Semileptonic $b \to c$ Form Factors

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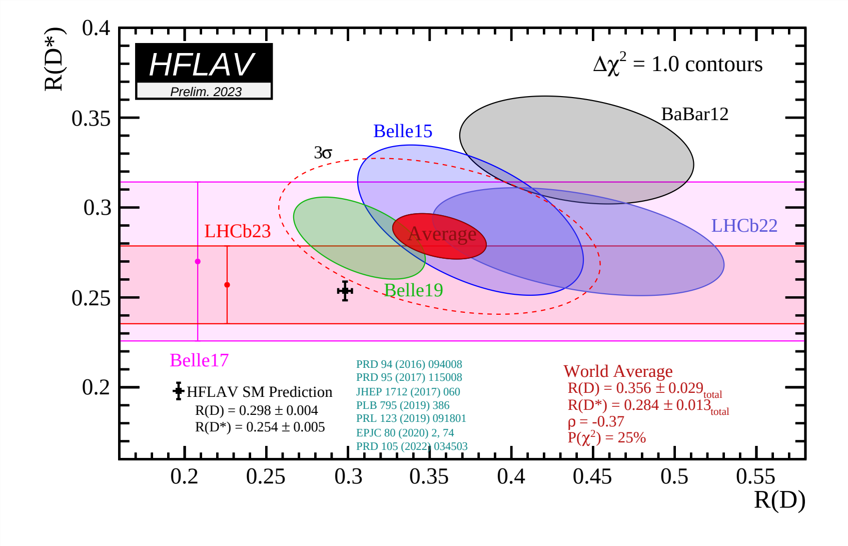
Test of Lepton-Flavour Universality (LFU)

[HFLAV 1909.12524; Winter '23 update]



$\overline{B} \to D^{(*)}$ Form Factors

- parametrize mismatch between free-quark processes and hadronic processes
  - scalar-valued functions of a single variable: momentum transfer $q^2 = m^2_{\ell\overline{\nu}}$

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  - $\overline{B} \to D$: 1 form factor; 1x vector current — available from EXP+TH
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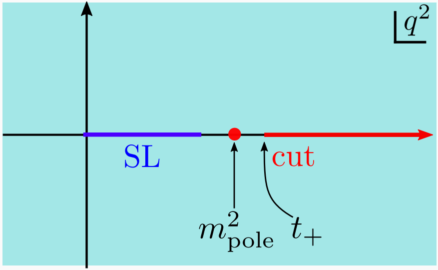
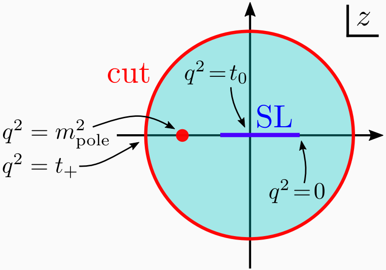
- for BSM interpretation in the Weak Effective Theory up to mass dimension six
  - $\overline{B} \to D$: +1 form factors; tensor currents — available from TH only
  - $\overline{B} \to D^*$: +3 form factors; tensor currents — available from TH only

- crossing symmetry relates hadronic form factors for $\overline{B} \to D^{(*)} \ell^- \overline{\nu}$ with form factors for $\ell^- \overline{\nu} \to \overline{B D}^{(*)}$ production [Boyd, Grinstein, Lebed 1997]
- integrated cross section (χ) can be computed in a local OPE
  - known to high precision: NNLO in α_s, power corrections small
- inspires a parametrization based on a conformal mapping of the first Riemann sheet of the form factor to the z unit disk
  - reproduces known analytical properties of the form factors
  - sets an absolute scale for any of the form factors, with bounded coefficients

$$f = \frac{1}{\sqrt{\chi}} \times \left[\sum_k a_k^f z^k \right] \times [\text{known things}]$$

dispersive bound : $\sum |a_k^f|^2 \leq 1$





- known poles are taken care of by so-called Blaschke factors
- in the semileptonic phase space $-|z| < 0.07$

- heavy-quark expansion very effective if both quark flavours b & c are heavy [Isgur,Wise '89]
  - simultaneous expansion in α_s up to NLO and $\Lambda_{\text{had}}/m_{b,c}$ up to 2nd power
    [Falk,Neubert hep-ph/9209268 & hep-ph/9209269]
  - yields parametric relations between form factors across both different currents and processes, as long as both initial and final state are elements of the same spin symmetry representation
  - relates BSM-only (tensor) FFs to SM FFs [Bernlochner,Ligeti,Papucci,Robinson 1703.05330]
- challenges available theory inputs in a global fit

heavy-quark expansion of any of the 10 $\overline{B} \to D^{(*)}$ form factors:

$$f = \left(A^f + \frac{\alpha_s}{\pi} B^f\right) \xi + \sum_{i=1}^{6} \left[\frac{\Lambda}{2m_b} C^f_{b,i} L_i + \frac{\Lambda}{2m_c} C^f_{c,i} L_i\right] + \frac{\Lambda^2}{4m_c^2} D^f \ell_i$$

$$+ \text{ higher order terms}$$

all 10 form factors connected by heavy-quark spin symmetry

- coefficients $A^f(q^2)$ to $D^f(q^2)$ are known to $\mathcal{O}(\alpha_s(\mu))$
- non-perturbative "Isgur-Wise" functions ξ, L_1 to L_6, and ℓ_1 to ℓ_6
  - equations of motion: only 10 independent functions
  - require parametrization (typical & adhoc: expand in z)

- power counting

ε^1	$\frac{1}{m_c}$	✓
ε^2	$\frac{1}{m_b}$, α_s, $\frac{1}{m_c^2}$	✓
ε^3	$\frac{\alpha_s}{m_c}$, $\frac{1}{m_b m_c}$, ...	...

- downside: no manifest dispersive bound

- express BGL coefficients a_k^f in terms of HQE parameters
  - commonly discussed CLN param is: HQE to $\mathcal{O}(1/m)$ + dispersive bound + simplifying assumptions

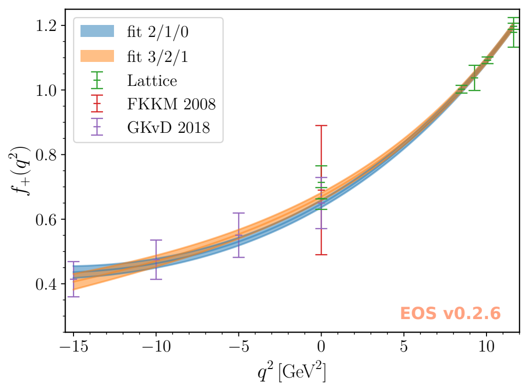
- upside: combination of dispersive bounds & HQE is more constraining than dispersive bounds in isolation [see e.g. Bigi, Gambino, Schacht]
  - HQE relates $\overline{B} \to D^{(*)}$ FFs to $\overline{B}^* \to D^{(*)}$ FFs, which are currently unavailable from (other) theory methods
  - strengthens dispersive bound by further constraining allowed parameter space

2019

Theory Inputs (2019)

- precise lattice QCD results for $\overline{B}_{(s)} \to D_{(s)}$ form factors [FNAL/MILC 1503.07237; HPQCD 1505.03925]
  - several synthetic data points for vector & scalar FFs
  - covering substantial parts of phase space with large q^2
- first lattice QCD results for $\overline{B}_{(s)} \to D^*_{(s)}$ form factors [FNAL/MILC 1403.0635; HPQCD 1711.11013]
  - one single data point for axial FF
  - clear need for $\mathcal{O}(1/m_c^2)$ [Jung,Straub 2018]
- QCD light-cone sum rule results [Gubernari,Kokulu,DvD 1811.00983; Bordone,Gubernari,Jung,DvD 1912.09335]
  - several synthetic data points for the full basis of form factors
  - covering $q^2 \leq 5\,\text{GeV}^2$ only

- nominal model is 3/2/1:
  - LP up to z^3
  - NLP up to z^2
  - NNLP up to z^1
- good fit: $\chi^2/\text{d.o.f} = 10/51$
- benefitting from large amount of information in $\overline{B} \to D$ FFs, transfered to $\overline{B} \to D^*$ FFs



Global HQE Fits (2019)

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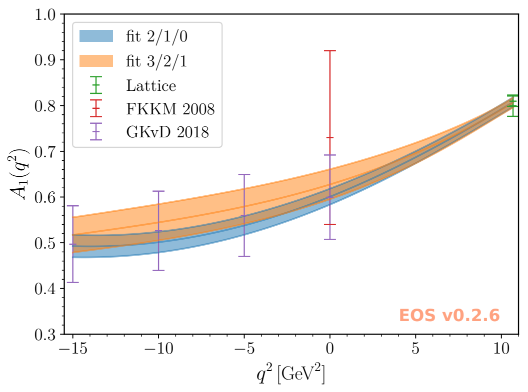

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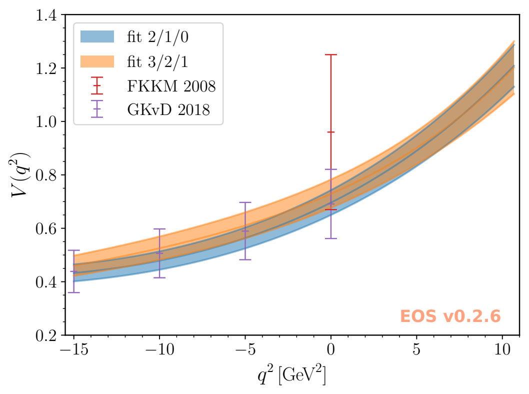

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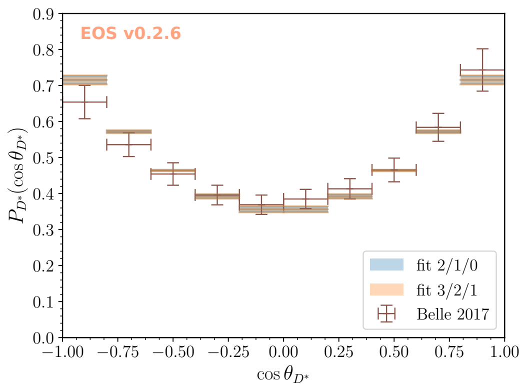
- compatible with Belle experimental data



2023

in $b \to c$ FFs:

- first lattice QCD results for $\overline{B} \to D^*$ form factors beyond $q^2 = q^2_{\text{max}}$

  [FNAL/MILC 2105.14019; JLQCD 2306.05657; HPQCD 2304.03137]

  - several synthetic data points for vector + 2x axial + pseudoscalar FFs

- no updated HQE fit yet, due to issues in BGL fits already

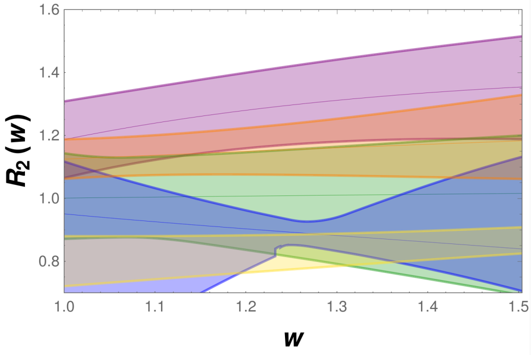
in FF parameters in general:

- BGL-like parametrization applicable with accurate dispersive bound for higher pair production thresholds [Gubernari,DvD,Virto 2011.09813]
- applied to $b \to s$ FFs ($\Lambda_b \to \Lambda$; $B \to K^{(*)}$ + $B \to \phi$) and $b \to u$ FFs ($\overline{B_s} \to K$)

  [Blake,Meinel,Rahimi,DvD 2205.06041; Gubernari,Reboud,DvD,Virto 2305.06301; RBC/UKQCD 2303.11280]

- ratios of FFs: R_0, R_1, R_2
- can be determined from
  - BGL fits to
    - FNAL/MILC 2021
    - HPQCD 2023
    - JLQCD 2023
  - 2019 HQE postdiction
  - Belle data

[M. Jung, Flavour@TH 2023]



Issues visible in Form Factor Ratios

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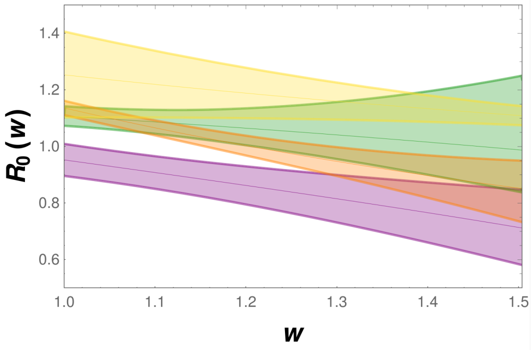

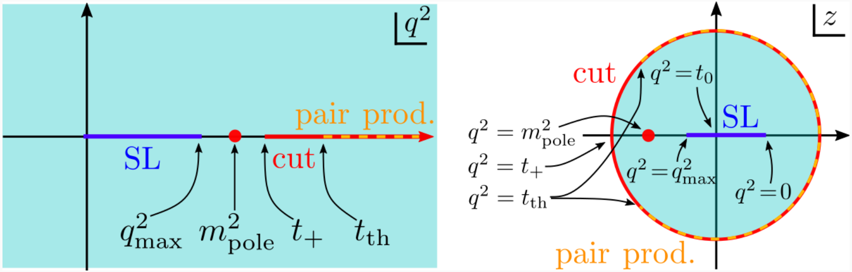

$\overline{B}_{(s)} \to D_{(s)}$:

- $t_+ = (M_{B_c} + M_\pi)^2$
- $t_{\rm th} = (M_{B_{(s)}} + M_{D_{(s)}})^2$

required changes

- $z^k \longrightarrow p_k(z)$: orthonormal polynomials w.r.t. scalar product on an arc of the unit circle

Quo Vadis?

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- dispersively bounded & HQE-based parametrization are both important tools in the FF basis
- HQE-based parametrization provides crucial cross check of theory inputs

  2019 excellent agreement, good fit
  2023 new lattice QCD inputs for $\overline{B} \to D^*$ at odds
    - with each other
    - with $\overline{B} \to D$
    - with Belle data
- update of global HQE fit desirable but currently not feasible until issues understood
- dispersively bounded parametrizations have seen improvements
  - no application to $\overline{B} \to D^{(*)}$ yet
  - not shown today: splitting of dispersive bounds by (virtual W) polarization

[Gubernari,Reboud,DvD,Virto 2305.06301]