{a} <u>on</u> 4*C^F* $\mathsf{S}(\mathsf{C})$ CO
n v it can be obtained from the comparison with data . $\qquad \qquad \qquad \qquad$ The result is proportional to the strong coupling constant;

When many very different measurements and analyses are combined, a rather consistent value of the strong coupling constant is obtained: $\alpha_s(m_Z) = 0.118 \pm 0.001$.

 $F = \frac{1}{2}$ cussed in the text. The yellow (light shaded) bands and dotted lines indicate the pre-average values of each sub-field. The dashed line and blue (dark shaded) band represent the final world average \sim value of *–s*(*m*² *^Z*). The "*" symbol within the "hadron colliders" sub-field indicates a determination precise determinations of the strong coupling constant. In fact, we will see in what follows Note that lattice computations play a very important role in this, providing one of the most that lattice calculations start playing decisive role in several precision measurements.

The electromagnetic coupling constant

$$
\alpha^{-1} = 137.035999174(35)
$$

$\alpha^{-1} = 137.035999174(35)$

The electromagnetic coupling constant determines the strength of two charges separated by a distance r. In Quantum Electrodynamics, the "constant" becomes a function of the distance. The distance scale where r-dependence becomes strong, is the Compton wave length of an electron. The sleatremerantie soupling senstant determines the dioderantiaghous deaphing conduitie actommnous If the distance scale where r-denendence hecomes discribed by a potential by a position the distance-dependent coupling. The reason for the distance-dependence is the electromagnetic coupling constant determines the strength of two charges separate
which is measured. international constant introduces the scale of the scale of the Computer of the Union of the Quality of the Co
Computer of the scale who was a dependence because a strong is the Computer we way to length at ziolariol
ron \circ a distance r. In Quantum Electrodynamics, the " The distance scale where r-dependence become e lectron. distance-dependence of the fine structure constant is very important. Hence, It is a more practical, thought the constant space. The constant becomes The distance scale where r-dependence becomes strong, is the Compton wave length of an ength of two ch 1 + ⇧(*q*²) s separated by nagnetic coupling constant determines the strength of two charges sep ie r. In Quantum Electrodynamics, the "constant" becomes a function of th

$$
V(r) = \frac{Q_1 Q_2 \alpha}{r} \qquad \qquad \lambda_e = 1/m \sim 10^{-12} \text{ m} \qquad \qquad r \gg \lambda_e \to \alpha(r) \to \alpha.
$$

$$
\alpha(q) = \frac{\alpha}{1 + \Pi_{\gamma\gamma}(q^2)} \qquad \qquad \Pi_{\gamma\gamma}(q^2 = 0) = 0 \qquad \qquad \Longrightarrow \qquad \alpha(0) = \alpha.
$$

In practice, measuring the Coulomb force isn't the best option; instead, one finds a quantity
that ean be measured and semputed with yers bigh presision. A quitable entien is the that can be measured and computed with very high electron's anomalous magnetic monient. $\begin{array}{c} \mathsf{L} & \mathsf{L} \mathsf{I} \\ \mathsf{I} & \mathsf{I} \end{array}$ that can be measured and computed with very high precision. A suitable option is the electron's anomalous magnetic moment. IN practice, moasuring the Coulomb force for the two cot option, motel
Heat ears les researches d'argel esperanted with very leigh researches. A sui instead, on find source and computed with very mean precision. A sur-
Instruction one can be precise the reserved. SENSITIVITY OF FINITY TO THE FINE STRUCTURE CONSTANT AND CAN BE CONSTANT AND CAN BE COMPUTED WITH A BE COMPUTED WITH α precision. One of such quantities is the celebrated anomalous magnetic moment of the latter when the measure of the measure value is the set of the set nedsl a ^{i} th v $[$ 2⇡ $\frac{1}{10}$ arecision A suitable option is t s anom

$$
H_e = -\vec{\mu} \cdot \vec{B}
$$

$$
\vec{\mu} = g\mu_0 \vec{s}
$$

$$
\mu_0 = \frac{e\hbar}{2mc}
$$

$$
g = 2(1 + a_e)
$$

$$
a_e = 1.15965218073(28) \times 10^{-3}
$$

$$
a_e = \frac{\alpha}{2\pi} + ...
$$

valid in "normal life" the valid in "normal life" the valid in "normal life" the vacuum polarization is irrele
The valid in the vacuum polarization is in the vacuum polarization in the vacuum polarization is in the vacuum

+ *..,* (3.44)

*l*2*{e,µ,*⌧*}*

↵¹

d computation gives: $\alpha^{-1}(m_z) = 128.89 \pm 0.09$ 1 GeV and a bottom quark with the mass 4*.*5 GeV. ⇧hadr(*m^z*) = 0*.*0045*|*⇢*,*!*,* ⁰*.*022*|*cont ⁼ 0*.*026*.* (3.53) event to the contract of the c
Putting the contract of the co where leptons and hadrons and hadrons and hadrons contribute in similar proportion to the increase in the increase in $w = \frac{1}{m} \cdot \frac{1}{m} = \frac{1}{28} \cdot \frac{1}{20}$ $\frac{1}{\sqrt{2}}$. Clearly, the above refined calculations and $\frac{1}{\sqrt{2}}$ $\alpha^{-1}(m_z) = 128.89 \pm 0.09$

(*m^z*) = 129*.*05*,* (3.54)

$$
\sigma_{e^+e^-\to V} = \frac{12\pi^2\Gamma_{V\to e^+e}}{m_V} \delta(s-m_V^2)
$$

contributions to the vacuum polarisation function. Contributions to precision electroweak observables constant at the scale $a = m_z$. At low values of a, one cannot use au **Rumpo** at the scale $q = m_z$. At Contributions to precision electroweak observ constant at the scale $q = m_z$. At low values of c the following equation holds

$$
\boxed{\Pi_{\gamma\gamma}^{\text{lept}}(m_z) = -\frac{\alpha}{3\pi} \sum_{l \in \{e,\mu,\tau\}} \ln\left(\frac{m_Z^2}{m_l^2}\right) - \frac{5}{3} \approx -0.032}
$$

$$
\left[\frac{1}{\prod_{\gamma\gamma}^{\text{hadr}}(m_z) = -0.0045|_{\rho,\omega,\phi} - 0.022|_{\text{cont}} = -0.026}\right]
$$

der hadronic vacuum polarization contribution contribution to the muon contribution to the muon contribution t
The muon contribution to the muon contribution to the muon contribution to the muon contribution to the muon c

 $\overline{\text{P}}$ and the governess formules $\overline{\text{C}}$ and $\overline{\text{C}}$ \over Γ thorough and accurate in Γ (m) Γ eta
Example and the setimate: $\frac{1}{2}$ From the above estimate: $\alpha^{-1}(m) = 120.05$ \sim 200 1011. \sim \sim $^{-1}$ (m) $-$ 100 Sulture above estimate: \Rightarrow α (π) $-$ 129.0 ate: $\Rightarrow \alpha^{-1}(m_z) = 129.05$ per the electric diverse continue.

² From the above estimate: From th *l* θ UNC ve estim

$$
\sigma_{\rm lept} = \sigma_{\rm point} \sqrt{1 - \frac{4m_l^2}{s}} \left(1 + \frac{2m_{\rm lept^2}}{s} \right)
$$

Contributions to precision electroweak observables must be expressed through fine structure constant at the scale $q = m_z$. At low values of q, one cannot use quarks to compute hadronic t low values of a one cannot use quarks to compute hadronic constructions in the commodate of the computer induity.
Analarisation function *l, q* point(*s*) *randager* and *see*
arks to compute ha point(*s*) *<u>. Architecture</u>* $\sum_{n=1}^{\infty}$ α ation function. l, q ions to precision electrowedk tobservables must be expressed through importance.

$$
\alpha(q) = \frac{\alpha}{1 + \Pi_{\gamma\gamma}(q^2)} \qquad \Pi_{\gamma\gamma}(q^2) = \frac{q^2}{\pi} \int_{s_0}^{\infty} \frac{ds}{s(s - q^2 - i0)} \operatorname{Im} \left[\Pi_{\gamma\gamma}(s) \right] \qquad \text{for } \gamma \to \infty
$$
\n
$$
\operatorname{Im} \Pi_{\gamma\gamma}(s) = \frac{e^2}{12} \left(R^{\text{lept}}(s) + R^{\text{hadr}}(s) + R^{\text{rest}}(s) \right) \qquad R^{\text{lept}}(s) = \frac{\sigma_{\text{lept}}(s)}{\sigma_{\text{point}}(s)}, \qquad R^{\text{hadr}}(s) = \frac{\sigma_{\text{hadr}}(s)}{\sigma_{\text{point}}(s)} \qquad \sigma_{\text{point}} = \frac{4\pi\alpha^2}{3s}
$$

 $\frac{1}{2}$ $Im\Pi_{\gamma}$

A simple test of the Standard Model

The muon decay in the Standard Model proceeds through the exchange of a W-boson. As we discussed, the mass of the W-boson has been precisely measured. Hence, we can compute the decay rate with high precision and compare the prediction with the result of the measurement.

We do this to test the consistency of the Standard Model and check for possible contributions of physics beyond the Standard Model to the muon decay.

 $\overline{\nu}_e$

MuLan experiment at PSI

^G^F = 1*.*¹⁶³⁹⁴ ⇥ ¹⁰⁵ GeV²

and then compare with the result of the result of the result of the experimental measurements or ϵ extract the findom decay fate is usually computed in the context of th
I life indom decay fate is usually computed in the context of th heavy electroweak physics are hidden in the Fermi constant. $A \cdot \mathcal{A}$ standard calculation of the life-time gives \mathcal{A} standard calculation of the life-time gives \mathcal{A} *^F m*⁵ ¹⁹²⇡³ *^f* tł nt = *^µ* = heory ¹⁹²⇡³ *^f* Then The muon decay rate is usually computed in the context of the Fermi theory. Then, e *,* (4.2) $\frac{1}{2}$ = 105088725 MeV^{, m}¹ m² MeV/, manufaction in the energy of the Fermi theory, There eled *F* he m $\frac{1}{2}$ ⌧*m*⁵ *^µf* (*m*² *e/m*² *µ*) The muon decay rate is usually computed in the context of the Fermi theory. Then, effects of heavy electroweak physics are hidden in the Fermi constant.

 $\mathcal{L}_{\mathcal{L}}$ and $\mathcal{L}_{\mathcal{L}}$ percent division to $\mathcal{L}_{\mathcal{L}}$ percent division the two $\mathcal{L}_{\mathcal{L}}$

$$
\mathcal{L}_{F} = \frac{G_{F}}{\sqrt{2}} \bar{e} \gamma_{\alpha} (1 - \gamma_{5}) \nu_{e} \bar{\nu}_{\mu} (1 - \gamma_{5}) \mu + h.c \qquad f(x) = 1 - 8x + 8x^{3} - x
$$

Using the PDG value of the muon lifetime, we easily find the muon lifetime, we easily $T_{\rm M/hv}$ there is a 3.5 nergent difference? and Eq. (4.10). In this respect, the first thing the first thing the first thing the first thing the two values
This respect, the first thing the first thing the two values of the two values of the two values of two values The test of the Standard Model consists in the construction of Eq. (4.8) with the construction of Eq. (4.8) with $\frac{1}{\sqrt{2}}$ Why there is a 3.5 percent difference? The test of the Standard Model consists in the comparison of Eq. (4.8) \Box

tree-diagram under the assumption *m^µ* ⌧ *M^W* , we easily find

z

A percent-level comparison requires us to consider radiative corrections. Traditionally, one distinguishes between QED corrections in the Fermi theory and weak corrections in the SM, although at the end of the day, all of them are just electroweak corrections. t to Eq. (4.1) that are not part of the Standard Model. Given the fact that π is the fact that π requires us to consider rudiditive corrections. Truditionally, one corrections in the Fermi theory and weak corrections in the SIVI, the SIVI, dy, all of them are just electrowedk corrections. to Eq. (4.1) that are not part of the Standard Model. Given the fact that are not part of the fact that \mathcal{A} as to consider rudiqued conections. Truditionidity, one
The Standard Model would be the Standard Model would be an analysis of the Standard Standard Standard Standard

first. I Say that we compute the enective Lagiangian, including corrections, and then use it to calculated the matrix element for uding long-distance QED corrections. In the modern language we will say that we compute the effective Lagrangian, including short-distance electroweak corrections, and then use it to calculated the matrix element for the muon decay including long-distance QED corrections. We compute the enective Lagrangian, including is, and then use it to calculated the matrix element for nce QED corrections.

In this factor in this factor in the calculation, we find that Eq. (4.8) shifts by about \mathbb{R} a fragmentive correction is way too small to reconcile two values of the state of the second in the set that i
Intermeters it works in the wrong direction, increasing the difference the Fermi constant (and, furthermore, it works in the wrong direction increasing the difference
keeping the set The PDG value of the muon lifetime is Hence, it follows that the QED radiative correction is way too small to reconcile two values of between them). e, it works in the wrong direction increasing the difference

$$
\frac{1}{\tau_{\mu}} = \Gamma_{\mu} = \frac{G_F^2 m_{\mu}^5}{192\pi^3} f\left(\frac{m_e^2}{m_{\mu}^2}\right) F(\alpha)
$$
\n
$$
\overline{v_e}
$$
\n
$$
\overline{v_e} = 1 + \frac{\alpha}{\pi} \left(\frac{25}{4} - \pi^2\right) + \mathcal{O}(\alpha^2) \approx 1 - 8 \times 10^{-3}
$$

However, not all is lost since, leaving the QED corrections aside, we find many electroweak oneloop diagrams that provide corrections to the muon decay that need to be analysed…

We will try to estimate the magnitude of the expected corrections by considering one of the box diagrams. Such diagrams renormalise the Fermi Lagrangian but the expected corrections are tiny; it is hard to argue that such corrections can reconcile the (measured and computed) Fermi constants unless an enhancement factor is found.

$$
\gamma_\mu (1-\gamma_5)\frac{1}{\hat{k}}\gamma_\nu (1-\gamma_5)\bigg]\otimes \bigg[\gamma_\mu\gamma_5\frac{1}{\hat{k}}\gamma_\nu (1-\gamma_5)\bigg]\;\frac{1}{k^2-m_W^2}\;\frac{1}{k^2-m_Z^2}
$$

$$
B \approx \frac{g^2}{16\pi^2 c_W^2} \mathcal{L}_F \approx \frac{\alpha}{4\pi s_W^2 c_W^2} \mathcal{L}_F \approx 3 \times 10^{-3} \mathcal{L}_F
$$

In, unphysical) weak $\begin{array}{cc} \n\mathsf{U}_F = \frac{1}{\sqrt{2} \, 4} \, \overline{m_{\nu}^2} - \overline{\Pi_M} \n\end{array}$ Note that go is still a bare (i.e. Lagrangian, unphysical) weak coupling.

It turns out that such enhanced corrections are hiding in the vacuum polarization functions of electroweak gauge bosons. We will now investigate how this happens and what is the enhancement factor. We will start with the discussion of the impact of the vacuum polarization contributions on the Fermi constant. aut that quah anhangood corrections are biding in the vaquum no larization but that such emianced corrections are mang in the vacuum polanzation runctions or
veak aquae bosons. We will now investigate how this happens and what is the WE CONSIDER A VECTOR IN A VILLE AND DENOTE ITS IN ANDER INTERNATION FUNCTION IN A
The construction of the construction of the construction of the production function function function and a variable *i* (*q*) *i d i* (*q*) *d i* (*d*) *c d islami*. We will say the *proportionally ignore all terms proportionally ignore and* α Lagrangian. This with the discussion of the impact of the vacuum polarization gauge-boson mass because of radiative corrections. *V,*⁰ is the so-called *bare mass* which is the mass parameter in the out that such enhanced corrections are hiding in the vacuum polarization functions of veak gauge bosons. We will now investigate how The mass of the vector boson – as is the mass of any other particle – is *m*² *V,*⁰ = *m*² *^V* + *m*² *^V ,* (4.13) ations on the Fermi constant. and choose the choose the choose that the choose the choose the choose that the choose the choose the choose that the choose the choos

$$
\sum_{V} w \qquad \qquad -i \Pi_{VV}(q^2) g^{\mu\nu} + \mathcal{O}(q^{\mu} q^{\nu})
$$
\n
$$
\sum_{V} w \qquad \qquad -ig_{\mu\nu} \qquad \qquad -ig_{\mu\nu} \qquad \qquad -ig_{\mu\nu} \qquad \qquad m_{V,0}^2 = m_V^2 + \delta m_V^2
$$
\n
$$
\sum_{V} w \qquad \qquad -ig_{\mu\nu} \qquad \qquad m_{V,0}^2 = m_V^2 + \delta m_V^2
$$
\n
$$
\frac{-ig_{\mu\nu}}{q^2 - m_V^2 + [\Pi_{VV}(q^2) - \Pi_{VV}(m_V^2)]} \qquad \qquad \delta m_V^2 = \Pi_{VV}(m_V^2)
$$
\n
$$
\sum_{V} \qquad \qquad \delta m_V^2 = \Pi_{VV}(m_V^2)
$$
\n
$$
\sum_{V} \qquad \qquad \delta m_V^2 = \Pi_{VV}(m_V^2)
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\n
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\sum_{V} \qquad \qquad \delta m_V^2 = \Pi_{VV}(m_V^2)
$$
\n
$$
\sum_{V} \qquad \qquad \delta m_V^2 = \Pi_{VV}(m_V^2)
$$
\n
$$
\sum_{V} \qquad \qquad \delta m_V^2 = \Pi_{VV}(m_V^2)
$$

In general since the SM is renormalisable theory, we have to express all quantities through physical parameters. In case of the Fermi constant, the relevant ones are the Z-mass, the Wmass and the fine structure constant. In general since the SM is renormalisable theory, we have to express all quantities through δ M is renormalisable theory, we have to express all quantity case $\frac{1}{2}$ *s*2 *W,*0 Fermi constant, the relevant one: i cablo *s*2 ress all quan
Cones are the *m*² Z-mass, the Wand where we used the relation between the weak mixing angle and gauge

$$
m_{z,0}^{2} = m_{z}^{2} + \delta m_{z}^{2}, \quad m_{w,0}^{2} = m_{w}^{2} + \delta m_{w}^{2}, \quad \alpha_{0} = \alpha(m_{z}) + \delta \alpha
$$
\n
$$
g_{0}^{2} = \frac{4\pi\alpha_{0}}{s_{W,0}^{2}} \qquad \qquad g_{W,0}^{2} = 1 - \frac{m_{w,0}^{2}}{m_{z,0}^{2}}
$$
\n
$$
\frac{\delta G_{F}}{G_{F}} = \frac{\delta \alpha}{\alpha} - \frac{\delta s_{W}^{2}}{s_{W}^{2}} - \frac{\delta m_{W}^{2}}{m_{W}^{2}} + \frac{\Pi_{WW}(0)}{m_{W}^{2}} \qquad \qquad \frac{\delta s_{W}^{2}}{s_{W}^{2}} = -\frac{c_{W}^{2}}{s_{W}^{2}} \left(\frac{\delta m_{w}^{2}}{m_{w}^{2}} - \frac{\delta m_{z}^{2}}{m_{z}^{2}}\right)
$$

. (4.20)

$$
\frac{\delta G_F}{G_F} = \frac{\delta \alpha}{\alpha} - \frac{c_W^2}{s_W^2} \left(\frac{\Pi_{ZZ}(m_Z^2)}{m_Z^2} - \frac{\Pi_{WW}(m_W^2)}{m_W^2} \right) + \frac{\Pi_{WW}(0) - \Pi_{WW}(m_W^2)}{m_W^2}
$$

ontinu *W,*⁰ = 1 *^m*² *G^F W m*² W bosons. *s*² *s*2 y corred *s*2 ✓*m*² *m*² he prop *m*² We will continue with the study of the the self-energy corrections to the propagators of Z and

◆

$$
T_1^{\mu\nu} + T_2^{\mu\nu} \qquad T_{AA}^{\mu\nu} = T_1^{\mu\nu} - T_2^{\mu\nu}
$$

$$
T_2^{\mu\nu} = m_1 m_2 \int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{\mathrm{Tr} \left[\gamma^{\mu} \gamma^{\nu} \right]}{(k^2 - m_1^2)((k - q)^2 - m_2^2)}
$$

We will focus on the fermionic contributions to vacuum polarisation functions of the gauge bosons — they are the important quantities; they involve vector and axial-vector currents. Z d*^d k* Tr ⇥ *^µ*(5)(*k* ˆ + *m*1)⌫(5)(*k* ˆ (2⇡)*^d* (*k*² *m*1)²((*q* + *k*)² *m*² $\frac{1}{2}$ ation functions of t
and axial-vector c α detailed the coupletion of the coupletion functions of the gouple ²) *,* (5.1) ies; they involve vector and axial-vector currents.

 \overline{S} <u>|</u> <u>Isin</u> 1e Feynman parameters and sl *, T ^µ*⌫ nentum in the denominat $|5.33\rangle$ *T ^µ*⌫ 2 DI DI DI DI DI
2 pa in 2 ta k Combining propagators using the Feynman parameters and shifting the loop momentum, we e find OMENTUM OMENTUM COMPUTE COMPUTE obtain to obtain the unshifted momentum in the denominator, we find momentum, compute the trace and find *n n a tor,* $\overline{\mathcal{M}}$ \cong 11114 Z d*^d k* α Combining propagators using the Feynman parameters (2⇡)*^d* (*k*) and the Feynman parameters
and momentum in the denomi ator, v

$$
T_1^{\mu\nu} = 4 \int [dx]_{12} \int \frac{d^d k}{(2\pi)^d} \frac{\frac{(2-d)}{d} g^{\mu\nu} k^2 - (2q^{\mu}q^{\nu} - q^2 g^{\mu\nu}) x_2 (1 - x_2)}{(k^2 - \Delta + i0)^2} \qquad T_2^{\mu\nu} = 4m_1 m_2 g^{\mu\nu} \int [dx]_{12} \int \frac{d^d k}{(2\pi)^d} \frac{1}{(k^2 - \Delta + i0)^2}
$$

$$
[dx]_{12} = dx_1 dx_2 \delta(1 - x_1 - x_2) \qquad \Delta = m_1^2 x_1 + m_2^2 x_2 - q^2 x_2 (1 - x_2)
$$

$$
\int \frac{d^d k}{(2\pi)^d} \frac{\text{Tr}\left[\gamma^{\mu}(\gamma_5)(\hat{k} + m_1)\gamma^{\nu}(\gamma_5)(\hat{k} - \hat{q} + m_2)\right]}{(k^2 - m_1^2)((k - q)^2 - m_2^2)} \qquad d = 4 - 2\epsilon
$$

 $\overline{}$ Integration over the loop momentum is performed using the foll Z d*^d k* (2⇡)*^d* (*k*² *m*² + *i*0)*ⁿ* $\overline{\mathcal{L}}$ *i*(1)*ⁿ*(*m*²) Integration over the (*k*² *m*² + *i*0)*ⁿ* \overline{O} rentum is performed us (4⇡)*d/*² med using the f [d*x*]¹² = d*x*¹ d*x*2(1 *x*¹ *x*2)*.* (5.5) Intervals over *kalled interviewership were adding* the rondwing (otal Integration over the loop momentum is performed using the following (standard) formula

$$
q \sim \sqrt{\sum_{k}^{\mu} \sum_{V} \mu_{V,AA} = \int \frac{d^d k}{(2\pi)^d} \frac{\text{Tr}\left[\gamma^{\mu}(\gamma_5)(\hat{k} + m_1)\gamma^{\nu}(\gamma_5)(\hat{k} - \hat{q} + m_2)\right]}{(\hat{k}^2 - m_1^2)((k - q)^2 - m_2^2)}
$$

$$
T_{VV}^{\mu\nu} = T_1^{\mu\nu} + T_2^{\mu\nu} \qquad T_{AA}^{\mu\nu} = T_1^{\mu\nu} - T_2^{\mu\nu}
$$

$$
T_{AA}^{\mu\nu} = T_1^{\mu\nu} - T_2^{\mu\nu}
$$

$$
T_2^{\mu\nu} = 4 \int [\text{dx}]_{12} \int \frac{d^d k}{(2\pi)^d} \frac{\frac{(2-\hat{q})}{d} g^{\mu\nu} k^2 - (2q^{\mu}q^{\nu} - q^2g^{\mu\nu})\chi_2(1 - \chi_2)}{(\hat{k}^2 - \Delta + i0)^2} \qquad T_2^{\mu\nu} = 4m_1 m_2 g^{\mu\nu} \int [\text{dx}]_{12} \int \frac{d^d k}{(2\pi)^d} \frac{1}{(\hat{k}^2 - \Delta + i0)^2}
$$

$$
[\text{dx}]_{12} = \text{dx}_1 \, \text{dx}_2 \delta(1 - \chi_1 - \chi_2) \qquad \Delta = m_1^2 x_1 + m_2^2 x_2 - q^2 x_2 (1 - x_2)
$$

$$
q \sim \sqrt{\sum_{j_2}^{\nu} \mu} \qquad T_{VV,AA}^{\mu\nu} = \int \frac{d^d k}{(2\pi)^d} \frac{\text{Tr}\left[\gamma^{\mu}(\gamma_5)(\hat{k} + m_1)\gamma^{\nu}(\gamma_5)(\hat{k} - \hat{q} + m_2)\right]}{(\hat{k}^2 - m_1^2)((k - q)^2 - m_2^2)}
$$

$$
T_{VV}^{\mu\nu} = T_1^{\mu\nu} + T_2^{\mu\nu} \qquad T_{AA}^{\mu\nu} = T_1^{\mu\nu} - T_2^{\mu\nu}
$$

$$
T_1^{\mu\nu} = 4 \int [dx]_{12} \int \frac{d^d k}{(2\pi)^d} \frac{\frac{(2 - d)}{d} g^{\mu\nu} k^2 - (2q^{\mu}q^{\nu} - q^2g^{\mu\nu})x_2(1 - x_2)}{(\hat{k}^2 - \Delta + i0)^2} \qquad T_2^{\mu\nu} = 4m_1 m_2 g^{\mu\nu} \int [dx]_{12} \int \frac{d^d k}{(2\pi)^d} \frac{1}{(\hat{k}^2 - \Delta + i0)^2}
$$

$$
[dx]_{12} = dx_1 dx_2 \delta(1 - x_1 - x_2) \qquad \Delta = m_1^2 x_1 + m_2^2 x_2 - q^2 x_2 (1 - x_2)
$$

$$
I_n(m^2) = \int \frac{d^d k}{(2\pi)^d} \frac{1}{(k^2 - m^2 + i0)^n} = \frac{i(-1)^n (m^2 - i0)^{d/2 - n} \Gamma(n - d/2)}{(4\pi)^{d/2}}
$$

di↵erent from four because *T ^µ*⌫ It is a $\frac{1}{2}$ **1 p** $\frac{1}{2}$ **p** $\frac{1}{2}$ Since *I*2() = *iN*✏ brtional \sim 4 7 December

where

(5.3)

 $\frac{1}{2}$ *x*2(1 *x*2)

ˆ

ˆ

where we have used α and α

⇤

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We will drop terms proportional to $\ q^\mu q^\nu$ because they lead to the mass-suppressed terms. α tional to $q^\mu q^\nu$ because they lead to the mass-suppress *V V,AA* can be written as follows \overline{a} to the γ dss-suppressed terms. $\overline{}$ $q^{\mu}q^{\nu}$ because they lead to the mass-suppressed term where we have used *d* = 4 2✏ and (1 + ✏)*µ*2✏

$$
T^{\mu\nu}_{VV}=T^{\mu\nu}_1+T^{\mu\nu}_2
$$

$$
T_1 = \frac{4N_{\epsilon}}{\epsilon} \int [dx]_{12} \left(\frac{\Delta}{\mu^2}\right)^{-\epsilon} \left[-\Delta(x_1, x_2) + q^2 x_2 (1 - x_2)\right] \qquad T_2 = \frac{4N_{\epsilon}}{\epsilon}
$$

 $\frac{1}{2}$ $\$ (*x*) discussed the polarization contributions to m lepton). of α also important to account for the fact that α

2*N*✏

Z

✓

◆✏

*T*² =

*m*1*m*²

[d*x*]¹²

$$
\int [dx]_{12} \left(\frac{\Delta}{\mu^2}\right)^{-\epsilon} \left[-\Delta(x_1, x_2) + q^2 x_2 (1 - x_2)\right] \n\qquad \qquad T_2 = \frac{4N_{\epsilon}}{\epsilon} m_1 m_2 \int [dx]_{12} \left(\frac{\Delta}{\mu^2}\right)^{-\epsilon} \n\qquad N_{\epsilon} = \frac{\Gamma(1 + \epsilon)\mu^{-2\epsilon}}{(4\pi)^{d/2}}
$$

. We compute the computer of the integration of the integration property in the integration of the integration $T = \frac{1}{2}$ photon, where only vector current contributes, we find ($\eta_f = N_c$ for a quark and $\eta_f = 1$ for a We can assemble the vacuum polarization contributions for different vector bosons. For the where only vector current contributes we find ($n_f = N_c$ for a quark and $n_f =$ emble the vacuum polarization contributions for different vector bosons ⇧*f* (*q*²) = *e*² *Q*2 *^f* ⌘*^f* (*T*¹ + *T*2)*|m*1=*m*2=*^m* = *q*² *e*2 *Q*2 for a quark and $\eta_f = 1$ [d*x*12] *x*2(1 *x*2)

$$
T^{\mu\nu}_{AA} = T^{\mu\nu}_{1} - T^{\mu\nu}_{2} \qquad T^{\mu\nu}_{1,2} = iT_{1,2}g^{\mu\nu}
$$

$$
\tilde{\Pi}_{\gamma\gamma}^{(f)}(q^2) = q^2 \Pi_{\gamma}^{(f)} \gamma(q^2) = e^2 Q_f^2 \eta_f (T_1 + T_2) |_{m_1 = m_2 = m} = q^2 e^2 Q_f^2 \eta_f N_{\epsilon} \left(\frac{4}{3\epsilon} - 8 \int_0^1 dx \, x(1 - x) \ln \frac{\Delta}{\mu^2} \right)
$$

We note that the result is proportional to q^2 , which implies that the photon remains massless (as it should be, of course). = *q*² *e*2 *Q*2 *^f* ⌘*^f* t⁽ $x\ln$ *µ*2 *x*² 11 UHD 11
. *f* α^2 , which implies that the photon remains massless *q*₂, windically provided the proton formations

where

4*N*✏

Z

✏

We will now analyse important features of this formula that will allow use in this formula that will allow use
We will allow use in this formula that will allow use in the will allow use in the will allow use in the will

*µ*2

(*x*1*, x*2) + *^q*²

*x*2(1 *x*2)

are of interest to us.

Next, we compute the Z-boson vacuum polarization. The interaction of Z-bosons and fermions

involves vector and vector-axial currents.

Note that there is a contribution that involves only T_2 , and that T_2 is proportional to the mass of the fermion in the loop squared. If the fermion is heavier that the Z-boson , this gives a significant enhancement. The same applies to the vacuum polarisation of the W-boson where T_1 appears.

$$
i\frac{g}{4c_W}\left(g_V^{(f)}\gamma^\mu+g_A^{(f)}\gamma^\mu\gamma_5\right)
$$

$$
g_{A,f}^{2}\big)\left(T_{1}+T_{2}\right)-2g_{A,f}^{2}T_{2}\big)\Big|_{m_{1}=m_{2}=m_{f}}
$$

$$
\Pi_{WW} = \frac{g^2}{8} V_{UD} V_{UD}^* \eta_f (T_1 + T_2 + T_1 - T_2)_{m_1 -}
$$

$$
= \frac{g^2}{4} V_{UD} V_{UD}^* \eta_f T_1 \Big|_{m_1 \to m_U, m_2 \to m_D}
$$

We will now use these results to compute the vacuum polarisation corrections to the Fermi constant making use of the fact that leading contributions to ZZ and WW vacuum polarisation functions are independent of q (since they are proportional to the heaviest (top) quark mass).

$$
\frac{\delta G_F}{G_F} = \frac{\delta \alpha}{\alpha} - \frac{c_W^2}{s_W^2} \left(\frac{\Pi_{ZZ}(m_Z^2)}{m_Z^2} - \frac{\Pi_{WW}(m_W^2)}{m_W^2} \right) + \frac{\Pi_{WW}(0) - \Pi_{WW}(m_W^2)}{m_W^2}
$$

$$
\frac{\delta G_F}{G_F} \approx \frac{\delta \alpha}{\alpha} - \frac{c_W^2}{s_W^2} \left(\frac{\Pi_{ZZ}^{tt}(0)}{m_Z^2} - \frac{\Pi_{WW}^{tb}(0)}{m_W^2} \right)
$$

$$
\frac{\Pi_{ZZ}(0)}{m_Z^2} = -\frac{g^2}{2c_W^2 m_Z^2} \frac{N_\epsilon}{\epsilon} \sum_f g_{A,f}^2 \eta_f m_f^2 \left(1 - \epsilon \ln \frac{m_f^2}{\mu^2}\right) \qquad g_{A,f} = 2T_L^{(3)} = \pm 1
$$

$$
\frac{\Pi_{WW}(0)}{m_W^2} = \frac{g^2}{m_W^2} \frac{N_\epsilon}{\epsilon} \sum_{f_u, f_d} V_{f_u, f_d} V_{f_u, f_d}^* \eta_f \left(-\frac{m_{f,u}^2 + m_{f,d}^2}{2} + \epsilon \int_0^1 dx \left(m_{f,u}^2 x + m_{f,d}^2 (1 - x)\right) \ln \frac{m_{f,u}^2 x + m_{f,d}^2 (1 - x)}{\mu^2}\right),
$$

$$
\frac{\Pi_{ZZ}^{tt}(0)}{m_Z^2} - \frac{\Pi_{WW}^{tb}(0)}{m_W^2}
$$

$$
\frac{1}{1} = \frac{g^2 m_t^2 N_c}{64\pi^2 m_W^2} \approx 0.02.
$$

⇡ ¹*.*¹⁶ ⇥ ¹⁰⁵ GeV²

.

⇢ *G^F* =

p24 cos o cos o cos o cos o cos o c

= *G^F ,* (5.23)

$$
G_F = \frac{4\pi\alpha}{\sqrt{24s_W^2 m_W^2}} \left(1 + \delta\alpha - \frac{c_w^2}{s_w^2} \left(\frac{\Pi_{zz}^{tt}(0)}{m_z^2} - \frac{\Pi_{WW}^{tb}(0)}{m_W^2}\right)\right) \approx 1.16 \times 10^{-5} \text{ GeV}^2
$$

INSTANT. Corrections enhanced by the square of the top quark mass and fairly large correction related to the 41 19 predicted values of the Ferring change in the fine structure constant help us reconcile measured and predicted values of the Fermi constant.

Another interesting quantity is the relative strength of charged and neuti low energies. It is characterised by the so-called rho-parameter. *F* in of charaed and neutral current interactions at Another interesting quantity is the relative strength of charged and neutral current interactions, at

$$
\mathcal{L}_W = \frac{g}{\sqrt{2}} W^+_\mu J^{\mu,+}_W + h.c. \qquad \mathcal{L} = \frac{g^2}{2m_W^2} J^{\mu,+}_W J^-_{W,\mu} + \frac{g^2}{2c_W^2 m_Z^2} J^{\mu}_{Z} J_{z,\mu} = \frac{g^2}{2m_W^2} \left[J^{\mu,+}_W J^-_{W,\mu} + \rho J^{\mu}_{Z} J_{z,\mu} \right]
$$
\n
$$
\mathcal{L}_Z = \frac{g}{\cos \theta_W} J^{\mu}_{Z} \qquad \rho = \frac{g_0^2}{2(m_{Z,0}^2 - \Pi_{ZZ}(0))c_{w,0}^2} \frac{2(m_{W,0}^2 - \Pi_{WW}(0))}{g_0^2} \qquad c_{W,0}^2 = \frac{m_{W,0}^2}{m_{Z,0}^2}
$$

$$
\mathcal{L}_W = \frac{g}{\sqrt{2}} W^+_\mu J^{\mu,+}_W + h.c. \qquad \mathcal{L} = \frac{g^2}{2m_W^2} J^{\mu,+}_W J^-_{W,\mu} + \frac{g^2}{2c_W^2 m_Z^2} J^{\mu}_Z J_{z,\mu} = \frac{g^2}{2m_W^2} \left[J^{\mu,+}_W J^-_{W,\mu} + \rho J^{\mu}_Z J_{z,\mu} \right]
$$

$$
\mathcal{L}_Z = \frac{g}{\cos \theta_W} J^{\mu}_Z \qquad \qquad \rho = \frac{g_0^2}{2(m_{Z,0}^2 - \Pi_{ZZ}(0))c_{w,0}^2} \frac{2(m_{W,0}^2 - \Pi_{WW}(0))}{g_0^2} \qquad c_{W,0}^2 = \frac{m_{W,0}^2}{m_{Z,0}^2}
$$

$$
\rho = 1 + \frac{\Pi_{ZZ}(0)}{m_Z^2} - \frac{\Pi_{WW}(0)}{m_W^2} \approx 1 + \frac{g^2 N_c m_t^2}{64\pi^2 m_W^2} \approx 1.02
$$

⇢ *G^F* =

*g*2

p24 cos que establecen el cos

$$
\mathcal{L}_Z = \frac{g}{\cos \theta_W} J_Z^{\mu}
$$

$$
\mathcal{L}_W = \frac{g}{\sqrt{2}} W^+_\mu J^{ \mu, +}_W + h.c. \qquad \mathcal{L} =
$$

For historical reasons, it was interesting to know the impact of the Higgs boson on the radiative corrections in the Standard Model.

The Higgs boson couples to fermions with the strength proportional to their masses; for currents that appear in the Fermi Lagrangian, all such masses are tiny. Hence, the only diagrams to study in this case are the ones where the Higgs boson couples to the propagator of the virtual W boson.

$$
T_3^{\mu\nu} = m^2 \int dx \int \frac{d^d k}{(2\pi)^d} \frac{g^{\mu\nu} - \frac{k^{\mu}k^{\nu}}{m^2}}{(k^2 - \Delta_H)^2} \qquad \Delta_H = m_H^2 x + m^2
$$

$$
T_3^{\mu\nu} = ig^{\mu\nu} T_3, \qquad T_3 = -i m^2 \int dx \int \frac{d^d k}{(2\pi)^2}
$$

 $\Delta_H = m_H^2 x + m^2 (1 - x) - q^2 x (1 - x)$. $\frac{x^2}{m^2}$ $\Delta_H = m_H^2 x + m^2 (1 - x) - q^2 x (1 - x)$

We will again compute a generic vacuum-polarisation contribution and then use it to derive corrections to various relevant quantities. We work in the unitary gauge where unphysical parts of the Higgs doublet are not present. We will again compute a generic vacuum-polarisation contribution and $T_3^{\mu\nu}$ $\frac{1}{3}^{\mu\nu} = m^2$ $\int d^d k$ $(2\pi)^d$ $g^{\mu\nu} - \frac{k^{\mu}k^{\nu}}{m^2}$ *m*² $k^2 - m^2$ 1 $(k + q)^2 - m_H^2$ *,* (5.26) where *m* is the mass of the electroweak gauge boson and *m^H* is the mass of *k q* $k + q$ *H* We will again compute a generic vacuum-polarisation contribut corrections to various relevant quantities -11.12 Z d*^d k* (2⇡)*^d* y ν $\frac{R}{r}$ $\frac{G}{r}$ $\frac{1}{m^2}$ $\frac{1}{(k+1)^2}$ $\frac{1}{12}$ = m^2 where *m* is the mass of the electroweak gauge boson and *m^H* is the mass of the Higgs boson. The reason for introducing *m*² factor will become later. aution ar corrections to various relevant quantities. We work in the unitary gauge where unphysical parts of the Higgs doublet are not present *^g^µ*⌫ *^kµk*⌫ *m*² $\frac{1}{2}$ where *m* is the mass of the electroweak gauge boson and *m^H* is the mass of $\frac{k^{\mu}k^{\nu}}{m^2}$ 1 and α factor α factor will be α factor will be α $\sqrt{(k+a)^2-m_{11}^2}$ and internal parameters, shift the internal parameters of $\sqrt{2}$ gration momentum and discard all the terms that are proportional to *q^µ*. We obtained and the contract of the
The contract of the contract We will again compute a generic vacuum-polarisation contribution and the mass of the mass $\mathcal{L}^{\mathcal{U}}$ $T_3^{\mu\nu} = m^2 \int \frac{d^4k}{(2\pi)^4} \frac{g^{\mu\nu} - \frac{\mu}{m^2}}{12\pi^2} \frac{1}{(1-\mu)^2}$ **g**
 g
 g
 g
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 *g***

***g*
 *g***

l

***g* the Higgs boson. The reason for introducing *m*² factor will become later. Nette vacaant polancation commander and then docte to denve
Intransmittiae War work in the unitary aquae where unnhysical narts of grading the terms that we momentum and discording and $k+q$ $\bigwedge^{\mathcal{A}}$ (2⇡)*^d* $\frac{H}{\sqrt{2\pi}}$ *m*² (*k*² *H*)² *,* (5.27)

$$
T_3^{\mu\nu} = m^2 \int \frac{d^d k}{(2\pi)^d} \frac{g^{\mu\nu} - \frac{k^{\mu}k^{\nu}}{m^2}}{k^2 - m^2} \frac{1}{(k+q)^2}
$$

the was a thermal propagators using Feyhman parameters, shifting the lock will be reasonable with a factor of the reason of the component of the state of the component of the component of the component of the component of terms that are proportional to the inorientum q in the numerator, we c WONDINING propagators asing Feynman parameters, shifting th
The was a the other was very a which all the the inter-second which also we are all Combining propagators using Feynman parameters, shifting the loop momentum and neglecting all terms that are proportional to the momentum q in the numerator, we arrive at e loop momenture (2⇡)*^d* and negled $Combini$ d*x* (2⇡)*^d* Feynman parameters, shifting the loop mo
the momentum q in the numerator, we arri ameters, shifting the loop momentum and neglecting all Averaging over directions of *k*, we find

$$
T_3 = -im^2 \int\limits_0^1 dx \int \frac{d^d k}{(2\pi)^d} \frac{1 - \frac{k^2}{dm^2}}{(k^2 - \Delta_H)^2}
$$

0

years and 3 divided with the theoretical predictions. It was decided to the theoretical predictions. It was decided to the theoretical predictions of the term of to move the move parameter and the move ring from BNL to Fand the muon storage reserves Tadpole diagrams fully cancel in the contribution to the rho parameter

 \bigcup $\ddot{}$ d*x* (*m*²

$$
\frac{1}{\sqrt{3}} = \frac{N_{\epsilon}}{4\epsilon} \left[\left(3m^2 - m_H^2 + \frac{q^2}{3} \right) - \epsilon \left(m^2 + m_H^2 - \frac{q^2}{3} \right) \right]
$$

$$
- 2\epsilon \int_0^1 dx \left(m^2 (1+x) - m_H^2 x + q^2 x (1-x) \right) \ln \left(\frac{\Delta_H}{\mu^2} \right) \right]
$$

$$
\rho = 1 + \frac{\Pi_{ZZ}(0)}{m_Z^2} - \frac{\Pi_{WW}(0)}{m_W^2}
$$

 $\overline{}$

Computing the contribution of the Higgs boson to the rho-parameter, we find a logarithmic senstitivity in the limit $m_H \gg m_{Z,W}$ $\overline{}$ the limit $m_H \gg m_{Z,W}$ Computing the contribution of the Higgs boson to the rho-parameter senstitivity in the li

$$
\Delta \rho_H = \frac{g^2 T_3(m_Z^2)}{\cos \theta_W^2 m_Z^2}\Big|_{q^2 \to 0} - \frac{g^2 T_3(m_W^2)}{m_W^2}\Big|_{q^2 \to 0}
$$

$$
P_{\text{e}} = \frac{g^2}{\Delta \rho_H = \frac{g^2}{(4\pi)^2 m_W^2} \frac{3}{4} \left(m_W^2 - m_Z^2 \right) \ln \frac{m_H^2}{m_Z^2} = -\frac{3\alpha}{16\pi} \frac{m_Z^2}{m_W^2} \ln \frac{m_H^2}{m_Z^2}
$$

*m*²

*^W ^m*²

H

z

H

⇢*^H* =

*^W m*²

*q*2!0

*m*²

i
I

*q*2!0

Because of the weak sensitivity to the Higgs mass, it was difficult to pinpoint its value from the precision electroweak fits, although there were clear indications that the mass is relatively low.

The muon anomalous magnetic moment: a two-decade saga

FNAL and the new experience there was started. Nothing epitomises
muon anomalous m experiment at BNL v results was observe

$$
a_{\mu}^{\rm exp} = 1
$$

*a*th $a_{\mu}^{\text{th}} = 116\,591\,846(63) \times 10^{-11}$ $\frac{1}{\mu}$ = 116 591 846(63) × 10⁻¹¹ $a^{\text{th}} = 116591846(63) \times 10^{-11}$ the theory side or it should be New Physics.

 \equiv ι $\Delta a_\mu = a_\mu^{\rm exp} - a_\mu^{\rm th} = (237 \pm 80) \times 10^{-11}$

and the theorem is to study the muon and not the electron magnetic anomaly \mathbb{R}^n of *g g* $\frac{1}{2}$ is $\frac{1}{2}$ in the stronger than the electron one. In fact, we have used the electron magnetic anomaly to fix the value of the fine structure
constant LAN IST US IN THE MUST CONTRIBUTION CONTRIBUTIONS TO THE MUST BE MUST Note that we like to study the muon and not the electron magnetic anomaly because New Physics contributions affect the muon anomalous magnetic moment 40.000 times stronger than the electron one. In fact, we have used constant…. a certain level, it is illustrated all the challenges that precision particle physics \mathbf{r} frote that we fixe to study the muon and not the efection magnetic
hecause New Physics contributions affect the muon anomalous m

$$
a_1^{\text{BSM}} \sim \frac{\alpha}{\pi} \frac{m_l^2}{M^2} \sim 60 \times 10^{-11}
$$
 $M \sim 200 \text{ GeV}$

precision SM physics program better than the muon anomalous magnetic moment moment. This measured in the dedicated experiment at BNL v σ \leq σ \sim \sim \sim ation between theoretical and experimental results was observe and this discrepancy, the storage ring was moved to INULTIMITY SPILUTIMES the current experimental provided with private program
Interview and the do muon anomalous measured in the current experimental results of the current exp experiment at BNL v $\begin{array}{cc} \gamma & \leq & \end{array}$ ation between theoretical a results was observe the results on the results are put to the results are put to results are put to results are $\frac{1}{\sqrt{2}}$. The contribution of a $\frac{1}{\sqrt{2}}$ bound state to the anomalous magnetic magneti

Several contributions need to be computed to arrive at the muon anomalous magnetic

Hadronic vacuum polarization, NLO $\begin{array}{c} | & -98.4 \\ \text{Hadronic light-by-light} & 105 \pm 26 \end{array}$

moment at the shown level of precision.

Hadronic vacuum polarization, LO 6949 \pm 37 *±* 41 + 27 *+* 28.4 Hadronic light-by-light

Deviation: 240

 $\mathsf{T}\vert_{\mathsf{c}}$, and c is are taken from Ref. c . c are taken from Ref. c . The magnitude of various contributions, in units of 10-11.

QED provides the largest contribution, by far. The current frontier is the five-loop QED; its calculation requires an extremely high degree of specialisation, outsiders cannot judge the correctness of these results. Nevertheless, one can argue that QED cannot be the

reason for the large g-2 discrepancy since in this case either the three-loop contribution or the enhanced four-loop contributions must be wrong. But all these contributions have been checked multiple times, so we are confident that QED is not a reason for the current discrepancy.

N.B. Since very high precision is required in this case, sometimes interesting side questions are being discussed that people have not thought about before (actually, people did think about it well before and forgot). For example, can QED bound states (positronium) contribute to g-2 and whether or not such contributions are "outside" of the conventional perturbative computations?

$$
\zeta(3) \qquad \qquad a_{\mu}^{\text{cont}} = -a_{\mu}^{\text{PS}} + \text{regular PT}
$$

Electroweak contributions start at one loop; the two-loop contributions are known and are significant. You would think that it is straightforward (i.e. difficult but straightforward) to compute them, but this is not the case.

 $\frac{1}{2}$ $\frac{1}{2}$ The reason is the "anomalous" contributions which exhibit strong sensitivity to infra-red physics where using quarks in the loops becomes invalid and hadrons are needed. Note that the result below contains hadron masses which is not something that is seen often when loop diagrams are calculated.

$$
\frac{G_F m_\mu^2}{8\pi^2 \sqrt{2}} \left(2 \ln \frac{m_\pi^2}{m_\mu^2} + 1 + \ln \frac{m_\rho^2}{m_\mu^2} - \frac{m_\rho^2}{m_{a_1}^2 - m_\rho^2} \ln \frac{m_{a_1}^2}{m_\rho^2} + \frac{3}{2}\right)
$$

^µ . The idea behind $-1940 \times$

 $a_{\mu}^{\text{two}} = (6949 \pm 37.2 \pm 21.0)$ $a_\mu^{\sf hvp} = (6949 \pm 37.2 \pm 21.0) \times 10^{-11}$

$$
a_{\mu}^{\pi\pi} = 400 \times 10^{-11} \qquad a_{\mu}^{\rho,\omega,\phi} = 5514 \times 10^{-11} \qquad a_{\mu}^{\text{cont}} = 1240 \times 10^{-11} \qquad \text{BABAR, KLO}
$$

$$
a_{\mu}^{\text{hvp,th}} = a_{\mu}^{\pi\pi} + a_{\mu}^{\rho,\omega,\phi} + a_{\mu}^{\text{cont}} \approx 7160 \times 10^{-11} \qquad \qquad a_{\mu}^{\text{bvp}} = (694)
$$

rcent. This moves the hadronic vacuum polarisation contribution up by about 150 x 10⁻¹¹ and up to proves the nuuronic vucuum polansution continuution up by upout 190 x 10 m.
2. at the discrepancy with the experimental result by more than 2 siama tho tho mo ⁰⁰ $\frac{1}{2}$ music energy scale for the recent recent of the ends of condictmentions and the fire the meson contribution of the distribution of the physics of the physi The recent result of the CMD-3 collaboration increased the rho-meson contribution by about 3 *z* the discrepancy with the experimental result b The problem, however, is that the problems to be known rather that the second term is the second to be known ra accurated, independence of the contribution of α , α is about α in α The recent result of the CMD-3 collaboration increased the rho-meson contribution by about 3 percent. This moves the hadronic vacuum polarisation contribution up by about 150 x 10-11 reducing the discrepancy with the experimental result by more than 2 sigma.

The next contribution we discuss is the hadronic vacuum polarization. It is a large contribution and it needs to be know to a precision of about 1 percent. The enhancement of the low-s region makes estimating it difficult (this is a big difference with respect to a similar contribution to the electromagnetic coupling constant). whe next contribution we discuss is the hadronic vacuum polarization and it poods to be know stramanatic coupling constant) 3⇡ *s R*had(*s*) *a*(1)(*s*)*,* (6.5) *<u>makes</u>* estimating it difficult (this is a big difference) the ore-loop and the one-loop and the state with an experiment that appears due to an experiment of the a t and one-loop anomalous magnetic moment that appears due to anomalous magnetic moment that appears due to an The next contribution we discuss is the hadron and it needs to be know to a precision of abou makes estimating it difficult (this is a big differe

-
-
- the "continuum" contribution a_{μ}^{cont}

By itself, the CMD-3 result would have remained controversial because other measurements consistently arrive at lower result for a hadronic vacuum polarization. However, a new lattice calculation by the BMW collaboration also arrived at the result that is close to that of CMD-3 and (maybe !) solves the g-2 problem fully.

For completeness: the last contribution is the hadronic light-by-light scattering one. It is small but very troubled contribution, that was often considered as a leading candidate for being wrong (primarily because there is no data that can be used to evaluate it and because it used to be a negative of its current value for a while). However, theorists seem to have gotten it right (within a fairly large error bars), as is also confirmed by the dedicated lattice calculations.

$$
\sum_{\mu}^{\nu} \sum_{\lambda}^{\nu}
$$

$a_\mu^{\rm hvp} = 7141(45) \times 10^{-11} \qquad {\rm BMW}$ lattice collaboration

$a_\mu^{\rm hIbl} = (105 \pm 26) \times 10^{-11}$

 \rightarrow Theory Initiative, Phys.Rept. 887 (2020) 1-166 (2020) 1-166 (2020) 1-166 (2020) 1-166 (2020) 1-166 (2020) 1-166

[Budapest–Marseille–Wuppertal-coll., Nature 593, 51 (2021) & 2407.10913] Z. Fodor, BMW collaboration, ICHEP 2024

T seeing the convergence of the theoretical results soon.
-Hence, after more than 20 years, thanks to the new result of the BWM collaboration, the new measurement of CMD-3 and work of many theorists, it seems that the muon magnetic anomaly crises is starting to ease. Of course, cross checks of these results will continue and, hopefully, we will start seeing the convergence of the theoretical results soon.

The unitarity of the CKM matrix

Io get access to these CKM matrix elements, we need to understand weak decays of light
parkspectledrens are always difficult and if we aim at testing the above relation with bigh from and the and the annual term of the down and to a down-to a down-to down-the mannitons.
Intension we should find a way to avoid the hadronic uncertainties impacting our predictions To get access to these CKM matrix elements, we need to understand weak decays of light hadrons. Hadrons are always difficult and, if we aim at testing the above relation with high precision, we should find a way to avoid the hadronic uncertainties impacting our predictions.

$$
|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1
$$

Since $|V_{ub}| \sim$ 10⁻⁵ we will drop it from the above equation. Hence, we will be checking the relation *reen cosine and sine of the Cabibbo angle.* between cosine and sine of the Cabibbo angle. not get into testing the above equation with 10⁵ accuracy, we can neglect *VILLE VIII VII*

$$
|V_{ud}|^2 +
$$

$$
^{2}+|V_{us}|^{2}=1
$$

As we have seen, the CKM matrix arises when generic Yukawa interactions are diagonalised by independent unitary transformations of left up-type and down-type quarks. The CKM matrix is unitary. How this can be checked? q

$$
V^{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \qquad J_W^{\mu,+} = \frac{1}{2} \sum_l \bar{\nu}_l \gamma^{\mu} (1 - \gamma_5) l + \frac{1}{2} \sum_{i,j} \bar{u}_i \gamma^{\mu} (1 - \gamma_5) l
$$

We will discuss tests of unitarity relations that involve light-quark CKM elements (1.61)

(1) Any. How this can be checked:
\n
$$
\mathcal{L}_W = \frac{g}{\sqrt{2}} W^+_\mu J^{ \mu, + }_\text{W} + h.c.
$$

$$
V^{CKM} = \begin{pmatrix} V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \qquad J^{u,+}_{W} = \frac{1}{2} \sum_{l} \bar{\nu}_{l} \gamma^{\mu} (1 - \gamma_{5}) l + \frac{1}{2} \sum_{i,j} \bar{u}_{i} \gamma^{\mu} (1 - \gamma_{5}) V_{ij} d_{j}
$$

 \mathbf{v} \mathbf{v} and \mathbf{v} an electron and a neutrino ⇡ ! ⇡⁰ + *e* + ¯⌫*^e* . This process is facilitated by

$$
m_{\pi}^0 + e + \bar{\nu}_e \qquad m_{\pi}^- = 139.57 \text{ MeV} \qquad m_{\pi}^0 = 134.98 \text{ MeV}
$$

Consider the decay $\pi^- \to \pi^0 + e + \bar{\nu}_e$ $m_{\pi}^- = 139.57$ $\cos \theta \to \pi^- \to \pi^0 + e + \bar{\nu}_e$ process reads $\sin(10 \text{eV})$ $\pi \rightarrow \pi^+ + e + \nu_e$ $m_{\pi}^- = 139.5$

$$
\begin{array}{c}\n\overbrace{\int_{\pi}^{W^-} \int_{\overline{\nu}_e}^{\overbrace{\pi}} \overbrace{\int_{\overline{\nu}_e}^{\overbrace{\pi}} \overbrace{\nu}_{dd}}^{\overbrace{\pi}^0} \\
\overbrace{\int_{\pi}^{0}}^{\pi} \overbrace{\int_{\pi}^{0}}^{\overbrace{\pi}^0} \overbrace{\int_{\pi}^{0}}^{\pi^0} \\
\overbrace{\int_{\pi}^{0}}^{\pi^0} \overbrace{\int_{\pi}^{0}}
$$

an electronic Lagrangian (Lagrangian Corporation)
Anno 1990 - Carlo Corporation (Lagrangian Corporation)

Jin symmetry does not restrict the form fac pin symmetry does not restrict the form fo p_{1} p_{2} p_{3} p_{4} and p_{5} are two properties that we refer to the set of the electromagnetic current. The isospin symmetry does not restric $\sum_{i=1}^{\infty}$ factor $f_1(a^2)$ the electromagnetic current. The isospin symmetry does not restrict the form factor $f_1(q^2)$. $\frac{1}{2}$ or momentum transfer $\frac{1}{2}$ ($\frac{1}{4}$) $-$ 0 is under the unit of $\frac{1}{4}$ is units units very smiller *i*dgriede current. The isospin symmetry does not restrict the form ructor $f_1(q)$ 1e electromagnetic c $\left(\begin{array}{ccc} 9 & 1 & \cdots \end{array} \right)$ pin symmetry does not restrict the form factor f_1

 ϕ decay $\pi^- \to \pi^0 + e + \bar{\nu}_e$ the momentum = *f*1(*q*² he decay $\pi^- \to \pi^0 + e + \bar{\nu}_e$ the momentum $f(1) = f(1)$ The important simplification arises because in the decay transfer a is very small $q^2 = (p_{\pi^-} - p_{\pi^0})^2 = m_{\pi^-}^2 + n_{\pi^-}^2$ Since masses of *u* and *d* quarks are very small in comparison with the QCD mplitication arises because in the decay $\pi \rightarrow \pi^{\circ} + e + \nu_e$ the mo $\begin{array}{ccc} \text{small.} & 2 & 12 & 2 & 2 & 2 \end{array}$ $\overline{\nu}_e$ the momentum The important simplification arises because in the decay $\pi^-\to\pi^0+e+\bar\nu_e\,$ the momentum transfer q is very small. $\pi^{-} \to \pi^{0} + e + \bar{\nu}_{e}$ value of an
Intaimplification arise 2 p applification arises because in the decay $\pi^- \to \pi^0 + e + \bar{\nu}$ It follows that *q*² h⇡0 (*p*⇡⁰)*|J*↵ *^q* (0)*|*⇡(*p*⇡)ⁱ ⁼ ^h⇡⁰ (*p*⇡⁰)*|u*¯↵*d|*⇡(*p*⇡)ⁱ $\pi^- \rightarrow \pi^0 + e + \bar{\nu}_e$ the momer $\rightarrow \pi^0 \perp \rho \perp \overline{\nu}$ the memontum $p^{2} = p^{2} + p^{2}$ and $F^{2} = p^{2} + p^{2}$

$$
q^2|_{\text{max}} = (m_{\pi^-} - m_{\pi^0})^2 \approx (5 \text{ MeV})^2
$$

(7.5)

 $(m_{\pi^0})^2 \approx (5 \text{ MeV})^2$ $\sqrt{q_{\text{max}}^2} \ll \Lambda_{\text{QCD}}$ *factors*. We note that the axial current does not contribute to this matrix $m_e^2 \sim (5 \text{ MeV})^2$ $\sqrt{q_{max}^2} \ll \Lambda_{\text{QCD}}$ $f = \frac{f}{\pi^0}$ \approx (3 IVIEV) v $\frac{f}{\pi}$ v $\frac{f}{\pi}$ $f^2 \approx (5 \text{ MeV})^2$ $\sqrt{q_{\text{max}}^2} \ll 1$ $= (m_{\pi^-} - m_{\pi^0})^2 \approx (5 \text{ MeV})^2$ $\sqrt{q_{\text{max}}^2} \ll \Lambda_{\text{QCD}}$ $_{\rm max}^2 \ll \Lambda_{\rm QCD}$ $\frac{m}{2} \approx (5 \text{ MeV})^2$ $\sqrt{a^2} \ll \Lambda_{\text{OCF}}$ $\lim_{\eta \to 0}$ \int $\lim_{\eta \to 0}$ \int $\frac{d\eta}{\eta}$ $\lim_{\eta \to 0}$ $\lim_{\eta \to 0}$ $\lim_{\eta \to 0}$ $\lim_{\eta \to 0}$ $\lim_{\eta \to 0}$ matrix element because pions are pseudoscalar particles. $(m_{\pi^0})^2 \approx (5 \text{ MeV})^2$ $V q_{\text{max}}^2 \ll \Lambda_{\text{QCD}}$

$$
q^2 = (p_{\pi^-} - p_{\pi^0})^2 = m_{\pi^-}^2 + m_{\pi^0}^2 - 2m_{\pi^-}E_{\pi^0}
$$

G^F

Here = *m*⇡ *m*⇡⁰ and the result is written as an expansion in *me/* ⇠ 0*.*1 will miss an important effect, related to the radiative corrections. However, if we do so, we will miss an important effect, related to the radiative corrections. and */m*⇡ ⇠ 0*.*03. Hence, we derived a prediction for a semileptonic decay We can immediately use this formula to extract Vud from the decay of a charged pion.

 $m = \frac{1}{2}$ $u'v' \cdot u'$ inplies that $u_1(v) = v$, i.e. the value of the u ompletely fixed! Hence, the only unknown parameter in the deck amplitude is the CKM matrix element V_{ud} .
G $\sqrt{2}$ is the conclusion of the contract particles. Since masses of *u* and *d* quarks are very small in comparison with the QCD s pulletic in the decuy Conservation of the isospin current $\bar u \gamma^\mu d$ implies that $f_1(0) = \sqrt{2}$, i.e. the value of the form factor at q = 0 is completely fixed! Hence, the only unknown parameter in the decay which implies that the amplitude is *fully fixed*! f the γ in the only unknown parameter in the demonstration of γ $=\sqrt{2}$, *i.e.* conservation of the isospirituation in a *f* a implies that $r_1(v) = v$ \geq *f*, i.e. the value of the decay form factor at q = 0 is completely fixed! Hence, the only unknown parameter in the decay *d* implies that $f_1(0) = \sqrt{2}$, i.e. the value of the *Vud f*1(0)(*p^µ* ⇡ ⁺ *^p^µ* ⇡⁰) ¯*u*(*p^l*)*µ*(1 5)*u*(*p*⌫)*.* (7.11) matrix element because punt because per la component de la component de la component de la component de la com

$$
\mathcal{M} = \frac{G_F}{\sqrt{2}} V_{ud} \langle \pi^0(p_{\pi^0}) | J_q^\alpha | \pi^-(p_{\pi^-}) \rangle \times \bar{u}_e(p_3) \gamma_\alpha (1 - \gamma_5) v_\nu(p_4)
$$

$$
\overbrace{\hspace{1.5cm}\int_{\bar{\nu}_e}^{\bar{\nu}_e}} \quad \langle \pi^0(p_{\pi^0}) | J_q^{\alpha}(0) | \pi^-(p_{\pi^-}) \rangle = \langle \pi^0(p_{\pi^0}) | \bar{u} \gamma^{\alpha} d | \pi^-(p_{\pi^-}) \rangle = f_1(q^2) (p_{\pi^0} + p_{\pi^-}).
$$
\n
$$
\int_{\pi^0}^{\pi^0} f_1(q^2) \approx f_1(0) \left(1 + \frac{q^2}{\Lambda_{\rm QCD}^2} \right) \to f_1(0) \qquad q = p_{\pi^0} - p_{\pi^-}
$$

$$
\mathcal{M} = -\frac{G_F}{\sqrt{2}} V_{ud} \; f_1(0) (p_{\pi^-}^\mu + p_{\pi^0}^\mu) \; \bar{u}(p_l) \gamma_\mu (1+\gamma_5) u(p_\nu)
$$

At the same time, *f*1(0) can be related to the "isospin" charge and found

$$
\Gamma = \frac{G_F|V_{ud}|^2 \Delta^5}{30\pi^3} \left(1 - \frac{5m_e^2}{\Delta^2} - \frac{3}{2}\frac{\Delta}{m_\pi}\right) \qquad \Delta = m_{\pi^-} - m_{\pi^0}
$$
$$
(x)J_{\text{lep},\alpha}(x) \t J_q^{\alpha} = \bar{u}\gamma^{\alpha}(1-\gamma_5)d \t J_{\text{lept}}^{\alpha} = \bar{e}\gamma^{\alpha}(1-\gamma_5)\nu_e
$$

 \widetilde{A} γ γ γ $(1 - \gamma_5)\gamma$ γ $g_{\rho\sigma}g_{\alpha\beta} = \kappa$ γ $(1 - \gamma_5)$ $\frac{\partial^2}{\partial \tau} \gamma^\alpha \gamma^\mu (1-\gamma_5) \gamma^\sigma \gamma^\beta q_{\alpha\sigma} q_{\alpha\beta} = k^2 \gamma^\mu (1-\gamma_5)$ $k^2 \gamma^{\mu} (1 - \gamma_5)$

This effect is a logarithmically-enhanced short-distance renormalisation of the Fermi Lagrangian that is relevant for describing decay of the charged pion. Our goal is to find diagrams that exhibit logarithmic sensitivity to the mass of the W boson which is much larger than the mass of the pion. Consider, as an example, the decay of a charged pion, and a neutral pion, and a neutral pion, and a neutral pio e renomiquation of the remin Lugiangian
Clenomiquation of the remin Lugiangian *^L* ⁼ *G^F ^Vud ^J^q* ↵(*x*)*J*↵ *^L* ⁼ *G^F ^Vud ^J^q* ↵(*x*)*J*↵ lep(*x*)*,* (7.3) ormalisation of the Fermi Lagrangian an electron and a neutrino and a neutrino in the set of th
Contracted by the set of the se an guan is tu in

^h⇡(*p*2)*|J*↵ *^q* (0)*|*⇡0(*p*1)ⁱ ⁼ ^h⇡(*p*2)*|u*¯↵*d|*⇡0(*p*1)ⁱ To simplify the calculation, it is useful to employ the Landau gauge for the photon propagator | since in this gauge the vertex corrections are ultraviolet finite.

$$
D_{\alpha\beta}(k) = \frac{-i}{k^2} \left(-g_{\alpha\beta} + \frac{k_{\alpha}k_{\beta}}{k^2} \right)
$$
\n
$$
M_{V} = e^2 Q_u Q_d (-i) \int \frac{d^4k}{(2\pi)^4} \frac{\gamma^{\alpha} \hat{k} \gamma^{\mu} (1 - \gamma_5) \hat{k} \gamma^{\beta}}{(k^2)^3} \left(g_{\alpha\beta} - \frac{k_{\alpha}k_{\beta}}{k^2} \right)
$$
\n
$$
d_{\alpha\beta} = \frac{\gamma^{\alpha} \hat{\sigma} \sigma_{\beta\alpha}}{k^2} u
$$
\n
$$
\gamma^{\alpha} \hat{k} \gamma^{\mu} (1 - \gamma_5) \hat{k} \gamma^{\beta} g_{\alpha\beta} \rightarrow \frac{k^2}{4} \gamma^{\alpha} \gamma^{\rho} \gamma^{\mu} (1 - \gamma_5) \gamma^{\sigma} \gamma^{\beta} g_{\rho\sigma} g_{\alpha\beta} = k^2 \gamma^{\mu} (1 - \gamma_5)
$$
\n
$$
\gamma^{\alpha} \hat{k} \gamma^{\mu} (1 - \gamma_5) \hat{k} \gamma^{\beta} \frac{k_{\alpha}k_{\beta}}{k^2} = k^2 \gamma^{\mu} (1 - \gamma_5)
$$
\n
$$
M_V \rightarrow 0
$$

M^B $= e^2 Q_e Q_d (-i)$

 $\gamma^{\mu}(1-\gamma_5)\hat{k}$ $\hat{k}\gamma^{\alpha}$ \otimes

$$
\frac{w_{\text{tot}}}{V_{\text{tot}}} = \frac{d}{v_{\text{tot}}} =
$$

If we put everything together, including estimates of long-distance QED corrections, we find $\pi^- \rightarrow \pi^0 + e + \bar{\nu}_e$ $\pi^ V_{ud}$ π^0 $W^ \bar{u} \sim \sqrt{d}$ *d e* $\bar{\nu}_e$ *Vud* stimates of long-distance QFD corrections, we find gen natuu on rung alutanud $S_{EW} = 1 + \frac{\alpha}{\pi}$ π $\ln \frac{m_W}{\Omega^2}$ 0*.*3 MeV ≈ 1.013 T the prediction for the prediction for the decay rate by about two percent two percent two percent two percent of P $=\frac{QF|Vud|}{2Q^2}\left(1-\frac{JHl_e}{\Lambda^2}-\frac{JH}{2}\right)S_{Edd}^2(1+\delta_{RC})$ \sum_{ν} and \sum_{ν} and \sum_{ν} include the include \sum_{ν} corrections, \sum_{ν} corrections, \sum_{ν} ± 0.00 The *Vus* is easier to obtain, because one can extract it by considering tions from distances *k>m^p* ⇠ *mn*. If we do that, we will find that Eq. (7.3) If we put everything together, including estimates of long-distan $\mathcal{L} = \frac{G_F}{\sqrt{2}} J^c$ $J(\$ $\frac{d}{dx}$ $\int_{\mathbb{R}^n}$ $\int J_{\mathrm{lep},\alpha}(x)$ $S_{EW} = 1 + \frac{\alpha}{\pi} \ln \frac{m_W}{\Omega 3 \text{ MAV}} \approx 1$ π ⁻ V_{ud} π ⁰ π ⁰ $\qquad \qquad$ \qquad $a = \frac{1 + 2a_1}{30\pi^3} \left(1 - \frac{e}{\Delta^2} - \frac{1}{2} \frac{E}{m_\pi}\right) S_{EW}^2 (1 + \delta_{RC})$ level. From this calcualtion refined to include long-distance QED corrections, $\mathbf{r} - \mathbf{r}$ $|V_{ud}| = 0.9739(29)$. S_E^2 $\chi^{'2}_{EW}(1+\delta_{RC})$ $\Gamma = \Gamma_{\pi^-,tot}$ Br $\Gamma_{\pi^-,tot} = (2.6033 \pm 0.0005) \times 10^{-8}$ sec⁻¹, Br = (1.036 ± 0.006) × 10⁻⁸. bosons are still not fully dynamical. Hence, we have to consider α have to consider α that the consider α tions from distances *k>m^p* ⇠ *mn*. If we do that, we will find that Eq. (7.3) $\pi^- \to \pi^{\circ} + e + \bar{\nu}_e$ \overline{v}_e $\mathcal{L} = \frac{\Delta F}{\sqrt{2}} J_q^{\alpha}(x) J_{\text{lep},\alpha}(x)$ $S_{EW} = 1 + \frac{\Delta F}{\pi} \ln \frac{W}{0.3 \text{ MeV}} \approx 1.013$ 0*.*3 MeV ⇡ 1*.*013*.* (7.13) $T=\frac{G_F|V_{ud}|^2\Delta^3}{1-\frac{5m_e^2}{1-\$ and, hence, potentially, hence, potentially, has an important important important important important important in Δ^2 in Δ^2 $\Gamma - \Gamma$ R_r Γ $=$ (2.6033 \pm 0.0005) \times 10⁻⁸ soc⁻¹ R_r $=$ (1.036 \pm 0.006) \times 10⁻⁸ we find the second terms.
We find the second terms of th $|V| = 0.9739(29)$ The *Vus* is easier to obtain, because one can extract it by considering tions from distances μ matrix μ matrix μ and μ will find that, we will find that, we will find that μ 0*.*3 MeV $\sqrt{2}$ \sim 1. From this calculation refined to include to include to include \sim Δ^2 $2 m_{\pi}$ ⁰ EW ^{(+ 1 O}RC) $\frac{0000 \pm 0.0000}{\sqrt{10}}$ **bod**, $\frac{1}{2}$ **b** $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ ^h0*|u*¯↵⁵ *^d|*⇡(*p*)ⁱ ⁼ *^f*⇡*p^µ ,* mentum transfers are moderate. The non-trivial problem is the ratio *f*⇡*/fK*. Γ $\rm R_{r}$ $\rm \Gamma$ $(9.6099 + 0.0005) \times 10^{-8}$ $\rm \Omega_{200}$ $^{-1}$ $\rm \Omega_{20}$ $(1.096 + 0.001)$ are radiative corrections to the this ratio but the set of π , where π is π $|V_{ud}| = 0.9739(29)$. We can invoke the *SU*(3)-flavor symmetry but it may have large corrections $\frac{1}{\sqrt{d}}$ $\Gamma = \frac{G_F |V_{ud}|^2 \Delta^3}{2R^3} \left(1 - \frac{5m_e^2}{\Delta^2} - \frac{3}{2} \frac{\Delta}{\Delta}\right) S_{\text{FUV}}^2 (1 + \delta_{RC})$ $\sum_{i=1}^{N}$ are $\sum_{i=1}^{N}$ $\sum_{i=1}^{N}$ *f* $\sum_{$ W_{tot} $W_{\pi^-,\text{tot}} = (2.6033 \pm 0.0005) \times 10^{-6} \text{ sec}^{-1}, \quad Br = (1.036 \pm 0.006) \times 10^{-6}$. tions that give G *F* $\overline{\sqrt{2}}$ $J^{\alpha}_q(x)J_{\text{lep},\alpha}(x)$ \overline{G} G_F $\mathcal{L} = \frac{G_F}{\sigma} J^{\alpha}(x) J_{\text{loc}}$ ⇡ ⁺ *^p^µ* ⇡⁰) ¯*u*(*p^l*)*µ*(1 5)*u*(*p*⌫)*.* (7.11) $\sqrt{2}$ $\Gamma =$ $G_F |V_{ud}|$ $2\Delta^{5}$ $30\pi^3$ $\sqrt{2}$ $1 - \frac{5m_e^2}{\Delta^2}$ *e* $\frac{5m_e^2}{\Delta^2} - \frac{3}{2}$ 2 Δ m_{π} ◆ $S_{\text{FW}}^2(1+\delta_{RC})$ $\Gamma_{\pi^-, \rm tot} = (2.6033 \pm 0.0005) \times 10^{-8}~{\rm sec}^{-1}, \quad {\rm Br} = (1.036 \pm 0.006)$ and */m*⇡ ⇠ 0*.*03. Hence, we derived a prediction for a semileptonic decay tions from distances *k>m^p* ⇠ *mn*. If we do that, we will find that Eq. (7.3) $\frac{a}{1}$ $\binom{n}{k}$ $\bm{U}_{\text{lep},\bm{\alpha}}(x)$ $S_{EW} = 1 + \frac{\alpha}{\pi} \ln \frac{m_W}{\Omega 3 \text{ MAV}} \approx 1.0$ $T_{V_{ud}}$ T_0 T_0 T_1 T_1 T_2 T_3 T_4 T_5 T_7 T_8 T_9 T_1 T_1 T_2 T_3 T_1 T_2 T $a = \frac{1 - \mu_0}{30\pi^3} \left(1 - \frac{e}{\Delta^2} - \frac{1}{2} \frac{S_{EW}}{m_{\pi}}\right) S_{EW}^2 (1 + \delta_{RC})$ level. From this calcualtion refined to include long-distance QED corrections, $\mathbf{r} = \mathbf{r} \pi^{-1}$

decays of *K* ! *l*⌫*l* and ⇡ ! *l*⌫*l*. Since ⇡ and *K* are pseudoscalars, these ve to rely on lattice computations. The *agreement* only one can extend the section of the section of the can extract in a can extract it by considering the section of the constant in the section of the constant in the constant in the constant in the constan decays to surflewfildt cusich but we fluve to fery off futtice computations. decays of the *K* is somewhat easier but we have to rely on lattice com decays processed via a matrix element of the axial current of the axial cur is somewhat easier but we have to rely on lattice computations \mathcal{V} will put approach. An alternative is to use lattice computation. Getting V_{us} is somewhat easier but we have to rely on lattice computations. = 1*.*1932(19)*.* (7.17) Let us note that if we really determine *Vud* from the above formula, there will be important to the first missing. The first one is completed is single field in the first one is single s The *Vus* is easier to obtain, because one can extract it by considering decays of *K* is and *K* in the casic inducedary of the computations.

|Vud |

f 2 ⇡

 $0.22121(AF)$ $= 0.23131(45)$ *v*_u 231: \overline{a} $\left\{ \begin{array}{c} 1 \\ 2 \end{array} \right\}$

² = 0*.*9739(29)*.* (7.14) Note that in this ratio all short distance electroweak e↵ects cancel out. There are radiative corrections to this ration but they are not very large because mo-

(7.15)

|Vud | (*K* ! *µ*⌫) *|Vus | k |Vud |* $|V_{ud}|$ Finally, we find:

$$
)J_{\text{lep},\alpha}(x) \qquad S_{\text{EW}} = 1 + \frac{\alpha}{\pi} \ln \frac{m_W}{0.3 \text{ MeV}} \approx 1.013
$$

$$
\frac{e|V_{ud}|^2\Delta^5}{30\pi^3}\left(1-\frac{5m_e^2}{\Delta^2}-\frac{3}{2}\frac{\Delta}{m_\pi}\right)S_{EW}^2(1+\delta_{RC})
$$

$$
\frac{\Gamma(\pi \to \mu \nu)}{\Gamma(K \to \mu \nu)} = \frac{|V_{ud}|^2}{|V_{us}|^2} \frac{f_{\pi}^2}{f_{k}^2} F(m_{\pi}, m_{K}, m_{\mu}) \qquad \frac{f_{K}}{f_{\pi}} = 1.1932(19)
$$

$$
|V_{ud}|^2 + |V_{us}|^2 = 0.9992 \pm 0.006
$$

(<mark>∴)</mark>
() *µ* () *µ*

Precision physics at the LHC

Physics at the LHC, so far, can be summarized as follows: discovery of the Higgs boson, no new particles or interactions, strong exclusion limits and many measurements of the SM cross sections which by and large show excellent agreement with high-precision theoretical predictions.

ATLAS Preliminary August 202 \sqrt{s} = 13 TeV **Model Signature** $\int \mathcal{L} dt$ [fb⁻¹] **Mass limit** Reference 140
140 2-6 iets $\frac{E_{T,\rm miss}^{\rm miss}}{E_{T,\rm miss}^{\rm miss}}$ 1.85 $\tilde{q}\tilde{q}$, $\tilde{q} \rightarrow q \tilde{\chi}_{1}^{0}$ $0e.u$ $m(\tilde{\chi}_1^0)$ <400 GeV 2010.14293 $1-3$ jets 0.9 2102.10874 mono-jet \tilde{q} [8x Degen.] $m(\tilde{q})-m(\tilde{\chi}_1^0)=5$ GeV 2-6 jets 140 2010.14293 $\tilde{g}\tilde{g}, \tilde{g} \rightarrow q\bar{q}\tilde{\chi}_{1}^{0}$ $0 e, \mu$ $E_T^{\rm miss}$ $m(\tilde{\chi}^0_1) = 0$ GeV Forbidden $1.15 - 1.95$ $m(\tilde{\chi}_1^0)$ =1000 GeV 2010.14293 2-6 jets 140 2.2 $m(\tilde{\chi}_1^0)$ <600 GeV 2101.01629 $\tilde{g}\tilde{g}$, $\tilde{g} \rightarrow q\bar{q}W\tilde{\chi}_{1}^{0}$ 1 e, μ $\tilde{g}\tilde{g}$, $\tilde{g} \rightarrow q\bar{q}(\ell\ell)\tilde{\chi}^0$ $ee, \mu\mu$ 2 jets $E_T^{\rm miss}$ 140 2204.13072 2.2 $m(\tilde{\chi}_1^0)$ < 700 GeV $\tilde{g}\tilde{g}$, $\tilde{g} \rightarrow qqWZ\tilde{\chi}_{1}^{0}$ 0 e,μ 7-11 jets 140
140 1.97 2008.06032 $m(\tilde{\chi}^0_1)<$ 600 GeV SS e,μ 6 iets 1.15 m($\tilde{\chi}$)-m($\tilde{\chi}^0_1$)=200 GeV 2307.01094 0-1 e , u $3b$ 140
140 $\tilde{g}\tilde{g}, \tilde{g} \rightarrow t\bar{t}\tilde{\chi}_{1}^{0}$ m $(\tilde{\chi}^0_1)$ <500 GeV 2211.08028 $E_T^{\rm m}$ SS e, μ 1.25 6 jets 1909.08457 $m(\tilde{g})-m(\tilde{\chi}_1^0)=300$ Ge $\tilde{b}_1\tilde{b}_1$ 0 e, μ $2b$ $E_T^{\rm miss}$ 140 1.255 $m(\tilde{\chi}_1^0)$ < 400 Ge) 2101.12527 10 GeV< $\Delta m(\tilde{b}_1, \tilde{\chi}^0_1)$ < 20 GeV 2101.12527 ∆m $(\tilde{\chi}_2^0, \tilde{\chi}_1^0)$ =130 GeV, m $(\tilde{\chi}_1^0)$ =100 GeV $\tilde{b}_1 \tilde{b}_1$, $\tilde{b}_1 \rightarrow b \tilde{X}_2^0 \rightarrow bh \tilde{X}_1^0$ $\frac{E_{T}^{\rm miss}}{E_{T}^{\rm miss}}$ $0.23 - 1.35$ 0 e,μ 6 *b* 140
140 Forbidden 1908.03122 0.13-0.85 $2b$ 2τ $(\tilde{\chi}^0)$ =130 GeV. m $(\tilde{\chi}^0)$ =0 Ge 2103.08189 ≥ 1 jet 140 $\tilde{t}_1 \tilde{t}_1, \tilde{t}_1 \rightarrow t \tilde{X}_1^0$ 0-1 e, μ $E_T^{\rm miss}$ 1.25 $m(\tilde{\chi}_1^0)$ =1 GeV 2004.14060.2012.03799 140 $\tilde{t}_1 \tilde{t}_1, \tilde{t}_1 \rightarrow Wb\tilde{\chi}^0_1$ 1 e, μ 3 jets/1 $$ 1.05 $m(\tilde{\chi}_1^0)$ =500 GeV 2012.03799, ATLAS-CONF-2023-043 $E_{\rm T}^{\rm miss}$ Forbidder 2 jets/1 b $E_{\rm}^{\rm miss}$ 1-2 τ 140 $m(\tilde{\tau}_1) = 800$ Ge) $\tilde{t}_1\tilde{t}_1, \tilde{t}_1 \rightarrow \tilde{\tau}_1 b\nu, \tilde{\tau}_1 \rightarrow \tau \tilde{G}$ Forbidden -1.4 2108.07665 36.1
140 $\tilde{t}_1 \tilde{t}_1, \tilde{t}_1 \rightarrow c \tilde{X}_1^0$ / $\tilde{c} \tilde{c}, \tilde{c} \rightarrow c \tilde{X}_1^0$ 0 e,μ $2c$ 0.85 $m(\tilde{X}_1^0)=0$ GeV 1805.01649 0.55 $0 e, \mu$ mono-jet $m(\tilde{t}_1,\tilde{c})-m(\tilde{X}_1^0)=5$ GeV 2102.10874 1-2 e, μ 1-4 b 140 $\tilde{t}_1\tilde{t}_1, \tilde{t}_1 \rightarrow t\tilde{\chi}_2^0, \tilde{\chi}_2^0 \rightarrow Z/h\tilde{\chi}_1^0$ $E_T^{\rm miss}$ 0.067-1.18 $m(\tilde{\chi}_2^0)$ =500 GeV 2006.05880 $\tilde{t}_2\tilde{t}_2, \tilde{t}_2 \rightarrow \tilde{t}_1 + Z$ $3 e, \mu$ 1_b $E_T^{\rm miss}$ 140 Forbidden 0.86 m(${\tilde\chi}_1^0$)=360 GeV, m(${\tilde\imath}_1$)-m(${\tilde\chi}_1^0$)= 40 GeV 2006.05880 $\tilde{\chi}^{\pm}_1 \tilde{\chi}^0_2$ via WZ Multiple ℓ /jets $\frac{E_T^{\rm miss}}{E_T^{\rm miss}}$ 140
140 0.96 2106.01676, 2108.07586 $m(\tilde{\chi}_1^0)$ =0. wino-bino 0.205 $ee, \mu\mu$ ≥ 1 jet n($\tilde{\chi}_1^\pm$)-m($\tilde{\chi}_1^0$)=5 GeV, wino-bir 1911.12606 $\tilde{\chi}^{\pm}_{1}\tilde{\chi}^{\mp}_{1}$ via WW $E_T^{\rm miss}$ 140 2 e,μ 0.42 m $(\tilde{\mathcal{X}}_1^0)$ =0, wino-bino 1908.08215 $E_T^{\rm miss}$ 140 Multiple ℓ /jets 2004.10894, 2108.07586 1.06 $\tilde{\chi}^{\pm}_{1} \tilde{\chi}^{0}_{2}$ via Wh $\tilde{\chi}^{\pm}$ / $\tilde{\chi}^0$ Forbidden m $(\tilde{\chi}^0_1)$ =70 GeV, wino-bino $\tilde{\chi}_1^{\pm} \tilde{\chi}_1^{\mp}$ via $\tilde{\ell}_L/\tilde{\nu}$ 2 e,μ $E_T^{\rm miss}$ 140 1908.08215 1.0 $m(\tilde{\ell},\tilde{\nu})=0.5(m(\tilde{\chi}_1^{\pm})+m(\tilde{\chi}_1^0))$ E_T^{miss} 140 2τ 0.48 ATLAS-CONF-2023-029 $\tilde{\tau} \tilde{\tau}, \, \tilde{\tau} \rightarrow \tau \tilde{\chi}_{1}^{0}$ $m(\tilde{\chi}_1^0)=0$ $\stackrel{\text{f} \text{miss}}{E_T^\text{miss}}$ 0 iets 140
140 $\tilde{\ell}_{\text{L},\text{R}}\tilde{\ell}_{\text{L},\text{R}},\tilde{\ell} \rightarrow \ell \tilde{\chi}^0_1$ 2 e,μ
ee,μμ 0.7 $m(\tilde{\chi}_1^0)=0$ 1908.08215 ≥ 1 jet 0.26 $m(\tilde{\ell})-m(\tilde{\chi}_{1}^{0})=10$ GeV 1911.12606 $0 e, \mu$ $\geq 3 b$ $E_{\text{miss}}^{\text{miss}}$
 $0 e, \mu$ 0 jets $E_{\text{T}}^{\text{miss}}$
 $0 e, \mu$ ≥ 2 large jets $E_{\text{T}}^{\text{miss}}$ $\tilde{H}\tilde{H}$, $\tilde{H} \rightarrow h\tilde{G}/Z\tilde{G}$ $BR(\tilde{\chi}^0_1 \rightarrow h\tilde{G}) = 1$
 $BR(\tilde{\chi}^0_1 \rightarrow Z\tilde{G}) = 1$ 140 0.94 To appear $\frac{140}{140}$ 0.55 2103.11684 $0.45 - 0.93$ $BR(\tilde{X}_1^0 \rightarrow Z\tilde{G}) =$ 2108.07586 ≥ 2 jets E_T^{miss} 140 0.77 2204.13072 $2 e, \mu$ $\mathsf{BR}(\tilde{\mathcal{X}}_1^0\rightarrow Z\tilde{G})\mathsf{=}\mathsf{BR}(\tilde{\mathcal{X}}_1^0\rightarrow h\tilde{G})\mathsf{=}0.5$ Direct $\tilde{\chi}^+_1 \tilde{\chi}^-_1$ prod., long-lived $\tilde{\chi}^\pm_1$ Disapp. trk $E_T^{\rm miss}$ 140 0.66 Pure Wind 2201.02472 1 jet 0.21 Pure higgsin 2201.02472 Stable \tilde{g} R-hadron pixel dE/d> 140 2205.06013 2.05 140 \tilde{g} $[\tau(\tilde{g}) = 10 \text{ ns}]$ pixel dE/dx $E_T^{\rm miss}$ $m(\tilde{\chi}_1^0)$ =100 GeV 2205.06013 Metastable \tilde{g} R-hadron, $\tilde{g} \rightarrow q q \tilde{X}_1^0$ 2.2 $E_T^{\rm miss}$ 140 $\tilde{\ell}\tilde{\ell},\ \tilde{\ell}{\rightarrow}\ell\tilde{G}$ Displ. lep 0.7 $\tau(\tilde{\ell}) = 0.1$ ns 2011.07812 0.34
 0.36 $\tau(\tilde{\ell}) = 0.1$ ns 2011.07812 pixel dE/dx E_T^{miss} 140 $\tau(\tilde{\ell}) = 10$ n 2205.06013 $\tilde{\chi}^{\pm}_{1}\tilde{\chi}^{\mp}_{1}/\tilde{\chi}^{0}_{1}$, $\tilde{\chi}^{\pm}_{1}\rightarrow$ $Z\ell \rightarrow \ell \ell \ell$ 140 1.05 2011.10543 $3 e, \mu$ Pure Wind 4 e,μ 0 jets 140 1.55 $\tilde{\chi}_1^{\pm} \tilde{\chi}_1^{\mp}/\tilde{\chi}_2^0 \rightarrow WW/Z \ell \ell \ell \ell \nu \nu$ $E_{\tau}^{\rm m}$ m $(\tilde{\mathcal{X}}^0_1)$ =200 GeV 2103.11684 $\lambda_{22} \neq 0, \lambda_{12} \neq 0$ $\tilde{g}\tilde{g}, \tilde{g} \rightarrow qq\tilde{\chi}_{1}^{0}, \tilde{\chi}_{1}^{0} \rightarrow qqq$ \geq 8 jets 140 2.25 Large $\lambda_1^{\prime\prime}$ To appear Multiple 36.1 0.55 1.05 ATLAS-CONF-2018-003 $\tilde{t}\tilde{t}, \tilde{t} \rightarrow t\tilde{X}_1^0, \tilde{X}_1^0 \rightarrow tbs$ $m(\tilde{\chi}_1^0)$ =200 GeV, bino-like $\tilde{t}\tilde{t}, \tilde{t} \rightarrow b\tilde{\chi}_{1}^{\pm}, \tilde{\chi}_{1}^{\pm} \rightarrow bbs$ $\geq 4b$ 140 0.95 $m(\tilde{\chi}_1^{\pm})$ =500 GeV 2010.01015 2 jets + 2 b 36.7 0.61 1710.07171 $\tilde{t}_1 \tilde{t}_1, \tilde{t}_1 \rightarrow q t$ $2e,\mu$
 1μ $2 b$
DV 36.1
136 $0.4 - 1.45$ $BR(\tilde{t}_1 \rightarrow be/b\mu) > 20^\circ$ 1710.05544 \rightarrow qu $=$ 100%, cos θ _i= 2003.11956 $\tilde{\chi}_{1}^{\pm}/\tilde{\chi}_{2}^{0}/\tilde{\chi}_{1}^{0}, \tilde{\chi}_{1}^{0}, \rightarrow tbs, \tilde{\chi}_{1}^{\pm} \rightarrow bbs$ 1-2 e, μ ≥6 jets 140 $0.2 - 0.32$ Pure higgsin 2106.09609

 $\overline{1}$

Mass scale [TeV]

ATLAS SUSY Searches* - 95% CL Lower Limits

*Only a selection of the available mass limits on new states or phenomena is shown. Many of the limits are based or

 10^{-1}

to move the result in the right direction and this leads to large radiative corrections. generation constant playe ar $\frac{1}{2}$ 0 0 0 0 0 0 1 0 1 1 0 0 0 1 0 1 1 0 0 1 1 1 0 0 1 1 1 0 0 1 1 1 0 0 1 1 1 0 0 1 1 1 0 0 1 1 1 0 0 1 1 1 0 0 1 1 1 0 0 1 1 1 0 0 1 1 1 0 0 1 1 1 0 0 1 1 1 0 0 1 1 1 0 0 1 1 1 0 0 1 1 0 1 1 0 1 1 0 1 1 0 1 1 0 1 1

In general, the theoretical foundation for describing processes with large momentum transfer in hadron collisions is the collinear factorisation formula. Since non-perturbative corrections are typically small, we need partonic cross sections with high enough precision and parton distribution functions. $W_{\rm eff}$ kind of precision can one hope for at the LHC. This, of course, is at the LHC? This, moving target. From a theory side, the limiting factor is our ability to predict our process. For most processes and observables, this term is irrelavant, but s with high enough precision and parton ; cross sections with nigh enough precision and parton

$$
d\sigma_{\text{hard}} = \sum_{ij \in \{q,g\}} \int dx_1 dx_2 f_i(x_1) f_j(x_2) d\sigma_{ij}(x_1, x_2), \{p_{\text{fin}}\}) O_J(\{p_{\text{fin}}\}) \left(1 + \mathcal{O}(\Lambda_{\text{QCD}}^n / Q^n)\right)
$$

$$
d\sigma_{ij} = d\sigma^{(0)} \left(1 + \frac{\alpha_s}{\pi} c_1 + \left(\frac{\alpha_s}{\pi}\right)^2 c_2 + \left(\frac{\alpha_s}{\pi}\right)^3 c_3 + \cdots\right)
$$

$$
\sigma \sim \alpha_{\rm s}(\mu)^n \rightarrow \alpha_{\rm s}(\mu_1)^n \left(1 - \frac{n\beta_0}{2} \frac{\alpha_{\rm s}(\mu_1)}{\pi} \ln \frac{\mu}{\mu_1}\right) \qquad \beta_0 = 11/3C_A - 2/3n_f
$$
\n
$$
\frac{\Delta \sigma_n}{\sigma_n} = n \frac{11}{6} C_A \frac{\alpha_{\rm s}}{\pi} \approx n \, 2C_A \frac{\alpha_{\rm s}}{\pi} \approx 0.24 \ n
$$
\nIntegrals, the integral of the system is given by the integral of the system σ_n is not always even.

$$
\frac{\Delta \sigma_n}{\sigma_n} = n \frac{11}{6} C_A \frac{\alpha_s}{\pi} \approx n 2 C_A \frac{\alpha_s}{\pi} \approx 0.24 n
$$
\n
$$
\frac{\Delta \sigma_n}{\sigma_n} = \frac{n \frac{11}{6} C_A \frac{\alpha_s}{\pi} \approx n 2 C_A \frac{\alpha_s}{\pi} \approx 0.24 n
$$
\n
$$
\frac{\alpha_{\text{topion}}}{\sigma_n} = \frac{n \frac{11}{6} C_A \frac{\alpha_s}{\pi} \approx n 2 C_A \frac{\alpha_s}{\pi} \approx 0.24 n
$$
\n
$$
\frac{\alpha_{\text{topion}}}{\sigma_n} = \frac{n \frac{11}{6} C_A \frac{\alpha_s}{\pi} \approx n 2 C_A \frac{\alpha_s}{\pi} \approx 0.24 n
$$

One would expect moderate higher-order effects: $c_1 \sim C_A$, $c_2 \sim C_A^2$ $\alpha_s/\pi \sim 0.04$. However, in practice large corrections have been observed for many processes; one reason is that if high powers of the strong coupling constant are involved, the scale dependence of the strong coupling constant plays an important role: if you start with a wrong scale, corrections try moderate higher-order effects: $c_1 \sim C_A$, $c_2 \sim C_A^2$ $\alpha_s/\pi \sim 0.04$. protons disintegrate and all momenta transfers are large. These processes can be ● A major role in such an understanding is played by parton-parton scattering that is n the right direction and this ledds to large radiative corrections. of the strong coupling constant are involved the scale dependence of the ONE PRYS CHANDIQUE ROS. IF YOU SERIE WITH A WISH'S SOUN, SE role: if you start with a wrong scale c O. I. Jou dealer when a mong dockey donctout a provider that is a property with the starts with the starts with $\frac{1}{2}$ enough power of ↵*^s* and you choose a scale in some way, then radiative oupling constant are involved, the scale dependence of the important role: if you start with a wrong so to move the result in the right direction and this leads to large radiative corrections. ✓ ↵*^s* (*µ*1) c, conc
DNS. ln *^µ* nd this leads to large re

Higgs production in gluon fusion

Consider the flagship LHC process — Higgs boson production in the gluon fusion — where the cross section is claimed to be measured to about ten percent precision. This is much worse than a percent precision that we have been entertaining at the electroweak sector.

We will discuss how accurately the Higgs boson production cross section in gluon fusion can be predicted theoretically and what it takes to reach the state-of-the-art precision.

At the LHC, Higgs bosons are mainly produced in the gluon fusion. The top quark loop gives the largest contribution At the LHC. Higgs bosons are mainly produced in the aluon fusion. The to

$$
\mathcal{L} = m_t \left(1 + \frac{h(x)}{v} \right) \overline{t} t
$$

↵*s* 4π

 $\frac{4}{3}$ of gluons with arbitrary number of Higgs bosons reads: $\frac{2\pi}{12\pi}$ Hence, the effective Lagrangian that describes interaction **T***R* = 1*/2*, the *^g*↵ ⁺ *^q*↵*q*) ln ✓ ◆ gradaniada
100 home

. (8.7)

◆

describes interaction
ggs bosons reads:
$$
\mathcal{L} = \frac{\alpha_s}{12\pi} G_{\mu\nu}^a G^{a,\mu\nu} \ln\left(1 + \frac{h(x)}{v}\right)
$$

VIIStiberger, Bonetti, Tancredi, K.W., Becchetti, Bonciani, Del Duca, Hirschi, Moriello, Czakon, ESCHINEIII, SCHENENDEIGEL, INIGGENEUL, FONCERT u, d, s, c ,
Eschment, Schellenberger , Niggetiedt, Poncelet u, d, s, c , Mistlberger, Bonetti, Tancredi, K.M., Becchetti, Bonciani, Del Duca, Hirschi, Moriello, Czakon,

$$
\underline{\text{pb}}^{+2.22\,\text{pb}\,(+4.56\%)}_{-3.27\,\text{pb}\,(-6.72\%)}\,\left(\text{theory}\right)\pm1.56\,\text{pb}\,(3.20\%)\,\left(\text{PDF}+\alpha_s\right).
$$

$$
\mathcal{L} = \frac{\alpha_s}{12\pi} G_{\mu\nu}^a G^{a,\mu\nu} \ln\left(1 + \frac{h(x)}{v}\right) \rightarrow \frac{\alpha_s}{12\pi v} G_{\mu\nu}^a G^{a,\mu\nu} h
$$

The limit of the large top quark mass allows us to "remove one loop" and calculate corrections in a theory with a point-like Higgsgluon-gluon vertex. This gives us O(3%) precision; once this is accomplished, all the "smallish" effects have to be evaluated anew. He rurge top quantifiass allows as to trefflove one loop tana calculate corrections in a theory with a politicine ringg.
Mertex, This gives us (28%) precision: once this is accomplished, all the "smallish" effects have to $\alpha^{''}$ and α *v* prodion, drive this is decomptioned, an the pritament choote have to be evaluated

Let us discuss what we can say from the knowledge of the high-precision Higgs production in gluon fusion and assume, for definiteness, that the LHC will, eventually, be able to measure Higgs production in gluon fusion to a 3 percent precision.

The charm Yukawa coupling is poorly known. Its contribution to Higgs production is about two ntributi
blings i *a*_p_{*a*}
Traja + *g*² + *a*₂ + *g*² + *a*₂ + *g*² + *a*₂ + *g*² + *a*² + *g*² + *g* if they ar , times large percent. Hence, one can constrain the Yukawa couplings if they are O(2) times larger than the θ *v* SM. 00000

 $\overline{}$ 3⇡ *v* the Higgs production ate *h* ◆ The Higgs self-coupling changes the Higgs production rate by about a percent. The effect is O(3) times larger than the SM. linear, so a three-percent measurement/theory prediction will constrain h^3 couplings that are

$$
\mathcal{L} = \frac{\alpha_s}{12\pi} G_{\mu\nu}^a G^{a,\mu\nu} \ln\left(1 + \frac{h(x)}{v}\right)
$$

\n
$$
\mathcal{M}_{gg \to H} = \mathcal{M}_{gg \to H}^{(0)} \left(1 + c_1 \frac{3m_h^2}{(4\pi)^2 v^2}\right) \approx \mathcal{M}_{gg \to H}^{(0)} \left(1 + 5c_1 \times 10^{-3}\right)
$$

$$
\frac{|\mathcal{A}_t + \mathcal{A}_c|^2}{|\mathcal{A}_t|^2} \approx 1 - \frac{3m_c^2}{m_H^2} \ln^2 \frac{m_c^2}{m_H^2} \approx 1 - 2 \times 10^{-2}
$$

The large effects in Higgs boson production in gluon fusion that we discussed are not atypical, although they are somewhat extreme. Furthermore, even in the simplest cases such as the total cross section for single vector boson production at the LHC, the perturbative expansion looks peculiar. At the same time, these results do not include N3LO QCD parton distribution functions, so it is not quite clear if there won't be any changes once higher-order PDFs become available.

Duhr, Dulat, Mistlberger

Sudakov logarithms

$$
\mathcal{M} = H^{\rho} \frac{-i}{s - m_V^2} L^{(0)}_{\rho} \qquad L^{(0)}_{\rho} = ig_{Z,e} \bar{u}_1 \gamma^{\rho} v_2 \qquad s \gg m_V^2 \qquad q
$$

\n
$$
L^{(1),\rho} = g_{Z,e} g_{Z,e}^2 \int \frac{d^4 k}{(2\pi)^4} \frac{\bar{u}_1 \gamma^{\mu} (\hat{p}_1 + \hat{k}) \gamma^{\rho} (\hat{k} - \hat{p}_2) \gamma_{\mu} v(p_2)}{(\hat{k} + p_1)^2 (\hat{k} - p_2)^2 (\hat{k}^2 - m_V^2)}
$$

\n
$$
\Delta = m_v^2 x_3 + P^2 - i0 = m_v^2 x_3 - s x_1 x_2 - i0 \qquad L^{(1),\rho} = -L^{(0),\rho} \frac{\alpha_g^2}{2\pi} \frac{1}{2} \ln^2 \frac{s}{m_V^2}
$$

At the LHC, electroweak corrections are usually less important than the QCD ones because of the relation between the couplings $\alpha_s \sim 0.1 \gg \alpha \sim 0.01$. But this is not always the case. \mathbb{R}^n electroweak corrections but since, typically, At the LH *H*⇢ and *L*(0) ⇢ are quark and lepton tree-level currents. The lepton one reads α and $\alpha_s \sim 0.1 \gg \alpha \sim 0.01$. But this is not a *u* less important than the QCD ones because of the *L*
L
L
L
L
L
l Z,e $\frac{1}{2}$ [d*x*]³ \sim $\gg \alpha \sim 0.01$. *m*² *^v x*² *sx*1*x*² 2CD ones be
<mark>ovs the case</mark> *m*²

$$
\mathcal{M} = H^{\rho} \frac{-i}{s - m_V^2} L_{\rho}^{(0)} \qquad L_{\rho}^{(0)} = ig_{Z,e} \bar{u}_1 \gamma^{\rho} v_2 \qquad s \gg m_V^2 \qquad q
$$

$$
L^{(1),\rho} = g_{Z,e} g_{Z,e}^2 \int \frac{d^4 k}{(2\pi)^4} \frac{\bar{u}_1 \gamma^{\mu} (\hat{p}_1 + \hat{k}) \gamma^{\rho} (\hat{k} - \hat{p}_2) \gamma_{\mu} v(\rho_2)}{k + \rho_1^2 (k - \rho_2)^2 (k^2 - m_V^2)}
$$

$$
L^{(1),\rho} = g_{Z,e} g_{Z,e}^2 \Gamma(3) \int [dx]_3 \int \frac{d^4 k}{(2\pi)^4} \frac{\bar{u}_1 \gamma^{\mu} (\hat{p}_1 + \hat{k}) \gamma^{\rho} (\hat{k} - \hat{p}_2) \gamma_{\mu} v(\rho_2)}{((k + P)^2 - \Delta)^3}
$$

$$
\Delta = m_v^2 x_3 + P^2 - i0 = m_v^2 x_3 - s x_1 x_2 - i0 \qquad L^{(1),\rho} = -L^{(0),\rho} \frac{\alpha_g^2}{2\pi} \frac{1}{2} \ln^2 \frac{s}{m_V^2}.
$$

$$
M_0 \left(1 - \frac{\alpha_g}{4\pi} \ln^2 \frac{s}{m_V^2} \right) \quad \Longrightarrow \quad \left[\quad d\sigma = d\sigma_0 \left(1 - \frac{\alpha_g}{2\pi} \ln^2 \frac{s}{m_V^2} \right) \right]
$$

$$
\mathcal{M} = H^{\rho} \frac{-i}{s - m_V^2} L^{(0)}_{\rho} \qquad L^{(0)}_{\rho} = ig_{Z,e} \bar{u}_1 \gamma^{\rho} v_2 \qquad s \gg m_V^2 \qquad q
$$

$$
L^{(1),\rho} = g_{Z,e} g_{Z,e}^2 \int \frac{d^4 k}{(2\pi)^4} \frac{\bar{u}_1 \gamma^{\mu} (\hat{p}_1 + \hat{k}) \gamma^{\rho} (\hat{k} - \hat{p}_2) \gamma_{\mu} v(p_2)}{(\hat{k} + p_1)^2 (\hat{k} - p_2)^2 (\hat{k}^2 - m_V^2)}
$$

$$
L^{(1),\rho} = g_{Z,e} g_{Z,e}^2 \Gamma(3) \int [dx]_3 \int \frac{d^4 k}{(2\pi)^4} \frac{\bar{u}_1 \gamma^{\mu} (\hat{p}_1 + \hat{k}) \gamma^{\rho} (\hat{k} - \hat{p}_2) \gamma_{\mu} v(p_2)}{((\hat{k} + P)^2 - \Delta)^3}
$$

$$
\Delta = m_v^2 x_3 + P^2 - i0 = m_v^2 x_3 - s x_1 x_2 - i0 \qquad L^{(1),\rho} = -L^{(0),\rho} \frac{\alpha_g^2}{2\pi} \frac{1}{2} \ln^2 \frac{s}{m_V^2},
$$

$$
\mathcal{M} = \mathcal{M}_0 \left(1 - \frac{\alpha_g}{4\pi} \ln^2 \frac{s}{m_V^2} \right) \quad \Longrightarrow \quad \boxed{\text{d}c}
$$

M = *M*⁰

, (9.18)

so that

Since also

 $\overline{}$

 $\overline{}$

In QED or QCD , enhanced virtual correction gets ca In QED or QCD, enhanced virtual correction gets cancelled with the real emission contributions. " anced virtual correction
 $\frac{d}{dt}$ $\overline{1}$ I correction gets cancelled with the real emission co **hanced virtual correction gets cancelled v** rection gets cance (*p*1*k*)(*p*2*k*) *s* $\frac{2}{3}$ Z ne real em mis $\frac{1}{2}$ *s*
 8 *V l* with the real emission contributions. ission contrik red virtual correction gets cancelled with the real emission contributions perturbative QCD to the LHC physics. The LHC physics of the LHC physics of the LHC physics. The LHC physics of
The LHC physics of the LHC physics. The LHC physics of the LHC physics. The LHC physics of the LHC physics of However, if we translate the above exercise to a question about elec-

Comparison with the virtual corrections, makes the $\qquad \qquad$ d*^R* = d⁰ *g*² *Z,e* Z *^k*] ²*p*1*p*² To compute the integral, we perform the Sudakov decomposition $x = 0.99$ of the *Z*. We then find of view are quite divided. The concellation obvious. of the *Z*. We then find Comparison with the virtual corrections, makes the cancellation obvious.

and pp -> X+ V (V=Z,W) are, typically, treated as different processes in express that \mathcal{L} [d³ and rugument is the under owner of these rectrowear
and the size | d₁₁₄
| that hat there are d *v* – *z*, *v v)*
double Ic le logarithmic el elect ⇣ with the collision's energy. To estimate the size of these corrections, we write the coupling for the $\,|\,$ I Z-boson and take $s = 1$ TeV. *said pp. All viventy, end, cypiddity, a dated ab amorone pro*
 s that there are double logarithmic electroweak corrections there Hence, for *s* = 1 TeV, we find Experience this cancellation does not quite work for electrown With the collision's energy. To estimate the size of these con JY. TO ESUITIULE LITE SIZE OF LITESE CO
-

$$
d\sigma_R = d\sigma_0 g_{Z,e}^2 \int [d^3k] \frac{2p_1p_2}{(p_1k)(p_2k)} \qquad \Longrightarrow \qquad C
$$

$$
M_R = H^{\rho} \frac{-i}{s - m_V^2} L_{\rho, R} \qquad L_R^{\rho} = -ig_B g_{Z,e} \bar{u}_1 \left[\frac{\hat{\epsilon}(\hat{p}_1 + \hat{k}) \gamma^{\rho}}{2p_1 k + m_V^2} + \frac{\gamma^{\rho}(-\hat{p}_2 + k)}{-2p_2 k + m_V^2} \right] v_2
$$

\n $k \sim m_V \ll \sqrt{s} \qquad \Longrightarrow \qquad L_R^{\rho} = -g_{Z,e} L^{\rho,(0)} \left(\frac{p_1 \epsilon_Z}{p_1 k} - \frac{p_2 \epsilon_Z}{p_2 k} \right)$
\n $d\sigma_R = d\sigma_0 g_{Z,e}^2 \int [d^3 k] \frac{2p_1 p_2}{(p_1 k)(p_2 k)} \qquad \Longrightarrow \qquad d\sigma_R = d\sigma_0 \frac{\alpha_g}{2\pi} \ln^2 \frac{s}{m_V^2}$
\nComparison with the virtual corrections, makes the cancellation obvious.

Work for electroweak corrections since process $pp\rightarrow X$ processes in experiment. Ini
a that are always negative ar (*p*1*k*)(*p*2*k*) they arow¹ To compute the integral, we perform the integral, we perform the Sudakov decomposition \mathcal{L} α parameters of the vert for electroweak corrections since process pp->X *.* (9.24) e size of these *v v COLLECTION* ections, we write te th ⇣ *e s* coupling for \cdot w eak correction \overline{U} $\ddot{}$ that there are double logarithmic electroweak corrections that are always negative and they grow \vert It is clear that such corrections become very large if *s m*² However, this cancellation does not quite work for electroweak corrections since process pp->X and pp -> X+ V (V=Z,W) are, typically, treated as different processes in experiment. This implies ↵ and pp -> X+ V (V=Z,W) are, typically, treated as different processes in experiment. This implies ec *s* that $\begin{array}{|c|c|c|c|c|}\n\hline\n\text{sses in experiment. This implies}\n\hline\n\end{array}$

$$
\alpha_g = \frac{\alpha}{s_w^2 c_w^2} (T_3 - Q \sin^2 \theta_W)^2
$$

$$
- \frac{\alpha_g}{2\pi} \ln^2 \frac{s}{m_V^2} \approx -0.16 (T_3 - Q \sin^2 \theta_W)^2
$$

Lindert, Mai $Lindert, Mai$

Large effects can indeed be seen in realistic (complete) computations. Below the electroweak corrections to Z+j production at the LHC are shown. NLLVI EW ffects can indeed be seen in NLOVI EW $\overline{1}$ 102
102
102 d*s p*T,Z [pb/GeV] λ is no to λ is no so duoting of the $|1|$ is no obey in NLLVI EW 9IOW I d \overline{C}

The top quark mass

l

Top quark mass extractions from cross sections rely on its strong sensitivity to m_t and on higherorder perturbative predictions for top quark pair production cross section. From this observable alone, the top quark mass was measured to about 700 MeV.

$$
\sigma_{t\bar{t}} = \sigma_0 \left(\frac{m_0}{m_t}\right)^5 \left[1 + c_{\rm np} \frac{\Lambda_{\rm QCD}}{m_t} + \ldots\right]
$$

$$
m_t \to m_t + \frac{c_{\rm np}}{5} \Lambda_{\rm QCD}
$$

To be certain that this works in the right way, we need to know whether there are linear nonperturbative power corrections to the cross section; otherwise they may impact the extracted value of the top quark mass.

> The existing theory of hard hadron collisions does not allow us to say with confidence whether such corrections exist or not; without this, it is not possible to trust the ultra-precise value of the top quark mass.

$$
d\sigma_{\text{hard}} = \sum_{ij \in \{q,g\}} \int dx_1 dx_2 f_i(x_i)
$$

 $dx_1 dx_2 f_i(x_1) f_j(x_2) d\sigma_{ij}(x_1, x_2)$, $\{p_{fin}\}) O_J(\{p_{fin}\})$ $\left(1+\mathcal{O}(\Lambda^n_{\mathrm{QCD}}/Q^n)\right)$

One possibility to explore this problem is to connect perturbative and non-perturbative computations, by checking the sensitivity of the former to "non-perturbative" (soft) momenta regions.

It is certainly possible to understand the famous (Kinoshita-Lee-Naunberg) cancellation of soft and collinear singularities in this way; it becomes particularly instructive if the gluon is given a mass.

 $d\sigma_{e^+e^- \to t\bar{t}} + d\sigma_{e^+e^- \to t\bar{t}}$

Infra-red safety implies that the sensitivity to long-distance physics, parameterised by the mass of the gluon, is absent, i.e. whatever this mass is, the result is the same, up to power corrections.

 $\bar{t}+g$ $\sim d\sigma_0$ $(1 + \alpha_s \mathcal{O}(\lambda^0))$

Power corrections $O(\lambda^n)$ are also interesting if our goal is high-precision predictions; these corrections basically tell us when perturbative predictions alone become insufficient.

In addition, it turns out that for this discussion, it is important to understand what is meant by the "top quark mass'' that one tries to measure.

 $+$ ${\cal O}(\lambda^2)$

We would like to know whether or not there are linear correction to top quark pair production at a hadron collider since, if it is there, it would have important implications for the extraction of the top quark mass with the highest precision.

$$
\mathrm{d}\sigma_{t\bar{t}}(E,\lambda)=\mathrm{d}\sigma_{t\bar{t}}^{(0)}+\lambda\;\mathrm{d}\sigma_{t\bar{t}}^{(1)}\; \text{-}
$$

To answer this question, one needs to compute loop corrections and real-emission contributions to top quark pair production cross section with the massive gluon and then expand the result in the small gluon mass up to the linear terms.

In quantum field theory , particle masses are inferred from poles of propagators. This cannot work for quarks beyond fixed-order perturbation theory (confinement). This issue is quite obscure for the top quark since it is very heavy and very unstable (top is the only quasi-free quark, as we often say).

- $\vec{k}|, m$) $w(|\vec{k}|, m) \approx 1, \quad |\vec{k}| \leq m$ $\alpha_s(|$ \overline{k} $|k|) \approx$ Λ_QCD^2 $\vec{k}^2 - \Lambda_{\rm QCD}^2$ \overline{k} $k|, m) \approx 1,$ | \overline{k} $|k| \leq m$
- The pole mass of a top quark cannot be determined with the precision better than $\delta m \sim \Lambda_{\rm QCD} \sim 300\,\,{\rm MeV}$.
- We must think about the top quark mass as a parameter of the Lagrangian and define it according to a chosen renormalization ``scheme''. Depending on the choice of the scheme and the renormalization scale, we get potential-subtracted, 1S etc.). On the other hand, these short-distance masses can be determined with a much

different mass parameters, that range from the MSbar to the low-scale short-distance masses (kinetic, higher precision than the pole mass, at least in theory.

$$
G(p,m) \sim \frac{1}{p^2 - m^2} \quad \Rightarrow \quad p^2 = m^2
$$

$$
m_{\text{pole}} = m_{\text{bare}} + \frac{4}{3} \int_{0}^{\infty} \frac{d^3 \vec{k}}{4\pi^2} \frac{\alpha_s(|\vec{k}|)}{\vec{k}^2} w(|\vec{k}|, m) \qquad w(|
$$

$$
m(\mu) = m_{\text{bare}} + \frac{4}{3} \int_{\mu}^{\infty} \frac{\mathrm{d}^3 \vec{k}}{4\pi^2} \frac{\alpha_s(|\vec{k}|)}{\vec{k}^2} w(|\vec{k}|, m) \qquad \qquad \Lambda_{\text{QCD}} \ll \mu \qquad \qquad m_{\text{pole}} = m(\mu) + \frac{4}{3} \int_{0}^{\mu} \frac{\mathrm{d}^3 \vec{k}}{4\pi^2} \frac{\alpha_s(|\vec{k}|)}{\vec{k}^2}
$$

$$
m_{\text{pole}} = m(\mu) + \frac{4}{3} \alpha_s(\mu) \mu
$$

p/t

N*p/^b*

where

d

@

*^m*²

^b Dp,µ

^T [*^V*] = ↵*sC^F*

p/t

N*p/^b*

⇡

Z

3

$$
\left[\begin{aligned}\n\sigma &= \sigma_{\text{LO}}(m_t) + \sigma_R + \sigma_V + \sigma_{\text{ren}} = \sigma_{\text{LO}}(\tilde{m}_t) + \delta \sigma_{\text{NLO}} &= \sigma_R + \sigma_V + \sigma_{\text{ren}} + \delta \sigma_{\text{mass}}^{\text{expl}} + \delta \sigma_{\text{mass}}^{\text{impl}} \\
\mathcal{T}_{\lambda} \left[\delta \sigma_{\text{mass}}^{\text{expl}} + \delta \sigma_{\text{mass}}^{\text{impl}} \right] &= \frac{C_F \alpha_s}{2\pi} \frac{\pi \lambda}{m_t} \int d\text{Lips}_{\text{LO}} \left[\frac{m_t^2}{p_d p_t} \left[1 + p_d^\mu \left(\frac{\partial}{\partial p_d^\mu} - \frac{\partial}{\partial p_t^\mu} \right) \right] F_{\text{LO}} - m_t \text{Tr} \left[1 \mathbf{N} \rlap{\psi}_b \bar{\mathbf{N}} \right] - m_t \text{Tr} \left[(\psi_t + m_t) \left(\frac{\partial \mathbf{N}}{\partial m_t} \rlap{\psi}_b \bar{\mathbf{N}} + \mathbf{N} \rlap{\psi}_b \frac{\partial \bar{\mathbf{N}}}{\partial m_t} \right) \right] \right], \\
\mathcal{T}_{\lambda} \left[\sigma_{\text{R}} \right] &= \frac{\alpha_s C_F}{2\pi} \frac{\pi \lambda}{m_t} \int d\text{Lips}_{\left(p_u, p_b; p_d, p_t, p_X \right)} \left[\left(\frac{3}{2} - \frac{m_t^2}{p_d p_t} - \frac{m_t^2}{p_t p_b} \right) - \frac{m_t^2}{p_d p_t} p_d^\mu \left(\frac{\partial}{\partial p_d^\mu} - \frac{\partial}{\partial p_t^\mu} \right) - \frac{m_t^2}{p_t p_b} p_b^\mu \left(\frac{\partial}{\partial p_t^\mu} + \frac{\partial}{\partial p_t^\mu} \right) \right] F_{\text{LO}} \right], \\
\mathcal{T}_{\lambda} \left[\sigma_V \right] &= -\frac{\alpha_s C_F}{2\pi} \frac{\pi \lambda}{m_t} \int d\text{Lips}_{\text{LO}} \left[\text{Tr} \left[\rlap{\psi}_t \mathbf{N} \rlap{\psi}_b \bar{\mathbf{N}} \right] + \left(\frac{2p_t p_b - m_t
$$

N¯

One can show that $\mathcal{O}(\lambda) \sim \mathcal{O}(\Lambda_{\text{QCD}})$ power corrections to top quark pair production cross section cancel provided that it is expressed through one of the short distance masses; however, such corrections are present if the cross section is written in terms of the pole mass. Below the way the cancellation works in case of single-top production at the LHC is shown. W_{∞} (W_{∞}) prought through one of the under much masses, nowe = LO(*mt*) + *^R* + *^V* + ren = LO(˜*mt*) + NLO*,* (6.1) WEIGHT TO TOP GUAIN PUI PIOUUOLION DI OU BOULION *do* we hat $\sigma(x) \sim \sigma(\mu(qc))$ power corrections to top quantipal production or the short distance masses thome me of the nole mase Relow the way where B LHU IS SHOWH. $y \cdot y = h \cdot y = h \cdot y = h \cdot y$ single-top production at the LHC is shown. *p*^{*p*} *,* (2.29) = LO(*mt*) + *^R* + *^V* + ren = LO(˜*mt*) + NLO*,* (6.1) brt distance masses; now *dt* 2*ptk* **b** *p p e mas*: $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ momentum mapping. **xpressed through one of the short di**

On the contrary, the expansion of 1*/d^b* is simple since the momentum *p^b* is not subject to

$$
\sigma = \sigma_{\text{LO}}(m_t) + \sigma_R + \sigma_V + \sigma_{\text{ren}} = \sigma_{\text{LO}}(\tilde{m}_t) + \delta \sigma_{\text{NLO}} \qquad \delta \sigma_{\text{NLO}} = \sigma_R + \sigma_V + \sigma_{\text{ren}} + \delta \sigma_{\text{mass}}^{\text{expl}} + \delta \sigma_{\text{mass}}^{\text{impl}})
$$
\n
$$
\mathcal{T}_{\lambda} \left[\delta \sigma_{\text{mass}}^{\text{expl}} + \delta \sigma_{\text{mass}}^{\text{impl}} \right] = \frac{C_F \alpha_s}{2\pi} \frac{\pi \lambda}{m_t} \int d\text{Lips}_{\text{LO}} \left[\frac{m_t^2}{p_d p_t} \left[1 + p_d^\mu \left(\frac{\partial}{\partial p_d^\mu} - \frac{\partial}{\partial p_t^\mu} \right) \right] F_{\text{LO}} - m_t \text{Tr} \left[1 \mathbf{N} \dot{p}_b \mathbf{\bar{N}} \right] - m_t \text{Tr} \left[(\dot{p}_t + m_t) \left(\frac{\partial \mathbf{N}}{\partial m_t} \dot{p}_b \mathbf{\bar{N}} + \mathbf{N} \dot{p}_b \frac{\partial \mathbf{N}}{\partial m_t} \right) \right] \right]
$$
\n
$$
\mathcal{T}_{\lambda} \left[\sigma_{\text{R}} \right] = \frac{\alpha_s C_F}{2\pi} \frac{\pi \lambda}{m_t} \int d\text{Lips}_{\nu} (p_u, p_b; p_d, p_t, p_X) \left[\left(\frac{3}{2} - \frac{m_t^2}{p_d p_t} - \frac{m_t^2}{p_t p_b} \right) - \frac{m_t^2}{p_d p_t} p_d^\mu \left(\frac{\partial}{\partial p_d^\mu} - \frac{\partial}{\partial p_t^\mu} \right) - \frac{m_t^2}{p_t p_b} p_b^\mu \left(\frac{\partial}{\partial p_b^\mu} + \frac{\partial}{\partial p_t^\mu} \right) \right] F_{\text{LO}} \right],
$$
\n
$$
\mathcal{T}_{\lambda} \left[\sigma_V \right] = -\frac{\alpha_s C_F}{2\pi} \frac{\pi \lambda}{m_t} \int d\text{Lips}_{\text{LO}} \left[\text{Tr} \left[\dot{p}_t \mathbf{N} \dot{p}_b \mathbf{\bar{N}} \right] + \left(\frac{
$$

$$
\begin{split}\n\sigma &= \sigma_{\text{LO}}(m_t) + \sigma_R + \sigma_V + \sigma_{\text{ren}} = \sigma_{\text{LO}}(\tilde{m}_t) + \delta \sigma_{\text{NLO}} \qquad \delta \sigma_{\text{NLO}} = \sigma_R + \sigma_V + \sigma_{\text{ren}} + \delta \sigma_{\text{mass}}^{\text{expl}} + \delta \sigma_{\text{mass}}^{\text{impl}} \\
\mathcal{T}_{\lambda} \left[\delta \sigma_{\text{mass}}^{\text{expl}} + \delta \sigma_{\text{mass}}^{\text{impl}} \right] &= \frac{C_F \alpha_s}{2\pi} \frac{\pi \lambda}{m_t} \int d\text{Lips}_{\text{LO}} \left[\frac{m_t^2}{p_d p_t} \left[1 + p_d^\mu \left(\frac{\partial}{\partial p_d^\mu} - \frac{\partial}{\partial p_t^\mu} \right) \right] F_{\text{LO}} - m_t \text{Tr} \left[1 \mathbf{N} p_b \mathbf{\bar{N}} \right] - m_t \text{Tr} \left[\left(\dot{\varphi}_t + m_t \right) \left(\frac{\partial \mathbf{N}}{\partial m_t} \dot{\varphi}_b \mathbf{\bar{N}} + \mathbf{N} \dot{\varphi}_b \frac{\partial \mathbf{N}}{\partial m_t} \right) \right] \right] \\
\mathcal{T}_{\lambda} \left[\sigma_{\text{R}} \right] &= \frac{\alpha_s C_F}{2\pi} \frac{\pi \lambda}{m_t} \int d\text{Lips}_{\{p_u, p_b; p_d, p_r, p_X\}} \left[\left(\frac{3}{2} - \frac{m_t^2}{p_d p_t} - \frac{m_t^2}{p_t p_b} \right) - \frac{m_t^2}{p_d p_t} p_d^\mu \left(\frac{\partial}{\partial p_d^\mu} - \frac{\partial}{\partial p_t^\mu} \right) - \frac{m_t^2}{p_t p_b} p_b^\mu \right] - \frac{m_t^2}{p_t p_b} p_b^\mu \left(\frac{\partial}{\partial p_b^\mu} + \frac{\partial}{\partial p_t^\mu} \right) \right] F_{\text{LO}} \\
\mathcal{T}_{\lambda} \left[\sigma_V \right] &= -\frac{\alpha_s C_F}{2\pi} \frac{\pi \lambda}{m_t} \int d\text{Lips}_{\text{LO}} \left[\text{Tr} \left[\dot{p}_t \mathbf{N} \dot{p}_b \math
$$

$$
\mathcal{T}_{\lambda} \left[\sigma_{\text{ren}} \right] = \frac{\alpha_s C_F}{2\pi} \frac{\pi \lambda}{m_t} \int d\text{Lips}_{\text{LO}} \left[\frac{3}{2} F_{\text{LO}} + m_t \text{Tr} \left[(\rlap{\,/} \psi_t + m_t) \frac{\partial \mathbf{N}}{\partial m_t} \rlap{\,/} \psi_b \bar{\mathbf{N}} \right] + m_t \text{Tr} \left[(\rlap{\,/} \psi_t + m_t) \mathbf{N} \rlap{\,/} \psi_b \frac{\partial \bar{\mathbf{N}}}{\partial m_t} \right] \right],
$$

Upon combining the approximate expressions for the matrix element squared and the

$$
\mathcal{T}_{\lambda} \left[\delta \sigma_{\mathrm{NLO}} \right] = \frac{\alpha_s C_F}{2\pi} \frac{\pi \lambda}{m_t} \int \mathrm{d} \mathrm{Lips}_{\mathrm{LO}} \, \left(F_{\mathrm{LO}} - \mathrm{Tr} \left[\rlap{/} \rlap{/} \rlap{/} \mathbf{N} \rlap{/} \rlap{/} \mathbf{p}_{b} \bar{\mathbf{N}} \right] - m_t \mathrm{Tr} \left[\mathbf{1} \mathbf{N} \rlap{/} \mathbf{p}_{b} \bar{\mathbf{N}} \right] \right) = 0
$$

t

 $\mathcal{O}(\lambda) \sim \mathcal{O}(\Lambda_{\rm QCD})$ $\frac{1}{2}$ that $\frac{1}{2}$ \sim $\mathcal C$ $Q(\Lambda_{\OmegaCD})$ @*m^t .* @*m^t* @*m^t* e the final result if the construction of a final result for the final result for the cross section in the cross section in the cross section in the cro rrections are present if the cross section is written in terms 6 The final result for the cross section W_{max} is relevant for the relevant formulae. We begin with the NLO cross section expressed for the NLO cross t sent in the cross section is written in terms of t One can show that $\rho(1) = \rho(1) = 0$ nower corrections to top quark pair production of *ptp^d* $\frac{1}{2}$ d $\overline{}$ $\overline{1}$ 2*ptk* **URRELLING**

STILLING

STILLING \overline{S} i *p*
p *p***d**
p d d rovided that it is expressed through one of corroctions are pro the cance **C**_F $\frac{1}{2}$ → $\frac{1}{2}$ 1 ا ⇡ η ^t Z a works in \mathbb{R}^n recent if the cross vorks in ca <u>ا</u> ⇡ \mathcal{T} Z Ω

$$
\sigma = \sigma_{\text{LO}}(m_t) + \sigma_R + \sigma_V + \sigma_{\text{ren}} = \sigma_{\text{LO}}(\tilde{m}_t) + \delta \sigma_{\text{NLO}}
$$

i

(*p/^t* + *mt*)

p/b

(*p/^t* + *mt*)N*p/^b*

1 + *p^µ*

d

@

◆ *^F*LO *^mt*Tr ^h

1N*p/^b*

N¯

Makarov, K.M. , Nason,Oczelik can be significant in the commutations. Although in the corresponding to the corresponding to the corresponding to α and α and large estimate to a region of the parameter which ends above the *p*^t above the *p*^t Interesting the shifts exhibition of the Makarov, K.M., Nason, Oczelik

Linear power corrections do exist in kinematic distributions independent of the mass parameter use. In general, shifts are not large but they become enhanced and reach a few percent close to edges of the allowed kinematic regions. Linear power corrections are not universal and exhibit non-trivial dependencies on the kinematic variables. \overline{r} p *ections do exist* in kinematic distributions independ *p*^{*p*} *p*^{*q*} *p*^{*q*} *pq*^{*d*} *pq*^{*d*} *pq*^{*d*} *pq pq*^{*d*} *pq* \overline{a} \overline{a} $\overline{0}$ *p p d p e g l o n* K l D atic variables. IONS
' *independent of the mass parameter* regions. Linear power corrections are not universal and e re kinen = ↵*s* ⇡ (2*C^F CA*⌧) 2(1 ⌧) *,* (6.3) doppard of the mass parameter ic distributions independent of the mass parameter *c* and reach ⌧!1 *yt* NP[*yt*] NP [*pt*?] In this section we compute the linear power corrections to three simple observables – the top v hil ear power corrections are not universal and exhibit ariables.

Results for the Tevatron where quark annihilation channel dominates.

Conclusions

The Standard "Model" of particle physics, supplemented with general theory of relativity, is the current version of the "ultimate theory of everything". The SM is not a model where you have enough knobs to turn to achieve an agreement with the observation.

We do this in the most natural way — by comparing the best measurements of various quantities and elementary particle's reactions with the best theoretical predictions that we have for them. This is called precision SM physics.

Treating it as such, we would like to find how far we should push it before it starts breaking, ideally in the controllable environment (colliders, low-energy experiments etc.).

In principle, since the SM is a renormalizable theory, we just need a few parameters to describe (or fail to describe) every single measurement that it out there.

In practice, we start seeing limits of what we can do with perturbative physics and that nonperturbative physics starts playing more and more important role in the deliberations about the validity of the SM.

