

Semi-leptonic decays on the lattice

Oliver Witzel

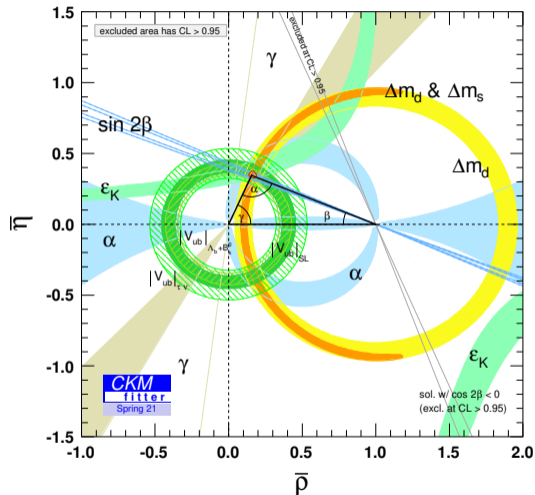


Heavy Flavours – Quo vadis?

Ardbeg, Islay, Scotland · Juni 22, 2023

Motivation: CKM unitarity triangle

- ▶ Combine several determinations to perform an over-constrained fit
- ▶ Use tree-level determinations of $|V_{ub}|$ and $|V_{cb}|$
 - Commonly used $B \rightarrow \pi l \nu$ and $B \rightarrow D^{(*)} l \nu$
 - Long standing $2 - 3\sigma$ discrepancy between exclusive ($B \rightarrow \pi l \nu$) and inclusive ($B \rightarrow X_u l \nu$)
 - $B \rightarrow \tau \nu$ has larger error



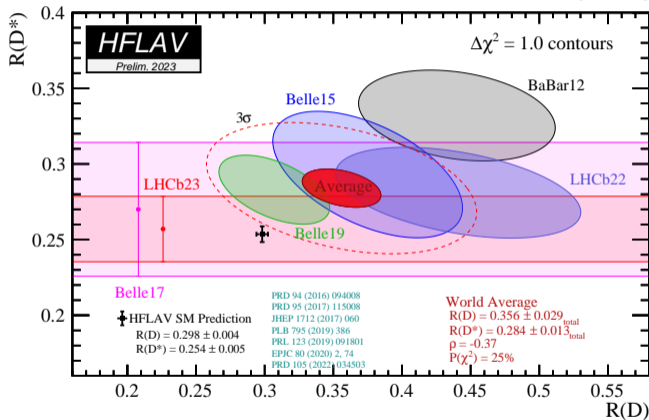
[<http://ckmfitter.in2p3.fr>]

Tension in $R_D^{(*)}$

► Testing universality of lepton flavors

$$R_{D^{(*)}}^{\tau/\mu} \equiv \frac{BF(B \rightarrow D^{(*)} \tau \nu_\tau)}{BF(B \rightarrow D^{(*)} \mu \nu_\mu)}$$

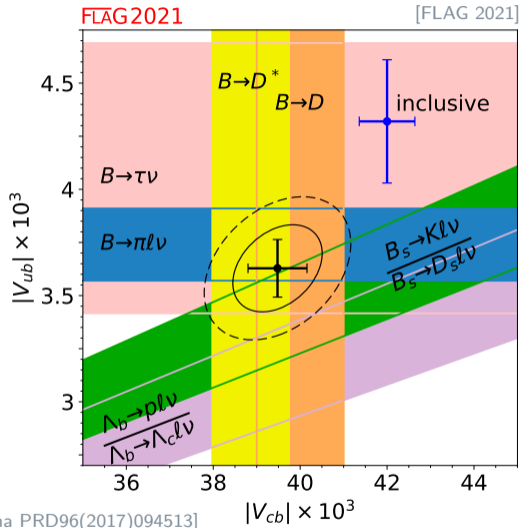
[HFLAV]



$|V_{ub}|$ and $|V_{cb}|$

- ▶ Leptonic decays $B_{(c)}^+ \rightarrow \ell^+ \nu_\ell$ experimentally difficult
 - Only $B^+ \rightarrow \tau^+ \nu_\tau$ measured (large error)
- ▶ Semileptonic decays preferred
 - Exclusive e.g. $B \rightarrow \pi \ell \nu$
 - Inclusive e.g. $B \rightarrow X_u \ell \nu$
 - B, B_s, Λ_b initial state
- ▶ Longstanding tension between exclusive and inclusive determinations
 - Novel ideas for inclusive lattice calculations

[Hashimoto PTEP(2017)053B03] [Hansen, Meyer, Robaina PRD96(2017)094513]
 [Bailas et al. PTEP(2020)043B07] [Gambino, Hashimoto PRL 125(2020)032001]
 [Barone et al. arXiv:2305.14092]. . .



Cabibbo-Kobayashi-Maskawa (CKM) matrix

$$\begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} = \begin{bmatrix} 0.97370(14) & 0.2245(8) & 0.00382(24) \\ 0.221(4) & 0.987(11) & 0.041(14) \\ 0.0080(3) & 0.0388(11) & 1.013(30) \end{bmatrix} \quad [\text{PDG, Workman et al. PTEP (2022) 083C01}]$$

$$\frac{|\delta V_{CKM}|}{|V_{CKM}|} = \begin{bmatrix} 0.014 & 0.35 & 6.3 \\ 1.8 & 1.1 & 3.4 \\ 3.8 & 2.8 & 3.0 \end{bmatrix} \%$$

- ▶ Heavy sector less well explored compared to light sector
- ▶ Large experimental efforts: LHCb, Belle II, BESIII, ...

$$\begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \leftrightarrow \begin{bmatrix} \pi \rightarrow l\nu & K \rightarrow l\nu & B \rightarrow \pi l\nu \\ & K \rightarrow \pi l\nu & B_s \rightarrow K l\nu \\ D \rightarrow l\nu & D_s \rightarrow l\nu & B_{(s)} \rightarrow D_{(s)} l\nu \\ D \rightarrow \pi l\nu & D \rightarrow K l\nu & B_{(s)} \rightarrow D_{(s)}^* l\nu \\ B_d \leftrightarrow \bar{B}_d & B_s \leftrightarrow \bar{B}_s & \end{bmatrix}$$

- ▶ Typical nonperturbative LQCD calculations to extract CKM matrix elements
- ▶ Why is the uncertainty for $|V_{ub}|$ and $|V_{cb}|$ so large?

Heavy flavors on the lattice

► Quark masses

up ~ 0.002 GeV

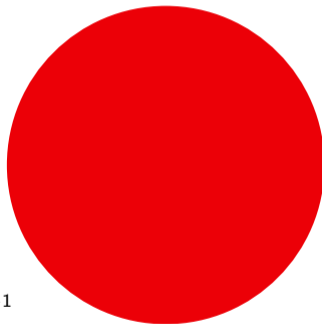


charm ~ 1.25 GeV

top ~ 175 GeV

down ~ 0.005 GeV

• strange ~ 0.095 GeV



bottom
 ~ 4.2 GeV

► Lattice simulations have a cutoff a^{-1}

→ Fully relativistic quarks require $am \ll 1$ i.e. $m \ll a^{-1}$

→ Typically $a^{-1} \gtrsim 2$ GeV $\Rightarrow m_{\text{charm}} \lesssim a^{-1} \lesssim m_{\text{bottom}}$

→ Charm but in particular bottom quarks require special considerations

$\times 10$

Simulating heavy flavors

- ▶ Traditionally: simulate charm and bottom using **effective actions**
 - Heavy quark effective Theory (HQET), Non-Relativistic QCD, Relativistic Heavy Quark (RHQ, Fermilab, Tsukuba)
 - Allows to simulate charm and bottom quarks on coarser lattices
 - Additional systematic uncertainties, partly perturbative renormalization, ...
 - Few percent total errors
- ▶ State-of-the-art: **fully relativistic** simulations at $a^{-1} > 2$ GeV
 - Heavy Highly Improved Staggered Quarks (HISQ), Heavy Domain-Wall Fermions (DWF), ...
 - Same action for light (up/down/strange) as for heavy (charm/bottom) quarks
 - ↪ Simulate heavier than charm and extrapolate
 - Fully nonperturbative renormalization straight-forward, reduced systematic uncertainties
 - Sub-percent precision feasible ↪ **QED effects** become relevant

Overview

- ▶ Semileptonic $b \rightarrow u$ decays
(hadronic pseudoscalar final states)

$$B \rightarrow \pi l \nu \text{ and } B_s \rightarrow K l \nu$$

- ▶ Semileptonic $b \rightarrow c$ decays
(hadronic vector final states)

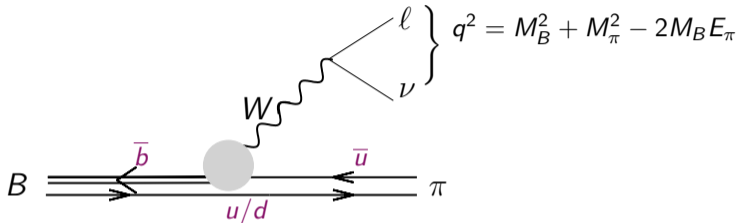
$$B \rightarrow D^* l \nu$$

- ▶ Summary

$$b \rightarrow u$$

(hadronic pseudoscalar final states)

Exclusive semi-leptonic decays: $B \rightarrow \pi \ell \nu$



► Conventionally parametrized placing the B meson at rest

$$\frac{d\Gamma(B \rightarrow \pi \ell \nu)}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{24\pi^3} \frac{(q^2 - m_\ell^2)^2 \sqrt{E_\pi^2 - M_\pi^2}}{q^4 M_B^2}$$

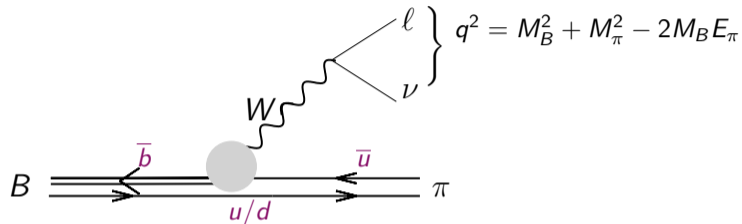
experiment

CKM

known

$$\times \left[\left(1 + \frac{m_\ell^2}{2q^2} \right) M_B^2 (E_\pi^2 - M_\pi^2) |f_+(q^2)|^2 + \frac{3m_\ell^2}{8q^2} (M_B^2 - M_\pi^2)^2 |f_0(q^2)|^2 \right]$$

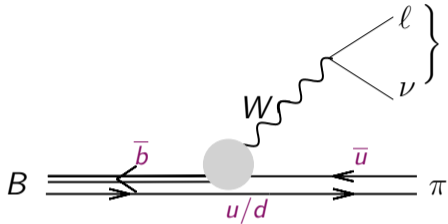
nonperturbative input

Exclusive semi-leptonic decays: $B \rightarrow \pi l \nu$ 

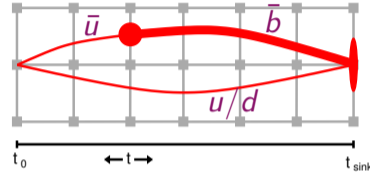
► Nonperturbative input

- Parametrizes interactions due to the (nonperturbative) strong force
- Use operator product expansion (OPE) to identify short distance contributions
- Calculate the flavor changing currents as point-like operators using lattice QCD

Exclusive semi-leptonic decays: $B \rightarrow \pi \ell \nu$



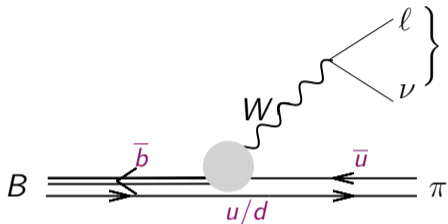
$$q^2 = M_B^2 + M_\pi^2 - 2M_B E_\pi$$



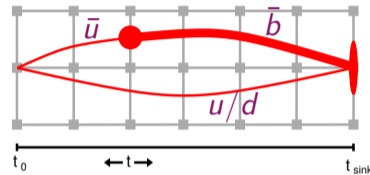
- Calculate hadronic matrix element for the flavor changing vector current V^μ in terms of the form factors $f_+(q^2)$ and $f_0(q^2)$

$$\langle \pi | V^\mu | B \rangle = f_+(q^2) \left(p_B^\mu + p_\pi^\mu - \frac{M_B^2 - M_\pi^2}{q^2} q^\mu \right) + f_0(q^2) \frac{M_B^2 - M_\pi^2}{q^2} q^\mu$$

Exclusive semi-leptonic decays: $B \rightarrow \pi \ell \nu$



$$q^2 = M_B^2 + M_\pi^2 - 2M_B E_\pi$$



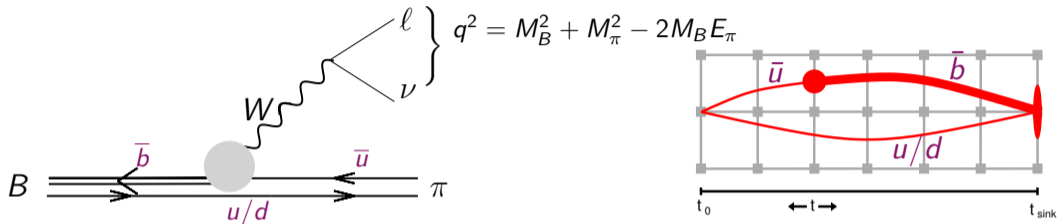
- ▶ Calculate hadronic matrix element for the flavor changing vector current V^μ in terms of the form factors $f_+(q^2)$ and $f_0(q^2)$
- ▶ On the lattice f_\perp and f_\parallel are directly proportional to 3-point functions

$$f_\parallel(E_P) = \langle P | V^0 | B_{(s)} \rangle / \sqrt{2M_{B_{(s)}}} \quad \text{and} \quad f_\perp(E_P) p_P^i = \langle P | V^i | B_{(s)} \rangle / \sqrt{2M_{B_{(s)}}}$$

$$f_0(q^2) = \frac{\sqrt{2M_{B_{(s)}}}}{M_{B_{(s)}}^2 - M_P^2} \left[(M_{B_{(s)}} - E_P) f_\parallel(E_P) + (E_P^2 - M_P^2) f_\perp(E_P) \right]$$

$$f_+(q^2) = \frac{1}{\sqrt{2M_{B_{(s)}}}} \left[f_\parallel(E_P) + (M_{B_{(s)}} - E_P) f_\perp(E_P) \right]$$

Exclusive semi-leptonic decays: $B \rightarrow \pi \ell \nu$



- ▶ Calculate hadronic matrix element for the flavor changing vector current V^μ in terms of the form factors $f_+(q^2)$ and $f_0(q^2)$

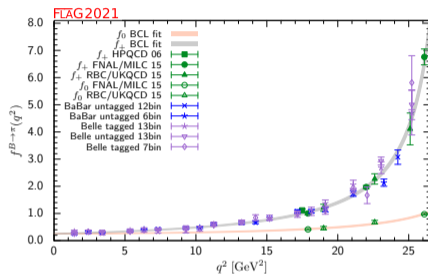
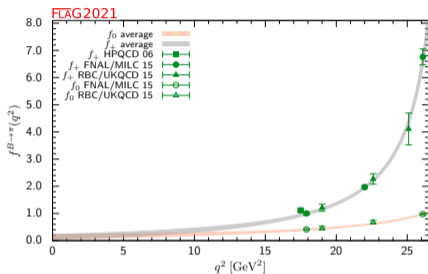
- ▶ On the lattice f_\perp and f_\parallel are directly proportional to 3-point functions

$$f_\parallel(E_P) = \langle P | V^0 | B_{(s)} \rangle / \sqrt{2M_{B_{(s)}}} \quad \text{and} \quad f_\perp(E_P) p_P^i = \langle P | V^i | B_{(s)} \rangle / \sqrt{2M_{B_{(s)}}}$$

- ▶ Alternatively, express form factors in terms of f_1 and f_2 with $v^\mu = p_B^\mu / M_B$ motivated by HQET

$$f_1(v \cdot p_\pi) + f_2(v \cdot p_\pi) = f_\parallel(E_\pi) / \sqrt{2} \quad \text{and} \quad f_2(v \cdot p_\pi) = f_\perp(E_\pi) \cdot (v \cdot p_\pi / \sqrt{2})$$

FLAG average [FLAG 2021]

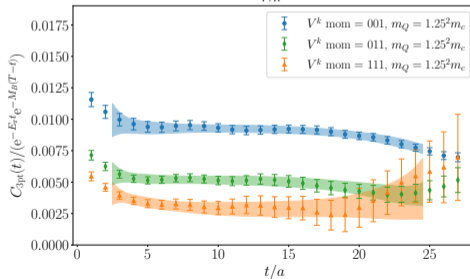
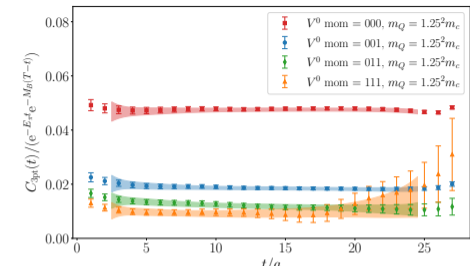


- ▶ FLAG average: Fermilab/MILC [Bailey et al. PRD92(2015)014024], RBC/UKQCD [Flynn et al. PRD 91 (2015) 074510]
 - Shown in addition HPQCD [Dalgic et al. PRD73(2006)074502][PRD75(2007)119906]
- ▶ Used effective actions only allowed determinations of form factors at large q^2
- ▶ Combined fit with experimental data gives $|V_{ub}^{\text{excl}}|$
 - [BaBar PRD 83 (2011) 032007][PRD 86 (2012) 092004] [Belle PRD 83 (2011) 071101][PRD 88 (2013) 032005]
- ▶ Shape of lattice data largely consistent with experimental data

JLQCD 2022: $B \rightarrow \pi \ell \nu$

[Colquhoun et al. PRD 106 (2022) 054502]

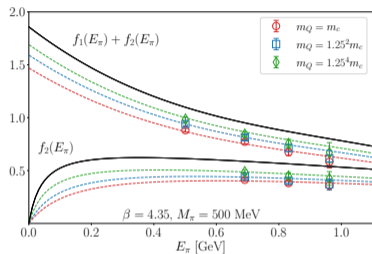
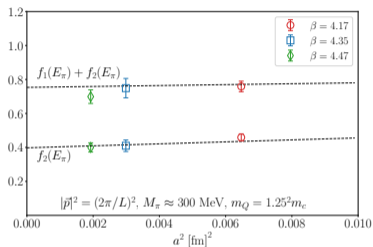
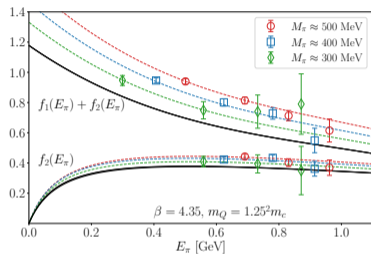
- ▶ Unitary setup
 - MDWF light/strange and heavy quarks with $am_c \leq am_Q \leq 2.44 \cdot am_c$
 - Additional extrapolation in the heavy quark mass to reach m_b
 - Fully nonperturbative renormalization
- ▶ $a \approx 0.044$ fm, 0.055 fm, 0.080 fm
- ▶ $M_\pi \gtrsim 230$ MeV
- ▶ Comparable stat. and sys. errors
 - Total errors: $f_+ \sim 10\%$, $f_0 \sim 6\%$



- ▶ $a \approx 0.080$ fm, $am_{u/d} = 0.007$,
 $am_Q = 0.68808$

JLQCD 2022: combined chiral, heavy-quark, continuum limit

[Colquhoun et al. PRD 106 (2022) 054502]



► Extrapolate in M_π

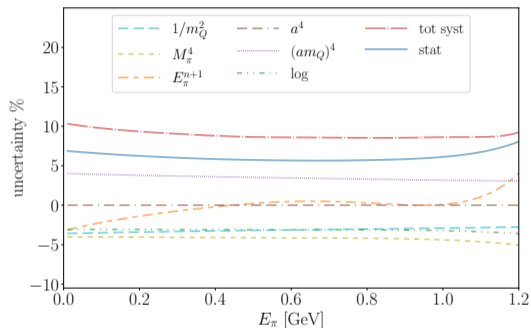
► Extrapolate in a^2

► Extrapolate in m_Q

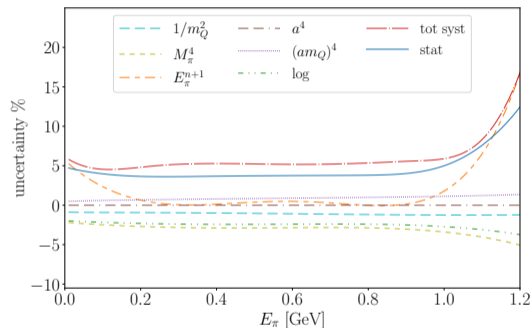
► Extrapolation over the simulated range of “high q^2 ”

► Extract “synthetic” data points for z-expansion from continuum-physical quark mass limit

JLQCD 2022: error budget [Colquhoun et al. PRD 106 (2022) 054502]



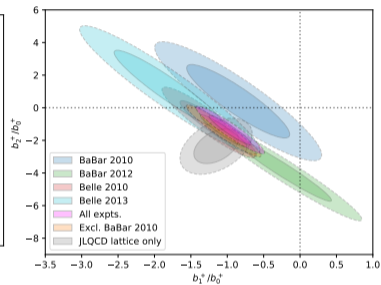
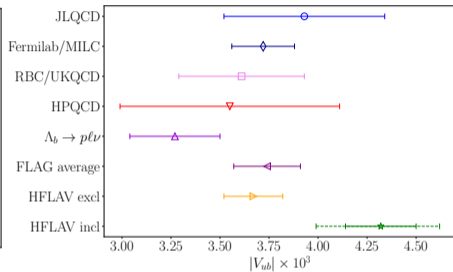
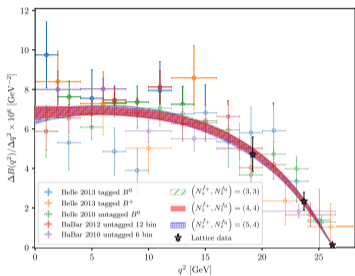
► Systematic uncertainties for f_+



► Systematic uncertainties for f_0

JLQCD 2022: $|V_{ub}|$ and comparisons

[Colquhoun et al. PRD 106 (2022) 054502]



► Joint fit to determine $|V_{ub}|$
 $\Rightarrow |V_{ub}| = (3.93 \pm 0.41) \cdot 10^{-3}$

► Updates from other collaborations
 expected relatively soon

► Shape parameters of BCL z-fit
 → Tension with BaBar 2010
 → Looking forward to
 new data from Belle II

What are the challenges calculating $B \rightarrow \pi \ell \nu$?

- ▶ Ratio of $m_{\text{bottom}}/m_{\text{up}}$ is worst
⇒ Signal-to-noise issue
- ▶ B meson are heavy (5279 MeV), pions are light (138 MeV)
 - Decay releases lots of energy \rightsquigarrow large range in q^2 to be covered
 - Requires simulations of pions with very high momenta (noisy)
- ▶ Experimentally clean environment of B factories (strongly) preferred
- ▶ Alternative B decay modes have their own theoretical/experimental challenges e.g. $B \rightarrow \rho(\rightarrow \pi\pi)\ell\nu$ on the lattice

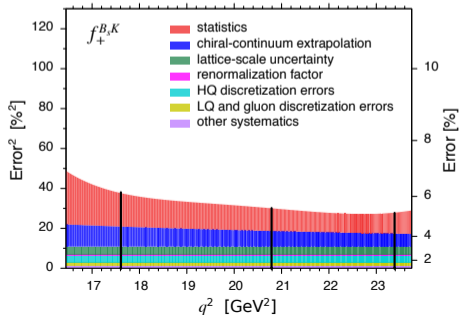
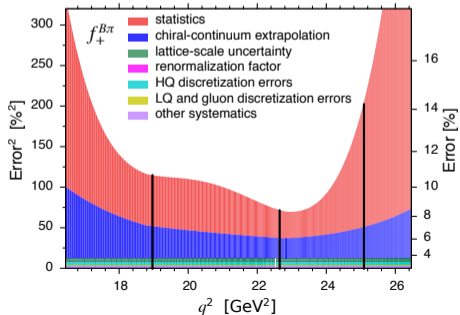
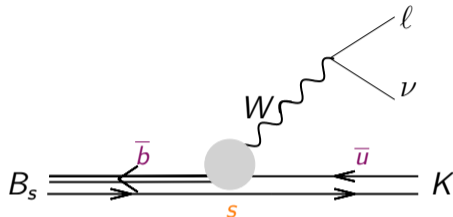
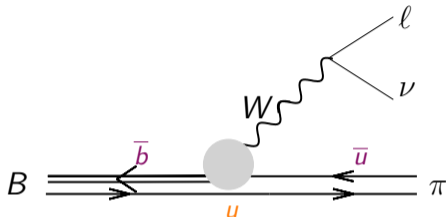
Alternative: $B_s \rightarrow K\ell\nu$ or $\Lambda_b \rightarrow p\ell\nu$

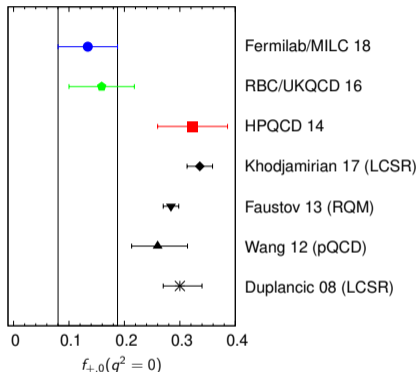
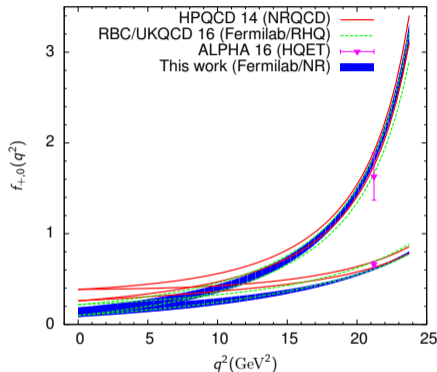
- ▶ Experimentally not ideal for B factories
 - Running at $\Upsilon(5s)$ is less efficient in creating $B_s\bar{B}_s$ pairs

- ▶ Abundantly created in pp collisions at the LHC \rightsquigarrow LHCb
 - Normalization not straight forward at LHCb, better to consider (double-)ratios
 - Determine $|V_{cb}|/|V_{ub}|$ from $B_s \rightarrow D_s\ell\nu/B_s \rightarrow K\ell\nu$
 or $\Lambda_b \rightarrow \Lambda_c\ell\nu/\Lambda \rightarrow p\ell\nu$ [Detmold, Lehner, Meinel, PRD92 (2015) 034503]

- ▶ Compare:
 - $M_B = 5279 \text{ MeV} : M_\pi = 138 \text{ MeV} \sim 38, q^2 \text{ range} \sim [m_\ell^2, 27] \text{ GeV}^2$
 - $M_{B_s} = 5367 \text{ MeV} : M_K = 494 \text{ MeV} \sim 11, q^2 \text{ range} \sim [m_\ell^2, 24] \text{ GeV}^2$
 - \rightsquigarrow cheaper and more precise to compute with LQCD

Comparison $B \rightarrow \pi l \nu$ vs. $B_s \rightarrow K l \nu$ [Flynn et al. PRD 91 (2015) 074510]



$B_s \rightarrow K l \nu$ tensions (< 2023)

▶ HPQCD, RBC-UKQCD, ALPHA, Fermilab/MILC

[Bouchard et al. PRD90(2014)054506] [Flynn et al. PRD91(2015)074510] [Bahr et al. PLB757(2016)473]
[Bazavov et al. PRD100(2019)034501]

▶ Lattice form factors differ at $q^2 = 0$

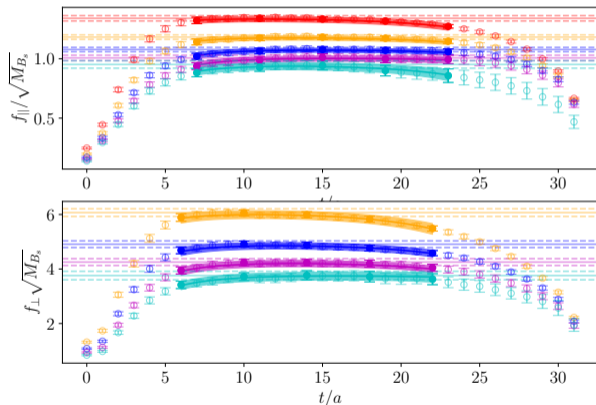
RBC/UKQCD 2023: Update $B_s \rightarrow K \ell \nu$ [Flynn et al. PRD 107 (2023) 114512]

→ J. Tobias Tsang, Andreas Jüttner, Jonathan Flynn, Ryan Hill, Amarjit Soni, OW

- ▶ Effective RHQ action for b quarks
 - SDWF light/strange
 - Nonperturbatively tuned RHQ parameters
 - Directly simulating physical b quarks
 - Mostly nonperturbative renormalization

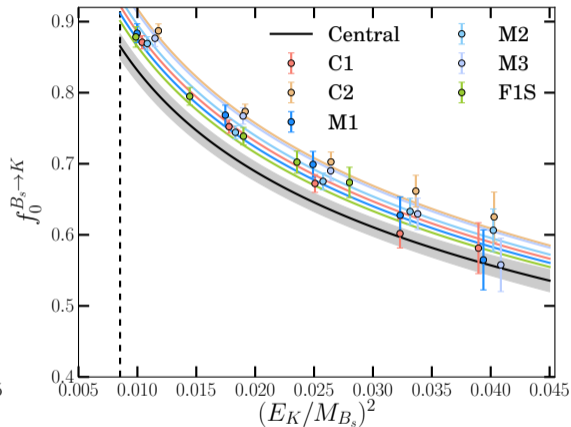
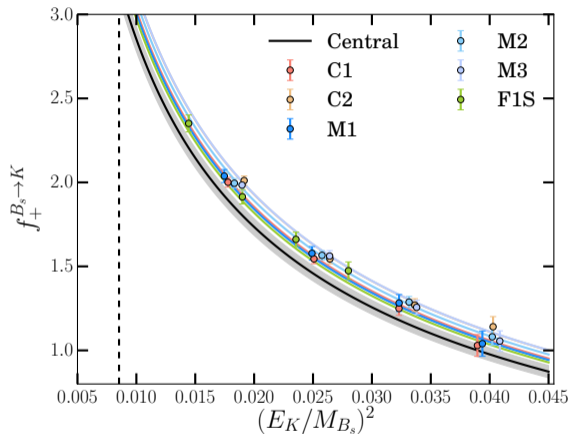
▶ $a \approx 0.11$ fm, 0.08, 0.07 fm

▶ $M_\pi \gtrsim 250$ MeV



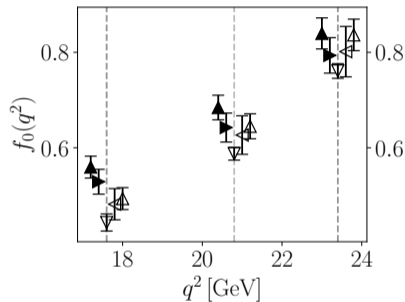
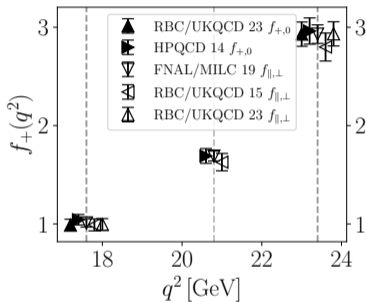
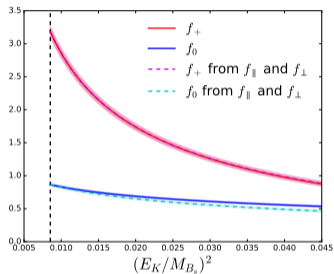
▶ $a \approx 0.07$ fm, $M_\pi = 250$ MeV

RBC/UKQCD 2023: Chiral-continuum extrapolation [Flynn et al. PRD 107 (2023) 114512]



► Chiral-continuum fit in terms of f_+ and f_0 over simulated range in q^2

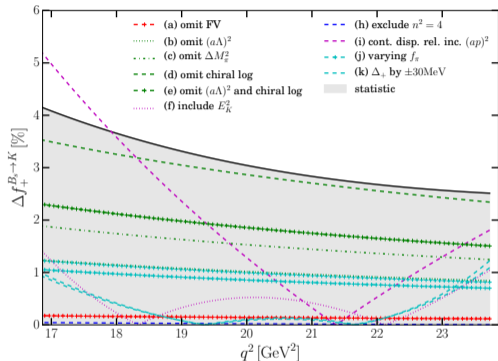
RBC/UKQCD 2023: f_+ and f_0 vs. f_{\parallel} and f_{\perp} [Flynn et al. PRD 107 (2023) 114512]



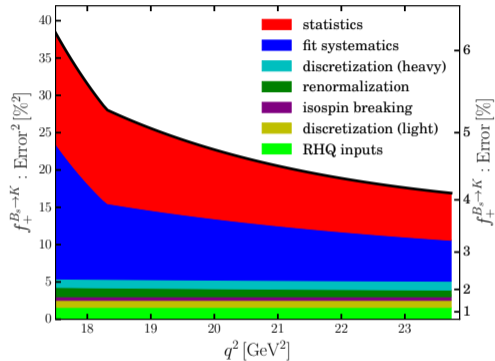
- ▶ Chiral-continuum fit in terms of f_+ and f_0 vs. fitting f_{\parallel} and f_{\perp} and then constructing f_+ and f_0

- ▶ Comparing literature results for $B_s \rightarrow K\ell\nu$
- ▶ No resolved effect for f_+ but shift for f_0

RBC/UKQCD 2023: Error budget [Flynn et al. PRD 107 (2023) 114512]

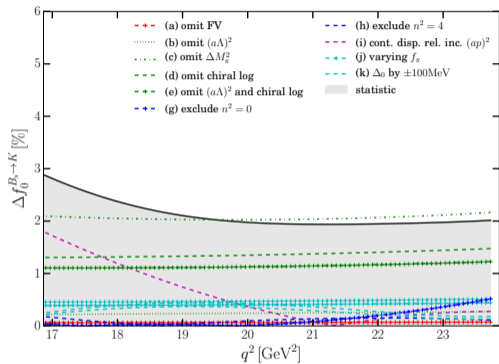


► Systematic of chiral-continuum fit f_+

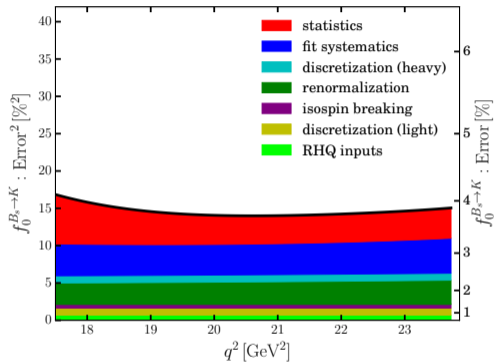


► Total error budget f_+

RBC/UKQCD 2023: Error budget [Flynn et al. PRD 107 (2023) 114512]



► Systematic of chiral-continuum fit f_0



► Total error budget f_0

Kinematical z -expansion (BGL) [Boyd, Grinstein, Lebed, PRL 74 (1995) 4603]

- ▶ Map complex q^2 plane with cut $q^2 > t_*$ onto the unit disk in z

$$z(q^2, t_*, t_0) = \frac{\sqrt{t_* - q^2} - \sqrt{t_* - t_0}}{\sqrt{t_* - q^2} + \sqrt{t_* - t_0}}$$

with

$$t_* = (M_B + M_\pi)^2 \quad (\text{two-particle production threshold})$$

$$t_\pm = (M_{B_s} \pm M_K)^2 \quad (\text{with } t_- = q_{max}^2)$$

$$t_0 \equiv t_{opt} = t_* - \sqrt{t_*(t_* - t_-)} \quad (\text{symmetrize range of } z)$$

- ▶ BGL express form factors $f_X = f_+, f_0$ as

$$f_X(q^2) = \frac{1}{B_X(q^2)\phi_X(q^2, t_0)} \sum_{n \geq 0} a_{X,n}(t_0) z^n$$

- ▶ With outer function $\phi_X(q^2, t_0)$ and Blaschke factors $B_X(q^2)$
- ▶ Account for cut differing from pair-production threshold [Gubernari, van Dyk, Virto JHEP02 (2021) 088]
[Gubernari, van Dyk, Reboud, Virto JHEP09 (2022) 133]

Kinematical z -expansion [Flynn et al. PRD 107 (2023) 114512] [Flynn, Jüttner, Tsang arXiv:2303.11285]

↪ Andreas Jüttner, J.Tobias Tsang, Jonathan Flynn

- ▶ Terms in the z expansion are limited:
 - Number of synthetic data points plus kinematic constraint: $K_+ + K_0 - 1 < N_+ + N_0$
- ▶ Truncation errors (e.g. large variations in $f_+(q^2 = 0)$ and $f_0(q^2 = 0)$) when
 - Varying q^2 values of synthetic points
 - Varying t_0 in z -transformation
- ▶ Avoid frequentist fit introducing systematic error
 - Perform Bayesian fit aiming to fit full z expansion (no truncation)
 - Use unitarity constraint to control higher-order coefficients

Bayesian inference for form factors [Flynn, Jüttner, Tsang arXiv:2303.11285]

- ▶ Compute z expansion coefficients as expectation values: $\langle g(a) \rangle = N \int da g(a) \pi(a|f, C_f) \pi_a$
- ▶ Probability for parameters given model and data

$$\pi(a|f, C_f) \propto \exp \left\{ -\frac{1}{2} \chi^2(a, f) \right\} \quad \chi^2(a, f) = (f - Za)^T C_f^{-1} (f - Za)$$

and prior knowledge from unitarity constraint $\pi_a \propto \theta(1 - |a_+|_\alpha^2) \theta(1 - |a_0|_\alpha^2)$

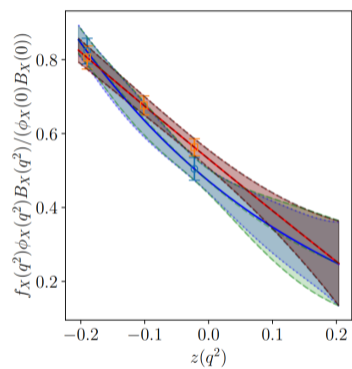
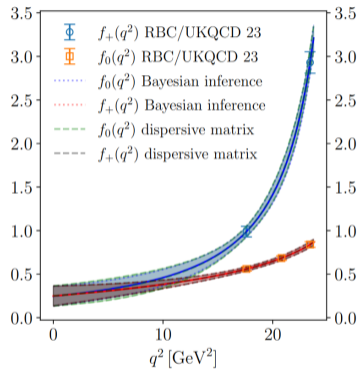
- ▶ Perform Monte Carlo integration using multivariate distribution a but drop samples incompatible with unitarity
- ▶ To increase probability modify expression and correct with an accept-reject step

$$\pi(a|f_p, C_{f_p}) \pi_a(a_p|M) \propto \theta(a) \exp \left\{ -\frac{1}{2} (f_p - Za)^T C_{f_p}^{-1} (f_p - Za) - \frac{1}{2} a^T \frac{M}{\sigma^2} a \right\}$$

with
$$p \leq \frac{\exp \{ -1/\sigma^2 \}}{\exp \{ -a^T \frac{M}{2\sigma^2} a \}}$$

RBC/UKQCD 2023: z-expansion

[Flynn et al. PRD 107 (2023) 114512] [Flynn, Jüttner, Tsang arXiv:2303.11285]



$q^2 = 0$

Duplancic, Melic	$0.30^{(+4)}_{(-3)}$
Khodjamirian, Rusov	$0.336(23)$
Faustov, Galkin	$0.284(14)$
Wang, Xiao	$0.26^{(+4)}_{(-3)}(2)$
RBC/UKQCD 23	$0.25(11)$
Fermilab/MILC 18	$0.13(5)$
HPQCD 14	$0.32(6)$

► Consistent with result of dispersive matrix method by Martinelli, Simula, Vittorio et al.

$$b \rightarrow c$$

(hadronic vector final states)

Determining $|V_{cb}|^{\text{excl}}$

- ▶ Heavy-to-heavy transition \rightsquigarrow HQET relations
- ▶ Available channels
 - $\rightarrow B \rightarrow D\ell\nu$
 - $\rightarrow B_s \rightarrow D_s\ell\nu$ pseudoscalar final states
 - $\rightarrow B \rightarrow D^*\ell\nu$
 - $\rightarrow B_s \rightarrow D_s^*\ell\nu$ vector final states
- ▶ D^* and D_s^* suitable for using the narrow width approximation
 - \rightarrow Treat as QCD-stable particle

Exclusive semi-leptonic decays: $B_{(s)} \rightarrow D_{(s)}^* \ell \nu$

$$\langle D_{(s)}^*(k, \varepsilon_\nu) | \mathcal{V}^\mu | B_{(s)}(p) \rangle = V(q^2) \frac{2i\varepsilon^{\mu\nu\rho\sigma} \varepsilon_\nu^* k_\rho p_\sigma}{M_{B_{(s)}} + M_{D_{(s)}^*}}$$

$$\langle D_{(s)}^*(k, \varepsilon_\nu) | \mathcal{A}^\mu | B_{(s)}(p) \rangle = A_0(q^2) \frac{2M_{D_{(s)}^*} \varepsilon^* \cdot q}{q^2} q^\mu$$

$$+ A_1(q^2) (M_{B_{(s)}} + M_{D_{(s)}^*}) \left[\varepsilon^{*\mu} - \frac{\varepsilon^* \cdot q}{q^2} q^\mu \right]$$

$$- A_2(q^2) \frac{\varepsilon^* \cdot q}{M_{B_{(s)}} + M_{D_{(s)}^*}} \left[k^\mu + p^\mu - \frac{M_{B_{(s)}}^2 - M_{D_{(s)}^*}^2}{q^2} q^\mu \right]$$

- ▶ Determine the four form factors $V(q^2)$, $A_0(q^2)$, $A_1(q^2)$, $A_2(q^2)$
or in HQE convention $h_V(w)$, $h_{A_0}(w)$, $h_{A_1}(w)$, $h_{A_2}(w)$
- ▶ Narrow-width approximation i.e. $D_{(s)}^*$ is treated as a QCD-stable particle

First lattice calculations over the full q^2 range

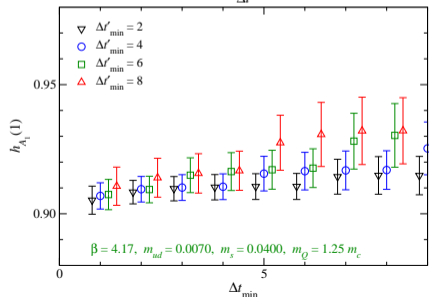
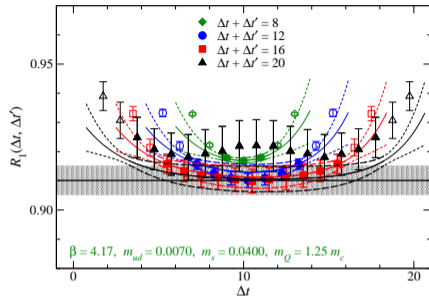
- ▶ $B \rightarrow D^* l \nu$
 - 2021 Fermilab/MILC [Bazavov et al. EPJC 82(2022)1141]
 - 2023 HPQCD [Harrison, Davies, arXiv:2304.03137]
 - 2023 JLQCD [Y. Aoki et al. arXiv:2306.05657]
 - Preliminary LANL/SWME [Jang et al. PoS Lattice2019 (2020) 056]

- ▶ $B_s \rightarrow D_s^* l \nu$
 - 2022 HPQCD [Harrison, Davies PRD105(2022).094506][arXiv:2304.03137]

- ▶ Some tension in the shape of the form factors
 - Further scrutiny required

JLQCD 2023: $B \rightarrow D^* \ell \nu$ [Y. Aoki et al. arXiv:2306.05657]

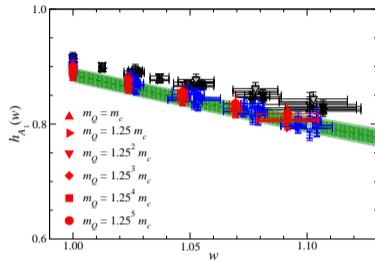
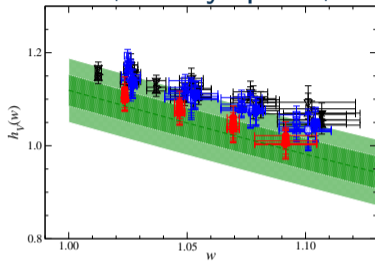
- ▶ Unitary setup
 - MDWF light/strange and heavy quarks with $am_c \leq am_Q \leq 2.44 \cdot am_c$
 - Additional extrapolation in the heavy quark mass to reach m_b
 - Fully nonperturbative renormalization
- ▶ $a \approx 0.044$ fm, 0.055 fm, 0.080 fm
- ▶ $M_\pi \gtrsim 230$ MeV
- ▶ Carefully checking for excited state contamination using multiple source sink separations (e.g. for h_{A_1})



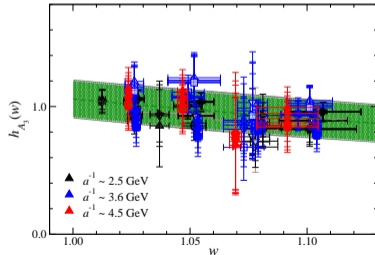
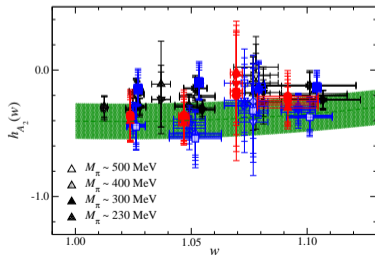
JLQCD 2023: combined chiral, heavy-quark, continuum limit

[Y. Aoki et al. arXiv:2306.05657]

- ▶ h_V : sys. error dominates
- ▶ h_{A1} stat. and sys. error similar
- ▶ $h_{A2,A3}$ stat. error dominates
- ↪ Different setup with moving B meson would help



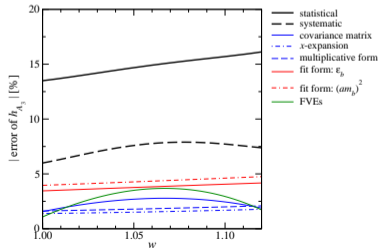
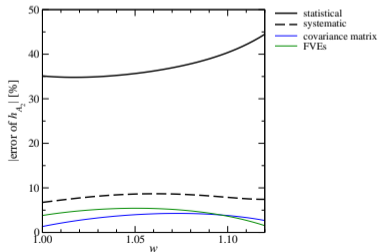
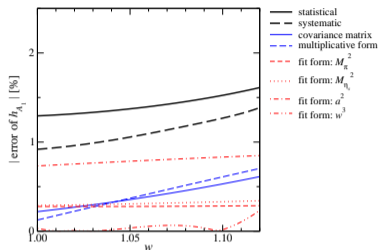
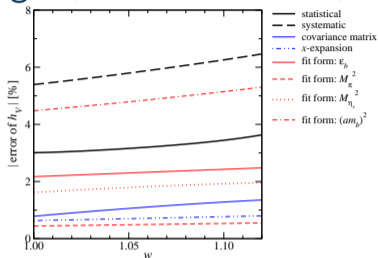
- ▶ Extrapolation over simulated range of q^2
- ▶ Predictions consistent with HQET-based parametrization



JLQCD 2023: error budget [Y. Aoki et al. arXiv:2306.05657]

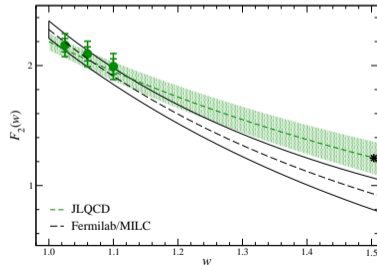
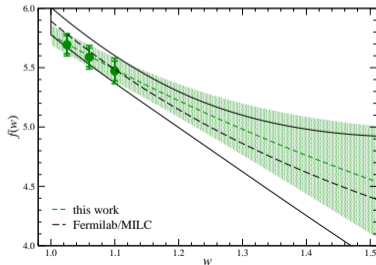
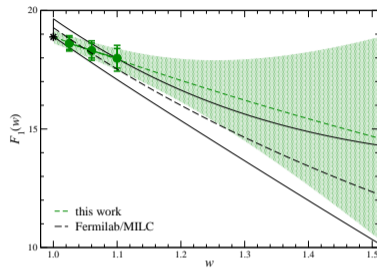
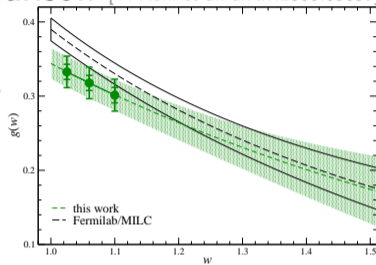
- ▶ h_V : sys. error dominates
- ▶ h_{A1} stat. and sys. error similar
- ▶ $h_{A2,A3}$ stat. error dominates
- ↪ Different setup with moving B meson would help

- ▶ Extrapolation over simulated range of q^2
- ▶ Predictions consistent with HQET-based parametrization



JLQCD 2023: comparison [Y. Aoki et al. arXiv:2306.05657]

- ▶ Convert $h_V, h_{A1}, h_{A2}, h_{A3}$ to relativistic convention $g, f, \mathcal{F}_1, \mathcal{F}_2$
- ▶ Synthetic data points in 2σ agreement with Fermilab/MILC
- ▶ Slopes/shapes differ

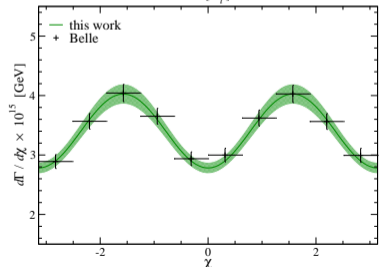
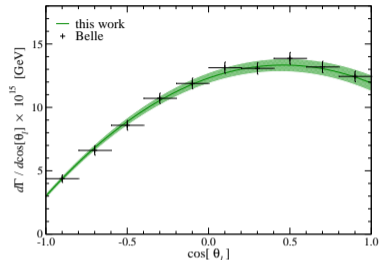
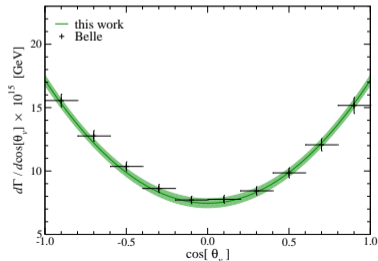
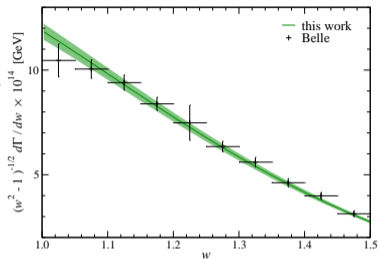


JLQCD 2023: comparison [Y. Aoki et al. arXiv:2306.05657]

- ▶ Convert $h_V, h_{A1}, h_{A2}, h_{A3}$ to relativistic convention $g, f, \mathcal{F}_1, \mathcal{F}_2$

- ▶ Good agreement with Belle 2019 data

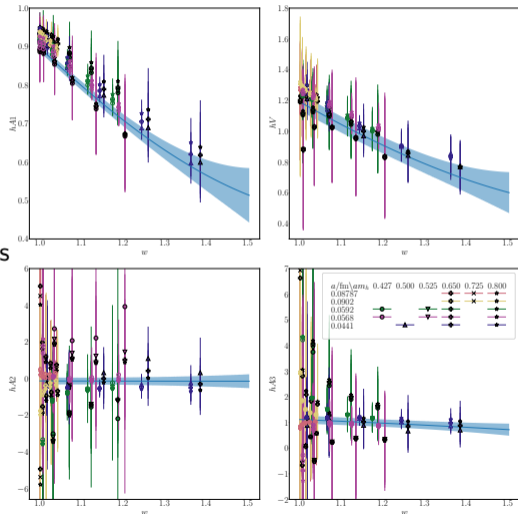
[Belle PRD 100 (2019) 052007]



HPQCD 2023: $B_{(s)} \rightarrow D_{(s)}^* \ell \nu$ [Harrison, Davies, arXiv:2304.03137]

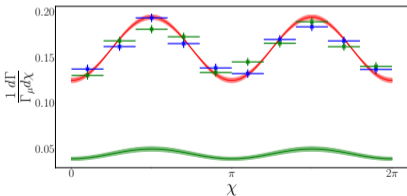
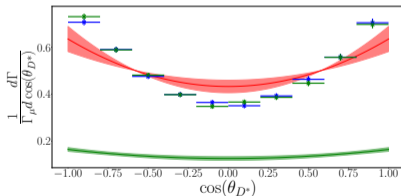
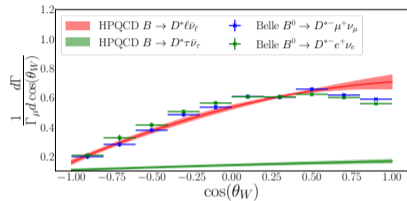
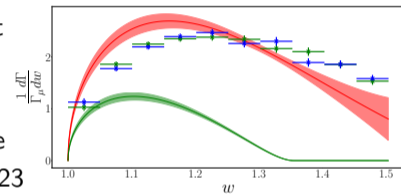
- ▶ All-HISQ setup
 - Updating [Harrison, Davies PRD105(2022).094506]
 - Fully non-perturbative renormalization
 - Simulate heavier-than-charm → close-to-bottom
 - Directly cover most of the allowed q^2 range at the finest lattice spacing
 - ↔ Parametrize pole mass for different charm masses in a combined chiral, heavy quark, continuum, kinematical extra-/interpolation
 - Also analyzing tensor BSM operators

▶ $h_V, h_{A1}, h_{A2}, h_{A3}$ for $B \rightarrow D^* \ell \nu$



HPQCD 2023: $B_{(s)} \rightarrow D_{(s)}^* \ell \nu$ [Harrison, Davies, arXiv:2304.03137]

- ▶ Shape largely consistent with Fermilab/MILC 21
- ▶ Some tension with Belle 2019 data and JLQCD 23



Summary

- ▶ Heavy flavors are challenging
 - Require to accommodate another scale on the lattice
 - Simulations with physical light quarks are even more challenging
 - Semi-leptonic decay processes cover a large range q^2
 - Leptonic decays experimentally difficult
- ▶ Puzzles in heavy flavor physics
 - Tension between $|V_{ub}|^{\text{excl}}$ vs. $|V_{ub}|^{\text{incl}}$ and $|V_{cb}|^{\text{excl}}$ vs. $|V_{cb}|^{\text{incl}}$
 - Shape comparisons of form factors